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Beta-Decay Matrix Elements of La^{140} and the Implications for Isobaric Analog States*

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Experimental data are used to determine the first-forbidden matrix elements for the 2.2-MeV β transition of La^{140}. The results show that the matrix-element ratio $(\int\vec{\alpha}/\int i\vec{r})$ does not agree with the usual theoretical prediction which assumes the Ahrens-Feenberg approximation. The size of the experimental vector-matrix-element ratio can be understood on the basis of significant off-diagonal contributions in the Coulomb Hamiltonian. The theory of conserved vector current is used to investigate impurities in the isobaric analog to the ground state of La^{140}. In addition, the contribution of third-forbidden matrix elements to the transition is investigated.

I. INTRODUCTION

For several years, the study of first-forbidden β transitions has been a useful method of learning some of the details of nuclear structure. Now that the conserved-vector-current (CVC) theory has been used to predict the ratio of two of the vector-type first-forbidden nuclear matrix elements $(\int\vec{\alpha}/\int i\vec{r})$, these experiments have taken on the additional value of being able to test the accuracy of the CVC predictions.

The formula which has been widely used to predict the matrix-element ratio was derived independently by Fujita[1] and Eichler.[2] More recently Damgaard and Winther[3] have modified the calculation to obtain an expression for the matrix-element ratio which does not depend on the assumption that the Coulomb Hamiltonian is diagonal. The Damgaard and Winther approach de-emphasizes the CVC ratio as a useful tool in simplifying nuclear matrix-element extraction and emphasizes its role in supplying direct information on nuclear structure.

Previous experiments[4] have shown that a measurement of the energy dependence of the β-γ circular-polarization correlation of La^{140} would be useful in determining the matrix-element ratio more precisely. This measurement has been performed by Ohlms, Bosken, and Simms.[5] Their results will be combined with other experimental data to show that the matrix-element ratio for La^{140} agrees with the Damgaard-Winther formulation, but it does not agree with the calculation of Fujita and Eichler.

The formulas used in the matrix-element analysis are those developed by Bühring.[6] In these formulas Bühring uses the exact electron radial wave functions and takes into account the finite nuclear size. In a later paper[7] he also treats the Coulomb screening of the nuclear charge by the atomic electrons. These formulas have been presented by Simms[8] in such a way that the importance of the

higher-order terms is more clearly demonstrated. The original matrix-element-extraction computer code developed by Simms has been partially revised to include the energy-dependence of the β-γ circular-polarization correlation and a more accurate treatment of the scalar nuclear matrix elements. The most important higher-order matrix elements have also been included in the analysis.

II. THEORY

In this section we wish to obtain an expression for the β-decay vector-matrix-element ratio using the CVC theory. The CVC theory allows one to extend certain relationships among electric-dipole γ-transition operators to vector (first-forbidden) β decay. These $E1$ γ-decay operator relationships will be discussed before the β-decay formulas are presented.

The same nuclear Hamiltonian will be used for both the β and γ transitions.

$$H = H_K + H_c + H_m + H_{NN}. \tag{1}$$

H_K is the kinetic energy; H_c is the Coulomb energy; H_m is the rest mass; and H_{NN} is the energy arising from the potentials generated by the mesonic nucleon-nucleon forces.

A. $E1$ γ-Transition Operators

The theory of electric dipole ($E1$) γ transitions is well known. In the dipole approximation the transition amplitude is proportional to the matrix element (see Fig. 1):

$$M_\gamma = \langle f | \vec{p}_\gamma | a \rangle. \tag{2}$$

The operator $\vec{p}_\gamma$ is shorthand notation for the expression

$$\vec{p}_\gamma = \sum_{j=1}^{A} \tfrac{1}{2}(1 - \tau_3{}^j)\vec{p}_j, \tag{3}$$

and $\vec{p}_j$ is the linear momentum of the radiating nucleon. The isospin convention used here is such that $\tau_3|p\rangle = -|p\rangle$ and $\tau_3|n\rangle = +|n\rangle$. Since $\vec{p}_j$ is proportional to the time derivative of $\vec{r}_j$ (the spatial coordinate of the radiating nucleon), one usually considers the commutator

$$C = [H, \vec{r}_\gamma], \tag{4}$$

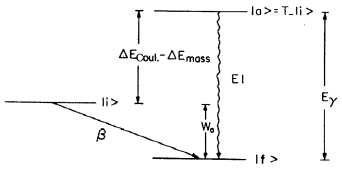

FIG. 1. Schematic of nuclear β and γ transition to the same final state showing notation and energy relationships for β^- decay.

where $\vec{r}_\gamma$ is shorthand notation for

$$\vec{r}_\gamma = \sum_{j=1}^{A} \tfrac{1}{2}(1 - \tau_3{}^j)\vec{r}_j. \tag{5}$$

From elementary quantum mechanics it is known that

$$[H_K, \vec{r}_\gamma] = i\hbar c\,\vec{\alpha}_\gamma. \tag{6}$$

The operator $c\vec{\alpha}_\gamma$ is the relativistic velocity operator defined by

$$c\vec{\alpha}_\gamma = \sum_{j=1}^{A} \tfrac{1}{2}(1 - \tau_3{}^j)c\vec{\alpha}_j. \tag{7}$$

The operator $\vec{r}_\gamma$ is a function of the same variables as H_c and H_m, and since these variables commute,

$$[H_c, \vec{r}_\gamma] = 0, \tag{8}$$

$$[H_m, \vec{r}_\gamma] = 0. \tag{9}$$

The Siegert theorem[9] for nuclear γ-transitions states that electric-multipole operators are not influenced by meson-exchange effects. Therefore,

$$[H_{NN}, \vec{r}_\gamma] = 0. \tag{10}$$

If Eqs. (6)-(10) are substituted into (4) and matrix elements are taken between initial and final states, the result is

$$\langle f | [H, \vec{r}_\gamma] | a \rangle = i\hbar c \langle f | \vec{\alpha}_\gamma | a \rangle. \tag{11}$$

The following definitions are made to simplify notation:

$$\langle \vec{\alpha} \rangle_\gamma = \langle f | \vec{\alpha}_\gamma | a \rangle, \tag{12a}$$

$$\langle i\vec{r} \rangle_\gamma = i \langle f | \vec{r}_\gamma | a \rangle. \tag{12b}$$

Furthermore if $|f\rangle$ and $|a\rangle$ are eigenstates of H with eigenvalues E_f and E_a, respectively, then Eq. (11) becomes

$$(\langle \vec{\alpha} \rangle / \langle i\vec{r} \rangle)_\gamma = (E_a - E_f)/\hbar c. \tag{13}$$

