Spin Assignments of States in Mg^{24} from an Analysis of Doppler-Broadened Gamma Rays *†

W. J. Stark,‡ P. M. Cockburn,§ and R. W. Krone
University of Kansas, Lawrence, Kansas 66044
(Received 7 November 1969)

The spins of the Mg^{24} states at 12.74-, 12.85-, and 12.97-MeV excitation, which are populated in the $Na^{23}(p, \alpha\gamma)Ne^{20}$ reaction, have been determined to be 2^+, 2^-, and 2^-, respectively, from α-γ correlation measurements. The correlation is measured simply by observing the 1.63-MeV γ ray at 0°, and analyzing the γ-ray line shape in terms of the possible spins of the Mg^{24} states.

INTRODUCTION

Several prominent resonances are observed in the $Na^{23}+p$ reaction corresponding to compound states in Mg^{24} which have unassigned spins and parities.[1] In the particular region of interest in Mg^{24} around 13-MeV excitation, there are several decay channels open. For example, decays through the p_0, p_1, γ, α_0, and α_1 channels are all energetically allowed. However, double-correlation measurements on these decay radiations have so far been unable to determine some of the Mg^{24} level spins.

The present paper describes a new technique which can be used in general to measure the triple angular correlations in $(\alpha, b\gamma)$ reactions as recently reported by Krone, Cockburn and Stark.[2] This technique has been applied to the study of the $Na^{23}(p, \alpha\gamma)Ne^{20}$ reaction and has resulted in the determination of previously unknown spins of the 12.854- and 12.966-MeV levels in Mg^{24}. Briefly, the method relies on the Doppler shift of the detected γ rays to determine the direction of the emitted particle. The relationship between the observed γ-ray line shape and the particle-γ correlation, or in this particular case, the α_1-γ correlation, is developed fully in the appendix.

EXPERIMENTAL METHOD AND ANALYSIS

Angular distribution measurements of the 1.63-MeV γ rays resulting from the $Na^{23}(p,\alpha\gamma)Ne^{20}$ reaction were made at each of the resonances studied. These measurements were carried out solely to limit the allowed ranges of the orbital and channel-spin mixing parameters that were later used in the line-shape analysis. The target for the angular distribution measurements consisted of Na^{23} evaporated onto a gold backing. A thin gold overlay on the target reduced the chemical reaction of the sodium with other elements. The target thickness was typically $8\,\mu g/cm^2$. The target chamber was designed to permit direct cooling of the target backing. The targets were mounted at 45° to the proton beam to reduce the angular dependence of the target absorption. The γ rays were detected with a 5 in.×5 in. collimated, shielded NaI(Tl) crystal, and the γ-ray yields were extracted using standard γ-ray-stripping computer techniques. The angular distributions were analyzed using the channel-spin formalism of Ferguson and Rutledge.[3]

For the Doppler-shape measurements, it was important to detect as much of the angular-correlation effect as possible. Since this effect will be diminished by any attenuation of the nuclear recoil, thin targets were constructed so that most of the nuclei recoil into vacuum. This was accomplished by evaporating NaBr onto a 10-$\mu g/cm^2$-thick foil. The resulting γ-ray line shape is affected by the slowing down of the recoiling nuclei in the target material and the carbon foil. This effect was small and was taken into account as described in the appendix. A 15-cc Ortec coaxial Ge(Li) detector was used and had the required energy resolution to observe the Doppler-broadeded γ rays. Its resolution was determined to be ~4.5 keV full width at half maximum (FWHM) for 1.333-MeV γ rays from Co^{60}. Conventional modular electronics were used for amplifying and storing the detector signals in a 4096-channel analyzer. A Co^{60} source was used to calibrate the spectrum to better than 0.2%. For the present measurements the detector was placed at 0° to the beam axis. As shown in the appendix, this choice simplifies the analysis of the experimental pulse-height distribution.

