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[1]M. Baranger, Phys. Rev. 120, 957 (1960).

[2]R. Arvieu, Ann. Phys. (Paris) 8, 407 (1963).

[3]L. S. Hsu, Nucl. Phys. A96, 624 (1967).

[4]K. Ikeda, T. Udagawa, and H. Yamaura, Progr, Theoret. Phys. (Kyoto) 33, 22 (1965).

[5]T. T. S. Kuo and G. E. Brown, Nucl. Phys. 85, 40 (1966); A92, 481 (1967); T. T. S. Kuo, *ibid.* A90, 199 (1967).

[6]M. Gmitro, J. Hendeković, and J. Sawicki, Phys. Letters 26B, 252 (1968), and Phys. Rev. 169, 983 (1968); M. Gmitro and J. Sawicki, Phys. Letters 26B, 493 (1968).

[7]K. Bleuler, M. Beiner, and R. de Toureil, Nuovo Cimento 52B, 45, 149 (1967); M. Beiner, private communication to J. Sawicki.

[8]K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. 126, 881 (1962).

[9]C. M. Shakin, Y. R. Waghmare, and M. H. Hull, Jr., Phys. Rev. 161, 1006 (1967); C. M. Shakin, Y. R. Waghmare, M. Tomaselli, and M. H. Hull, Jr., *ibid.* 161, 1015 (1967).

[10]F. Tabakin, Ann. Phys. (N.Y.) 30, 51 (1964).

[11]M. Gmitro, A. Rimini, J. Sawicki, and T. Weber, Phys. Rev. 173, 964 (1968).

PHYSICAL REVIEW C VOLUME 1, NUMBER 5 MAY 1970

# Energy-Dependent Beta-Gamma Circular Polarization of La $^{140}$ †

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The $\beta$-$\gamma$ circular polarization $P_\gamma$ has been measured as a function of the $\beta$-particle energy for $La^{140}$ and for $Co^{60}$. The $Co^{60}$ results confirm that $P_\gamma$ is proportional to $v/c$. The results for the 2.2-MeV transition of $La^{140}$ show that the original prediction for the vector-matrix-element ratio is not correct for this transition. However, the results are consistent with predictions based on the conserved-vector-current (CVC) theory if the approach suggested by Damgaard and Winther is followed. This interpretation yields information on third-forbidden matrix elements.

## I. INTRODUCTION

For several years first-forbidden $\beta$ transitions have been used to study details of nuclear structure. These measurements have taken an additional significance because the ratio of two of the $\beta$-decay matrix elements ($\int\vec{\alpha}/\int i\vec{r}$) can be predicted by the conserved-vector-current (CVC) theory.[1,2] A more recent application of the CVC theory by Damgaard and Winther[3] shows that contributions from third-forbidden matrix elements can be very important in determining the CVC ratio. Therefore a measurement of the first-forbidden matrix elements may be used to determine the contribution of a third-forbidden matrix element to the first-forbidden transition.

Previous measurements of the spectrum shape,[4] the energy dependence of the directional correlation coefficient,[5-8] and the angular dependence of the $\beta$-$\gamma$ circular polarization[9,10] have restricted the matrix elements for the 2.2-MeV $3^- - 2^+$ $\beta$ transition in $La^{140}$. It has also been shown that a measurement of the energy dependence of the $\beta$ circularly-polarized $\gamma$ correlation should be of great help in restricting the possible value of the vector-matrix-element ratio.[10]

This measurement has been performed in our laboratory. The $\gamma$-ray circular polarization was measured using the technique of forward Compton scattering from magnetized iron.[11] The effect is very small, less than 1%, so it was necessary for the electronic system to have three special features: (1) Gain stabilization was used for both the $\beta$ and $\gamma$ detectors; (2) the electronics associated with the $\beta$ detector was designed to accept high counting rates so that adequate statistical accuracy could be obtained in a reasonable period of time; and (3) since a strong source was used to obtain high counting rates, it was necessary to record true and chance coincidences simultaneously.

