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Effect of the Mott-Schwinger Interaction on Neutron-Proton Scattering

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The influence of the interaction between the magnetic moment of a neutron and the Coulomb field of a target proton is investigated. For this interaction, channel spin is not conserved. A formalism is developed which includes the resulting singlet-triplet mixing. The scattering amplitude, or M matrix, is presented, and its elements are related to the coefficients of the most general form of the M matrix. Expressions for the nine most common nucleon-nucleon-scattering observables are obtained, including the effects of the singlet-triplet mixing. Coupled radial wave equations are derived and solved to reveal the phase-shift modifications. Calculations are performed over an energy range from 25 to 210 MeV using nuclear phase shifts previously determined from best χ^2 fits to the scattering data. Our calculations indicate that all nine of the scattering observables considered are significantly influenced by the Mott-Schwinger interaction, but only for very small ($<5°$) scattering angles.

I. INTRODUCTION

IN a previous paper,[1] the effect of the interaction between the magnetic moment of a neutron and the Coulomb field of a target nucleus on the polarization and the differential-scattering cross section was evaluated. The influence of this Mott-Schwinger (MS) interaction on the aforementioned observables was sufficiently large to suggest that the evaluation of the effect of including this interaction in the neutron-proton scattering problem would also be of interest.

The modification resulting from the inclusion of the MS force in the nucleon-nucleon scattering problem has received some attention. Garren[2] included the MS effect in an approximate relativistic calculation of high-energy p-p scattering by use of the Born approximation. Breit and co-workers have considered the problem in connection with high-energy p-p scattering[3–5] as well as in a more general context.[6,7] In the more recent works,[6,7] Breit pointed out that the wave distortion produced by the nuclear interaction invalidates the Born-approximation treatments for low l waves.[6] In addition, the n-p problem was discussed and differences between n-p and p-p scattering introduced by the MS interaction were pointed out. It was found that for p-p scattering at 147 MeV, the polarization $P(\theta)$, the correlation coefficient $C_{KP}(\theta)$, and the triple scattering parameter $A(\theta)$ are appreciably affected at small scattering angle although it is doubtful that measurements of $C_{KP}(\theta)$ and $A(\theta)$ can be made with sufficient accuracy to detect this influence. In the case of $P(\theta)$, however, a noticeable improvement in agreement with experimental data was achieved in the range of 8°–20° by inclusion of the MS interaction. The basic approximation used in all these treatments was the plane-wave Born approximation.

With the exception of the references just cited, the effects of the MS interaction on the analysis of nucleon-nucleon scattering data have been largely ignored. Since its approximate inclusion has produced better agreement with experiment and since its inclusion in the neutron-nucleus problem gave some interesting results, it would seem very desirable to develop more accurate techniques for treating the MS interaction and for systematically evaluating the domains of importance of this interaction.

The remainder of this paper is divided into three sections. Section II deals with the theory of neutron-proton scattering, and with the development of a formalism for including the MS interaction. In Sec. III, the formalism is applied to determine the regions of importance of the MS interaction. Section IV contains the summary and conclusions. The treatment is nonrelativistic.

II. GENERAL FORMALISM FOR NEUTRON-PROTON INTERACTION

A. Preliminaries

The MS interaction in the nucleon-nucleon scattering problem requires a somewhat different treatment than in the nucleon-nucleus problem. The essential difference is because of the occurrence of the target spin operator in the nucleon-nucleon Hamiltonian. This occurrence has three consequences. (i) Since

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[1] W. S. Hogan and R. G. Seyler, Phys. Rev. **177**, 1706 (1969).
[2] A. Garren, Phys. Rev. **101**, 419 (1956).
[3] G. Briet, Phys. Rev. **99**, 1581 (1955).
[4] M. E. Ebel and M. H. Hull, Phys. Rev. **99**, 1596 (1955).
[5] G. Breit, Phys. Rev. **106**, 314 (1957).
[6] G. Breit and H. M. Ruppel, Phys. Rev. **127**, 2123 (1962).
[7] G. Breit, Rev. Mod. Phys. **34**, 766 (1962).

the total spin can exceed the spin of the neutron, the dimensionality of the scattering matrices is increased; (ii) the presence of the nuclear tensor interaction leads to transitions between states whose orbital angular momenta differ by two; and (iii) the spin operator in the MS interaction need not be the total spin of the system. For p-p scattering, since the magnetic moment of each particle is acted on by the Coulomb field of the other, the MS interaction involves the operator $\mathbf{l}\cdot\mathbf{s}$, where $\mathbf{s}$ is the total spin of the system, whereas for n-p scattering the analogous term is $\mathbf{l}\cdot\mathbf{s}_n$ which does not commute with the total spin and will therefore lead to transitions between states of different total spin. The first consequence is a rather trivial complication, but the second and third lead to systems of coupled equations unlike the situation for nucleon-nucleus scattering.

The question of interest here is whether an accurate treatment of the MS interaction might alter the results drawn from an analysis of nucleon-nucleon scattering data. The standard method of analyzing such data (cross section, polarization, depolarization, etc.) is to attempt to find that set of phase shifts which best represents the entirety of experimental results. The best result here is defined in terms of χ^2 minimization.[8] Restated the question then is to what extent does inclusion of the MS force alter these phase shifts.

Inclusion of the MS term in the p-p scattering problem would bring about no changes in the general formalism. That is, the forms for the scattering amplitudes would not be changed, and of course, no change in the χ^2 minimized phase shifts would result, except possibly for the relatively minor influence of large l partial waves, where the usual practice is to include their contribution through an analytical treatment of the one-pion exchange potential. Breit and Ruppel[6] have included the large l partial wave effect of the MS force in the p-p scattering problem and found it to be small, but not negligible. Although the charge-independent nuclear Hamiltonian introduces no spin mixing, inclusion of the MS force in n-p scattering mixes the singlet and triplet spin states (consequently, the two isotopic spin states are also mixed) and does change the interpretation of the phase shifts since new scattering amplitudes are introduced. The usual practice is to regard this coupling between spin states to be negligibly weak and to neglect the MS interaction entirely.[6,9] There are good reasons for neglecting this effect in the phase-shift analyses. First, in contrast with p-p scattering, there are insufficient n-p scattering data to allow one to effect a reliable independent determination of the phase shifts.[10] Further, essentially all the data are at scattering angles somewhat larger than where one expects the MS influence to be evident. Since the nuclear spin-orbit force is of relatively short range and therefore less important at lower energies,[11] one might expect the spin-orbit force-dependent effects to be most influenced by the relatively weak MS force in this lower-energy range.

The consideration that the inclusion of the MS force introduces no changes in the p-p scattering formalism coupled with the considerations that the large forward angle p-p cross section tends to obscure the MS force influence, and that the use of Coulomb wave functions introduces additional numerical complexities, led to a decision to investigate the influence of the MS force on only n-p scattering.

In what follows in this section, the n-p scattering formalism is developed including the MS force, and the appropriate phase shifts are calculated. In Sec. III, the effect of the MS interaction on the scattering observables is calculated and compared to experimental results.

B. Coupled Radial Equations

Consider the scattering of a neutron and a proton with total angular momentum projection m. Let the quantization axis lie along the linear momentum vector of the neutron beam. We express the wave function as

$$\psi^m = \sum_{ljs} \mathcal{Y}_{jls}{}^m(\theta,\phi) R_{jls}(r), \tag{1}$$

where $R_{jls}(r)$ represents the radial dependence, and $\mathcal{Y}_{jls}{}^m$ gives the spin and angle dependence

$$\mathcal{Y}_{jls}{}^m = \sum_{\lambda\sigma} (ls\lambda\sigma \mid jm) Y_l{}^\lambda(\theta,\phi)\chi_s{}^\sigma. \tag{2}$$

In Eq. (2), $(ls\lambda\sigma \mid jm)$ represents a Clebsch-Gordan coefficient,[12] $Y_l{}^\lambda(\theta,\phi)$ represents the well-known spherical harmonic,[12] and $\chi_s{}^\sigma$ represents the spin-wave function with total spin s and z-projection σ. Since $\mathbf{l}\cdot\mathbf{s}_n$ does not commute with s^2, it follows that an eigenfunction of s^2 (e.g., $\mathcal{Y}_{jls}{}^m$) is not an eigenfunction of $\mathbf{l}\cdot\mathbf{s}_n$. It is, therefore, clear that the $\mathbf{l}\cdot\mathbf{s}_n$ will mix singlet and triplet spin states and from parity conservation considerations, these will be states where $j=l$. In order to derive the appropriate radial-wave equations, it is necessary to evaluate the expression

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[8] R. A. Arndt and M. H. MacGregor, in *Methods in Computational Physics* (Academic Press Inc., New York, 1966), Vol. 6, p. 253.