The 1.63-MeV Doppler-broadened γ-ray peak was extracted from the pulse-height distribution with the aid of a computer program. The output from this program was the γ-ray line shape after background subtraction, together with the corresponding error spectrum. This background was determined by fitting a polynomial up to third order in the channel number to the background on both sides of the peak. The procedure used to analyze the Doppler-broadened γ rays involved computation of the line shape for various possible spin assignments of the Mg^{24} compound state and then fitting the experimental data in terms of χ^2. The line shape was computed using Eq. (A9), and the detector resolution was taken into account by folding in a Gaussian peak about each point in the theoretical

TABLE I. Summary of results for the angular correlation measurements. δ_0, δ_1, and δ_2 are the mixing ratios for channel spin and orbital angular momentum of the proton and α particle, respectively. The multipolarity mixing ratio δ_3 for this case is zero.

Resonance (keV)	Possible assignment from angular distribution	Ratio χ^2_{min}/χ^2 from Doppler shape analysis	$\tan^{-1}\delta_0$ (deg)	$\tan^{-1}\delta_1$ (deg)	$\tan^{-1}\delta_2$ (deg)	Present assignment	Previous assignment
1091	1^-	0.25		0	−2.5	2^+	2^+
	2^+	1.0					
1210	2^-	1.0	50		10	2^-	
	3^+	0.05					
1254	1^+	1.0	20			1^+	1^+
	2^-	0.61					
	3^+	0.23					
	4^-	0.01					
1328	1^+	0.008	45		0	2^-	
	2^-	1.0					
	3^+	0.008					
	4^-	0.02					
1458	3^-	1.0		5	−30	3^-	3^-
	4^+	0.09					

pulse-height distribution. Equation (A9) contains the angular correlation in terms of the coefficients A_k, which were computed for each given spin sequence. Only those spin sequences and those values of the channel spin and orbital-angular-momentum mixings of the protons and α particles were considered that were consistent with the angular distribution measurements. For each permissible spin sequence, the mixing parameters were varied through the allowed range of values to find the parameters giving the best fit to the Doppler-broadened γ-ray line shape.

For all the resonances investigated, the orbital angular momenta of the protons and α particles are limited to small l values from a comparison of the reduced widths to the Wigner-limit value. This was taken into account in the analysis of both the angular distribution and triple-correlation data by restricting the possible orbital-angular-momentum mixing parameters to small values.

RESULTS

The states studied in the Mg^{24} compound nucleus vary considerably in their properties, and therefore can be described best by discussing each of the resonances separately. The results for the five resonances studied are summarized in Table I.

1091-keV Resonance

Decay at this resonance proceeds mainly through the p_1, α_0, and α_1 channels. Previous studies of the α_0 angular distribution[4] failed to identify the spin of this state because of interference between the 1091- and the 1136-keV resonance. Angular distribution studies using the $Ne^{20}(\alpha,\gamma)Mg^{24}$ reaction[5] and the $Ne^{20}(\alpha,\alpha_1\gamma)Ne^{20}$ reaction[6] have more recently resulted in an assignment $J^\pi = 2^+$ for a state at $E_x = 12.740$ MeV in Mg^{24}. The decay through the α_0 channel requires this state to have natural parity. The present angular distribution measurements eliminate the possibilities $J^\pi = 3^-$ or 4^+. The best fit to the Doppler line shape resulted, when assuming $J^\pi = 2^+$, in agreement with the $Ne^{20} + \alpha$ reactions.