In natural units, where $m_e = \hbar = c = 1$, the result is

$$(\langle \alpha \rangle / \langle i\vec{r} \rangle)_\gamma = E_a - E_f = E_\gamma. \tag{14}$$

B. First-Forbidden "Electric Multipole" Operators

The β-decay "electric multipole" operators will be treated formally in the same way as their γ-transition counterparts. The β operators are defined below for $\beta^\pm$ decay:

$$i\vec{r}_\beta = \sum_{j=1}^{A} i\tau_\pm{}^j \vec{r}_j, \tag{15}$$

$$\vec{\alpha}_\beta = \sum_{j=1}^{A} \tau_\pm{}^j \alpha_j. \tag{16}$$

The objective of this section is to derive a relationship between the matrix elements of $\vec{\alpha}_\beta$ and $i\vec{r}_\beta$ similar to the relationship expressed in Eq. (14). The first step is to consider the matrix elements of the commutator $[H, i\vec{r}_\beta]$ (see Fig. 1):

$$\langle f|[H, i\vec{r}_\beta]|i\rangle = -W_0\langle f|i\vec{r}_\beta|i\rangle . \tag{17}$$

The commutation (6) still holds for β operators:

$$[H_K, i\vec{r}_\beta] = -\hbar c\vec{\alpha}_\beta . \tag{18}$$

The nucleon-rest-mass part of the Hamiltonian no longer commutes with $i\vec{r}_\beta$, because the $\tau_\pm$ in $i\vec{r}_\beta$ does not commute with the τ_3 in H_m. This result is a reflection of the fact that the nucleon's rest mass is changed in β decay.

$$[H_m, i\vec{r}_\beta] = \pm(M_n - M_p)i\vec{r}_\beta,$$
$$= \pm 2.5 m_e i\vec{r}_\beta , \tag{19}$$

for $\beta^\pm$ decay.

It is not obvious that the nucleon-nucleon part of the Hamiltonian still commutes with $i\vec{r}_\beta$. However, Fujita[1] and Eichler[2] have used CVC theory to extend the Siegert theorem to vector first-forbidden β decay. Therefore, it is expected that

$$[H_{NN}, i\vec{r}_\beta] = 0 . \tag{20}$$

To evaluate the remaining term in the commutator $[H, i\vec{r}_\beta]$, matrix elements are taken between the initial and final states, and the expansion theorem is applied by inserting complete sets of states.

$$C_c \equiv \langle f|[H_c, i\vec{r}_\beta]|i\rangle = \sum_{f'}\langle f|H_c|f'\rangle\langle f'|i\vec{r}_\beta|i\rangle - \sum_{i'}\langle f|i\vec{r}_\beta|i'\rangle\langle i'|H_c|i\rangle \tag{21}$$

or

$$C_c = [\langle f|H_c|f\rangle - \langle i|H_c|i\rangle]\langle f|i\vec{r}_\beta|i\rangle + \{\sum_{f'\neq f}\langle f|H_c|f'\rangle\langle f'|i\vec{r}_\beta|i\rangle - \sum_{i'\neq i}\langle f|ir_\beta|i'\rangle\langle i'|H_c|i\rangle\} . \tag{22}$$

The term in the square brackets is just the difference in the Coulomb energies of the initial and final nuclei, generally written as[10]

$$\Delta E_c = 2.4(\alpha Z/2\rho) \tag{23}$$

where α is the fine-structure constant, and Z and ρ are the charge and radius of the daughter nucleus. Z is positive (negative) for negatron (positron) emission.

The term in the curly brackets is a correction term arising from the nondiagonality of the Coulomb Hamiltonian, H_c. The Ahrens-Feenberg approximation[11] assumes that the diagonal elements of H_c are much larger than the nondiagonal elements, so the correction term in (22) is negligible. If this approximation is valid,

$$\langle f|[H_c, i\vec{r}_\beta]|i\rangle = 2.4\frac{\alpha Z}{2\rho}\langle f|i\vec{r}_\beta|i\rangle . \tag{24}$$

Substitution of Eqs. (18), (19), (20), and (24) into (17) yields an expression for the vector-matrix-element ratio in natural units:

$$\Lambda_{CVC}{}^0 \equiv (\langle\vec{\alpha}\rangle/\langle i\vec{r}\rangle)_\beta = W_0 + [2.4(\alpha Z/2\rho) \mp 2.5] \tag{25}$$

for $\beta^\mp$ decay. In writing Eq. (25), the following shorthand notation was used:

$$\langle\vec{\alpha}\rangle_\beta = \langle f|\vec{\alpha}_\beta|i\rangle , \tag{26a}$$

$$\langle i\vec{r}\rangle_\beta = \langle f|i\vec{r}_\beta|i\rangle . \tag{26b}$$

Equation (25) for β decay differs from Eq. (14) for γ decay in the extra terms present, as a result of the neutron-proton substitution.

C. Isobaric-Analog-State Approach to the Vector-Matrix-Element Ratio

The matrix elements $\langle\vec{\alpha}\rangle_\beta$ and $\langle i\vec{r}\rangle_\beta$ for the β transition from $|i\rangle$ to $|f\rangle$ (cf. Fig. 1) can be related by isobaric symmetry to the matrix elements $\langle\vec{\alpha}\rangle_\gamma$ and $\langle i\vec{r}\rangle_\gamma$ of the correspinding electromagnetic transition from $|a\rangle$ to $|f\rangle$, where $|a\rangle$ is the isobaric analog of $|i\rangle$:

$$|a\rangle = T_-|i\rangle . \tag{27}$$

The operator T_- is the isospin lowering operator.

The CVC theory relates $i\vec{r}_\beta$ (containing $\tau_\pm$) to $i\vec{r}_\gamma$ (containing τ_3) in the following way:[12]

$$i\vec{r}_\beta = -[T_-, i\vec{r}_\gamma] . \tag{28}$$

Taking matrix elements of both sides of (28) yields

$$\langle i\vec{r}\rangle_\beta = -\langle f|[T_-, i\vec{r}_\gamma]|i\rangle . \tag{29}$$

If $|a\rangle$ is the *exact* isobaric analog of $|i\rangle$, then

$$\langle i\vec{r}\rangle_\beta = \langle f|i\vec{r}_\gamma T_-|i\rangle . \tag{30}$$

Under the usual assumption that the isospin of a ground state is equal to T_3 for that state, it is easy to show that

$$\langle i\vec{r}\rangle_\beta = (2T_i)^{1/2}\langle i\vec{r}\rangle_\gamma . \tag{31}$$

A similar relationship exists between $\langle\vec{\alpha}\rangle_\beta$ and $\langle\vec{\alpha}\rangle_\gamma$, leading to the result

$$\langle\vec{\alpha}\rangle_\beta = (2T_i)^{1/2}\langle\vec{\alpha}\rangle_\gamma . \tag{32}$$