[9] M. H. MacGregor, R. A. Arndt, and A. A. Dubow, Phys. Rev. **135**, B628 (1964).

[10] R. A. Arndt and M. H. MacGregor, Phys. Rev. **141**, 873 (1966).

[11] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Co., Reading, Mass., 1962), p. 107.

[12] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, England, 1957).

$(\mathbf{l}\cdot\mathbf{s}_n)\mathcal{Y}_{jis}{}^m$ for the four possible states. We find that

$$(\mathbf{l}\cdot\mathbf{s}_n)\mathcal{Y}_{l+1,l,1}{}^m=\tfrac{1}{2}l\mathcal{Y}_{l+1,l,1}{}^m,$$
$$(\mathbf{l}\cdot\mathbf{s}_n)\mathcal{Y}_{l-1,l,1}{}^m=-\tfrac{1}{2}(l+1)\mathcal{Y}_{l-1,l,1}{}^m, \qquad (3)$$
$$(\mathbf{l}\cdot\mathbf{s}_n)\mathcal{Y}_{ll1}{}^m=-\tfrac{1}{2}\mathcal{Y}_{ll1}{}^m+\tfrac{1}{2}[l(l+1)]^{1/2}\mathcal{Y}_{ll0}{}^m,$$
$$(\mathbf{l}\cdot\mathbf{s}_n)\mathcal{Y}_{ll0}{}^m=\tfrac{1}{2}[l(l+1)]^{1/2}\mathcal{Y}_{ll1}{}^m,$$

which leads to the following four equations for the radial-wave functions outside the range of the nuclear force $(r>r_c)$:

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{dR_{ll0}}{d\rho}\right)+\left(1-\frac{l(l+1)}{\rho^2}\right)R_{ll0} -(\gamma/2\rho^3)[l(l+1)]^{1/2}R_{ll1}=0, \qquad (4)$$

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{dR_{ll1}}{d\rho}\right)+\left(1-\frac{l(l+1)}{\rho^2}\right)R_{ll1} +(\gamma/2\rho^3)R_{ll1}-(\gamma/2\rho^3)[l(l+1)]^{1/2}R_{ll0}=0, \qquad (5)$$

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{dR_{l+1,l,1}}{d\rho}\right)+\left(1-\frac{l(l+1)}{\rho^2}\right)R_{l+1,l,1} -(\gamma/2\rho^3)lR_{l+1,l,1}=0, \qquad (6)$$

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{dR_{l-1,l,1}}{d\rho}\right)+\left(1-\frac{l(l+1)}{\rho^2}\right)R_{l-1,l,1} +(\gamma/2\rho^3)(l+1)R_{l-1,l,1}=0, \qquad (7)$$

where $\gamma=(e^2/mc^2)k\,|\,\mu_n\,|$.

It should be noted that within the range of the nuclear force the tensor-force couples the two states $l=j\pm1$. This coupling must be accounted for in expressing scattering amplitudes in terms of phase shifts. The effect of this latter coupling on the matrix elements is well known, and is readily available in the literature.[13]

C. *M* Matrix

Since coupled wave functions such as R_{ll0} and R_{ll1} imply a sharing of flux between the states even though the initial flux might have existed in only one state, the phase shifts associated with R_{ll0} and R_{ll1} of Eqs. (4) and (5) are complex. These can be converted to two real phase shifts and a mixing parameter through the use of the well-known recipes of either Blatt and Beidenharn[14] or Stapp.[15] The Blatt-Biedenharn type of phase shift will be used for the derivation of the M matrix, although Stapp phase shifts will be used later in the calculations. The choice is arbitrary, for given one type of phase shift, one can always find the other.

Taking a linear combination of states, one may express the asymptotic form of the wave function as

$$\psi^m\sim\sum_{tlj}\sum A_{jlt}\phi_{jlt}{}^mF_{jlt}(\rho), \qquad (8)$$

where

$$\phi_{l\pm1,l1}{}^m=\mathcal{Y}_{l\pm1,l1}{}^m,$$
$$\phi_{ll+}{}^m=\mathcal{Y}_{ll1}{}^m+\tan\epsilon_l\mathcal{Y}_{ll0}{}^m,$$
$$\phi_{ll-}{}^m=\mathcal{Y}_{ll1}{}^m-\cot\epsilon_l\mathcal{Y}_{ll0}{}^m.$$

In this notation t ranges over the indices 1, +, and −, where the + and − subscripts correspond to the signs preceding the tan or cot term on the right-hand side of the last two expressions. The quantity ϵ_l represents the above-mentioned mixing parameter and $F_{jlt}(\rho)$ represents the radial dependence consistent with the ϕ functions and is normalized so as to differ asymptotically from $j_l(\rho)$ only by a phase shift.

If the incident neutron direction is selected as the quantization axis, the asymptotic form of the wave function can be expressed alternatively as

$$\psi^m\sim\exp(i\rho\cos\theta)\chi_s{}^m+(e^{i\rho}/r)\sum_{m'}M_{s'm'sm}\chi_{s'}{}^{m'}, \qquad (9)$$

where the unprimed indices refer to the incident beam and the primed indices to the scattered beam. The quantities $M_{s'm'sm}$ represent the amplitudes for scattering from the initial state to a final state with spin s' and projection m' and constitute the elements of the M matrix. As there are four possible values of the pair (s, m), the M matrix is square and of order four.

Equating Eqs. (8) and (9) and making use of the spherical Bessel function $[j_l(\rho)]$ expansion of the plane wave and the asymptotic forms of $j_l(\rho)$ and $F_{jlt}(\rho)$, explicit forms for the $M_{s'm'sm}$ can be determined. For the case of no spin-state mixing, these forms (including the $l=j\pm1$ state mixing) are readily available in the literature.[16]

The inclusion of the spin-state mixing MS interaction leads to the following for the $j=l$ elements of the S matrix,

$$S_{l00}{}^l=\frac{\tan\epsilon_l\exp(2i\delta_{ll+})+\cot\epsilon_l\exp(2i\delta_{ll-})}{\cot\epsilon_l+\tan\epsilon_l},$$
$$S_{l11}{}^l=\frac{\cot\epsilon_l\exp(2i\delta_{ll+})+\tan\epsilon_l\exp(2i\delta_{ll-})}{\cot\epsilon_l+\tan\epsilon_l}, \qquad (10)$$
$$S_{l01}{}^l=S_{l10}{}^l=\frac{\exp(2i\delta_{ll+})-\exp(2i\delta_{ll-})}{\cot\epsilon_l+\tan\epsilon_l},$$

where the δ's are the $j=l$ eigenphase shifts. The reader

[13] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon Press, Oxford, England, 1965), 3rd ed., Chap. 10.

[14] J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. **24,** 258 (1952).

[15] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105,** 302 (1957).

[16] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Ann. Rev. Nucl. Sci. **10,** 291 (1960).

TABLE I. Observables in terms of Wolfenstein coefficients.