1210-keV Resonance

The reaction proceeds through all channels except the α_0 channel. The proton capture cross section is very small with the measured resonance strength $\omega\gamma$ less than 0.8 eV.[1] No previous attempts have been made, therefore, to measure the angular distribution of the capture γ rays. A tentative assignment of $J^\pi = 3^+$ has resulted from a study of elastic scattered protons[7] from Na^{23}. The uncertainty of the assignment was subject to question, because it was difficult to resolve the 1210- and the 1204-keV resonances in the elastic scattering curve. Reexamination of the original data of Ref. 7 indicates that both resonances have an equally low yield for proton-elastic scattering. This, combined with possible interference effects, makes an unambiguous interpretation difficult. This problem does not arise in the case of the $(p,\alpha\gamma)$ reaction, for which the 1210-keV resonance shows a much higher yield than the 1204-keV resonance. The absence of decay through the α_0 channel suggests that this state has unnatural parity. Analysis of the present measurements shows that assignments $J^\pi = 1^+$ and 4^- are incompatible with

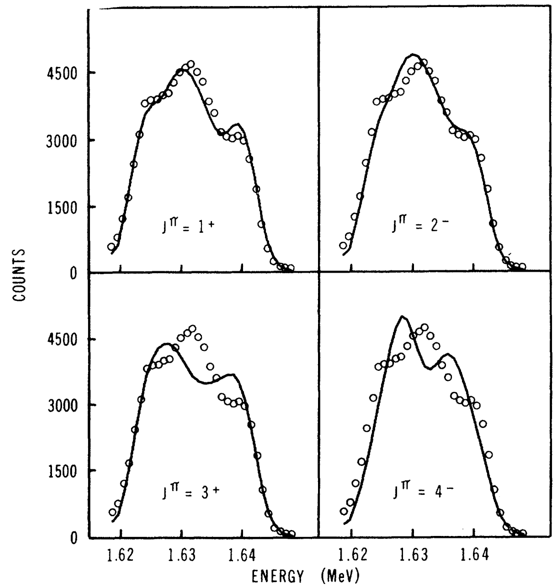

FIG. 1. Doppler-broadened γ-ray line shapes at E_p = 1254 keV. The solid curves are the theoretically calculated line shapes for various J^π of the Mg^{24} compound state. The circles represent the experimental points, with the diameter representing the uncertainty of the data.

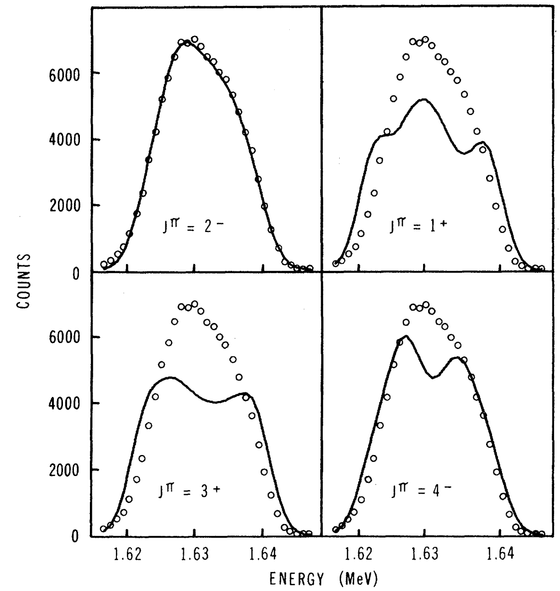

FIG. 2. Same as Fig. 1 at E_p = 1328 keV.

the angular distribution measurements. A good fit to the Doppler-broadened γ-ray line shape resulted only for the $J^\pi = 2^-$ choice.

1254-keV Resonance

This resonance has a known value $J^\pi = 1^+$. This assignment was first made by analyzing the α-γ angular correlation,[8] later confirmed by the study of proton elastic scattering[7] from Na^{23}. The absence of α_0 decay indicates the state likely to be one with unnatural parity. None of the possible values of J^π could be eliminated from the angular distribution analysis. It was, however, possible to place restrictions on the channel-spin mixing for $J^\pi = 2^-$, 3^+, and 4^-. Figure 1 shows the results of the Doppler shape analysis. The best fit to the data results with $J^\pi = 1^+$, in good agreement with previous results.