Therefore, *if* the state $|a\rangle$ is the *exact* isobaric analog of $|i\rangle$, the following equality holds:

$$(\langle\vec{\alpha}\rangle/\langle i\vec{r}\rangle)_\beta = (\langle\vec{\alpha}\rangle/\langle i\vec{r}\rangle)_\gamma . \tag{33}$$

Using Eq. (14) it is also evident that

$$(\langle\vec{\alpha}\rangle/\langle i\vec{r}\rangle)_\beta = E_\gamma . \tag{34}$$

From Fig. 1 it is seen that the energy of the

state $|a\rangle$ differs from the energy of the state $|i\rangle$ because of the change in Coulomb energy and the neutron-proton mass difference. The γ-transition energy E_γ may therefore be written as

$$E_\gamma = W_0 + [\Delta E_c - (M_n - M_p)]. \tag{35}$$

Using the expression for ΔE_c given in Eq. (23) leads at once to the relation (25) for the vector-matrix-element ratio.

Fujita[13] has used these arguments to corroborate the result [Eq. (23)] obtained with the commutator relationships in Sec. B.

D. Damgaard and Winther Formulation of the Vector-Matrix-Element Ratio

The validity of the Ahrens-Feenberg approximation was challenged by Damgaard and Winther.[3] They argued that while the off-diagonal matrix elements of H_c might in fact be small, there is no reason to assume that the corresponding nondiagonal elements of $i\vec{r}_\beta$ will also be small [cf. Eq. (22)]. In fact, since $i\vec{r}_\beta$ is linking nuclear wave functions which in general have different numbers of radial nodes, its off-diagonal matrix elements can vary by orders of magnitude. As a result, the correction term in Eq. (22) attains sufficient size to contribute to the value of the commutator $[H_c, i\vec{r}_\beta]$, and the Ahrens-Feenberg approximation is invalid.

To obtain a more realistic result for the commutator, Damgaard and Winther suggested that the matrix elements of $[H_c, i\vec{r}_\beta]$ actually be calculated, using a realistic form for H_c. If the form for H_c is chosen to be

$$H_c = \sum_{j=1}^{A} \tfrac{1}{2}(1 - \tau_3) e \Phi_{\text{Coul}}(\vec{r}_j), \tag{36}$$

where $\Phi_{\text{Coul}}(\vec{r}_j)$ is the Coulomb potential of a sphere of uniform charge Ze and radius ρ,

$$\begin{aligned} \Phi_{\text{Coul}}(\vec{r}_j) &= (Ze/2\rho)[3 - (r/\rho)^2] \qquad r \leq \rho, \\ &= Ze/r \qquad r \geq \rho, \end{aligned} \tag{37}$$

the commutator C_c then becomes

$$\langle f | [H_c, i\vec{r}_\beta] | i \rangle = (\alpha Z/2\rho)(3 - \lambda)\langle i\vec{r} \rangle_\beta, \tag{38}$$

where the parameter λ is

$$\lambda \equiv \frac{\langle f | i\vec{r}_\beta (r/\rho)^2 | i \rangle}{\langle f | i\vec{r}_\beta | i \rangle}. \tag{39}$$

The vector matrix-element ratio can now be expressed as

$$\begin{aligned} \Lambda_{\text{CVC}} &= (\langle \vec{\alpha} \rangle / \langle i\vec{r} \rangle)_\beta \\ &= W_0 + [(\alpha Z/2\rho)(3 - \lambda) \mp 2.5 m_e], \end{aligned} \tag{40}$$

or in terms of the ratio obtained using the Ahrens-Feenberg approximation [Eq. (25)],

$$\Lambda_{\text{CVC}} = \Lambda_{\text{CVC}}{}^0 + (\alpha Z/2\rho)(0.6 - \lambda).$$

In summary, the relationship for $\Lambda_{\text{CVC}}{}^0$, derived by Fujita and Eichler is a special case of the more general expression for Λ_{CVC} suggested by Damgaard and Winther. When the contributions due to the off-diagonal terms of H_c are negligible, λ is approximately equal to 0.6, and the two expressions are identical.

E. β - Decay Theory

1. *Introduction*

The theoretical formulas for first-forbidden β decay contain many higher-order nuclear matrix elements which are frequently neglected. Even though the results of a matrix-element extraction are usually not sensitive to the higher-order matrix elements, it is certainly not a good general practice to neglect them. When the theoretical formulas are arranged properly, it is not difficult to determine the relative importance of the higher-order terms.

The expressions for the observables can be simplified by following the procedure of Kotani and Ross,[14] where parameters are used to represent the matrix elements. However, a change will be made in their definition of these parameters so that it will be easier to estimate the relative sizes of the various terms.

In order to understand the notation which is used, it is necessary to review the reasons that first-forbidden decays have smaller transition probabilities than allowed decays. The relativistic first-forbidden matrix elements are reduced in size because the relativistic operators mix the large and small components of the nuclear wave functions. The usual estimate for this effect is that these first-forbidden matrix elements are a factor (v_{nucl}/c) smaller than allowed matrix elements. The quantity (v_{nucl}/c) is usually assigned an upper limit of 0.1. It will be convenient to define all of the first-forbidden matrix element *parameters* so that they are the same size as allowed *matrix elements*. By following this procedure, one will more easily see why first-forbidden transition probabilities are smaller than allowed transition probabilities. Also, the relative contributions of the various terms to the transition probability will be evident. For example, the parameter v will be used to represent the matrix element $\int \gamma_5$:

$$D'v \equiv C_A \int \gamma_5. \tag{42}$$

The effect of mixing the large and small components of the nuclear wave functions is contained in

the factor D' which would be approximately equal to v_{nucl}/c (≈ 0.1).

This definition of the parameters would be adequate for comparing first-forbidden and allowed transitions where in both cases there was a large degree of overlap between the initial and final nuclear-state wave functions. It is well known, however, that there are many factors in the nuclear structure which can cause the matrix elements to be much smaller than those observed for the ideal superallowed transitions. As far as the relative comparison of terms is concerned, one does not care about the absolute size of the matrix elements. Thus, the effect of the absolute size is removed by modifying the definition so that the matrix-element parameter is the ratio of the matrix element to a "size factor", η.

$$D'v = \frac{C_A}{\eta}\int \gamma_5 . \tag{43}$$

The parameter η can be determined from the observed transition probability. If ηv is approximately one, then there is a high degree of overlap between the wave functions of the initial and final nuclear states. If ηv is much smaller than one, then there is some property of the nuclear states which causes the matrix elements to be reduced.