Observable	Expression
$d\sigma/d\Omega\,\vert_u$	$\vert a\vert^2+\vert m\vert^2+2[\vert b\vert^2+\vert c\vert^2+\vert g\vert^2+\vert h\vert^2]$
$d\sigma/d\Omega\,\vert_u\mathbf{P}$	$-2\,\mathrm{Re}[c^*(a+m)+b^*(a-m)]\mathbf{n}_2$
$d\sigma/d\Omega\,\vert_u(1-D)$	$4\vert g\vert^2+4\vert h\vert^2$
$d\sigma/d\Omega\,\vert_uR$	$[\vert a\vert^2-\vert m\vert^2-4\,\mathrm{Re}(cb^*+gh^*)]\cos\frac{1}{2}\theta-2\,\mathrm{Re}i[a^*(c+b)-m^*(c-b)]\sin\frac{1}{2}\theta$
$d\sigma/d\Omega\,\vert_uR'$	$[\vert a\vert^2-\vert m\vert^2-4\,\mathrm{Re}(cb^*-gh^*)]\sin\frac{1}{2}\theta+2\,\mathrm{Re}i[a^*(c+b)-m^*(c-b)]\cos\frac{1}{2}\theta$
$d\sigma/d\Omega\,\vert_uA$	$[-\vert a\vert^2+\vert m\vert^2+4\,\mathrm{Re}(cb^*+gh^*)]\sin\frac{1}{2}\theta-2\,\mathrm{Re}i[a^*(c+b)-m^*(c-b)]\cos\frac{1}{2}\theta$
$d\sigma/d\Omega\,\vert_uA'$	$[\vert a\vert^2-\vert m\vert^2-4\,\mathrm{Re}(cb^*-gh^*)]\cos\frac{1}{2}\theta-2\,\mathrm{Re}i[a^*(c+b)-m^*(c-b)]\sin\frac{1}{2}\theta$
$d\sigma/d\Omega\,\vert_uC_{nn}$	$2[\vert c\vert^2-\vert b\vert^2+\vert h\vert^2-\vert g\vert^2+\mathrm{Re}\,am^*]$
$d\sigma/d\Omega\,\vert_uC_{KP}$	$-4\,\mathrm{Re}i[ch^*+bg^*]$

is referred to Eq. (3.14) of Ref. 14 for the connection between the M- and S-matrix elements.

D. General Considerations and the *M* Matrix

The requirement that the scattering matrix be invariant under rotation, time inversion, and space reflection allows only six terms to be retained in the general representation. All this is well known, and was first developed by Wolfenstein and Ashkin.[17] It happens that if one assumes charge independence of the interaction, then only five terms are allowed. Since the nuclear forces are believed to be charge-independent, the usual treatment of n-p scattering do not include the consequences of the sixth term.

One way of expressing the general six-term M matrix is as follows:

$$M=A+D\sigma_n^{(2)}+C\sigma_n^{(1)}+B\sigma_n^{(1)}\sigma_n^{(2)}+E\sigma_P^{(1)}\sigma_P^{(2)}+F\sigma_K^{(1)}\sigma_K^{(2)}. \quad (11)$$

In Eq. (12),

$$\mathbf{n}=\frac{\mathbf{k}_{\mathrm{in}}\times\mathbf{k}_{\mathrm{out}}}{|\mathbf{k}_{\mathrm{in}}\times\mathbf{k}_{\mathrm{out}}|},\qquad \mathbf{P}=\frac{\mathbf{k}_{\mathrm{out}}+\mathbf{k}_{\mathrm{in}}}{|\mathbf{k}_{\mathrm{in}}+\mathbf{k}_{\mathrm{out}}|},$$

and

$$\mathbf{K}=\frac{\mathbf{k}_{\mathrm{out}}-\mathbf{k}_{\mathrm{in}}}{|\mathbf{k}_{\mathrm{out}}-\mathbf{k}_{\mathrm{in}}|},$$

where $\mathbf{k}_{\mathrm{in}}$ and $\mathbf{k}_{\mathrm{out}}$ represent the wave-propagation vectors before and after the scattering. The superscripts (1) and (2) are used to distinguish between the spin spaces of the neutron and proton, respectively.

The coefficients A, B, C, D, E, and F are, in general, complex functions of the energy and scattering angle.

An alternative equivalent form of Eq. (12) is frequently seen in the literature,[16,18]

$$M=a+b(\sigma_n^{(1)}-\sigma_n^{(2)})+c(\sigma_n^{(1)}+\sigma_n^{(2)})+m\sigma_n^{(1)}\sigma_n^{(2)}+g(\sigma_P^{(1)}\sigma_P^{(2)}+\sigma_K^{(1)}\sigma_K^{(2)})+h(\sigma_P^{(1)}\sigma_P^{(2)}-\sigma_K^{(1)}\sigma_K^{(2)}). \quad (12)$$

If one postulates charge independence, then b would vanish and only five parameters would be necessary. In Eq. (11), this same condition (charge independence) would require that $C=D$.

The Wolfenstein coefficients a, c, m, g, and h are related to the coefficients A through F and to the M-matrix elements in Ref. 16.

The charge-dependent MS interaction introduces the parameter b which is related to other quantities of interest as

$$b=\tfrac{1}{2}(C-D)=(i/\sqrt{2})M_c, \quad (13)$$

where $M_c=M_{1100}=M_{1-100}=-M_{001-1}=-M_{0011}$.

E. Scattering Observables

This subsection will be concerned with expressing the quantities usually observed in scattering experiments in terms of the Wolfenstein coefficients, a, b, c, g, h, and m.

The observables that have been measured in nucleon-nucleon double- and triple-scattering experiments are listed in Table I along with the appropriate expression in terms of the Wolfenstein coefficients.

The reader is referred to Ref. 16 for a description of these observables and for a discussion of the procedure employed in obtaining these relations. If the coefficient b is set equal to zero, corresponding to the

[17] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).

[18] L. Wolfenstein, Phys. Rev. **96**, 1654 (1954).

case of a charge-independent interaction (no singlet-triplet coupling), the expressions reduce to those given in Ref. 16.

F. Phase-Shift Calculations

In order to evaluate the effect of the MS force on the observables, it is convenient to find its effect on the phase shifts. To accomplish this, it is necessary to know the phase shifts due to the nuclear interaction alone. It will be assumed that those phase shifts determined from the experimental data are the nuclear phase shifts. [The nuclear phase shifts were taken to be the isospin triplet phase shifts as determined from *p-p* data without taking into account the *p-p* MS interaction. The MS correction to these phase shifts might be expected to be somewhat larger than the *n-p* corrections considered in this work (a factor of 2 larger as a first approximation). However, it turns out that even a correction of this magnitude is negligible.] This is a very good approximation since the effects of the MS force turn out to be evident only at small angles, whereas most of the experimental data have been taken at relatively large scattering angles. If the effect of the MS interaction on the phase shifts had turned out to be significant, a self-consistent solution would have required the assumption of nuclear phases different from those determined from the experimental data such that the phases due to the total (nuclear plus MS) interaction agreed with experiment.

Equations (4)–(7) are pertinent to a determination of the phase shifts. In Eqs. (6) and (7), the radial wave functions do not correspond to pure l states because of the tensor force. The phase shifts associated with these coupled wave functions are real provided the mixing is done properly, and the MS term will modify them, just as in the case of nucleon-nucleus scattering.[1] These mixed-state phase shifts are not unique since one has some choice in the way the mixing is described. The coupling scheme in general use is that of Stapp, and the phase shifts are referred to as Stapp, or sometimes, bar-phase shifts. This scheme is presented in a number of places and will not be repeated here (see, e.g., Ref. 13).

Equations (4) and (5) show that outside the nuclear force range the radial wave functions for the two $j=l$ states are coupled through the MS force and hence the modification of the phase shifts associated with these states is more difficult. We focus our attention on the calculation of the complex phase shifts associated with the functions R_{ll0} and R_{ll1}.

We shall solve Eqs. (4) and (5) to first order in γ. Then by invoking unitarity, the imaginary part of the phase shifts (these are of second order in γ) will be found. As mentioned earlier the phase shifts δ_{l0} and δ_{l1}, determined from the observables,[10] will be regarded as resulting from the nuclear interaction alone. Subsequent results will show that this assumption is justified.