1328-keV Resonance

The reaction proceeds mainly through the p_1 and α_1 channel. Virtually no information is available about the properties of this state, since previous investigations were restricted chiefly to the study of processes other than the decay through the above two channels. Also, the absence of α_0 decay indicates that this state probably has unnatural parity. As in the case of the 1254-keV resonance, the angular distribution analysis could only place restrictions on the possible channel-spin mixing ratios for $J^\pi = 2^-$, 3^+ and 4^-. Figure 2 shows the results of the Doppler shape analysis, showing that $J^\pi = 1^+$, 3^+, and 4^- can unquestionably be eliminated.

1458-keV Resonance

The $Na^{23}(p, p)$ reaction[7] has shown a state at E_x = 13.091 MeV to have $J^\pi = 3^-$. Recently, a study of several states in Mg^{24}, using the $Ne^{20}(\alpha, \gamma)Mg^{24}$ reaction,[5] has identified a 2^+ state at E_x = 13.089 MeV, suggesting the existence of a doublet. The present study shows that assignments of $J^\pi = 1^-$ and 2^+ can be eliminated from the analysis of the angular distribution measurements. Figure 3 shows the experimental values for the Doppler-broadened γ rays with the fits obtained with $J^\pi = 3^-$ and 4^+. The best fit to the data resulting with $J^\pi = 3^-$ is in agreement with the proton elastic scattering data. The theoretical fit to the data is less convincing in this case, which may signify the presence of interference effects caused by the proximity of a nearby 2^+ state.

DISCUSSION

The method of obtaining spin and parity information about an excited state by analyzing the Doppler-broadened γ-ray pulse-height distribution has proved to be a useful approach. When applicable, this approach has several advantages over the more conventional methods used. For example, no normalization errors are introduced, since the correlation information results from a single measurement of the Doppler-broadened line shape.

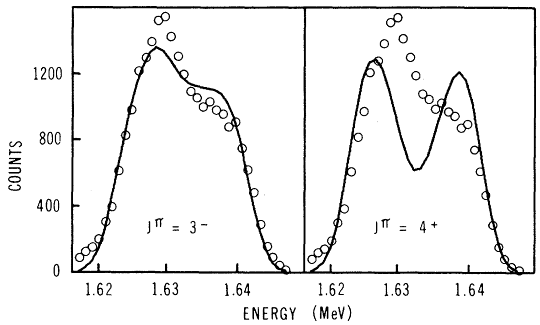

FIG. 3. Same as Fig. 1 at $E_p = 1458$ keV.

The experimental arrangement of the detector and associated electronics is relatively simple, requiring no coincidence circuitry.

There are several ways by which the present method can be extended. For example, the measurements reported were carried out with the detector placed at 0° with respect to the beam axis. In many cases, additional information may result by using a different detector geometry. Reactions other than $Na^{23}(p, \alpha\gamma)Ne^{20}$ may prove to be more sensitive to studies of this kind. Reactions having larger Q values and a more energetic γ ray would result in a relatively larger broadening. Moreover, the long lifetime of the first excited state in Ne^{20} requires a correction for the attenuation of recoil nuclei velocities in the target and supporting foil. Finally, any improvements in the resolution and efficiency of Ge(Li) detectors will result in a greater sensitivity of the technique to the reaction angular correlation.