The extraction of the matrix elements was carried out using an improved version of a method developed by Simms.[12] The details of this procedure will be discussed in the following paper.[13]

## II. EXPERIMENTAL METHODS

A block diagram of the electronic system is shown in Fig. 1. It is essentially a fast-slow coincidence system which uses a time-to-amplitude converter (TAC) to set the fast-coincidence requirement. Linear signals from the $\beta$ detector are processed so that they can be accepted by a multichannel analyzer (MCA) when they are in coincidence with $\gamma$ rays of the proper energy. The $\beta$ pulse is routed into the upper or lower half of the MCA depending on the direction of the magnetic field in the polarization analyzing magnet. In a particular memory half, the $\beta$ pulse is routed into an upper or lower quadrant depending on whether it corresponds to a true-plus-accidental or a pure-accidental coincidence. Simple subtractions are all that are required to get the relative difference in the number of true coincidences for the two field directions for each channel. This relative difference is related to the $\gamma$-ray circular polarization by the efficiency of the analyzing magnet.

One of the primary problems with this type of experiment is that the probability of detecting a $\gamma$ ray is rather small. Therefore it is necessary to use a strong source in order to obtain reasonable coincidence counting rates. Of course this means that the $\beta$ singles counting rate will be large, and the accidental coincidences will be significant. The procedure for performing linear pulse-height analysis with high $\beta$ counting rates will be considered first.

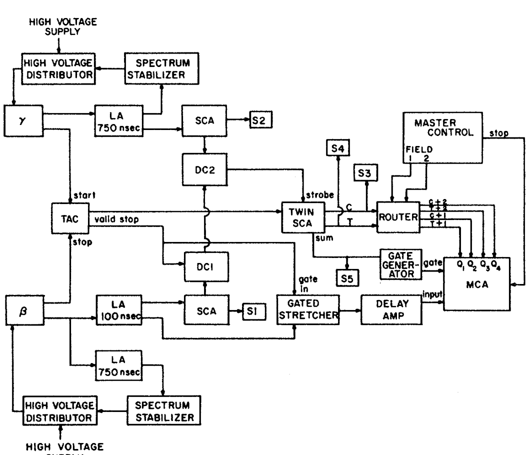


FIG. 1. Block diagram of the electronic system.

Linear pulses from the $\beta$ detector go to a fast-rise-time (20 nsec) double-delay-line clipped linear amplifier (DDLA)[14] (clipping time 100 nsec). The output of this amplifier is connected to a gated pulse stretcher[14] and a fast single-channel analyzer (SCA),[14] which sets the acceptable energy range for the $\beta$ particles. The linear gate in the stretcher is opened whenever a fast coincidence is recorded by the TAC. The stretched pulse is reshaped to bipolar form, and then it is presented to the MCA. There are two advantages in this arrangement. The large $\beta$ singles rate can be measured accurately with the 100-nsec DDLA and fast SCA. This is necessary in order to check the stability and magnetic field sensitivity of the $\beta$ detector. The fast stretcher permits the coincidence $\beta$ pulses to be recorded in a conventional MCA.

The coincidence system has several advantages over the conventional fast-slow arrangement. The output pulses from the TAC go to twin single-channel analyzers (TSCA).[14] This instrument is designed so that the two single-channel analyzers that it contains can conveniently be set to have identical window widths. One window is set to contain the true-coincidence peak in the time distribution. Therefore this SCA will respond to true and accidental coincidences. The other SCA is set on the accidental-coincidence part of the time distribution so that it will respond to an equal number of accidental coincidences. This system was checked in several ways, and it was found that the accidental coincidences could be measured with better than 1% accuracy. The TSCA provides a gate pulse and two route pulses for the MCA.