First, we express the solution to Eqs. (4) and (5) for $\rho>\rho_c$ in powers of γ,

$$R_i=R_i^{(0)}+\gamma R_i^{(1)}+\gamma^2R_i^{(2)}+\cdots, \tag{14}$$

where $i=0$ or 1, with $R_0\equiv R_{ll0}$ and $R_1\equiv R_{ll1}$.

The zero-order solutions are clearly just the solutions external to the nucleus ($\rho>\rho_c$) with no MS scattering present;

$$R_i^{(0)}=A_{li}[\cos\delta_{li}\,j_l(\rho)-\sin\delta_{li}\,n_l(\rho)]. \tag{15}$$

The first-order solutions involve integrals of the zero-order solutions;

$$R_0^{(1)}=C_lh_l^{(1)}(\rho)+E_lh_l^{(2)}(\rho)-\tfrac{1}{2}i[l(l+1)]^{1/2}\times\left(h_l^{(1)}(\rho)\int_{\rho_c}^{\rho}j_l(\rho')R_1^{(0)}(\rho')d\rho'/\rho'+j_l(\rho)\int_{\rho}^{\infty}h_l^{(1)}(\rho')R_1^{(0)}(\rho')d\rho'/\rho'\right), \tag{16}$$

$$R_1^{(1)}=D_lh_l^{(1)}(\rho)+F_lh_l^{(2)}(\rho)+\tfrac{1}{2}i\left(h_l^{(1)}(\rho)\int_{\rho_c}^{\rho}j_l(\rho')\times\{R_1^{(0)}(\rho')-[l(l+1)]^{1/2}R_0^{(0)}(\rho')\}d\rho'/\rho'+j_l(\rho)\int_{\rho}^{\infty}h_l^{(1)}(\rho')\{R_1^{(0)}(\rho')-[l(l+1)]^{1/2}R_0^{(0)}(\rho')\}d\rho'/\rho'\right). \tag{17}$$

In order to evaluate some of the constants in Eqs. (16) and (17), we use the logarithmic derivative (evaluated at $\rho=\rho_c$) which, to first order in γ, is

$$\rho_c\frac{R_i'(\rho_c)}{R_i(\rho_c)}=\rho_c\frac{R_i^{(0)\prime}(\rho_c)}{R_i^{(0)}(\rho_c)}\times\left[1+\gamma\left(\frac{R_i^{(1)\prime}(\rho_c)}{R_i^{(0)\prime}(\rho_c)}-\frac{R_i^{(1)}(\rho_c)}{R_i^{(0)}(\rho_c)}\right)+O(\gamma^2)\right], \tag{18}$$

or $L=L_i^{(0)}+\gamma L_i^{(1)}+O(\gamma^2)$, where

$$L_i^{(0)}=\rho_c[R_i^{(0)\prime}(\rho_c)/R_i^{(0)}(\rho_c)],$$

$$L_i^{(1)}=\rho_c\frac{R_i^{(1)}(\rho_c)}{R_i^{(0)}(\rho_c)}\left(\frac{R_i^{(1)\prime}(\rho_c)}{R_i^{(1)}(\rho_c)}-\frac{R_i^{(0)\prime}(\rho_c)}{R_i^{(0)}(\rho_c)}\right). \tag{19}$$

Rewriting the last equation, we have

$$R_i^{(0)}(R_i^{(1)\prime}-1/\rho_cL_i^{(1)}R_i^{(0)})=R_i^{(0)\prime}R_i^{(1)}. \tag{20}$$

Since the MS potential inside the nucleus (of order $\gamma l\rho_c^{-3}$) is much less than the nuclear potential, it is reasonable to expect that the logarithmic derivative at $\rho=\rho_c$ could be represented to sufficient accuracy through use of nuclear phase shifts only. To test this, we determined the minimum square-well depth (with range $r_c=1.7$ F) that gave the correct nuclear phase shift and then computed the change in the logarithmic derivative due to the addition of the MS force for $r<r_c$. This was done for all partial waves and all energies for which phase shifts are given in Ref. 10, and in all cases, the effect of the MS interaction on the logarithmic derivative was found to be negligible. As a consequence, in Eq. (20) one may ignore $L_i^{(1)}$. Substituting Eqs. (15)–(17) into Eq. (20) and using the Wronskian relation between j_l and n_l leads to the equations

$$i\cos\delta_{l0}(C_l-E_l)+\sin\delta_{l0}\bigg(C_l+E_l-\tfrac{1}{2}i[l(l+1)]^{1/2}A_{l1}\times\int_{\rho c}^{\infty}h_l^{(1)}(\rho')[\cos\delta_{l1}j_l(\rho')-\sin\delta_{l1}n_l(\rho')]d\rho'/\rho'\bigg)=0 \tag{21}$$

and

$$i\cos\delta_{l1}(D_l-F_l)+\sin\delta_{l1}\bigg(D_l+F_l+\tfrac{1}{2}iA_{l1}\times\int_{\rho c}^{\infty}h_l^{(1)}(\rho')[\cos\delta_{l1}j_l(\rho')-\sin\delta_{l1}n_l(\rho')]d\rho'/\rho' -\tfrac{1}{2}i[l(l+1)]^{1/2}A_{l0}\int_{\rho c}^{\infty}h_l^{(1)}(\rho')\times[\cos\delta_{l0}j_l(\rho')-\sin\delta_{l0}n_l(\rho')]d\rho'/\rho'\bigg)=0. \tag{22}$$

For the case where only a singlet wave is incident ($A_{l1}=F_l=0$), Eq. (22) gives

$$D_l=\tfrac{1}{2}A_{l0}[l(l+1)]^{1/2}\exp(i\delta_{l1})\sin\delta_{l1}I_{l0},$$

where

$$I_{l0}=\int_{\rho c}^{\infty}h_l^{(1)}(\rho')[\cos\delta_{l0}j_l(\rho')-\sin\delta_{l0}n_l(\rho')]d\rho'/\rho',$$

and Eq. (21) yields,

$$E_l=C_l\exp(-2i\delta_{l0}).$$

The asymptotic forms for the wave functions are then

$$\rho R_0\sim i^{l+1}[\tfrac{1}{2}A_{l0}\exp(-i\delta_{l0})+\gamma E_l]\times[e^{-i\rho}-(-)^l\exp(2i\delta_{l0})e^{i\rho}], \tag{23}$$

$$\rho R_1\sim\tfrac{1}{2}\gamma A_{l0}i^{-(l+1)}[l(l+1)]^{1/2}\times[\exp(i\delta_{l1})\sin\delta_{l1}I_{l0}-iU_{l0}]e^{i\rho}, \tag{24}$$

where

$$U_{l0}=\int_{\rho c}^{\infty}j_l(\rho')[\cos\delta_{l0}j_l(\rho')-\sin\delta_{l0}n_l(\rho')]d\rho'/\rho'.$$

Equations (23) and (24) yield two elements of the S matrix as

$$S_{l00}{}^l\equiv S_{00}=(-)^l\exp(2i\delta_{l0}),$$
$$S_{l01}{}^l\equiv S_{01}=(-)^l i\gamma[l(l+1)]^{1/2}\exp[i(\delta_{l0}+\delta_{l1})]B_l^{01}, \tag{25}$$

where the subscripts 0 and 1 denote channel spin and

$$B_l^{01}=a_l(\rho_c)\cos\delta_{l0}\cos\delta_{l1}-b_l(\rho_c)\sin(\delta_{l0}+\delta_{l1})+c_l(\rho_c)\sin\delta_{l0}\sin\delta_{l1}.$$

In the expression for B_l^{01},

$$b_l(\rho_c)=\int_{\rho c}^{\infty}j_l(\rho')n_l(\rho')d\rho'/\rho';$$

$a_l(\rho_c)$ and $c_l(\rho_c)$ are similar integrals involving $j_l^2(\rho')$ and $n_l^2(\rho')$, respectively.

