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Direct Nuclear Reactions Leading to Unbound Residual Nuclei

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The distorted-wave matrix element for direct reactions leading to three-body final states is written in terms of convergent integrals. The resulting expression can be evaluated in the Butler approximation, which leads to a diffractive model of the competition of, and interference between, direct breakup and sequential decay reaction mechanisms. Using Amado's methods, I demonstrate the general presence of such interference in processes of interest. Moreover, it should be possible to normalize unstable-state spectroscopic factors by measuring this interference.

Direct nuclear reactions leading to particle-unstable residual nuclei are in principle no harder to deal with than those leading to stable states. However, the continuum wave functions characterizing the final states of such reactions are not localized in configuration space, so that the ordinary computational technology associated with approximation schemes such as the Butler theory[1] or the distorted-wave Born approximation[2] (DWBA) cannot be used straightforwardly. The difficulty is connected with the presence of divergent integrals which appear when the residual nucleus becomes particle-unstable. Consider, for example, the zero-range DWBA matrix element for the reaction

$$d+A \to n+p+A \ ,$$

where for simplicity A will be considered a static, structureless, spinless nucleus[3]:

$$M_{fi} = (2\pi)^{3/2} g \int d^3r\, \psi_{\vec{n}'}^{(-)*}(\vec{r})\, \psi_{\vec{p}'}^{(-)*}(\vec{r})\, \psi_{\vec{K}}^{(+)}(\vec{r}) \ . \qquad (1)$$

All of the symbols have their usual meanings: $\psi_{\vec{k}}^{(\pm)}$ is a wave function for scattering either d, p, or n with momentum $\vec{k}$ from A, and g is the (virtual) amplitude for $d \to n+p$ at zero energy, $\lim_{q\to 0} g(q)$, where

$$g(q) = \int d^3r\, (2\pi)^{-3/2} e^{i\vec{q}\cdot\vec{r}}\, V_{np}(r)\phi_d(r) \ . \qquad (2)$$

The $+(-)$ superscripts refer to outgoing (incoming) scattered-wave boundary conditions. We presume that the deuteron, the neutron, and the proton interact with A through appropriate, known interactions. Since each wave function in (1) has the asymptotic form $e^{i\vec{k}\cdot\vec{r}} + O(r^{-1})$, the integral (1) diverges. The most strongly divergent part is well known to be proportional to $\delta(\vec{n}'+\vec{p}'-\vec{K})$, which vanishes identically as a result of energy conservation. That is, cancellations between successively larger contributions of alternating sign eventually yield the correct, finite matrix element. Any computational scheme which relies on these cancellations obviously places a great premium on the accuracy with which the functions are evaluated at arbitrarily large distances and should be avoided: Clearly, a decent theory should not be sensitive to what takes place outside the interaction region.

How should we evaluate matrix elements such as (1)? Recalling that the divergence difficulties arose from the unscattered parts of the wave functions, let us write each wave function in (1) as the sum of scattered and unscattered portions: In momentum space this decomposition takes the form

$$\psi_{\vec{k}}^{(\pm)}(\vec{k}'') = \delta(\vec{k} - \vec{k}'') + \langle \vec{k}'' | T(E_k \pm i\epsilon) | \vec{k} \rangle (E_k \pm i\epsilon - E_{k''})^{-1} , \tag{3}$$

where T is the off-shell two-body scattering amplitude appropriate to a particular wave function. The end result (in an obvious notation and where we have ignored the vanishing δ function) is