APPENDIX

The energy E of the γ radiation emitted from an ensemble of nuclei recoiling through vacuum can be expressed as

$$E = E_0[1 + (v_0'/c)\cos\theta_2'], \tag{A1}$$

where v_0' is the recoil velocity and θ_2' is the angle between the γ ray and the recoil nucleus as shown in Fig. 4. The primed coordinates refer to the laboratory system. Using the relationship between the velocity in the laboratory system and the velocity in the c.m. system, namely,

$$v_0'\cos\theta_2' = -(v_0\cos\theta_2 - v_c), \tag{A2}$$

we may write Eq. (A1) as

$$E = E_0[1 - (v_0\cos\theta_2 - v_c)/c], \tag{A3}$$

where v_c is the velocity of the center of mass and θ_2 is the c.m. angle between the α particle and the γ ray emitted at 0° to the beam axis. The yield observed by the γ-ray detector from events where the α particles are emitted into the solid angle $d\Omega$

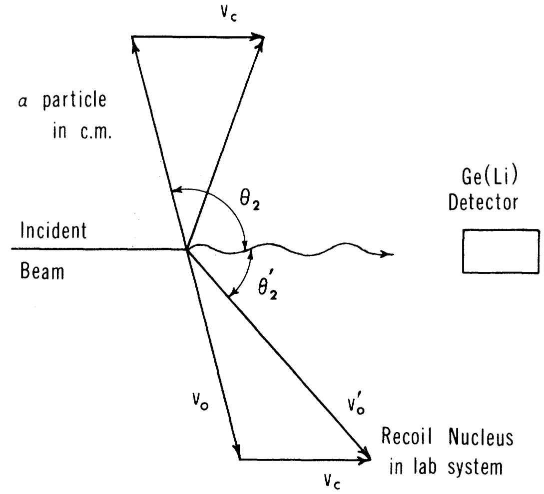

FIG. 4. Schematic representation of the $(p, \alpha\gamma)$ reaction. All primed symbols refer to the lab system.

about $(\theta\phi)$, and the recoiling nucleus decays with the velocity between v' and $v' + dv'$, can be expressed as

$$\frac{d^2N(v', \theta_2, \theta_3, \phi)}{dv'd\Omega} = \frac{dN(v')}{dv'} W(\theta_2, \theta_3, \phi), \tag{A4}$$

where dN/dv' is the velocity distribution function of the recoiling nuclei and $W(\theta_2, \theta_3, \phi)$ is the triple angular correlation function with θ_2, θ_3, and ϕ as defined in Fig. 5. For the geometry under consideration $\theta_3 = 0°$. If the γ-ray emitting nuclei recoil into vacuum, $v' = v_0'$, making $dN/dv' = 1$, which simplifies Eq. (A4) to

$$\frac{dN}{2\pi d(\cos\theta_2)} = N_0 W(\theta_2), \tag{A5}$$

where N_0 is proportional to the total number of γ rays emitted and $W(\theta_2)$ is the angular correlation

$$W(\theta_2) = \textstyle\sum_k A_k P_k(\cos\theta_2). \tag{A6}$$

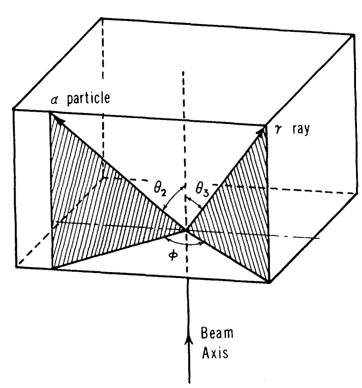

FIG. 5. Schematic representation of angular correlation variables.

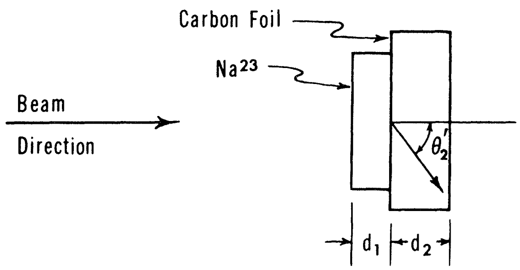

FIG. 6. Enlarged view of target and supporting foil.