The radial first-forbidden matrix elements such as $\int i\vec{r}$ present a different problem of notation. These matrix elements are generally a factor of 100 smaller than the allowed matrix elements, because the order of magnitude of the nuclear radius is typically 10^{-2} in natural units. Thus, in order to be consistent with the definition of the relativistic matrix-element parameters, the radial matrix elements should be normalized by dividing by the nuclear radius ρ. For example:

$$x = \frac{C_V}{\eta}\int \frac{i\vec{r}}{\rho}. \tag{44}$$

Then the radial matrix element $C_V \int i\vec{r}$ must be represented in the formulas by $\rho x \eta$.

The 15 most important first-forbidden nuclear matrix elements and their parameters are given in Table I. The six matrix elements which are usually used in the analysis of first-forbidden transitions are listed on the left side of the table. Nine more matrix elements which arise from third-forbidden terms and finite-size corrections are listed on the right side of the table. The parameters associated with these matrix elements are identified by a prime mark.

Since the radial matrix elements are generally an order-of-magnitude smaller than the relativistic ones, it might seem that the radial matrix element would play a minor role in first-forbidden transitions. Their contribution, however, is increased by the effect of the Coulomb force on the emitted β particle. In several terms of the transition probability, this causes the radial matrix elements to be multiplied by a parameter $\xi = \alpha Z/2\rho$ where Z is the charge of the daughter nucleus and α is the fine-structure constant. This parameter usually increases the contribution of the radial matrix elements by about a factor of 10, so that the Coulomb-enhanced radial matrix elements have approximately the same strength as the relativistic matrix elements.

TABLE I. Matrix elements and matrix-element parameters for first-forbidden β decay.

For $J=0$	$\Delta\pi=$yes
$D'v = \frac{C_A}{\eta}\int \gamma_5$	$D'v' = \frac{1}{\eta} C_A \int \gamma_5 \left(\frac{r}{\rho}\right)^2$
$w = -\frac{1}{\eta} C_A \int \frac{i\vec{\sigma}\cdot\vec{r}}{\rho}$	$w' = -\frac{1}{\eta} C_A \int \frac{i\vec{\sigma}\cdot\vec{r}}{\rho}\left(\frac{r}{\rho}\right)^2$
For $J=1$	**$\Delta\pi=$yes**
$D'y = \frac{1}{\eta} C_V \int \vec{\alpha}$	$D'y' = \frac{1}{\eta} C_V \int \vec{\alpha}\left(\frac{r}{\rho}\right)^2$
$x = \frac{1}{\eta} C_V \int \frac{i\vec{r}}{\rho}$	$x' = \frac{1}{\eta} C_V \int \frac{i\vec{r}}{\rho}\left(\frac{r}{\rho}\right)^2$
$u = \frac{1}{\eta} C_A \int \frac{\vec{\sigma}\times\vec{r}}{\rho}$	$u' = \frac{1}{\eta} C_A \int \frac{\vec{\sigma}\times\vec{r}}{\rho}\left(\frac{r}{\rho}\right)^2$
	$D's' = \frac{1}{\eta} C_V \int \frac{(\vec{\alpha}\cdot\vec{r})\vec{r} - \frac{1}{3}\vec{\alpha} r^2}{\rho^2}$
For $J=2$	**$\Delta\pi=$yes**
$z = -\frac{1}{\eta} C_A \int \frac{iB_{ij}}{\rho}$	$z' = -\frac{1}{\eta} C_A \int \frac{iB_{ij}}{\rho}\left(\frac{r}{\rho}\right)^2$
	$D'r' = -\frac{1}{\eta} C_A \int \gamma_5 \frac{R_{ij}}{\rho^2}$
	$D't' = -\frac{1}{\eta} C_V \int i\gamma_5 \frac{T_{ij}}{\rho^2}$

The parameter ξ is not really useful when the normalized matrix elements are used. It is more convenient to replace $\xi\rho x$ by $(\frac{1}{2}\alpha Z)x$. Thus a new parameter is defined to indicate the presence of the Coulomb enhancement:

$$D \equiv \pm\tfrac{1}{2}\alpha Z + W_0\rho \quad \text{for } \beta^{\mp} \text{ decay}. \tag{45}$$

The small added term in D, which contains the transition energy W_0, is present in order to simplify the final formulas.

It is important to remember that, when a matrix-element parameter is multiplied by ρ rather than D, the matrix element is not enhanced by the Coulomb field; and its contribution to the transition will usually be small.

It is now necessary to reexpress the vector-matrix-element ratio in terms of our parameters:

$$\Lambda_{CVC}{}^{0}=\frac{\langle\vec{\alpha}\rangle}{\langle i\vec{r}/\rho\rangle}=2.4\,\frac{\alpha Z}{2}+(W_0\mp 2.5)\rho\,, \tag{46}$$

and

$$\Lambda_{CVC}=\Lambda_{CVC}{}^{0}+\tfrac{1}{2}\alpha Z(0.6-\lambda)\,. \tag{47}$$

From Table I it is seen that

$$\Lambda_{CVC}=\frac{D'y}{x}\,. \tag{48}$$

2. *ft Values and Absolute Scaling*

The matrix-element scaling factor η is determined from the ft value of the β transition. If the direction of emission and the spin of the β particle and neutrino are not observed, the transition probability $1/\tau$ is

$$\frac{1}{\tau}=\frac{1}{\pi^3}\int_1^{W_0}F_0(Z,W)pWq^2S(W)dW\,. \tag{49}$$

$S(W)$ is the shape-correction factor in terms of the actual matrix elements – *not* the matrix-element parameters. For first-forbidden transitions, the following definition of the Fermi integral is most consistent with the definition for allowed transitions:

$$f=(\bar{S})^{-1}\int F_0(Z,W)pWq^2S(W)dW\,, \tag{50}$$

where the average value of S is defined as

$$\bar{S}=\frac{1}{W_0-1}\int_1^{W_0}S(W)dW\,. \tag{51}$$

The coupling constants can be removed from S by setting $C_V=1.0$ and $C_A=-1.2$ and using the experimental value[15] for the vector coupling constant. Then

$$ft=\frac{6150}{\bar{S}}\,. \tag{52}$$

The scale for the first-forbidden matrix elements is defined relative to the Fermi matrix element for superallowed transitions, i.e., $(\int 1)=\sqrt{2}$. Thus

$$\eta^2=6150[t\int_1^{W_0}F_0(Z,W)pWq^2C(W)dW]^{-1}\,, \tag{53}$$

where $C(W)$ is the shape correction factor defined in terms of the matrix-element *parameters*.

$$C(W)=S(W)/\eta^2\,. \tag{54}$$

In these formulas, p, W, q, and the Fermi function $F_0(Z,W)$ are in natural units, and the partial half-life t is in seconds.

