Vandenbosch formulation, which assumes a compound-nucleus mechanism,[8] is not necessarily valid even when it fits the data.

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PHYSICAL REVIEW C VOLUME 1, NUMBER 5 MAY 1970

$E0$ Transitions from $0_2^+ \rightarrow 0_1^+$ States in the $Z = 82$ Region*

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Energy spectra of internal-conversion electrons following $(p, 2n)$ reactions leading to even-even final nuclei in the $Z \simeq 82$ region were measured. $E0$ transitions leading from the 0^+ member of the two-phonon triplet to the 0^+ ground state were found, and the systematics of their transition rates was studied. They compete poorly with $E2$ transitions to the first excited (2^+) state, and the $E0$ transition rapidly becomes relatively less probable with decreasing A in this region.

INTRODUCTION

Throughout the Periodic Table, there is a scarcity of known $E0$ transitions, although there should be an $E0$ transition in nearly every even-even vibrational nucleus between the 0^+ member of the two-phonon triplet and the 0^+ ground state. Four different reasons can be given for this lack of $E0$ transitions. First, there is the strong competition from the decay of the 0^+ level via the $E2$ transition to the 2^+ level. Secondly, most experiments leave an excited nucleus with large amounts of angular momentum, and thus the decay chain will go through high-spin states and not a low-spin state, such as a 0^+. Thirdly, most experiments observe γ rays, and none are emitted in an $E0$ transition.

Finally, even when the experimenter is observing internal-conversion electrons, the conversion coefficient for all other transitions above 1 MeV is so low that rarely are data taken in the region of interest here, which is above 1 MeV.

The excited levels of even-even nuclei can be populated by $(p,2n)$ reactions on odd-A nuclei. In the present experiment, the excited levels of Po, Pb, and Hg were investigated from the reactions $Bi^{209}(p,2n)Po^{208}$, $Tl^{203}(p,2n)Pb^{202}$, $Tl^{205}(p,2n)Pb^{204}$, and $Au^{197}(p,2n)Hg^{196}$. $E0$ transitions were observed in Po^{208}, Pb^{204}, and Hg^{196}. There is a large variation in the intensity of the $E0$ transitions relative to the decay to the first 2^+ level, and systematic trends in the $Z=82$ region are discussed.

EXPERIMENTAL

The experiments were performed using the University of Pittsburgh three-stage tandem Van de Graaff accelerator. The conversion electrons were observed in a six-gap "orange"-type spectrometer like that originally designed by Kofoed-Hansen.[1] Figure 1 shows a vertical cross section of the spectrometer. The beam from the accelerator enters the magnet above a coil and passes through a soft-iron pipe in order to prevent its deflection by the field of the magnet. The position and shape of the beam on the target were fixed by an insulated tantalum slit. After exciting the magnet, the beam was stopped in a carbon block. The high (p,n) threshold in C^{12} reduced the neutron background produced by the proton beam. The γ-ray background from the Faraday cup was reduced by a thick shield of iron and lead blocks. The detection system is shown schematically in Fig. 2. The pulses from the detector passed through an amplifier into a single-channel analyzer, the output of which was routed into a 1024-channel group of a Nuclear Data 4096-channel analyzer set up in a 1024×4 mode. Routed into another of the groups was the output of a monitor system. The monitor was a sodium iodide crystal, which detected elastically scattered particles from the target. In addition, the output of the Faraday cup was converted into digital pulses, and these pulses were routed into a third group of the analyzer. A motor-driven device uniformly advanced the current in the spectrometer as well as the channel into which the detector, monitor, and Faraday cup pulses were routed. Although the magnet current and channel advance move at a constant rate, the amount of incident beam per channel may not be constant because of fluctuations in the beam and melting and/or breaking of targets in the beam. These effects are cancelled by the monitor, which gives an accurate measurement of the product of beam intensity and effective target thickness. Since the monitor and detector counts are routed into the same channel number in their respective groups, it was possible to normalize for the above effect by simply dividing the counts in the detector channel by the counts in the equivalent monitor channel. In addition, this method corrects for dead time in the analyzer.

The only reactions which can be induced by 17-MeV protons with any reasonable probability in the $Z=82$ region are (p,n) and $(p,2n)$, and the latter are expected to have, by far, the larger cross section. In order to ascertain which of these two reactions was the source of any particular conversion-electron group, the probability for its occurrence as a function of bombarding energy was studied at a few energies; for (p,n) reactions this probability decreases, while for $(p,2n)$ reactions it increases rapidly with increasing bombarding energy.