Expressing the γ-ray pulse-height distribution as

$$\left(\frac{dN}{dE}\right)dE=\frac{dN}{d(\cos\theta_2)}\frac{d(\cos\theta_2)}{dE}dE\,, \qquad \text{(A7)}$$

one obtains, using Eqs. (A3), (A5), and (A6),

$$\left(\frac{dN}{dE}\right)dE=-\frac{2\pi N_0}{\Delta E_0}\sum\nolimits_k A_k P_k(\cos\theta_2)dE\,, \qquad \text{(A8)}$$

where ΔE_0 is the full Doppler shift $E_0 v_0/c$. This can be evaluated as a function of the γ-ray energy by solving Eq. (A3) for $\cos\theta_2$ in terms of E. An estimate of the contribution to the pulse-height distribution resulting from γ rays emitted by nuclei being attenuated in the target can be approximated in the following manner: assume that the reaction takes place in the center of the target and supporting foil as shown in Fig. 6. The range of the recoil nuclei in the target material and supporting foil defines two angles, θ_{2i}, where $i=1$ or 2 for the target or backing material, respectively, within which all recoil nuclei are assumed to come to rest before emitting the γ ray. These pulses can be approximated by a δ function at $E=E_0$. The magnitude of this δ function can be estimated by integrating Eq. (A8) over the energy limits defined by the two angles. The pulse-height distribution then becomes

$$\left(\frac{dN}{dE}\right)dE=\frac{2\pi N_0}{\Delta E_0}\sum\nolimits_k A_k P_k(\cos\theta_2)dE+B\delta(E-E_0)\,, \qquad \text{(A9)}$$

where

$$B\simeq 2\pi N_0\sum_k\frac{b_k}{k+1}[(\cos\theta_{21})^{k+1}-(\cos\theta_{22})^{k+1}]\,. \qquad \text{(A10)}$$

The coefficients b_k in Eq. (A10) are the expansion coefficients of $W(\theta_2)$ in terms of $(\cos\theta_2)^k$. Since the value of B gives only an approximation to the attenuation of the recoil nuclei in the target, this value was allowed to vary by ±15% in the program to compute an optimum fit to the pulse-height distribution. When attenuation of the recoil nuclei is taken into account, the Doppler shift equation used changes slightly to

$$E=E_0\left[1-(-1)^i\frac{d_i}{\alpha_i c}-\frac{v_0\cos\theta_2-v_c}{c}\right], \qquad \text{(A11)}$$

where d_i and α_i are the thickness and the slowing down time in the medium i. As a result of this change, the FWHM of the Doppler line shape computed from Eq. (A9) becomes slightly smaller than in the case of no attenuation.

---

*Part of this work was submitted by one of us (W.J.S.) in partial fulfillment for the requirements of the Ph.D. degree at the University of Kansas, Lawrence, Kansas.

†This work was supported by the United States Atomic Energy Commission under Contract No. AT(11-1)-1120.

‡Present address: Braddock, Dunn, & McDonald, McLean, Virginia.

§Present address: Laboratorium fur Kernphysik, Eidg. Technische Hochschule, Zurich, Switzerland.

[1]P. M. Endt and C. Van der Leun, Nucl. Phys. A105, 1 (1967).

[2]R. W. Krone, P. M. Cockburn, and W. J. Stark, in Proceedings of the International Conference on Properties of Nuclear States, Montreal, Canada, 1969 (Presses de l'Université de Montréal, Montréal, Canada, 1969). Abstract No. 4.17, p. 104.

[3]A. J. Ferguson and A. R. Rutledge, Atomic Energy of Canada, Limited Report No. AECL-420, 1957 (unpublished).

[4]P. H. Stelson, Phys. Rev. 96, 1584 (1954).

[5]C. J. Highland and T. T. Thwaites, Nucl. Phys. A109, 163 (1968).

[6]R. H. Spear and I. F. Wright, Australian J. Phys. 21, 307 (1968).

[7]N. P. Bauman, F. W. Prosser, Jr., W. G. Read, and R. W. Krone, Phys. Rev. 104, 376 (1956).

[8]T. Seed, Phil. Mag. 44, 921 (1953).