The time-to-amplitude converter (Ortec model 437) was slightly modified for use in this experiment. In its original form the instrument had a reset period of 4 $\mu$sec for every start pulse that it accepted. This period was reduced to 0.6 $\mu$sec without degrading the performance of the instrument for time ranges up to 800 nsec. A second modification was made so that a logic pulse was available whenever a time conversion occurred. This pulse, called "valid stop," is timed from the stop input of the TAC. It was used to open the gated stretcher and to provide the input to the $\beta$ double-coincidence circuit (DC1).[15]

DC1 has two functions. It selects TAC events that correspond to $\beta$ particles which have the correct energy. It also serves to reject pileup in the $\beta$ amplifier. The valid-stop pulse is timed from the leading edge of the $\beta$-detector pulse. (That is, the stop channel of the TAC is triggered directly from the anode of the photomultiplier tube.) The fast SCA uses zero-crossing timing. Thus the leading-edge pulse is delayed (in DC1) and placed in coincidence with the zero-crossing pulse. It is well known that if pileup occurs in a DDLA, the zero-crossing point will be shifted so that a coincidence will not occur. This reduces the period in which pileup can occur without being recognized from the 100-nsec clipping period to 25 nsec (the resolving time of DC1).

The output pulse from DC1 goes to DC2 where a requirement on the $\gamma$-ray energy is imposed. The resolving time of this circuit is not critical because both of the input counting rates are low. The output pulse of DC2 goes to strobe the TSCA. Thus MCA gate and route pulses are not generated unless all of the energy, time, and pile-up conditions have been met.

Both detectors were stabilized by a system employing a Spectrastat (Cosmic Radiation Laboratory, Bellport, New York). This device varies the photomultiplier tube high voltage to keep the gain of the system constant. In order for the Spectrastat to work properly, there must be a well-defined peak in the detector spectrum. The $\gamma$ detector was stabilized by using the 136-keV $\gamma$ ray of $Fe^{57}$. The $\beta$ detector was stabilized by using a small NaI crystal doped with $Am^{241}$, an $\alpha$ emitter (Harshaw Chemical Co., Cleveland, Ohio). This crystal acted as a light pulser and provided a peak which was much higher than the end point of the $\beta$ spectrum. The Spectrastat provides stabilization against long-term drifts and also against gain shifts resulting from reversal of the field in the analyzing magnet. In order to obtain adequate magnetic field stabilization, it was necessary to mount the light pulser so that its light was emitted back into the plastic scintillator (see Fig. 2). The light was collected by reflection in the same way that light from the plastic scintillator is collected. Thus the photocathode was uniformly illuminated by the light pulser. This is important because the magnetic field affects electrons ejected from different parts of the photocathode in very different ways. Since the maximum current available from the Spectrastat was too small for the voltage divider used with the photomultipliers, a buffer unit was constructed. The Spectrastat controlled the output voltage of the buffer, but the current was supplied by a separate high-voltage supply.

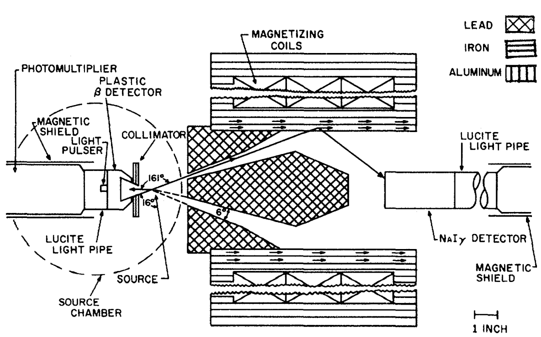


FIG. 2. Cross-section drawing showing the detectors and analyzing magnet.

The $\beta$ detector was calibrated by using conversion-electron sources. Unless precautions are taken to reduce gain shift in the photomultiplier tube, the calibration would be useless when the high-rate $\beta$ source was introduced. The gain shift was first minimized by using emitter followers to supply the current to the last four dynodes of the photomultiplier tubes. With this aid, the Spectrastat could use the light pulser to hold the gain constant for a very wide range of $\beta$ counting rates.

The detector geometry and analyzing magnet are shown in Fig. 2. The $\beta$ detector was NE102 plastic scintillator (Nuclear Enterprises, Los Gatos, California) machined into a well shape to reduce backscattering. A light pipe was used with the $\beta$ detector to provide a method of mounting the light pulser. The light pipe and $\beta$ detector were covered with titantium dioxide reflecting paint to reduce light loss. All optical contacts were made with Araldite 502 bonding agent (CIBA Products, Fairlawn, New Jersey). The energy resolution with a RCA 8575 photomultiplier tube was 15% full width at half maximum for the 624-keV conversion electron of $Cs^{137}$. (The same detector gave 13% resolution when the light pipe and pulser were not used.) One disadvantage of this shape for the detector is that the opening to the well is small (5/8 in.) even though a 2–in. photomultiplier tube is used. Other well shapes were used which had a larger opening and which still provided adequate reduction of the backscattering. Unfortunately, they gave poorer energy resolution.