The principal source of error in determining K/L

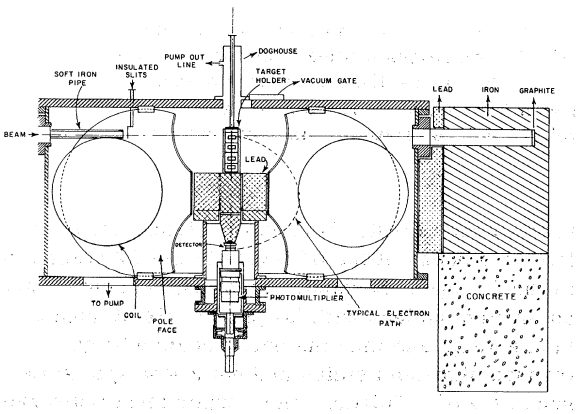

FIG. 1. A vertical section of the spectrometer.

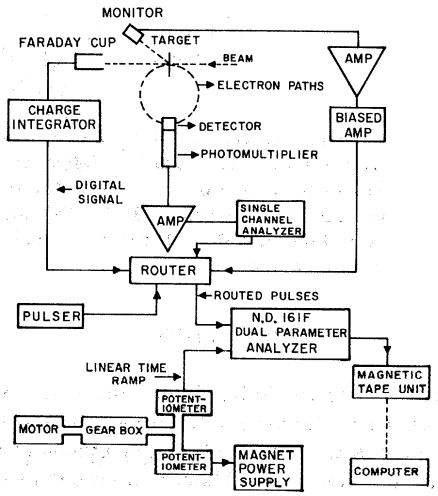

FIG. 2. Schematic diagram of the "orange" spectrometer.

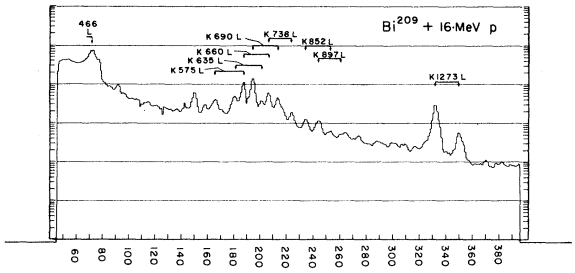

FIG. 3. Internal-conversion electrons from the $Bi^{209}(p, 2n)Po^{208}$ reaction.

ratios is background subtraction. The values obtained should be accurate to within about 15%.

RESULTS

The results from the bombardment of Bi^{209} with 16-MeV protons are shown in Fig. 3 and tabulated in Table I. The peaks observed are transitions in Po^{208} caused by the $(p, 2n)$ reaction on Bi^{209}. The most obvious thing about the electron spectrum is the strong transition at 1273 keV. The evidence for this being an $E0$ transition lies in the fact that if this peak were, for example, an $E2$ transition its transition intensity would be greater than that of the first 2^+ to the ground state, and the latter is normally, by far, the strongest transition. In addition, no γ rays were observed at 1273 keV in the $Pb^{208}(\alpha, 4n)Po^{208}$ reaction leading to the same states in Po^{208} in experiments reported by Yamakazi *et al.*[2] This means that the e_K/γ ratio is very high, which, at this energy, implies an $E0$ transition. The K/L ratio agrees within the experimental uncertainty with the theoretical value for an $E0$ transition, but at this energy the K/L ratio of most transition types is almost the same. The existence of an $E0$ transition in Po^{208} is not surprising, since there is a known one in both Po^{210} and Po^{212}.

The next odd-Z nuclei lighter than Bi^{209} are $Tl^{203,205}$. When a target of natural Tl (Tl^{NAT}) was bombarded, two small peaks appeared near 1.5 MeV, as shown in Fig. 4. The results are summarized in Table II. At first, it was thought that these peaks represented $E0$ transitions in Pb^{202} and Pb^{204} from $(p, 2n)$ reactions on the two isotopes. However, when the experiment was repeated with an isotopic target of Tl^{205}, both peaks were observed with the same intensity ratio, indicating that they arise from transitions in Pb^{204}. The identification of these peaks as K and L lines from $E0$ transitions follow the same argument as with Po^{208}, except now the lower intensities make the identification a little less certain. However, if this transition is not $E0$, its transition rate would be approximately one-half that of the $2^+ \rightarrow$ ground, and its existence would be quite difficult to explain. The

TABLE I. Electron lines from $Bi^{209}(p, 2n)Po^{208}$, $E_p = 16$ MeV. K and L refer to electron shell.