3. *Treatment of Higher-Order Terms*

The complete forms for the matrix-element combination used in this analysis are presented by Bühring[6] and also by Simms.[8] The parameters V (for $\Delta J=0$) and Y (for $\Delta J=1$) are defined to represent the two combinations of matrix elements which are most important.

$$DV\equiv D'[v+v'(a-D\hat{q}+\hat{q}^2)]+D(w+dw')\,, \tag{55}$$

$$DY\equiv D'[y+y'(a+\tfrac{1}{3}D\hat{q}-\tfrac{1}{3}\hat{q}^2]-D[(x+dx')+(u+du')]\,, \tag{56}$$

where:

$$a=-\tfrac{1}{6}[(W\rho+\tfrac{3}{2}\alpha Z)^2-\rho^2]\,, \tag{57}$$

$$d=-(1/5D)[\tfrac{1}{2}\alpha Z-a(3D+2\hat{q})]\,, \tag{58}$$

$$\hat{q}=\tfrac{1}{3}q\rho\,, \tag{59}$$

$$\hat{p}=\tfrac{1}{3}p\rho\,. \tag{60}$$

W and p are the β-particle energy and momentum, respectively, and q is the neutrino momentum. If there is no selection rule operating and there is no internal cancellation, the combinations V and Y are defined so that their order of magnitude is unity.

All of the matrix-element combinations $M^{(K)}_{k_e k_\nu}$ used by Bühring in his formulas for the transition probability are listed below. K is the tensor rank of the combination, and k_e, k_ν are the quantum numbers for the electron and neutrino partial wave.

$$(1+a)M_{11}^{(0)}=-DV\,, \tag{61}$$

$$(1+a)m_{11}^{(0)}=-\tfrac{1}{3}\rho(w+\tfrac{3}{5}aw')\,, \tag{62}$$

$$(1+a)M_{11}^{(1)}=DY+2\hat{q}[(u+au')-D's'(D-\hat{q})]\,, \tag{63}$$

$$(1+a)m_{11}^{(1)}=-\tfrac{1}{3}\rho[(x+\tfrac{3}{5}ax')+(u+\tfrac{3}{5}au')]\,, \tag{64}$$

$$(1+a)M_{12}^{(1)}=(1/\sqrt{2})\hat{q}[2(x+ax')+(u+au')+(D-\hat{q})D'(s'+\tfrac{4}{3}y')]\,, \tag{65}$$

$$(1+\tfrac{3}{5}a)M_{21}^{(1)}=(1/\sqrt{2})\hat{p}[2(x+\tfrac{3}{5}ax')-(u+\tfrac{3}{5}au')+\tfrac{9}{5}s'D'(D-\hat{q})]\,, \tag{66}$$

$$(1+a)M_{12}^{(2)}=\tfrac{1}{2}\sqrt{3}\,\hat{q}[(z+az')+2r'D'(D-\tfrac{2}{5}\hat{q})-t'D'(D-\tfrac{8}{5}\hat{q})]\,, \tag{67}$$

$$(1+\tfrac{3}{5}a)M_{21}^{(2)}=\tfrac{1}{2}\sqrt{3}\,\hat{p}[(z+\tfrac{3}{5}az')+2r'D'(\tfrac{3}{5}D+\tfrac{2}{5}\hat{q})-t'D'(\tfrac{3}{5}D-\tfrac{8}{5}\hat{q})]\,, \tag{68}$$

$$(1+a)m_{12}^{(2)}=\tfrac{1}{2}\sqrt{3}\,\hat{q}[\tfrac{1}{3}\rho D'(2r'-t')]\,, \tag{69}$$

$$(1+\tfrac{3}{5}a)m_{21}^{(2)}=\tfrac{1}{2}\sqrt{3}\,\hat{p}[\tfrac{1}{5}\rho D'(2r'-t')]\,. \tag{70}$$

We consider first the treatment of the third-forbidden parameters r', s', and t'. The contribution of these parameters is reduced by factors of DD' (≈ 0.01) and $D'\rho$ (≈ 0.001) from the contributions of the first-forbidden parameters. Thus, in order for r', s', and t' to contribute to the matrix-element combinations, they must be approximately a

factor of 100 times larger than the first-forbidden parameters. This would be possible only if the first-forbidden parameters were reduced in size, and η were small, because η times *any* matrix-element parameter must be less than $\sqrt{2}$. Since the contribution of these third-forbidden parameters is usually small, it is reasonable to assume that they can be neglected. However, once η is determined in the analysis, the validity of this assumption must be considered.

The remaining higher-order terms can be handled in a different way. For example, consider the parameter x'. This parameter occurs in the formulas as

$$\frac{x+ax'}{1+a},\quad \frac{x+\frac{3}{5}ax'}{1+\frac{3}{5}a},\quad \text{and}\ \frac{x+\frac{3}{5}ax'}{1+a}.$$

Therefore, the following notation is introduced:

$$x_0 \equiv \frac{x+\frac{4}{5}ax'}{1+\frac{4}{5}a}. \tag{71}$$

The parameter x_0 now serves as an average representation of the contributions made by the combinations of x and x' listed above. The parameters u_0, v_0, z_0, w_0, and y_0 are introduced in the same manner:

$$u_0 = \frac{u+\frac{4}{5}au'}{1+\frac{4}{5}a}, \tag{72}$$

$$v_0 = \frac{v+av'}{1+a}, \tag{73}$$

$$z_0 = \frac{z+\frac{4}{5}az'}{1+\frac{4}{5}a}, \tag{74}$$

$$w_0 = \frac{w+\frac{3}{5}aw'}{1+a}, \tag{75}$$

$$y_0 = \frac{y+ay'}{1+a}. \tag{76}$$

Because of Coulomb enhancement, the parameter u' and x' make a larger contribution in DY than elsewhere. Thus they cannot be represented in DY by an average value, using x_0 and u_0. Since the operators in u and x have the same radial form, it is reasonable to assume that the ratio of u' and u is nearly the same as the ratio of x' and x. This ratio is just the parameter λ [cf. Eq. (39)].