The $\gamma$ detector used a $3\times1.5$–in. NaI(Tl) crystal with an RCA 8575 photomultiplier tube. A Lucite light pipe (7 in. long) was placed between the crystal and the photomultiplier tube so that the photomultiplier tube would be outside of the magnetic field. The light pipe had an adverse effect on the energy resolution of the $\gamma$ detector (18% resolution for the $Cs^{137}$ $\gamma$ ray). However, good energy resolution is not necessary for the scattered $\gamma$ rays, and the light pipe helped reduce the effect of the magnetic fields on the photomultiplier tube. The effect of the magnetic field was further reduced by using magnetic shields and by placing a compensating coil around the photomultiplier tube. The coil was wired so that the magnetic field it produced was opposite to the field produced by the analyzing magnet on the photomultiplier tube.

The $Co^{60}$ source used in the testing of the instrument was prepared by the New England Nuclear Corp. The source was electroplated on $\frac{1}{4}$-mil

Mylar which had been coated with a 50-$\mu$g/cm$^2$ film of copper. The source was then covered with a 20-$\mu$g/cm$^2$ film of aluminum. Lanthanum was obtained from the Nuclear Science and Engineering Co., Pittsburg, Pennsylvania. The sources were prepared by evaporation to dryness from HCl solution under a heat lamp. The source strength was about 0.5 mCi, and the maximum source thickness was estimated to be less than 3 mg/cm$^2$. A new lanthanum source was used every other day.

In the $Co^{60}$ part of the experiment, the $\gamma$ single channel accepted all scattered $\gamma$ rays above 350 keV. $\beta$ pulses for energies between 30 and 350 keV were accepted. The decay scheme of $La^{140}$ is shown in Fig. 3. $La^{140}$-scattered $\gamma$ rays above 450 keV were accepted. All $\beta$ energies above 100 keV were accepted. For both isotopes, $\gamma$-$\gamma$ coincidences were possible. These events were measured by stopping the $\beta$ particles with an aluminum shield which was placed over the well detector opening.

There are two different types of data to be processed in the experiment; the number of counts recorded on various scalers, and the analyzer output. Every 10 min, scalers recording $\beta$ singles, $\gamma$ singles, pure-accidental coincidences, and true-plus-accidental coincidences were read out, and the direction of the field was switched. A typical run would consist of 15 h of $\beta$-$\gamma$ measurements followed by 8 h of $\gamma$-$\gamma$ measurements. The MCA was read out after the $\beta$-$\gamma$ and after the $\gamma$-$\gamma$ measurements.

The scaler outputs were monitored to give an indication of the stability and magnetic field dependence of the instrument. The $\beta$ and $\gamma$ singles counting rates should be field independent, as should the $\gamma$-$\gamma$ coincidence rate. To test for stability, the data were first corrected for source decay. Then the data were checked for slow drifts with time. The drift was usually less than 0.03% per day. The random variations around the average rate were also checked. A $\chi^2$ test was used for the coincidence counting rates. The stability of the instrument was quite good even though the $\beta$ singles counting rate was frequently higher than $5\times10^5$ counts/sec. The average deviation of the singles counting rates was typically less than 0.1%. The difference in the average singles rates for the two field positions was less than 0.1%. The true-to-chance coincidence ratio was approximately 10 : 1. Since the chance-coincidence rate was measured simultaneously with an accuracy of better than 1%, the accidental coincidences did not limit the accuracy of the experiment.

