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Cusps, Threshold States, and ^{16}N†

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States of ^{16}N have been observed whose excitation energies appear to be correlated with thresholds for neutron emission to excited states of ^{15}N. It is pointed out that similar correlations occur in ^{11}B and ^{12}C, and the correlation is discussed in terms of known threshold effects.

I. INTRODUCTION

A number of states in ^{16}N have been observed in a detailed study of the ^{18}O$(d,\alpha)^{16}$N reaction, which is discussed elsewhere.[1] Of particular interest are the states at $E_x(^{16}\mathrm{N}) = 8.819 \pm 0.015$, 9.794 ± 0.015, and 10.055 ± 0.015 MeV, because they all occur within 10 keV of thresholds for neutron emission by ^{16}N to excited states of ^{15}N (the third, fifth, and sixth excited states, respectively). In this paper we discuss the apparent correlation of these states with the neutron thresholds.

II. EXPERIMENTAL DETAILS

The general experimental procedures and results have been discussed in the previous paper, so we shall confine ourselves here to those details relevant to the states of interest. Each of the states has been observed at two bombarding energies ($E_d = 10.6$ and 11.2 MeV) and at several angles for each energy. The kinematic variation of the energy of the corresponding α groups allows them to be unambiguously identified as being produced by the ^{18}O$(d,\alpha)^{16}$N reaction. The energies of these groups were extracted from each spectrum where the group was resolved from contaminant groups using the method of energy calibration discussed in the previous paper.[1] This method was found to reproduce the excitation energies of the 7.97- and 8.47-MeV levels of ^{14}N and the 3.59-MeV level of ^{10}B to within 10 keV. The resulting excitation energies are presented in the table along with the energies of the corresponding thresholds based on the ^{15}N excitation energies of Warburton, Olness, and Alburger[2] and the neutron binding energy of 2.487 MeV taken from Mattauch, Thiele, and Wapstra.[3] The tentatively identified state at 9.66 MeV has been included because it might correspond to the threshold for the fourth excited state. However, since its identification is uncertain, because it is usually obscured by a contaminant group, we will not place much emphasis on this state.

As one can see from the table, the energies of the states of interest are all within about 10 keV of the corresponding thresholds. The average spacing between levels for all the observed levels between 7.5- and 10.3-MeV excitation is about 130 keV.[1] It is therefore quite improbable that three levels should be found, each within 10 keV of one of the three specified thresholds, if the levels were randomly distributed in this region of excitation energy. The correlation of the levels to the thresholds thus suggests a systematic threshold effect.

Before discussing the possible explanations for such a systematic effect, it may be worthwhile noting a few other experimental observations. The first is that the α groups all have widths equal to the experimental resolution of 70–80 keV so that their actual widths are probably less than half of this value, i.e., they are rather narrow ($\Gamma \leq 40$ keV).

It should also be noted that there are at least two other observed cases of a similar correlation of scattering anomalies with excited-state thresholds. Cusson,[4] in his study of ^{11}B levels via the ^{7}Li(α,α) and ^{7}Li(α,α') channels observed anomalies which he was reluctant to identify as ^{11}B states because they all coincided within their widths with the thresholds for neutron emission to the first five states of ^{10}B. In addition, the states of ^{12}C at 18.71, 23.04, and 23.51 MeV[5] occur within 10 keV of the thresholds for neutron emission to the ground, second, and third excited states of ^{11}C.

III. DISCUSSION

In considering how such threshold correlations could occur, we will be somewhat indirect, at least as far as the ^{16}N states are concerned, in that we will consider how these states might occur in the ^{15}N$(n,n)^{15}$N reaction rather than the ^{18}O$(d,\alpha)^{16}$N reaction. In so doing, we try to avoid the complication of the three-body problem which characterizes the present experiment, since the possibility of neutron emission means, that the reaction is actually ^{18}O$(d,\alpha n)^{15}$N. However, to the extent that the reaction is sequential via states of ^{16}N, the peaks in the experimental spectra correspond to states of ^{16}N which could also be observed by neutron scattering on ^{15}N.