Following a similar calculation with only a triplet-wave incident, one finds $S_{10}=S_{01}$ and

$$S_{11}=(-)^l\exp(2i\delta_{l1})(1+i\gamma B_l^{11})\equiv(-)^l\exp(2i\delta_{l1}{}^M), \tag{26}$$

where

$$B_l^{11}=a_l(\rho_c)\cos^2\delta_{l1}-b_l(\rho_c)\sin^2\delta_{l1}+c_l(\rho_c)\sin^2\delta_{l1}.$$

This result for S_{11} is consistent with the first-order correction to δ_{l1} as determined using the formalism of Ref. 1. We remark that if in Eq. (20) we had not neglected the logarithmic derivative correction $L_i^{(1)}$, we would have found in place of Eqs. (25) and (26) the same forms but with δ_{li} replaced by $\delta_{li}-\gamma L_i^{(1)}T_i$, where $T_i\equiv\rho_c[\cos\delta_{li}j_l(\rho_c)-\sin\delta_{li}n_l(\rho_c)]^2$. By approximating the nuclear interaction (for each partial wave) by a square well, adjusted to fit the "known" nuclear phase shift of that partial wave, we found that the phase-shift correction $\gamma L_i^{(1)}T_i$ was always less than 0.001° (we examined all the energies for which data is given in Ref. 10). A more realistic nuclear interaction might give a slightly larger correction but would not alter the conclusion that the effect of the *internal* region of the MS interaction is negligible, as expected.

The S-matrix elements of Eqs. (25) and (26) have been determined to first order in γ, and we see that unitarity is satisfied to this same order, e.g., $|S_{00}|^2+|S_{01}|^2=1+O(\gamma^2)$. By invoking unitarity to second order in γ, we can determine the imaginary parts of the phase shifts. Writing $S_{00}=(-)^l\exp[2i(\delta_{l0}+i\beta_{l0})]$ and $S_{11}=(-)^l\exp[2i(\delta_{l1}{}^M+i\beta_{l1})]$ and requiring unitarity to second order gives $\beta_{l0}=\beta_{l1}\equiv\beta_l$, where

$$\exp(-4\beta_l)=1-\gamma^2l(l+1)(B_l^{01})^2 \tag{27}$$

or

$$\beta_l\approx\tfrac{1}{4}\gamma^2l(l+1)(B_l^{01})^2 \tag{28}$$

explicitly demonstrating the second-order dependence on γ.

The connection between the complex phase shifts determined here, and the real phase shifts and mixing parameters which were previously introduced (i.e., the quantities δ_{ll+}, δ_{ll-}, $\cot\epsilon_l$ used to characterize the spin mixing scattering problem), are readily given through comparison of the scattering matrix elements expressed in the two forms [cf. Eq. (10)].

$$\exp[2i(\delta_{l0}+i\beta_l)] = \frac{\tan\epsilon_l \exp(2i\delta_{ll+})+\cot\epsilon_l \exp(2i\delta_{ll-})}{\tan\epsilon_l+\cot\epsilon_l}, \quad (29)$$

$$\exp[2i(\delta_{l1}{}^M+i\beta_l)] = \frac{\cot\epsilon_l \exp(2i\delta_{ll+})+\tan\epsilon_l \exp(2i\delta_{ll-})}{\tan\epsilon_l+\cot\epsilon_l}. \quad (30)$$

These two complex equations lead to four expressions, of which only three are independent, the remaining equation being given by the first three and the unitarity condition. These three independent equations are sufficient to determine δ_{ll+}, δ_{ll-}, and ϵ_l in terms of δ_{l0}, $\delta_{l1}{}^M$, and β_l.

By carrying out the appropriate algebra in Eqs. (29) and (30), one obtains the desired relations.

Defining

$$\alpha_1=\exp(-2\beta_l)(\cos2\delta_{l0}+\cos2\delta_{l1}{}^M),$$

$$\alpha_2=\exp(-2\beta_l)(\sin2\delta_{l0}+\sin2\delta_{l1}{}^M), \quad (31)$$

we find that

$$\delta_{ll+}=\tfrac{1}{2}\left(\cos^{-1}\frac{\alpha_1}{(\alpha_1{}^2+\alpha_2{}^2)^{1/2}}+\cos^{-1}[\tfrac{1}{2}(\alpha_1{}^2+\alpha_2{}^2)^{1/2}]\right), \quad (32)$$

$$\delta_{ll-}=\tfrac{1}{2}\left(\cos^{-1}\frac{\alpha_1}{(\alpha_1{}^2+\alpha_2{}^2)^{1/2}}-\cos^{-1}[\tfrac{1}{2}(\alpha_1{}^2+\alpha_2{}^2)^{1/2}]\right), \quad (33)$$

$$\epsilon_l=\tan^{-1}\left[\left(\frac{\exp(-2\beta_l)\cos2\delta_{l0}-\cos2\delta_{ll-}}{\cos2\delta_{ll+}-\exp(-2\beta_{l0})\cos2\delta_{l0}}\right)^{1/2}\right]. \quad (34)$$

Thus, this calculation together with the experimentally-determined singlet and triplet phase shifts provide the means for estimating the influence of the MS force on n-p scattering. Phase shifts determined in this manner include the effects of wave distortion, even for small l, in contrast to those determined by the plane-wave Born approximation as discussed by Breit and Ruppel.[6] Interestingly, however, we found that the use of the Born approximation for the calculation of the MS effect gave no significant difference (when compared with the first-order treatment) in the calculated observables. The Born-approximation calculation for the matrix element M_c agrees well with the first-order treatment at small angles, where the MS influence is strong, and although at large angles M_c is not well represented by the Born-approximation result, it is of no consequence since the MS influence on the observables is negligible at large angles. Thus, for n-p scattering, just as we found for neutron-nucleus scattering (Ref. 1), the Born approximation gives sufficiently accurate results in assessing the influence of the MS interaction, assuming the wave distortion effects to be negligible for large l. This assumption is discussed briefly in Ref. 6.

III. RESULTS OF CALCULATIONS

A. Scattering Calculations

For convenience and completeness, a brief summary of the procedure followed in performing the n-p scattering calculations is given here.

First, phase shifts obtained by fitting experimental data were taken from Ref. 10 and assumed to be the result of the nuclear interaction only. For the phase shifts corresponding to $l=j\pm1$ partial waves, the MS effect on the phase shifts was estimated through use of Eq. (36) of Ref. 1. For the $l=j$ partial waves, the imaginary part of the phase shift β_l was determined from Eq. (27) of the present work, and the real part was taken equal to the nuclear phase shift, in the case of the singlet wave, and equal to the nuclear phase shift as modified by Eq. (36) of Ref. 1 [equivalent to using Eq. (26)], in the case of the triplet wave.

The matrix-element expressions $M_{s'm'sm}$ were evaluated with $\varphi=0$ (y axis along $\mathbf{n}$). For those cases where the Born approximation to the MS term gives a nonzero scattering amplitude, the high l-wave contributions to the matrix elements were included in the same manner as in the neutron-nucleus problem of Ref. 1. That is, the series was truncated at an appropriate l, and a correction ΔM was added to those matrix elements where it applied. The correction is given by

$$\Delta M=\pm\frac{\gamma}{2\sqrt{2}}\left(\cot\tfrac{1}{2}\theta-\sum_{l=0}^{l_c}\frac{2l+1}{l(l+1)}P_l{}^1(\cos\theta)\right), \quad (35)$$

where the positive sign goes with those elements below the principal diagonal, the negative sign with those above. It happens that only those matrix elements which contain the $P_l'(\cos\theta)$ factors in the sums have nonzero Born amplitudes. Hence, it can be seen that the only Wolfenstein coefficients that are altered by the MS term as calculated by the Born approximation are b and c. More explicitly, the corrected matrix elements can be written as $M_c-\Delta M$, $M_{1011}+\Delta M$ and $M_{1110}-\Delta M$, where it is understood that the sums involved are truncated at $l=l_c$ (chosen equal to 5 here).