$$\begin{aligned} M_{fi} = g\Big(& \langle \vec{n}' + \vec{p}' | T_{dA}^{(+)}(E_d) | \vec{K} \rangle [E_d + i\epsilon - E_d(|\vec{n}' + \vec{p}'|)]^{-1} + \langle \vec{n}' | T_{nA}^{(+)}(E_{n'}) | \vec{K} - \vec{p}' \rangle [E_{n'} + i\epsilon - E_n(|\vec{K} - \vec{p}'|)]^{-1} \\ & + \langle \vec{p}' | T_{pA}^{(+)}(E_{p'}) | \vec{K} - \vec{n}' \rangle [E_{p'} + i\epsilon - E_p(|\vec{K} - \vec{n}'|)]^{-1} + \int d\vec{n}'' \langle \vec{n}' | T_{nA}^{(+)}(E_{n'}) | \vec{n}'' \rangle (E_{n'} + i\epsilon - E_{n''})^{-1} \\ & \times \{ \langle \vec{p}' | T_{pA}^{(+)}(E_{p'}) | \vec{K} - \vec{n}'' \rangle [E_{p'} + i\epsilon - E_p(|\vec{K} - \vec{n}''|)]^{-1} + \langle \vec{n}'' + \vec{p}' | T_{dA}^{(+)}(E_d) | \vec{K} \rangle [E_d + i\epsilon - E_d(|\vec{n}'' + \vec{p}'|)]^{-1} \} \\ & + \int d\vec{n}'' \int d\vec{p}'' \langle \vec{p}' | T_{pA}^{(+)}(E_{p'}) | \vec{p}'' \rangle (E_{p'} + i\epsilon - E_{p''})^{-1} \langle \vec{n}'' + \vec{p}'' | T_{dA}^{(+)}(E_d) | \vec{K} \rangle [E_d + i\epsilon - E_d(|\vec{n}'' + \vec{p}''|)]^{-1} \\ & \times \{ \delta(\vec{n}' - \vec{n}'') + \langle \vec{n}' | T_{nA}^{(+)}(E_{n'}) | \vec{n}'' \rangle (E_{n'} + i\epsilon - E_{n''})^{-1} \} \Big) . \end{aligned} \tag{4}$$

There are no divergent integrals anywhere in this expression, merely Cauchy singularities which are straightforward to handle numerically.

A particle-unstable state of the residual nucleus shows up as a resonance in n-A scattering. In the vicinity of a narrow resonance, with orbital angular momentum l, the n-A off-shell scattering matrix is given approximately by

$$\langle \vec{n}' | T_{nA}^{(+)}(E_{n'}) | \vec{k} \rangle \simeq \sum_{\mu = -l}^{l} \langle \vec{n}' | B^*; l\mu \rangle \langle B^*; l\mu | \vec{k} \rangle (E_{n'} - E_r + \tfrac{1}{2} i \Gamma_r)^{-1} , \tag{5}$$

where E_r and Γ_r are the position and width of the resonance B^*, and $\langle \vec{n}' | B^*; l\mu \rangle$ is its vertex function (decay amplitude). (When B^* is a broad resonance, the factorable form of the residue of the pole persists, but the Breit-Wigner denominator must be replaced with a more faithful representation of the energy dependence.) Inserting the approximation (5) into the expression (4) gives the DWBA matrix element for transitions to unstable residual nuclei, including the terms arising from direct breakup as well as those from sequential decay (SD). In view of the presence already of the double integral in (4), there is no additional complexity involved in a finite-range approximation: The necessary modification is just to replace the constant g by $g(\frac{1}{2} |\vec{n}'' - \vec{p}''|)$ and to bring this resulting (vertex) function under the relevant integral signs.

The complexity of Eq. (4) relative to the usual DWBA matrix element encountered in bound-state stripping encourages us to evaluate Eq. (1) in the Butler approximation. That is, we assume the absorption and distortion of the deuteron and proton wave functions effectively eliminate the contribution from radii smaller than R; moreover, we replace $\psi_{\vec{K}}^{(+)}(\vec{r})$ and $\psi_{\vec{p}'}^{(-)}(\vec{r})$ by plane waves outside R. This leads to

$$M_{fi} = (2\pi)^{-3/2} g \int_{r \geq R} d^3 r \, \psi_{\vec{n}'}^{(-)*}(\vec{r}) \, e^{i \vec{r} \cdot (\vec{K} - \vec{p}')} . \tag{6}$$

Let us add and subtract in (6) the integral over $r < R$ and replace $\psi_{\vec{n}'}^{(-)*}(\vec{r})$ by its equivalent expression in terms of T_{nA}, obtained by taking the Fourier transform of Eq. (3) with respect to $\vec{k}''$. Using Eq. (5) to express T_{nA}, we obtain the "neo-Butler" amplitude ($\vec{Q} = \vec{K} - \vec{p}' - \vec{n}'$, $\vec{q} = \vec{K} - \vec{p}'$):

$$\begin{aligned} M_{fi} = & -g[R^2/(2\pi^2 Q)] j_1(QR) + g[(2l+1)/4\pi] P_l(\hat{n}' \cdot \hat{q}) [E_{n'} - E_r + \tfrac{1}{2} i \Gamma_r]^{-1} \langle n' | B^*, l \rangle \\ & \times \{ \langle B^*, l | q \rangle [E_{n'} - E_q]^{-1} + (2m/\hbar^2) n' (i)^l (2/\pi)^{1/2} \int_0^R dr \, r^2 j_l(qr) \int_0^\infty dr'' r''^2 \langle B^*, l | r'' \rangle j_l(n' r_<) h_l^{(+)}(n' r_>) \} . \end{aligned} \tag{7}$$

The first term of (7) is associated with the diffractive dissociation (DD) of the deuteron, and the second term is the contribution from Butler stripping to the unstable residual nucleus, followed by its decay. They are represented graphically in Figs. 1(a) and 1(b), respectively. In obtaining the DD term of (7), it was unnecessary to make the zero-range approximation; so that we should replace the factor g by $g(\frac{1}{2} |\vec{n}' - \vec{p}'|)$, exactly as described in the preceding paragraph with reference to Eq. (4).