Electron energy (keV)	Electron intensity (arbitrary)	Transition energy (keV)	K/L	Transition intensity	Comments
156 ± 3	290 ±10	173L		1150 ($E2$)	$6_1^+ \rightarrow 4_1^+$
217 ± 4	13 ± 4				
426 ± 7	16 ± 2				
451 ± 7	4.2 ± 1.0				
482 ± 7	12 ± 1.5	575K		750 ($E2$)	$0_2^+ \rightarrow 2_1^+$
542 ± 8	16 ± 1.0	635K	1.5 ± 0.1	1350 ($E2$?)	
568 ± 8	34 ± 0.9	661K + 575L	3.5 ± 0.1	3400 ($E2$)	$4_1^+ \rightarrow 2_1^+$
597 ± 9	52 ± 1.0	690K	3.3 ± 0.1	5200 ($E2$)	$2_1^+ \rightarrow 0_1^+$
621 ± 9	11 ± 0.6	635L + ?			
643 ± 9	36 ± 0.6	661L + 738K	3.3 ± 0.1[a]	~1400 ($E2$)	$2_2^+ \rightarrow 2_1^+$
672 ± 10	16 ± 0.5	690L			
716 ± 10	4.0 ± 0.5	738L			
759 ± 11	2.3 ± 0.4	852K			
804 ± 11	3.0 ± 0.4	892K			
842 ± 12	weak	852L			
876 ± 12	doublet	892L + ?			
890 ± 12	7.6 ± 0.3				
932 ± 13	0.4 ± 0.2				
1180 ± 16	14 ± 0.3	1273K	5.7 ± 0.3	14 ($E0$)	$0_2^+ \rightarrow 0_1^+$
1253 ± 16	2.4 ± 0.09	1273L			
1342 ± 16	0.06 ± 0.03				

[a] 738K is assumed to have an intensity of 13 by assuming a K/L of 3.5 for the 660-keV transition and subtracting out the 660L line.

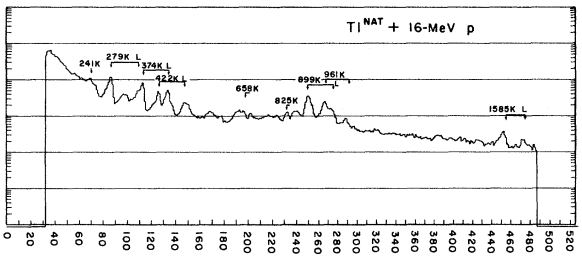

FIG. 4. Internal-coversion electrons from the $Tl^{NAT}(p,2n)Pb^{202,204}$ reactions.

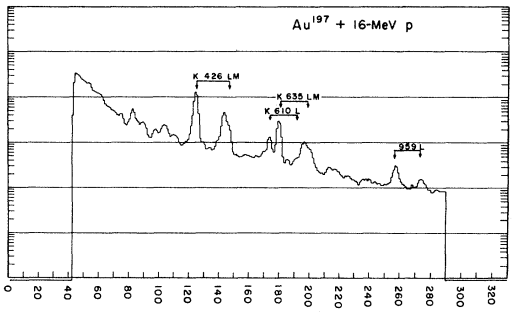

FIG. 5. Internal-coversion electrons from the $Au^{197}(p,2n)Hg^{196}$ reactions.

fact that no γ rays at this energy have been observed in Pb^{204} makes the identification more certain.

The observation of electrons from the $(p,2n)$ reaction on Au^{197} has been reported by Sakai, Yamazaki, and Ejiri,[3] and a transition at 959 keV (the approximate energy of an $E0$ transition, if it exists) was found. It was tentatively identified as the decay of the third 2^+ to the first 2^+ ($2_3^+ \rightarrow 2_1^+$) state by the fact that a state was known at about 959 keV above the 2^+ level in Hg^{196}. Our data from this reaction are shown in Fig. 5 and in Table III. The transition from the 2^+ to the first 2^+ state should be at least an order of magnitude weaker than those from the third to the second or from the second to the first, since the selection rules for transitions between members of a vibrational band require that the phonon number change by one unit; there is abundant experimental verification of this rule. If the 959-keV transition is taken to be $2^+ \rightarrow 2^+$, its total transition rate is found to be about equal to that from the first 2^+ to the ground state ($2_1^+ \rightarrow 0_1^+$), so the Sakai assignment is very questionable. Thus the same arguments as those used with the Bi and Tl reactions apply for identifying the 959-keV transition as $E0$. The intensities for well-identified transitions other than $E0$ are almost an order of magnitude less than that for $2_1^+ \rightarrow 0_1^+$, so any assignment for the 959-keV transition other than $E0$ would be very difficult. Moreover, the existence of analogous $E0$ transitions following the Bi and Tl reactions leads us to expect such a transition here.