As in the case of scattering by nuclei (Ref. 1), the purpose of this study was not to obtain the best fit to experimental data, but to find the general effects of the MS force. Customarily, in determining phase shifts through analysis of experimental data, the large l-wave contributions have been included by assuming the nuclear potential acting on these partial waves is the one-pion-exchange potential (OPEP).[19] The effects of the OPEP on the amplitude matrix elements are

[19] P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Strapp, Phys. Rev. 114, 880 (1957).

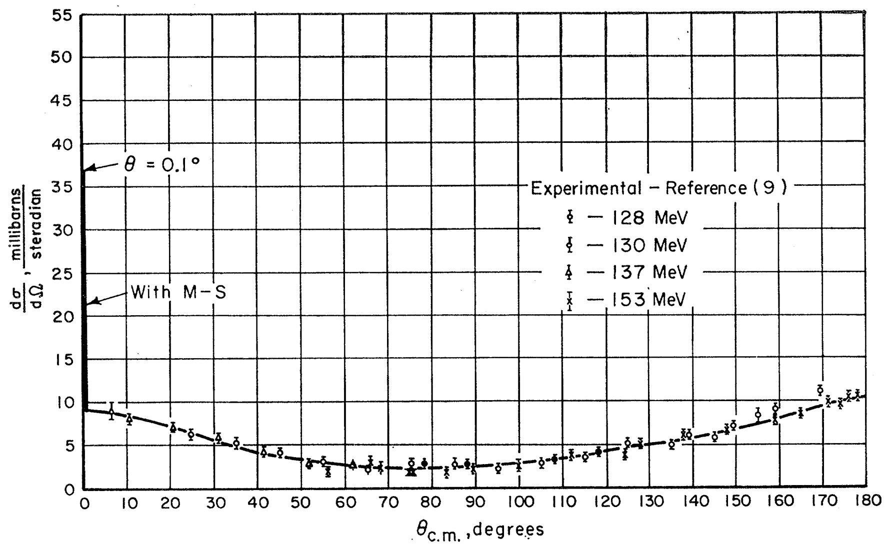

FIG. 1. *n-p* differential scattering cross section at 142 MeV (the solid line represents the calculated results of the present work).

normally included in the Born approximation in much the same spirit as large *l*-wave contributions to the MS scattering have been included here. This one-pion-exchange contribution (OPEC) has not been included in these calculations. Since it has a rather small effect on the observables,[20] and would not contribute anything to the evaluation of the MS effects, it seemed to be an unnecessary complication. (To be more precise, it is possible that coherent contributions from the OPEP could alter the details of the small-angle results of the MS calculation. However, it is improbable that the conclusions derived from this work would be altered.) Including the OPEC for large *l* waves produces small changes in the calculated observables. These changes, however, would not depend on whether the MS force were included or not.

Nuclear phase shifts from Ref. 10 at energies of 25, 50, 95, 142, and 210 MeV were used to evaluate the effect of the MS force on the nine scattering observables of Table I. The calculations were performed on the Battelle CDC-6400 digital computer. Typical results are shown in Figs. 1–8, together with the experimental data used for the nuclear phase-shift determinations reported in Ref. 10 and taken from Refs. 9 and 20. In all cases, significant MS effects are confined to small angles and hence do not exert

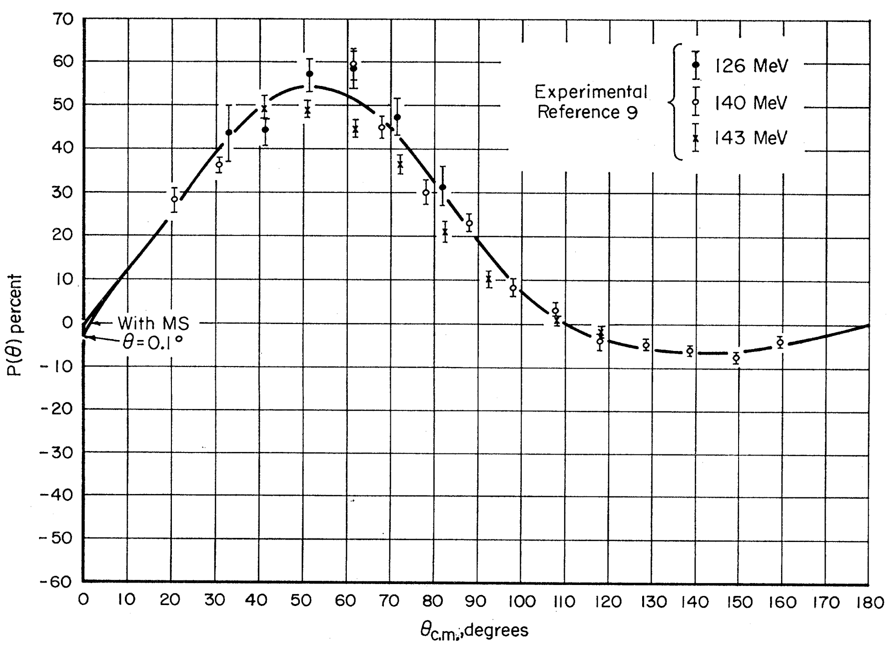

FIG. 2. Neutron polarization in *n-p* scattering at 142 MeV.

[20] H. P. Noyes, D. S. Baily, R. A. Arndt, and M. H. MacGregor, Phys. Rev. **139,** B380 (1965).

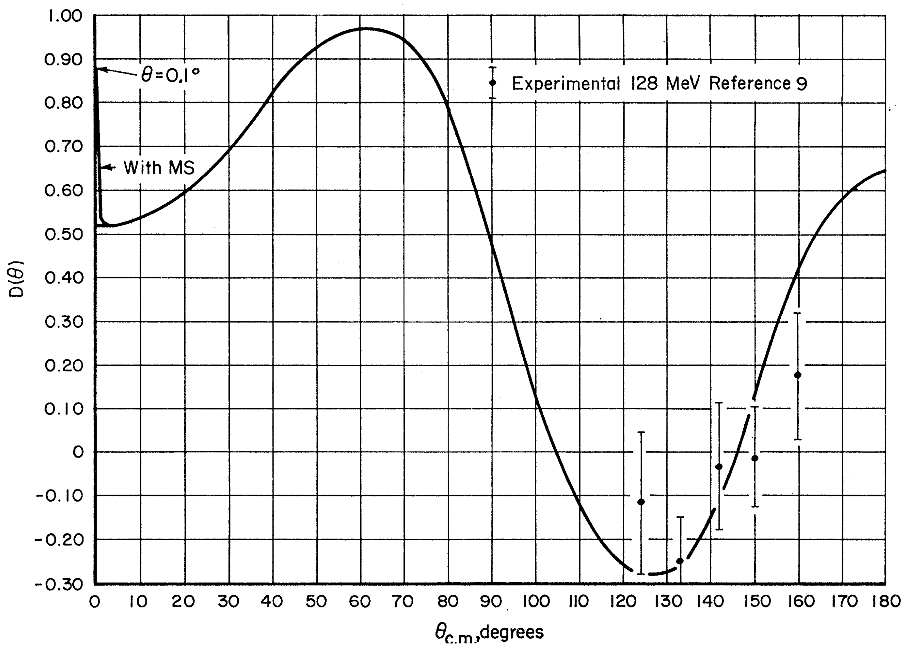

FIG. 3. Neutron depolarization in n-p scattering at 142 MeV.

any influence on the phase-shift determinations. The relatively good agreement of the calculated results with the experimental data would seem to justify omission of the OPEC in this application. Results at other energies and for other scattering observables follow the pattern of those shown. It should be noted that the cut-off radius for the proton (r_c) was taken at 1.7 F and that a 30% variation around this value was found to have an insignificant influence on the results.

A measure of the smallness of the MS effect on phase shifts is illustrated in Table II, where the eigen phase shifts, mixing parameters, and the imaginary part of the complex phase shifts are listed for the 142-MeV case.