The DD and SD processes give rise to entirely different sorts of angular correlations. The first correlates the outgoing neutron direction with that of the proton, whereas SD gives rise to a neutron with a definite symmetry about $\hat{q}$, the direction of the momentum transferred to the residual nucleus B^*. This latter correlation agrees with our preconceptions derived from the behavior of the sequential $(p, p'\gamma)$ reactions.

An extremely interesting question, difficult to answer in general, is whether the two types of processes can interfere noticeably. It is a remarkable fact that at least a part of the SD term is of the same order of magnitude as the DD term. We can see this by considering the partial-wave decomposition of the neutron wave function in momentum space:

$$\psi_{\vec{n}'}^{(-)*}(\vec{k}'') = \sum_{l=0}^{\infty}\left(\frac{2l+1}{4\pi}\right)P_l(\hat{n}'\cdot\hat{k}'')[\delta(n'-k'')/n'^2 + \langle n'|T^l_{nA}(E_{n'}+i\epsilon)|k''\rangle(E_{n'}+i\epsilon-E_{k''})^{-1}] \; . \tag{8}$$

As Amado has shown,[4] the imaginary part of the Green's function, $-i\pi\delta(E_{n'}-E_{k''})$, multiplied by the on-shell T matrix T^l_{nA} gives a term which has the same magnitude as the unscattered part of the partial-wave n-A wave function, when the phase shift in that partial wave is nearly resonant. That is,

$$\delta(n'-k'')/n'^2 - i\pi\delta(E_{n'}-E_{k''})\langle n'|T^l_{nA}(E_{n'})|k''\rangle \equiv \delta(n'-k'')\,e^{i\delta_l}(\cos\delta_l)/n'^2 \; , \tag{9}$$

where δ_l is the phase shift at energy $E_{n'}$. Moreover, a study of resonant final-state interactions by Amado and Noble[5] concluded that when the production interaction (in this case DD) has a range in configuration space comparable to the range of the interaction producing the final-state resonance (in this case B^*), there is likely to be substantial interference between the direct-production and rescattering terms (analogous to SD). Thus it is possible (under suitable conditions) for the proton differential cross section, defined by

$$\frac{d^2\sigma}{d\Omega_{p'}dE_{p'}} = \frac{2(2\pi)^4 m^2 p'}{\hbar^4 K}\int d^3n'\,|M_{fi}|^2\delta[\epsilon_d + \hbar^2(n'^2+p'^2-\tfrac{1}{2}K^2)/2m] \; , \tag{10}$$

to deviate substantially from a pure stripping pattern, even for quite narrow states, B^*. We should note, however, that the method of data analysis commonly employed tends to minimize this deviation: The proton angular distribution is usually plotted for the integrated area under the proton peak corresponding to the state B^*, corrected for "background." The background which is subtracted is usually estimated by interpolating the events on either side of the peak. By integrating over the peak, the interference term between direct and sequential processes, whose phase varies by π over the resonance (and whose average is consequently zero) is averaged out. By subtracting the nonresonant background, we ignore the contribution from direct production (DD) alone. Thus, for analyzing angular distributions as usually extracted from experiment, the sequential decay term of (7) is adequate. We should realize, however, that by throwing away the entire DD contribution, we neglect a possibly valuable way to normalize the spectroscopic information we hope to acquire, since the DD term is independent of the behavior of the $B^* \to n + A$ amplitude. It will therefore be very interesting to reexamine the data from direct reactions leading to unstable residual nuclei, keeping this in mind.

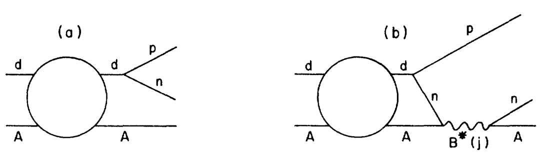

FIG. 1. (a) Diagrammatic representation of diffractive dissociation mechanism. (b) Diagrammatic representation of sequential decay following stripping.

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