With the above simplifications, the matrix-element combinations can now be written in the following form:

$$DV = D'v_0 + D(w + dw'), \tag{77}$$

$$DY = D'y_0 - D[(1+d\lambda)/(1+a)](x+u), \tag{78}$$

$$M_{11}^{(0)} = -DV, \tag{79}$$

$$m_{11}^{(0)} = -\tfrac{1}{3}\rho w_0, \tag{80}$$

$$M_{11}^{(1)} = DY + 2\hat{q}u_0, \tag{81}$$

$$m_{11}^{(1)} = -\tfrac{1}{3}\rho(x_0+u_0), \tag{82}$$

$$M_{12}^{(1)} = (1/\sqrt{2})\hat{q}(2x_0+u_0), \tag{83}$$

$$M_{21}^{(1)} = (1/\sqrt{2})\hat{p}(2x_0-u_0), \tag{84}$$

$$M_{12}^{(2)} = \tfrac{1}{2}\sqrt{3}\,\hat{q}z_0, \tag{85}$$

$$M_{21}^{(2)} = \tfrac{1}{2}\sqrt{3}\,\hat{p}z_0. \tag{86}$$

4. *The Correlation Function*

The general β-γ angular-correlation function is presented below.

$$N(W,\theta,S) = \sum_{n=0}^{3} S^n A_n(W) P_n(\theta), \tag{87}$$

$$A_n(W) = \sum_{k\le k'} b_{kk'}^{(n)}(W) G_{kk'}^{(n)}. \tag{88}$$

$P_n(\theta)$ are the Legendre polynomials, and $G_{kk'}^{(n)}$ are combinations of 3-j and 6-j symbols which are defined and tabulated for some important spin sequences by Kotani.[16] The particle parameters $b_{kk'}^{(n)}(W)$ are defined in terms of the matrix-element combination $M_{k e k_\nu}^{(K)}$ by Simms.[8] The helicity factor s is +1 (−1) for right- (left-) handed circularly-polarized γ rays.

III. ANALYSIS PROCEDURE

The 2.2-MeV $3^- \to 2^+ \to 0^+$ β transition of La^{140} was analyzed by a set of computer codes originally developed by Simms.[8] These programs used the energy dependence of the β-γ directional correlation coefficient $[A_2(W)/A_0(W)]$,[4] the energy dependence of the β-γ circular-polarization correlation $[P_\gamma(W_\beta, \theta_{\beta\gamma})]$,[5] and the spectrum shape factor $[C(W)]$[17] to determine the size of the nuclear matrix elements.

The computer program has been improved in order to treat the parameter w more accurately. Usually the matrix-element combination DV is much more important than w[(see Eqs. (79) and (80)]. That is,

$$\frac{M_{11}^{(0)}}{M_{11}^{(0)}} = \frac{DV}{w/3\rho} \approx \frac{3D}{\rho} \approx 30. \tag{89}$$

However, if there is internal cancellation in DV, then the experimental observables can become sensitive to w.

The β-γ circular-polarization correlation is given by

$$P_\gamma(W,\theta) = \frac{N(W,\theta,1) - N(W,\theta,-1)}{N(W,\theta,1) + N(W,\theta,-1)} \tag{90}$$

which, with Eq. (87), reduces to

$$P_\gamma(W,\theta) = \frac{A_1(W)P_1(\theta) + A_3(W)P_3(\theta)}{A_0(W) + A_2(W)P_2(\theta)}. \tag{91}$$

When an actual measurement of the β-γ circular-polarization correlation is made, there is attenuation due to the efficiency of the circular-polarization analyzer and geometrical factors such as finite solid angle. The effect is represented by:

$$N_{\text{obs}}(W,\theta,S)=\sum_{n=0}^{3}S^{n}\epsilon_{n}A_{n}(W)P_{n}(\theta), \tag{92}$$

where the ϵ_n are the attenuation factors for the various multipole orders. The expression (91) for the degree of circular polarization of the γ ray becomes the expression for the *observed effect*,

$$\delta=\frac{\epsilon_1 A_1(W)P_1(\theta)+\epsilon_3 A_3(W)P_3(\theta)}{A_0(W)+\epsilon_2 A_2(W)P_2(\theta)}. \tag{93}$$

The relationship between P_γ and δ is obviously very complicated. In fact it is impossible to obtain P_γ from an energy-dependent measurement of δ. The relative sizes of A_3 to A_1 and A_2 to A_0 must be known in order to determine P_γ; but in order to get the A's, the matrix elements must be extracted. Therefore, the analysis program was coded to use δ instead of P_γ to restrict the values of the matrix-element parameters.

IV. RESULTS

The analysis of La^{140} shows considerable deviation from the Fujita-Eichler CVC prediction for the vector-matrix-element ratio. With $\lambda=0.6$, the experimental vector-matrix-element ratio and the theoretical prediction are as follows:

$$\Lambda_{\text{CVC}}^{\text{th}}(\lambda=0.6)=0.5524, \tag{94a}$$

$$\Lambda_{\text{CVC}}^{\text{th}}(\lambda=0.6)=0.336\pm0.049. \tag{94b}$$

The analysis was repeated with variation in the value of the parameter λ. For $\lambda=2.45$, agreement between theory and experiment was obtained.

$$\Lambda_{\text{CVC}}^{\text{th}}(\lambda=2.45)=0.160, \tag{95a}$$

$$\Lambda_{\text{CVC}}^{\text{exp}}(\lambda=2.45)=0.160\pm0.019. \tag{95b}$$

The extracted matrix elements and other pertinent parameters for La^{140} are presented in Table II for $z_0=1.0$.

TABLE II. Extracted matrix-element parameters for La^{140}. $D=0.2397$, $a=-0.0781$, $\rho=0.0158$.

Parameter	Value
$D'y_0$	0.5456 ± 0.2102
x	3.243 ± 1.260
u	2.043 ± 0.783
Y	-0.284 ± 0.066
z_0	1.0000
η	0.0439 ± 0.0114
$\Lambda_{\text{CVC}}^{\text{exp}}=D'y_0/x$	0.160 ± 0.019
λ	2.45 ± 0.20

The experimental result of $\lambda=2.45$ provides a measurement of the higher-order matrix-element parameters u' and x'. The *extracted* vector-matrix-element ratio, $\Lambda_{\text{CVC}}^{\text{exp}}$, is actually $D'y_0/x$ instead of $D'y/x$. Thus there is an uncertainty in Λ_{CVC} due to the uncertainty in the size of y'. This error is only a few percent if y' is about the same size as y. In the worst possible case for La^{140}, y' can be no larger than $\sqrt{2}/\eta_{\min}$, which is 44. For the largest allowable value of $D'y_0$ and the smallest value of x obtained from Table II, the ratio $D'y/x$ is only 0.50. Thus even for the most extreme set of values for the matrix elements, agreement with the Fujita-Eichler result is still not possible.

Table II shows that the limits of error on $\Lambda_{\text{CVC}}^{\text{exp}}$ and Y are smaller than one might expect from the errors given for $D'y_0$, x, and u. This is true because the extreme values of $D'y_0$, x, and u do not occur in the same acceptable set of matrix elements. Therefore, as the limits of error on the matrix elements are being determined, the computer records the maximum and minimum values of $\Lambda_{\text{CVC}}^{\text{exp}}$ and Y that occur.