Our results show that, in general, the MS influence on the scattering observables is confined to small scattering angles. In fact, for all observables except the polarization and the correlation coefficient $C_{KP}(\theta)$ any significant influence is restricted to angles less

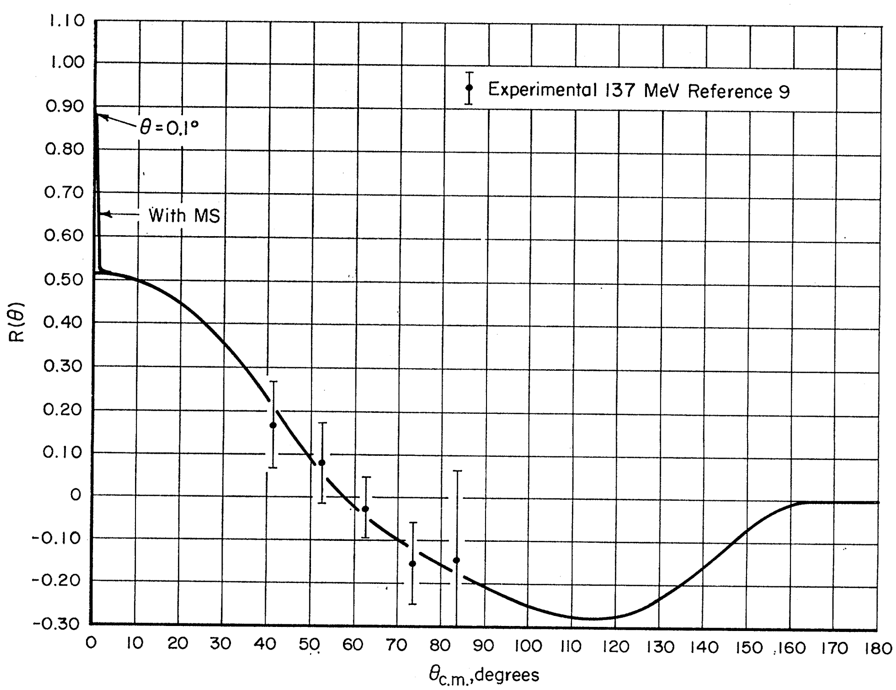

FIG. 4. $R(\theta)$ for n-p scattering at 142 MeV.

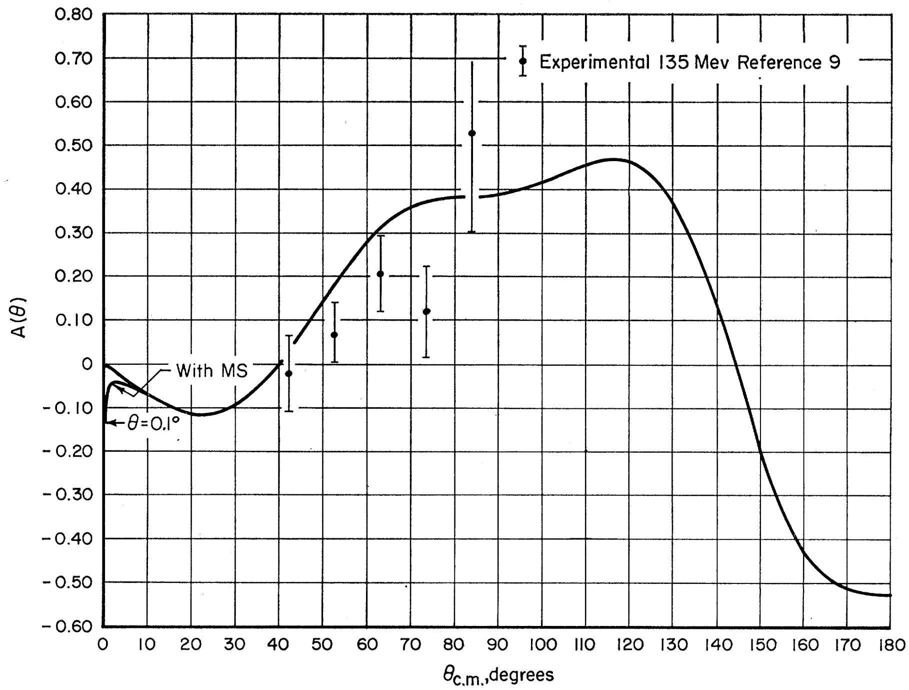

FIG. 5. $A(\theta)$ for *n-p* scattering at 142 MeV.

than 5°. For $P(\theta)$ and $C_{KP}(\theta)$ the MS influence is present at larger angles. At 25 MeV the MS influence on $C_{KP}(\theta)$ extends out to about 30° and on the polarization out to about 15° (Fig. 8). However, $C_{KP}(\theta)$, at 25 MeV is small, being of the order of 10^{-3} at 30°, and it has not yet been measured at any energy in the *n-p* case. At higher energies, where $C_{KP}(\theta)$ becomes larger, the influence is restricted to smaller angles as can be seen by examining the polarization curves at various energies. We found that the MS effect on polarization also becomes confined to smaller angles at the higher energies. A comparison of our *n-p* results with the *p-p* results of Breit and Ruppel[6] reveals that the MS influence on polarization extends to slightly larger angles in the case of *p-p* scattering; this result is not unexpected since the MS interaction

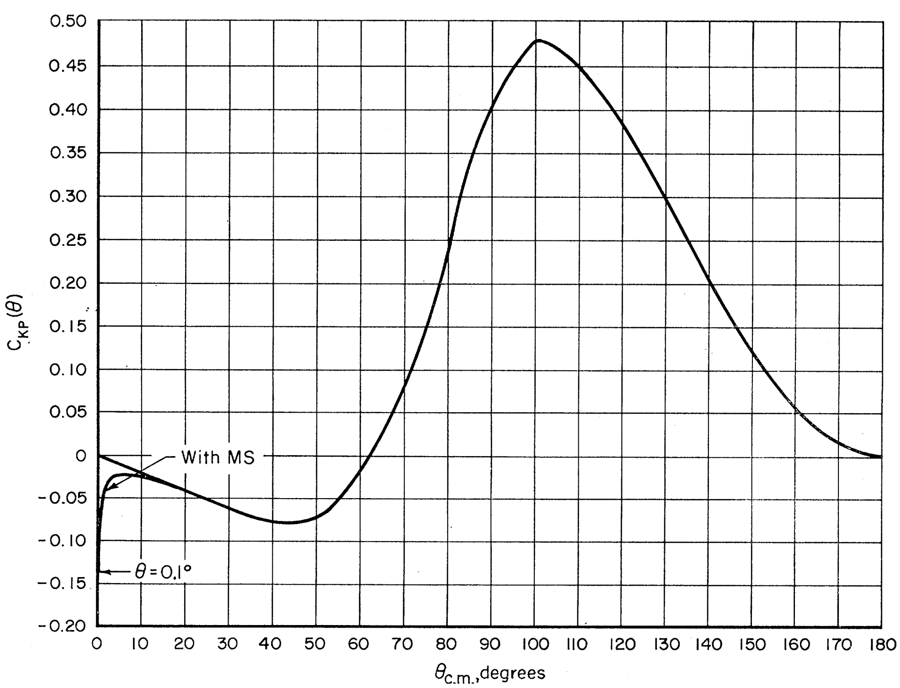

FIG. 6. Correlation coefficient C_{KP} in *n-p* scattering at 142 MeV.

TABLE II. Eigenphase shifts and mixing parameters for 142 MeV (all angles in radians).

MS force included	l	δ_{ll+}	δ_{ll-}	ϵ_l	β_l
no	0	0.28693	0	$\pi/2$	0
yes	0	0.28693	0	$\pi/2$	0
no	1	−0.29653	−0.27192	0	0
yes	1	−0.29639	−0.27192	0.00825	0.407×10^{-7}
no	2	0.41521	0.08552	0	0
yes	2	0.41534	0.08552	0.00119	0.148×10^{-6}
no	3	−0.03770	−0.01606	0	0
yes	3	−0.03762	−0.01605	0.01172	0.638×10^{-7}
no	4	0.07295	0.01099	0	0
yes	4	0.07301	0.01099	0.00406	0.632×10^{-7}
no	5	−0.01099	0	0	0
yes	5	−0.01096	2.7×10^{-6}	0.01558	0.292×10^{-7}

for p-p scattering involves the magnetic moments of both nucleons, whereas for n-p scattering only the neutron's magnetic moment enters.