There have been several threshold-related effects discussed in the literature. One of these is the so-called "threshold state" which corresponds to an enhanced probability near threshold for states with large reduced widths to the threshold channel. In spite of the fact that this phenomenon was originally proposed[6,7] to explain the correlation between charged-particle thresholds and states in ^{5}He and ^{5}Li, there is relatively little experimental evidence[7,8] for the case of neutron thresholds where the enhancement is expected to be strongest. However, since the effect only leads to an enhancement in the region of the threshold, it does not appear that it can explain the consistent occurrence of states so precisely at the threshold.

Another threshold phenomenon which is well known and which occurs precisely at the threshold is the Wigner cusp effect,[9,10] whereby the cross sections in previously open channels have infinite energy derivatives at thresholds for neutral-particle emission. In particular, we could expect to see anomalies in the scattering of neutrons by ^{15}N at the thresholds for inelastic scattering to excited states of ^{15}N. However, since the cusp effect occurs only in those partial waves which permit s-wave neutron emission in the opening channel, we could expect a significant effect only if one of those partial waves were dominant. The fact that the effect has been only occasionally observed[11-14] in nuclear-physics experiments is presumably due to the presence of sufficient amounts of other partial waves, so that the effect is a very small one on a large background. However, Frazer and Hendry[15] have studied the possibility of obtaining an enhanced cusp effect and find that there is a strong cusp effect if there is a virtual state S-matrix pole near the threshold. In this context, the term virtual state means a state such as the singlet deuteron, i.e., an almost-bound s-wave state which appears to have a negative energy. Such states occur only for the s-wave state of a neutral particle and have been discussed quite extensively by McVoy.[16] They have also been discussed by Weidenmüller[17] in connection with the "threshold states" of Baz,[6] Inglis,[7] and Barker.[8] If the interaction between the neutron and the residual excited state is such as to produce a virtual state, then we could expect a strong effect at threshold.

To have the regular occurrence of large cusps as suggested by the data, it is necessary to have a mechanism for the systematic occurrence of virtual states. This mechanism is essentially the same as that responsible for the "threshold states," namely, the relative insensitivity of the binding energy to the potential parameters when the binding energy is near zero. As a result there is a considerable range of potential strengths which allow virtual states. Hence, if we have a potential which produces a virtual state, moderate perturbations can still be expected to produce virtual states. This feature is evident in Fig. 1, which gives the binding energy of the $2s$ state in a square well as calculated from the work of Nussensveig.[18] Figure 1 is similar to figures given and discussed by McVoy[16] and Weidenmüller.[17] The present figure has been calculated for a well depth of 58 MeV which reproduces the binding energy of the $2s$ neutrons bound to the ^{15}N ground state when used with a radius of $a = 1.3A^{1/3}$ fm. The choice of a particular well depth affects only the normalization of the energy scale, E being proportional to V_0.

Therefore one of the most likely regions in which to expect a strong cusp enhancement is the $1p$-shell nuclei (i.e., $A = 5$–16) where the $2s_{1/2}$ neutron has small binding energies. Barker,[8] in looking for evidence of threshold states, picked out the states in this mass region that have large s-wave neutron reduced widths to the ground state of the nucleus of mass $A-1$, calculated the binding energy with respect to these ground states,

TABLE I. Excitation energies of states in ^{16}N and corresponding thresholds for neutron emission to excited state of ^{15}N.

E_x (MeV)	Number of spectra n[a]	Corresponding $N^{15}+n$ threshold (MeV)	Corresponding N^{15} state[b] (MeV)
8.819 ± 0.0025[c]	34	8.810 ± 0.005	6.323
9.794 ± 0.0055	12	9.787 ± 0.004	7.300
10.055 ± 0.0049	17	10.050 ± 0.004	7.563
(9.656) ± 0.026	2	9.641 ± 0.004	7.154

[a]The number of spectra with different combinations of E_d and θ is $\geq 0.75n$.
[b]Energies taken from Ref. 2.
[c]Uncertainty shown is standard error = (standard deviation of distribution)/$(n)^{1/2}$. This does not include the uncertainty in the calibration of the α-particle energy scale which is 10–15 keV.