Figures 2, 3, and 4 show the data that were used to extract the matrix elements. Also plotted in these figures are the theoretical results for each of the observables using a typical set of matrix elements which agree with the data. Several β transitions contribute to the circular polarization, so the experimental points in Fig. 4 are not expected to agree with the theoretical curve below $W=3.2$. The variation of the experimental limits on $\Lambda_{\text{CVC}}^{\text{exp}}$ and the theoretical prediction for Λ_{CVC}

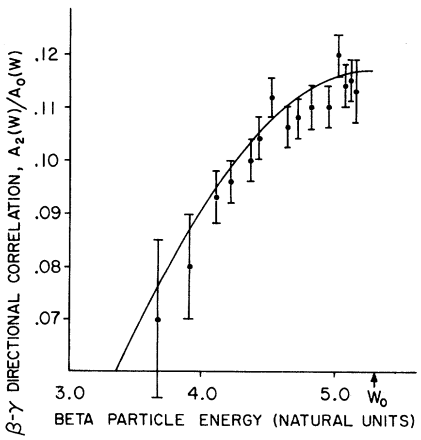

FIG. 2. The shape correction factor, $C(W)$, as a function of β energy, measured by Langer and Smith (Ref.17). The solid curve is the theoretical shape correction factor for the matrix-element set $D'y_0=0.567$, $x=3.53$, $u=2.32$, $z_0=1.0$, and $\lambda=2.45$.

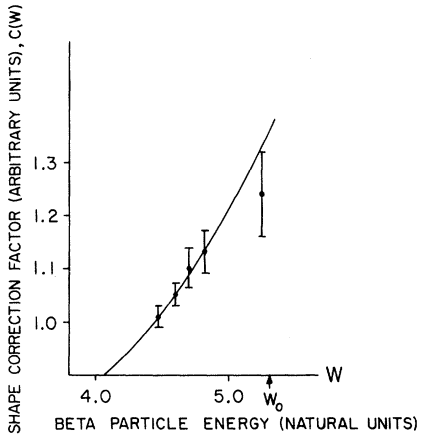

FIG. 3. The β-γ directional correlation coefficient, $A_2(W)/A_0(W)$, as a function of β energy, measured by Steffen *et al.* (Ref. 4). The solid curve is the theoretical directional correlation coefficient for the matrix-element set $D'y_0=0.567$, $x=3.53$, $u=2.32$, $z_0=1.0$, and $\lambda=2.45$.

are shown as a function of the parameter λ in Fig. 5. The difficulty in obtaining P_γ from δ has been discussed in Sec. III. The general nature of the circular polarization is illustrated in Fig. 6 by plotting P_γ for the set of matrix elements used to calculate the other theoretical curves.

In order for the neglect of the contributions of the third-forbidden parameters r', s', and t' to affect our results, these parameters must be at least $1/DD'$ ($\approx$30 for La^{140}) times as great as the largest of the first-forbidden parameters, as pointed out in Sec. II. E3. Since the smallest value of η consistent with its limits of error is 0.032, the largest possible value of the third-forbidden parameters is $\sqrt{2}/\eta$ or 44. In the absence of any known mechanism which would be expected to reduce the first-forbidden matrix elements without

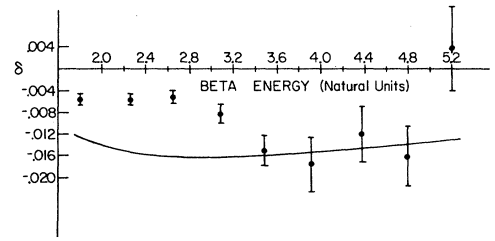

FIG. 4. The measured β-γ circular polarization, δ, as a function of β-particle energy, measured by Ohlms, Bosken, and Simms (Ref. 5). The solid curve is the theoretical prediction for the raw effect for matrix-element set $D'y_0=0.567$, $x=3.53$, $u=2.32$, $z_0=1.0$, and $\lambda=2.45$. Several β transitions contribute to the circular polarization, so the experimental points are not expected to agree with the theoretical curve below $W=3.2$.

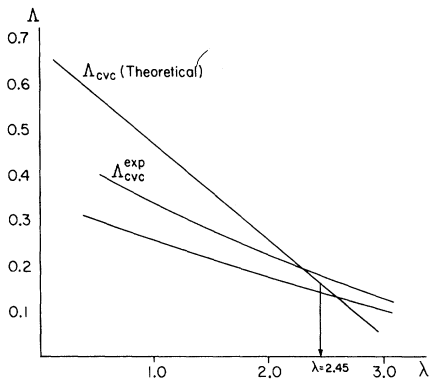

FIG. 5. The theoretical vector-matrix-element ratio, Λ_{CVC}, compared to the upper and lower limit on the extracted vector-matrix-element ratio, $D'y_0/x$, as a function of the parameter λ.

also reducing the third-forbidden matrix elements, it is extremely unlikely that the third-forbidden matrix elements r', s', and t' can influence our results.

A recent measurement[18] of the β-γ directional correlation differs slightly from the one used in this analysis.[4] Matrix elements were therefore extracted using these newer data. The primary result was that $\Lambda_{CVC}{}^{exp}$ was permitted to have even lower values, but its upper limit was essentially unchanged. The extracted value for the parameter λ was 2.76, so the disagreement with the Fujita-Eichler prediction still persists.

V. DISCUSSION

There are several interesting features about the matrix elements of La^{140}. The matrix elements are reduced in size by more than an order of mag-

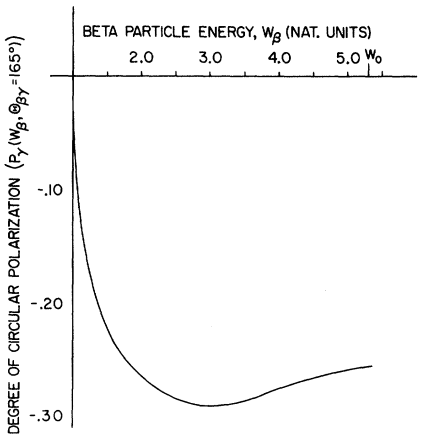

FIG. 6. The theoretical prediction for the true β-γ circular polarization, P_γ, as a function of β-particle energy for the matrix-element set $D'y_0=0.567$, $x=3.53$, $u=2.32$, $z_0=1.0$, and $\lambda=2.45$.

nitude relative to their maximum possible physical size. This diminished size is reflected in the large logft value (logft = 9.1) for the decay. The large logft is also a result of the substantial cancellation among the parameters x, u, and $D'y_0$. This cancellation reduces Y by about an order of magnitude.