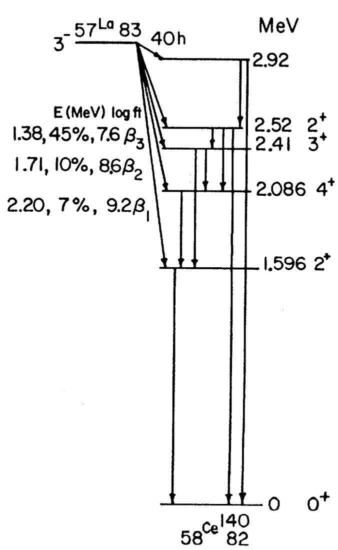


FIG. 3. Decay scheme of $La^{140}$.

The analyzer output was treated in a straightforward manner. The data were corrected for source decay and for the unequal amount of time that $\beta$-$\gamma$ and $\gamma$-$\gamma$ measurements were made. Accidental coincidences were removed from the $\beta$-$\gamma$ and $\gamma$-$\gamma$ numbers. Then the $\gamma$-$\gamma$ background was subtracted from the $\beta$-$\gamma$ result. Finally, the relative difference in the number of actual $\beta$-$\gamma$ coincidences was found for each channel. Statistical uncertainties were propagated in the standard fashion.

The $\beta$-$\gamma$ angular-correlation function for first-forbidden transitions has the following form:

$$N(W,\theta,S)=A_0(W)+SA_1(W)P_1(\theta)+A_2(W)P_2(\theta)+SA_3(W)P_3(\theta) \qquad (1)$$

where $W$ is the total energy of the $\beta$ particle and $\theta$ is the angle between the directions of emission of the $\beta$ particle and $\gamma$ ray. The helicity parameter $S$ is +1 and −1 for right-hand and left-hand circular polarization, respectively. The coefficients $A_l(W)$ depend on the nuclear matrix elements, and $P_l(\theta)$ are Legendre polynominals. The $\beta$-$\gamma$ circular polarization is defined by the expression

$$P_\gamma=\frac{N(+S)-N(-S)}{N(+S)+N(-S)}=\frac{A_1P_1+A_3P_3}{A_0+A_2P_2}. \qquad (2)$$

In its simplest form the relative difference in counting rate $\delta$ is related to the circular polarization $P_\gamma$ by the efficiency of the analyzing magnet:

$$\delta=\epsilon P_\gamma.$$

However, this expression becomes much more complicated when the finite solid angle of the $\beta$ detector and the circular-polarization analyzing magnet are included:

$$\delta(W,\varphi)=\frac{C_+(W,\varphi)-C_-(W,\varphi)}{C_+(W,\varphi)+C_-(W,\varphi)}$$

$$=\frac{\epsilon_1 A_1(W)P_1(\varphi)+\epsilon_3 A_3(W)P_3(\varphi)}{A_0(W)+\epsilon_2 A_2(W)P_2(\varphi)}\,. \qquad (3)$$

The notation of Steffen and Fraunfelder[16] has been used here except that a factor of 2 has been dropped in the definition of $\delta$ to make this expression consistent with the definition of $P_\gamma$. $C$ is the corrected coincidence counting rate, and $\varphi$ is the angle between the symmetry axis of the $\beta$ detector and the symmetry axis of the analyzing magnet. The efficiency parameters $\epsilon_1$ and $\epsilon_3$ are functions of the geometry of the experiment and the size of the spin-dependent part of the Compton scattering cross section relative to the spin-independent part. The parameter $\epsilon_2$ depends only on the geometry of the experiment.

The efficiency of the analyzing magnet was measured by observing the circular polarization of the $\gamma$ rays emitted in the decay of $Co^{60}$. For this allowed $\beta$ transition, $A_2$ and $A_3$ are zero, and it is theoretically predicted that $A_1/A_0=-\frac{1}{3}v/c$. This theoretical prediction for $P_\gamma$ has been confirmed by a large number of experimental measurements.[16] Therefore the value of $\epsilon_1$ was determined from the average value of the relative difference in counting rate divided by $v/c$ with $\varphi$ equal to 180°:

$$\epsilon_1=3\langle\delta/(v/c)\rangle=0.0252\pm0.001\,.$$

After the efficiency of the analyzing magnet was determined, the $Co^{60}$ data were analyzed to check the ability of the instrument to measure the energy dependence of $P_\gamma$. A plot of the parameter $D$ as a function of the $\beta$-particle energy $W$ is shown in Fig. 4:

$$D=\frac{\delta}{(v/c)\epsilon_1}\,.$$

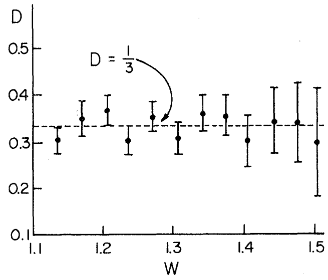


FIG. 4. Results for the energy-dependent $\beta$-$\gamma$ circular polarization of $Co^{60}$. The quantity $D$ is the circular polarization divided by $v/c$.

Since the expected energy dependence $(v/c)$ has been removed, $D$ should be constant. The data were least-square-fitted to a function of the form

$$D=A+BW.$$

It was found that $A=0.32$ and $B=0.01$ gave the best fit to the data. This is in very good agreement with the prediction that the energy dependence of $P_\gamma$ is $v/c$ for allowed $\beta$ decay.

The efficiency factor obtained from the $Co^{60}$ measurements cannot be used directly in evaluating the $La^{140}$ results. The Compton scattering cross section is a function of the energy of the incident $\gamma$ ray. Since the energy of the $La^{140}$ $\gamma$ ray ($E=1.6$ MeV) is different from the energy of the $Co^{60}$ $\gamma$ rays ($\langle E\rangle=1.24$ MeV), the efficiency will be slightly different. A computer program was used to calculate the efficiency of the analyzing magnet by performing a numerical integration over the scattering angles which were permitted by the geometry of the experiment. The calculated efficiency gave good agreement with the measured efficiency, so the calculation was used to determine how the measured efficiency would change as a function of the energy of the incident $\gamma$ ray. The same program was used to calculate $\epsilon_2$ and $\epsilon_3$ for use in the analysis of the $La^{140}$ data.

There is another difficulty which occurs in determining $P_\gamma$ from $\delta$ for a first-forbidden transition. Experiments are frequently reported where $P_\gamma$ has been obtained by simply dividing $\delta$ by $\epsilon_1$. It is obvious by comparing Eqs. (2) and (3) that this procedure is not correct unless $A_3$ is much smaller than $A_1$, and $A_2$ is much smaller than $A_0$. The ratio of $A_2$ to $A_0$ can be determined by measuring the $\beta$-$\gamma$ directional correlation. The ratio of $A_1$ to $A_3$ can be determined by measuring $\delta$ as a function of $\varphi$. However, when this measurement is evaluated, the uncertainty in the ratio $A_1/A_3$ must be combined with the uncertainty in $\delta$ in order to set limits of error on $P_\gamma$. Therefore the best procedure for extracting nuclear matrix elements is to use Eq. (3) and compare the theoretical prediction with $\delta$ rather than with $P_\gamma$.

## III. RESULTS

The measured values of $\delta$ as a function of $\beta$ energy are shown in Fig. 5. The following values of the efficiency factor $\epsilon_i$ must be used to compare these results with theoretical calculations:

$$\epsilon_1=0.0276,\quad \epsilon_2=0.724,\quad \epsilon_3=0.0168\,.$$

The parameter $\epsilon_3$ is much smaller than $\epsilon_1$ because the solid-angle correction makes the average value of $P_3$ much smaller than the average value of $P_1$. It is evident that a large error can be made

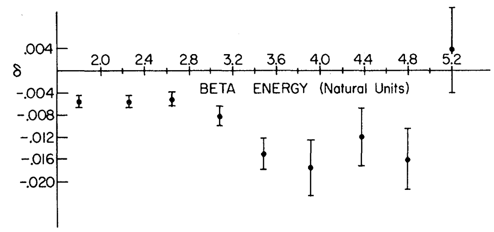


FIG. 5. Results for the energy-dependent $\beta$-$\gamma$ circular polarization of $La^{140}$. The quantity $\delta$ is the relative difference in the $\beta$-$\gamma$ coincidence counting rate. The difficulty in obtaining the circular polarization from $\delta$ for a first-forbidden transition is discussed in the text.

in determining $P_\gamma$ from $\delta$ if the relative size of $A_1$ and $A_3$ is not known. The data for $\beta$ energies above 1.5 Mev were used in the matrix-element extraction. This excludes $\beta$ particles from all the lower-energy transitions except for a minor contribution from the transition with an end-point energy of 1.7 MeV.