The fact that $P(\theta)$ and $C_{KP}(\theta)$ are the most strongly influenced quantities can be explained by the fact that they depend most strongly on the Wolfenstein coefficients b and c, and that these are the only Wolfenstein coefficients that are influenced by the large l-wave correction term ΔM. These considerations suggest that two of the four correlation coefficients pertinent to scattering of polarized beams[16,21] would be rather strongly modified by the MS force. These coefficients are defined in a manner similar to C_{nn} and C_{KP} but have never been measured and so they were not included in the calculations. As in the case of the Table I entries, these coefficients are described in Ref. 16. The following equations exhibit their dependence on the Wolfenstein parameters:

$$(d\sigma_2/d\Omega)_u C_{nnn} = -2\,\mathrm{Im}i[(c-b)(a^*+m^*)], \quad (36)$$

$$(d\sigma_2/d\Omega)_u C_{nKP} = -2\,\mathrm{Im}[(a-m)g^*+(a+m)h^*], \quad (37)$$

$$(d\sigma_2/d\Omega)_u C_{n\times k\,Kn} = -2\,\mathrm{Re}[a(c^*-b^*) + m(c^*+b^*)]\cos\tfrac{1}{2}\theta - 2\,\mathrm{Im}[ma^*+2bc^*]\sin\tfrac{1}{2}\theta, \quad (38)$$

$$(d\sigma_2/d\Omega)_u C_{n\times k\,nP} = 2\,\mathrm{Im}[g^*(a+m) + h^*(a-m)]\cos\tfrac{1}{2}\theta - 4\,\mathrm{Re}[cg^*+bh^*]\sin\tfrac{1}{2}\theta. \quad (39)$$

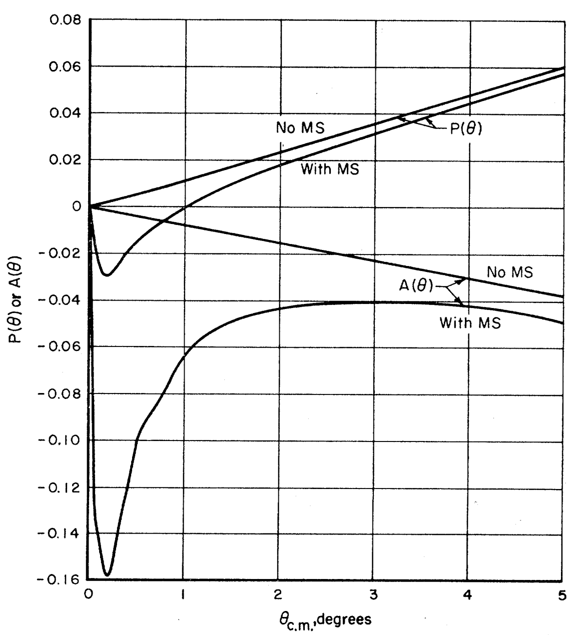

FIG. 7. Small angle scattering parameters in n-p scattering at 142 MeV.

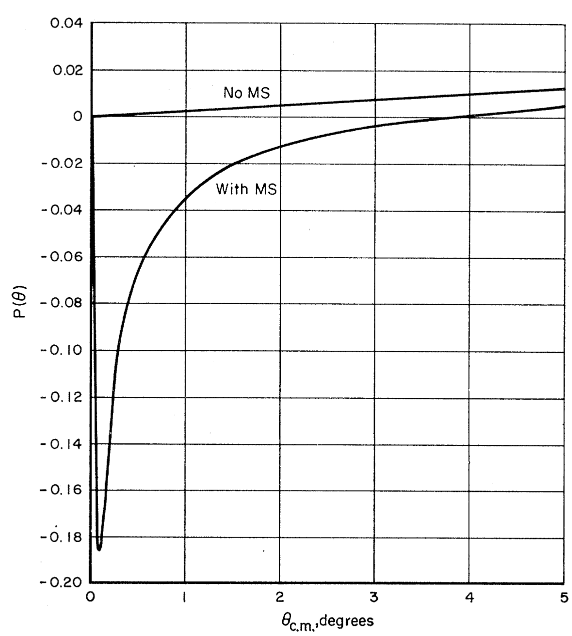

FIG. 8. Small angle polarization in n-p scattering at 25 MeV.

[21] L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).

These forms indicate a relative strong MS influence on C_{nnn} and $C_{n\times k\,Kn}$ and a weak influence on the remaining two coefficients.

IV. SUMMARY AND CONCLUSIONS

To summarize, a formalism was developed to include the MS interaction in the neutron-proton scattering problem. This procedure is of general interest ecause the MS force causes channel-spin nonconservation. The scattering amplitude matrix was derived, including this singlet-triplet mixing, and its elements were related to the Wolfenstein coefficients. The nine most common scattering observables were also related to these coefficients. The effect of the MS interaction on phase shifts was evaluated through the use of a perturbation calculation for low l waves, and a Born-approximation calculation for high l waves. Calculations for energies from 25–210 MeV were performed to find the influence of the MS force on the scattering observables where it was assumed that the phase shifts determined by the lowest χ^2 fits to the scattering data for the nucleon-nucleon problem were due to the nuclear interaction only.

From this study it can be concluded that all nine scattering observables are markedly influenced by the MS interaction, but for all but the lowest energies this influence is confined to small scattering angles ($<5°$). At low energy, the influence on polarization and the correlation coefficient C_{KP}, and possibly the correlation coefficients C_{nnn} and $C_{n\times k\,Kn}$, extends to beyond 10°. Thus, phase shifts as determined from scattering data and reported in the literature will not be altered by inclusion of the MS force in the formalism unless small angle measurements are included in the data.

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Self-Consistent Calculation of p-Shell Hypernuclear Binding Energies*

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Calculations of the binding energies of p-shell hypernuclei have been carried out treating all the nucleons and the Λ self-consistently. The Tabakin potential is used for the NN interaction and a simple central force which fits ΛN scattering is used for the Λ-N system. The binding energies obtained exceed those observed. The dynamical aspects of the hypernuclear system are also discussed, as well as various correction terms. Finally, a comparison is made with other hypernuclear calculations.

I. INTRODUCTION

BINDING energies of Λ hypernuclei in the p shell may provide valuable information about certain properties of the Λ-N interaction, which have not been or cannot be measured in free Λ-N scattering. Indeed Λ-N scattering data is quite sparse and is subject to a wide variety of interpretations. If it is assumed that the Λ-N potential is central then the binding energies of s-shell hypernuclei exceed the experimental values, independent of the particular model employed.[1] This has led to the suggestion that the Λ-N interaction is severely suppressed in a hypernucleus relative to the free Λ-N force. Alternatively, this difficulty might be overcome by including a tensor component or an exchange part in a phenomenological Λ-N interaction. Such attempts have so far been unsuccessful. The binding energy of the Λ in $_{\Lambda}He^5$, for example, was found[2] to be insensitive to an additional short-range tensor force and an exchange interaction would have little, if any, effect on a calculation for this system. It is likely that such additional components of the interaction will have a more pronounced effect in p-shell hypernuclear calculations but so far the problem of overbinding remains for $_{\Lambda}He^5$ and also, probably, for the $A=4$ hypernuclei.

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[1] R. C. Herndon and Y. C. Tang, Phys. Rev. **159**, 853 (1967); S. Ali, M. E. Grypeos, and L. P. Kok, Phys. Letters **24B**, 543 (1967); R. K. Bhaduri, Y. Nogami, and W. VanDijk, Phys. Rev. **155**, 1671 (1967).

[2] J. Law, M. R. Gunye, and R. K. Bhaduri, in *Proceedings of the International Conference on Hypernuclear Physics, Argonne National Laboratory, 1969* (to be published).