In addition to Au^{197}, Sakai also bombarded Ir^{191} and Ir^{193} leading to final states in Pt^{190} and Pt^{192}. No transition was found in the region investigated (up to 900 keV) that could be identified as $E0$; however, in Pt^{194} an $E0$ transition is known at 1.2674

TABLE II. Electron lines from $Tl^{NAT}(p,2n)Pb^{202,204}$, $E_p = 16$ MeV. K and L refer to electron shell.

Electron energy (keV)	Electron intensity (arbitrary)	Transition energy (keV)	K/L	Transition intensity	Comments
198 ± 3	21	286K	3.4	59	$4_2^+ \rightarrow 4_1^+ Pb^{204}$
237 ± 4	6.3				
277 ± 5	6.2	286L			
286 ± 5	8.3	374K		230	$4_1^+ \rightarrow 2_1^+ Pb^{204}$
322 ± 5	4.6				
362 ± 6	7.0				
411 ± 6	3.2	500K	4.9		
492 ± 8	0.66	500L			
572 ± 8	0.22	660K		20	$4_2^+ \rightarrow 2_1^+ Pb^{202,204}$
586 ± 8	0.4				
597 ± 9	0.4	685K		40	$0_2^+ \rightarrow 2_1^+ Pb^{204}$
624 ± 9	0.21	712K		22	$2_2^+ \rightarrow 2_1^+$?
742 ± 11	0.2				
812 ± 11	3.1	899K	3.7 ± 0.4	510	$2_1^+ \rightarrow 0_1^+ Pb^{204}$
873 ± 12	1.9	961K		350	$2_1^+ \rightarrow 0_1^+ Pb^{202}$
893 ± 12	0.83	899L			
948 ± 13	0.13	961L			
1497 ± 18	0.13	1585K	3.4 ± 1.6	0.13	$0_2^+ \rightarrow 0_1^+$
1585 ± 18	0.04	1585K			

TABLE IV. Ratios of transition intensities.

Nucleus	(1) $\frac{2_1^+ \rightarrow 0_1^+}{0_2^+ \rightarrow 0_1^+}$	(2) $\frac{0_2^+ \rightarrow 2_1^+}{0_2^+ \rightarrow 2_1^+}$ (a) Measured	(b) Relative S.P.	(3) $\frac{2_1^+ \rightarrow 0_1^+}{(0_2^+ \rightarrow 2_1^+) + (0_2^+ \rightarrow 0_1^+)}$
Po^{214}	125	2.5	1.2	38
Po^{212}	170			
Po^{208}	370	50	0.7	7
Pb^{204}	1450	140	1.6	10
Hg^{196}	1600	≤ 30	0.5	>80
Pt^{194}	30 000	3300	3.0	9

MeV. If $E0$ transitions did exist in Pt^{190} and Pt^{192}, they should appear in the energy region investigated.

DISCUSSION

A general summary of $E0$ transitions in the $Z = 82$ region is shown in Table IV. The numbers in the first column are the ratios of transition intensities of the first $2^+ \rightarrow$ ground state to that of the $E0$ transition. These intensities are the actual experimental results divided by $\alpha/(1+\alpha)$, where α is the internal conversion coefficient (unity in the case of $E0$). The results for Po^{214}, Po^{212}, and Pt^{194} are taken from the literature,[4] and the rest are from the present work. The results from Ref. 4 were obtained from radioactive-decay work, while the present results were, of course, from charged-particle reactions [$(p, 2n)$ exclusively]. This could be a factor in analyzing the ratios presented in Table IV, and for this reason the analysis is not detailed but only an order-of-magnitude result.