There have been previous measurements of the circular polarization for this transition by Singru, Simms, and Steffen[10] and by Estulin and Petushkov.[9] Both of these results are expressed in terms of the circular polarization of the $\gamma$ ray, $P_\gamma$. The $P_\gamma$ was obtained by dividing the observed difference in counting rate by $\epsilon_1$. It was possible to estimate the relative sizes of $A_1$ and $A_3$ from Singru's data since the measurement was made at several angles. However, the measurement of Estulin and Petushkov was made at only one angle, so it was impossible for them to estimate the relative sizes of $A_1$ and $A_3$. If the results of the present experiment are treated in the same fashion as the other measurements by dividing by $\epsilon_1$, our observed "$P_\gamma$" is somewhat larger than those quoted previously. However, the statistical errors are large enough to allow the three measurements to overlap.

## IV. CONCLUSIONS

Results for the nuclear matrix elements were extracted[13] by using the energy-dependent circular-polarization data of this experiment, the energy dependence of the shape correction factor,[4] and the energy dependence of the directional correlation parameter.[5] Fujita[2] has used the CVC theory to provide a simple method for calculating the ratio of two of the first-forbidden matrix elements:

$$\Lambda_{\mathrm{CVC}} = \frac{\int \vec{\alpha}}{\int (i\,\vec{r})/\rho} = \pm 1.2\alpha Z + (W_0 \mp 2.5)\rho \quad \text{for } \beta^{\mp}. \tag{4}$$

In this expression natural units are used; $\alpha$ is the fine-structure constant; $W_0$ is the end-point energy of the $\beta$ decay; $\rho$ is the nuclear radius; and $Z$ is the atomic number of the daughter nucleus. $Z$ is taken as a positive number, and the upper and lower signs are used for negatron emission and positron emission, respectively. For the $La^{140}$ transition being condidered here. Fujita's prediction gives the following value for $\Lambda_{\mathrm{CVC}}$ in natural units:

$$\Lambda_{\mathrm{CVC}}(\text{theoretical}) = 0.55\,.$$

The results of previous experiments[10] were consistent with values of $\Lambda_{\mathrm{CVC}}$ which varied from 0.55 to 0.36. Therefore it was previously impossible to confirm or reject Fujita's method for calculating $\Lambda_{\mathrm{CVC}}$. When our energy-dependent circular-polarization results are combined with the directional-correlation and spectrum-shape measurements which were used in the previous analysis,[10] much better limits can be set on the CVC ratio:

$$0.287 \leq \Lambda_{\mathrm{CVC}}(\text{experimental}) \leq 0.385.$$

Therefore it is now clear that Fujita's formula does not agree with the experimental vector-matrix-element ratio for this transition in $La^{140}$. Even though Fujita's method is very appealing because of its simplicity, this experiment demonstrates that it cannot be used to predict the ratio of first-forbidden matrix elements.

The failure of Fujita's formula does not necessarily mean that the CVC theory cannot be used to interpret the ratio of $\int \vec{\alpha}$ to $\int (i\,\vec{r})/\rho$. Damgaard and Winther[3] modified the calculation of Fujita by using a more general treatment for the Coulomb part of the Hamiltonian. An expression can be obtained[13] which depends on a third-forbidden matrix element:

$$\Lambda'_{\mathrm{CVC}} = \Lambda_{\mathrm{CVC}} \pm (0.6-\lambda)\tfrac{1}{2}\alpha Z \quad \text{for } \beta^{\mp}, \tag{5}$$

where

$$\lambda \equiv \frac{\int \vec{r}(r/\rho)^2}{\int \vec{r}}\,.$$

It is evident that if $\lambda$ is a positive number greater than 0.6, the theoretical CVC ratio will be reduced from the value predicted by Fujita. In the case of the $La^{140}$ transition considered here, it was found that for $\lambda = 2.45$ the experimental and theoretical values of the vector-matrix-element ratio agree. The matrix-element extraction and analysis which lead to this result will be discussed in more detail in the following paper.[13]

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[1]J. Eichler and S. Wahlborn, Phys. Letters 4, 344 (1963).