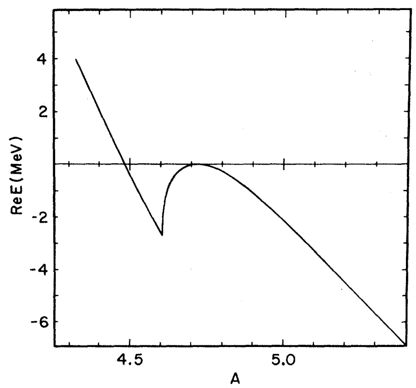

FIG. 1. The real part of the complex energy of the S-matrix pole for a $2s$ state in a 58-MeV-deep square well as a function of the potential-strength parameter $A = (2mV_0a^2/\hbar^2)^{1/2}$. Virtual states occur for $4.49 \leq A \leq 4.71$.

and plotted it against A. Instead of showing the expected clustering near zero binding energy, his Fig. 3 shows that the binding energy varies fairly smoothly from −1 MeV (i.e., unbound) for $A = 8$ through zero near $A = 9$ to about 3 MeV at $A = 17$. Thus, we could expect strong cusps corresponding to nearly bound virtual states for $A = 7$–9. Indeed one of the few strong cusp effects which has been observed is seen in the $^7Li(p,p)^7Li$ and $^7Li(p,p'\gamma)^7Li^*$ reactions[11,12] at the $^7Be + n$ threshold.

While it is not clear why the interaction between the neutron and the excited states should form virtual states for the cases of ^{11}B, ^{12}C, and ^{16}N when the s-wave neutron and the ground state of ^{10}B, ^{11}C, and ^{15}N form states which are bound by approximately 2 MeV, it is clear that if one of the excited states provides a sufficiently different potential so as to form a virtual state, then because of the relatively wide range of potentials which can form virtual states, several other excited states can be expected to form virtual states. We might note that nine of the eleven apparent threshold states in ^{11}B, ^{12}C, and ^{16}N occur at excited-state thresholds rather than at the ground-state thresholds which were examined by Inglis[7] and by Barker[8] in seeking evidence for threshold states.

The preceding discussion suggests that we may be able to identify the observed states of ^{16}N at 8.819, 9.794, and 10.055 MeV with nearly bound virtual states of a $2s_{1/2}$ neutron and the corresponding (third, fifth, and sixth) excited states of ^{15}N. In addition, the first and third excited states of ^{16}N are generally understood (see, e.g., Ref. 19) as being predominantly of the configuration $2s_{1/2}$ neutron bound to the ^{15}N ground state. One might wonder where the states corresponding to the first and second excited states of ^{15}N occur. There unfortunately is no evidence of these states available, other than the slightly suggestive energy spacing of 36 keV between the 7.640- and 7.676-MeV states[19] in ^{16}N, which agrees fairly well with the spacing[2] of 29 keV between the two ^{15}N levels. If this pair of ^{16}N levels did correspond to the $2s_{1/2}$ neutron coupled to the first and second excited states of ^{15}N, the neutron would be bound relative to the excited states by 110–120 keV. Figure 2 summarizes these suggested identifications and includes the predicted spin and parity of the ^{16}N states based on the coupling of a $2s_{1/2}$ neutron to the spin and parity of the corresponding ^{15}N states. These identifications should be amenable to experimental test, both through spin-parity determinations and because the presence of a virtual state or a weakly bound state should be evident in the shape of the neutron-production cross section in the new channel just above threshold.[15] The fitting