A ratio of the $2_1^+ \rightarrow 0_1^+$ to the $E0$ transition strengths shows a dramatic increase as A decreases. Since the strength of the former transition is relatively slowly varying with A, the cause must be assumed to be variations in the strength of the $E0$ transitions because of the competition of the $E2$ transition to the first 2^+ level ($0_2^+ \rightarrow 2_1^+$). This competition becomes very evident in column 2(a) of Table IV, where the ratios of the strength of this transition to that of the $E0$ are listed for the different nuclei. This ratio also shows a dramatic increase, implying that the competition of the $E2$ transitions improves as we go to lighter nuclei. The extent to which this effect may be due to mass and energy differences may be seen in column 2(b), which shows the ratios of single-particle (S.P.) rates on a relative basis. We see that these effects are quite minor in comparison with the observed differences.

Since the 0_2^+ level can decay by two different methods, it is interesting to calculate the ratio of

TABLE III. Electron lines from $Au^{197}(p, 2n)Hg^{196}$, $E_p = 16$ MeV. K and L refer to electron shell.

Electron energy (keV)	Electron intensity (arbitrary)	Transition energy (keV)	K/L	Transition intensity	Comments
194 ± 4	11.7				
224 ± 4	5.5				
252 ± 4	3.3				
272 ± 5	7.6				
286 ± 5	doublet				
345 ± 6	38	426K	2.7		$2_1^+ \rightarrow 0^+$
414 ± 7	14	426L			
421 ± 7	4	426M			
513 ± 8	0.8	600K			
526 ± 8	2.66	610K		205	$2_2^+ \rightarrow 2_1^+$
552 ± 8	7.7	635K		600	$4_1^+ \rightarrow 2_1^+$
599 ± 10		610L			
618 ± 10	triplet	635L			
631 ± 10		635M			
876 ± 12	0.84	956K	4.7	0.84	$0_2^+ \rightarrow 0_1^+$
946 ± 13	0.18	956L			

the $2_1^+ \rightarrow 0_1^+$ strength to the sum of all decays from the 0_2^+ level. This ratio is shown in column 3 of Table IV. Although it shows variation from one nucleus to the next, within an order of magnitude the numbers are constant. This is not surprising, since the formation of both levels is due to a complex statistical process, whence large differences in cross sections for exciting analogous levels in neighboring nuclei are not expected. It is somewhat surprising that in Hg^{196} the $0_2^+ \rightarrow 2_1^+$ transition was not seen; that is why only a lower limit is given in column 3 for that nucleus.

The main conclusion that can be reached from the results is that the strengths of the $E0$ transitions from $0_2^+ \rightarrow 0_2^+$ in the nuclei studied are primarily determined by the degree of competition with $E2$ transitions to the first 2^+ level. The general trend in going from Po^{214} to Pt^{194} is to increasing dominance by the latter transition. It may be noted that in a simple vibrational model, the $E0$ transition is forbidden.

In view of the sharp variations with A in the transition-rate ratio found here, it would be interesting to investigate these ratios in other mass regions.

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Shell-Model Continuum in Nuclear Bound States

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Shell-model calculations are performed for bound 0^+ states of Ca^{42} and Ni^{58}. The neutron configurations $1f_{7/2}$, $2p_{3/2}$, $2p_{1/2}$, $1f_{5/2}$, and $1g_{9/2}$ are included, as well as states in which one of the neutrons is in the shell-model continuum. These continuum contributions have little effect on energy eigenvalues, but modify the wave functions in the vicinity of the nuclear surface. The calculated wave functions yield one-particle-transfer form factors whose logarithmic derivatives at large radius are consistent with the neutron separation energies. Comparison is made with other procedures for calculating form factors within the framework of the ordinary shell model. It is found that the conventional well-depth procedure underestimates the (p,d) cross sections for populating highly excited states, relative to low-lying states, by about 50%. Two-neutron-transfer cross sections are also calculated and are found to be in agreement with those yielded by the well-depth procedure. However, some of our form factors exhibit an extra node at 8 F, showing that they do not decay like a Hankel function.

I. INTRODUCTION

When we do nuclear shell-model calculations, we have in mind two Hamiltonians: the shell-model Hamiltonian and the "true" Hamiltonian. The true Hamiltonian has a spectrum consisting of discrete states (the bound states of the system) and a continuum (scattering states). If the shell-model potential has finite range and depth, then the shell-model Hamiltonian also has a spectrum with discrete states and a continuum. Shell-model spectroscopic calculations generally attempt to express bound eigenstates of the true Hamiltonian in terms of bound eigenstates of the shell-model Hamiltonian. However, it is clear that an expansion of a bound eigenstate of the true Hamiltonian in terms of eigenstates of the shell-model Hamiltonian, will involve continuum shell-model eigenstates as