[2]J. I. Fujita, Phys. Rev. 126, 202 (1962).

[3]J. Damgaard and A. Winther, Phys. Letters 23, 345 (1966).

[4]L. Langer and D. Smith, Phys. Rev. 119, 1308 (1960).

[5]J. Alberghini and R. M. Steffen, Phys. Letters 7, 85 (1963).

[6]S. Bhattacheyee and S. Mitra, Phys. Rev. 131, 2611 (1963).

[7]R. Newsome and H. Fishbeck, Phys. Rev. 133, B273 (1964).

[8]M. Subotowicz, Acta Phys. Polon. 30, 943 (1966).

[9]I. Estulin and A. Petushkov, Nucl. Phys. 36, 334 (1962).

[10]R. M. Singru, P. C. Simms, and R. M. Steffen, Phys. Rev. 141, 1078 (1966).

[11]H. Schopper, Nucl. Instr. 3, 158 (1958).

[12]P. C. Simms, Phys. Rev. 138, B784 (1965).

[13]H. A. Smith and P. C. Simms, following paper, [Phys. Rev. C 1, (1970)].

[14]P. C. Simms (to be published).

[15]P. C. Simms, Nucl. Instr. Methods 70, 311 (1969).

[16]R. Steffen and H. Fraunfelder, in Alpha-, Beta-, and Gamma-Ray Spectroscopy, edited by K. Seigbahn (North-Holland Publishing Company, Amsterdam, The Netherlands, 1966), p. 1465.

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# Beta-Decay Matrix Elements of $La^{140}$ and the Implications for Isobaric Analog States*

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Experimental data are used to determine the first-forbidden matrix elements for the 2.2-MeV $\beta$ transition of $La^{140}$. The results show that the matrix-element ratio $(\int\vec{\alpha}/\int i\vec{r})$ does not agree with the usual theoretical prediction which assumes the Ahrens-Feenberg approximation. The size of the experimental vector-matrix-element ratio can be understood on the basis of significant off-diagonal contributions in the Coulomb Hamiltonian. The theory of conserved vector current is used to investigate impurities in the isobaric analog to the ground state of $La^{140}$. In addition, the contribution of third-forbidden matrix elements to the transition is investigated.

## I. INTRODUCTION

For several years, the study of first-forbidden $\beta$ transitions has been a useful method of learning some of the details of nuclear structure. Now that the conserved-vector-current (CVC) theory has been used to predict the ratio of two of the vector-type first-forbidden nuclear matrix elements $(\int\vec{\alpha}/\int i\vec{r})$, these experiments have taken on the additional value of being able to test the accuracy of the CVC predictions.

The formula which has been widely used to predict the matrix-element ratio was derived independently by Fujita[1] and Eichler.[2] More recently Damgaard and Winther[3] have modified the calculation to obtain an expression for the matrix-element ratio which does not depend on the assumption that the Coulomb Hamiltonian is diagonal. The Damgaard and Winther approach de-emphasizes the CVC ratio as a useful tool in simplifying nuclear matrix-element extraction and emphasizes its role in supplying direct information on nuclear structure.

Previous experiments[4] have shown that a measurement of the energy dependence of the $\beta$-$\gamma$ circular-polarization correlation of $La^{140}$ would be useful in determining the matrix-element ratio more precisely. This measurement has been performed by Ohlms, Bosken, and Simms.[5] Their results will be combined with other experimental data to show that the matrix-element ratio for $La^{140}$ agrees with the Damgaard-Winther formulation, but it does not agree with the calculation of Fujita and Eichler.

The formulas used in the matrix-element analysis are those developed by Bühring.[6] In these formulas Bühring uses the exact electron radial wave functions and takes into account the finite nuclear size. In a later paper[7] he also treats the Coulomb screening of the nuclear charge by the atomic electrons. These formulas have been presented by Simms[8] in such a way that the importance of the