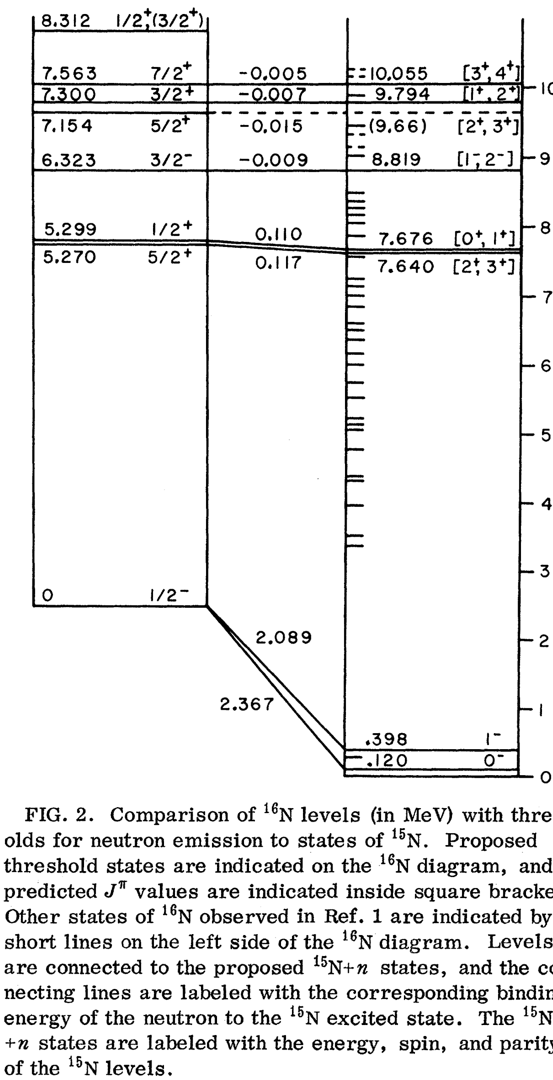

FIG. 2. Comparison of ^{16}N levels (in MeV) with thresholds for neutron emission to states of ^{15}N. Proposed threshold states are indicated on the ^{16}N diagram, and predicted J^π values are indicated inside square brackets. Other states of ^{16}N observed in Ref. 1 are indicated by short lines on the left side of the ^{16}N diagram. Levels are connected to the proposed $^{15}N+n$ states, and the connecting lines are labeled with the corresponding binding energy of the neutron to the ^{15}N excited state. The $^{15}N+n$ states are labeled with the energy, spin, and parity of the ^{15}N levels.

of this shape should allow determination of the magnitude of the "binding energy" of the responsible state relative to the new channel. A virtual state should cause a cusp effect in the other channels while a state weakly "bound" relative to the new channel should produce a resonance at the appropriate energy below threshold.

Assuming that these identifications are, for the most part, correct, we see that the binding energies decrease with increasing ^{15}N excitation in the manner that we would expect from Fig. 1, if we assumed that the strength of the ^{15}N well decreases uniformly with excitation energy. That is, the binding energy decreases more rapidly with the potential strength for large binding energies than for small binding energies, and after reaching zero binding energy there follows a large region of virtual states. If we use Weidenmüller's curve for the $2s_{1/2}$ neutron in a square well, we find that the potential strength $(VR^2)^{1/2}$ should decrease by 6% in going from the ^{15}N ground state to approximately 6-MeV excitation.

IV. CONCLUSION

States have been observed in ^{16}N which occur at the threshold energies for neutron emission to excited states of ^{15}N, and it has been noted that similar states occur in ^{11}B and ^{12}C. The apparent correlation between these states and the corresponding thresholds is based solely on the closeness of the energies involved, and it is clear that more data are necessary in order to establish the nature of the correlation. Especially desirable would be information on the spin and parity of the states and the neutron-production cross sections in the regions immediately above thresholds.

The discussions of threshold states by Baz,[6] Inglis,[7] and Barker[8] indicate why there might be states preferentially near neutron thresholds, but seem insufficient to account for the presence of these states so precisely at threshold. The presence of virtual states, which has been suggested here, does place the threshold states precisely at threshold. It is not, however, completely clear why such virtual states occur in the cases under discussion. On the other hand, the observation of the present ^{16}N states together with the similar states in ^{11}B and ^{12}C does suggest a threshold effect based on excited-state thresholds and as such is worthy of further study.

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