The reduced size of the matrix elements cannot be explained by the standard selection rules. In the La^{140} nucleus ($Z = 57$, $N = 83$) the shell model predicts a nucleon configuration with one hole in the proton $g_{7/2}$ shell and one neutron in the $f_{7/2}$ shell above the filled $N = 82$ shell. Since the transforming nucleon is expected to have the same total angular momentum before and after the β transition, a j selection rule would not be expected to reduce the vector-type matrix elements relative to the B_{ij} matrix element. The K selection rule applies only to deformed nuclei. Since the neutron configuration in La^{140} is very close to that of a filled shell, a K selection rule seems very unlikely.

The point of greatest interest in the analysis of La^{140} is the serious disagreement between the measured value for the vector-matrix-element ratio and the value predicted by the Fujita-Eichler formula [Eq. (25)]. For La^{140} the measured value of λ is not 0.6 but 2.45. This information on the sizes of the matrix-element parameters x' and u' is useful new data for nuclear-structure studies.

The disagreement with the Fujita-Eichler prediction can be understood in light of the discussion in Secs. II B, C, and D. The off-diagonal matrix elements of H_c are usually considered to be at least an order-of-magnitude smaller than the diagonal ones. However, as is shown in Eq. (22), the size of the correction term also depends on additional matrix elements of $i\vec{r}_\beta$ which do not contribute to the β transition. Damgaard and Winther[3] have suggested that these additional matrix elements $\langle f'|i\vec{r}_\beta|i\rangle$ ($f' \neq f$) and $\langle f|i\vec{r}_\beta|i'\rangle$ ($i' \neq i$) can be an order-of-magnitude larger than $\langle f|i\vec{r}_\beta|i\rangle$. When the additional matrix elements are large, the correction term in Eq. (22) cannot be neglected even if the off-diagonal matrix elements of H_c are small.

Of course this explanation is not acceptable unless it is physically possible for the matrix elements $\langle f'|i\vec{r}_\beta|i\rangle$ and $\langle f|i\vec{r}_\beta|i'\rangle$ to be much larger than $\langle f|i\vec{r}_\beta|i\rangle$. This can happen only if $\langle f|i\vec{r}_\beta|i\rangle$ is much smaller than $\sqrt{2}$, since none of the matrix elements of $i\vec{r}_\beta$ can be larger than $\sqrt{2}$. The matrix element $\langle f|i\vec{r}_\beta|i\rangle$ is ηx which has a typical value of 0.14 for La^{140}. Therefore, it is possible for La^{140} that the large deviation from the Fujita-Eichler prediction is caused by $\langle f'|i\vec{r}_\beta|i\rangle$ and $\langle f|i\vec{r}_\beta|i'\rangle$ being an order-of-magnitude larger than $\langle f|i\vec{r}_\beta|i\rangle$ rather than by the off-diagonal matrix elements H_c being large.

The discussion in Sec. II C shows that a simple relation [Eq. (34)] exists between the vector-matrix-element ratio and the energy of the exact isobaric analog to the initial state in La^{140}. When one considers the large amount of systematic evidence which is available on the energy of isobaric analog states, it seems unlikely that the analog state would exist at the very low energy (3.1 MeV above the initial state) which would be required by the measured vector-matrix-element ratio. (An effort has been made[19] to locate this analog state, but the results were not conclusive.) Furthermore, if the explanation given above for the La^{140} vector-matrix-element ratio is correct, then the mixing in the analog state due to the Coulomb field need not be large. The essential point is that when the β-decay matrix element $\langle f|i\vec{r}_\beta|i\rangle$ is small, the vector-matrix-element ratio can be quite sensitive to the off-diagonal matrix elements of H_c. The impurities in the analog state which would be produced by small off-diagonal matrix elements would not necessarily cause the energy of the analog state to be significantly different from the observed systematic trend.

This nucleus offers a significant challenge for nuclear model calculations. It would be necessary to explain the relatively small size of $\langle f|i\vec{r}_\beta|i\rangle$, and also permit some of the matrix elements $\langle f'|i\vec{r}_\beta|i\rangle$ and $\langle f|i\vec{r}_\beta|i'\rangle$ to be much larger than $\langle f|i\vec{r}_\beta|i\rangle$. The nuclear-model calculation could then be used to estimate the size of the off-diagonal matrix elements of H_c.

The Fujita-Eichler expression for predicting the vector-matrix-element ratio can still be quite useful in matrix-element extractions. If the experimental data indicate that the β-decay matrix element $\langle f|i\vec{r}_\beta|i\rangle$ is not small, it is unlikely that the correction term in Eq. (22) will be significant. Then the Fujita-Eichler expression can be used to limit the values of the matrix elements $\langle\vec{\alpha}\rangle$ and $\langle i\vec{r}\rangle$ which are acceptable.

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Search for Spontaneously Fissioning Elements in Nature

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The failure to observe fission tracks in old Pb-rich and Au-rich minerals restricts the spontaneous fission rates of Au and Pb to $\lesssim 1\times10^{-25}$ per year per Au atom and $<3\times10^{-24}$ per year per Pb atom (at 95% confidence level). No fission-track evidence for superheavy elements in these or other minerals rich in heavy elements has been found. These negative results are not compatible with recent proportional-counter data and track data of Flerov and co-workers.

Although it is not even clear whether superheavy elements can be made in known astrophysical processes,[1] the recent prediction that nuclides with $Z\approx110$ and $A\approx184$ may have[2] half-lives $\gtrsim10^8$ yr has stimulated searches and suggestions for where to search for "element X" in terrestrial bodies,[2-4] in cosmic rays,[5-7] and in meteorites.[8-10] In a recent report of work prior to publication, Flerov *et al.*[11] claim to have established the existence of a long-lived spontaneously fissioning element that is present in lead glasses and in the mineral galena (PbS), and that may be element 114. In a proportional counter they recorded large pulses, at rates ranging from $\sim10^{-3}$ to 10^{-2} per h, which they claim were not due to induced fission of the lead or thorium impurities, or to spontaneous fission of uranium impurities in the samples. The measured spontaneous fission rate per lead atom was found to be 10^{-21} to 10^{-20} per yr. Recently Cieslak has found spontaneous-fission tracks in lead glasses that support this rate.[12]

The implications of their interpretation of their result, if it is correct, are profound and far-reaching. Although our own search for element X is incomplete, we believe it is important to present our negative results, since they are already in conflict with the remarkable conclusions drawn by Flerov *et al.*[11]

Our method of establishing the existence of a long-lived spontaneously fissioning superheavy element is far more sensitive than the counter method, or the method based on tracks in man-made glasses; we seek fission tracks stored over *millions of years* in ancient minerals.[13] These tracks should be distinguishable on a statistical basis from those caused by either spontaneous or induced fission of known heavy elements. Their average length should be 25 to 40% greater (because of the much greater kinetic energy of the fragments),[10] and a detectable fraction of the tracks should be three-pronged and indicative of ternary fission.[10]