Photodisintegration of Three-Particle Nuclei*

I. M. Barbour† and A. C. Phillips‡

Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

We show that the He^3 and H^3 photodisintegration reactions are to a large extent independent of the detailed properties of the three-nucleon ground state, but depend crucially on the nuclear interaction in the final continuum states. The bound-state properties considered are the analytic form of the principal S state, the asymptotic behavior, the S' state, and the two-nucleon short-range repulsion. Since the rms radius of the bound state determines the over-all magnitude of the cross section, predictions of different wave functions with the same radius are compared. The final states are described by the Faddeev equations in the separable approximation. If the final-state interactions are correctly included, then the low-energy cross sections for the $He^3(\gamma, p)d$ reaction are enhanced by 20–25%, and the cross sections for $He^3(\gamma, n)2p$ are reduced by approximately 100%. A simultaneous but rough agreement is possible for the total two- and three-body breakup cross sections and the charge form factors of the three-particle nuclei; but, within the theoretical framework adopted, the two-body differential cross sections at 90° and the charge-form-factor data cannot be reconciled.

1. INTRODUCTION

In this article, the photodisintegration of a three-particle nucleus is considered as a three-body problem. The primary purpose is to examine the general features of the reaction and to explore its usefulness as a source of information on the structure of three-nucleon states.[1]

Early calculations of He^3 photodisintegration using simple bound-state wave functions and neglecting interactions between the nucleons in the continuum states met with moderate success in accounting for the experimental data for the two-body break-up channel,

$$\gamma+He^3=p+d.$$

However, similar approaches to the three-body channel,

$$\gamma+He^3=p+p+n,$$

showed a sharp discrepancy between experiment and theory, the calculated cross sections being as much as three times the experimental values.[2] Subsequent investigations have not drastically changed this situation. In general, the emphasis has been on the elaboration of the production mechanism for the reactions, using more sophisticated bound-state wave functions, higher multipole transitions, and retardation corrections; and the effects of the interaction between the nucleons in final states have usually been neglected or occasionally included by approximations of unknown quality.[3–9] We shall show that it is more appropriate to consider He^3 photodisintegration as a reaction with a simple and to a large extent model-independent production mechanism together with significant and varied rescattering effects in final continuum states.

The simplicity of the production mechanism is a consequence of the dominance of electric dipole transitions. In the two-body photodisintegration reaction, the angular distributions[2] show a marked deviation from the electric dipole $\sin^2\theta$ distribution, which can be ascribed to an interference of the electric dipole and quadrupole amplitudes.[3] However, electric dipole transitions account for at least 90% of the total two-body break-up cross section for photon energies below 40 MeV.[2,5,6,10] In particular, the magnetic dipole transitions which are theoretically intractable, contribute less than 1%, mainly because the dominant principal S state of He^3 is an eigenstate of the nuclear-spin magnetic dipole operator, and magnetic dipole transitions can only occur by virtue of magnetic-interaction currents, retardation corrections, and symmetry states other than the principal S state. This suppression of magnetic dipole transitions should also occur in the three-body photodisintegration reaction. Here the data is less accurate but, within the experimental errors of 10–20%, the angular distributions are consistent with electric dipole dominance.[2] Further

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† Department of Natural Philosophy, Glasgow University, Scotland.

‡ Present address: The School of Mathematical and Physical Sciences, University of Sussex, Falmer, England.

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evidence for electric dipole dominance comes from the bremsstrahlung-weighted sum rule, which relates the charge radii of the three-particle nuclei and the total photon absorption cross sections. Invoking charge symmetry for the He^3 and H^3 wave functions, one finds that the electric dipole contribution to the experimental integrated cross section,[2]

$$\int_Q^{170\ \mathrm{MeV}} \sigma_T(E_\gamma)\,(dE_\gamma/E_\gamma)=2.76\pm0.18\ \mathrm{mb},$$

amounts to 2.28 ± 0.28 mb; the error is mainly due to the uncertainty in the neutron-charge form factor.

The dominance of electric dipole transitions has the important consequence in that a clear cut separation of the electromagnetic and nuclear interactions is possible. In general, the existence of charge exchange, momentum-dependent, or nonlocal components of the nuclear interaction imply that the position of a nucleon within a nucleus cannot be considered as a point of constant charge, and electromagnetic interaction currents, which depend explicitly on the nuclear interaction, are necessary in order to allow for charge conservation. However, Siegert's theorem asserts that the interaction current contributions to electric multipole transitions, in the nonretarded approximation, may be eliminated provided that the dynamics of the nuclear system can be described in terms of the coordinates of the nucleons alone.[11]

Thus, with adequate representations of the three-nucleon bound and continuum states, electric dipole transitions alone are expected to account for the main features of the total cross sections for both the two-body and three-body channels of the photodisintegration of He^3. However, electric dipole radiation is not a particularly good probe of the nuclear structure. The form of the operator, $\mathbf{E}\cdot\mathbf{r}$, implies that the details of the interior of the bound-state wave functions are relatively unimportant, and possibly any reasonable wave function with the correct rms radius is sufficient. We shall show that the analytic and asymptotic form of the wave function, short-range correlations, and the S'-state admixture all have minor effects on the total cross section, and moreover these effects often depend on the treatment of the final-state interaction.

Although the electric dipole disintegration of He^3 is not useful in investigating the structure of the ground state, it does reveal significant properties of the continuum states. The form of the electric dipole operator, coupled with the large radius of He^3 implies a long-range production mechanism which samples a large volume of the final-continuum-state wave function. At first, one would expect that the short-range two-nucleon interactions in the final state could produce only moderate changes in the average value of the three-nucleon wave function taken over a large volume.[12] However, in any multiparticle system long-range effects are possible: The range of processes which correspond to the successive interactions between different pairs of particles is not necessarily the range of the two-particle forces, but instead may be given by the spatial extent of a virtual, correlated, two-body system when a member of this subsystem interacts with a third particle. For example, the interaction generated by the exchange of a nucleon between two alternative deuteron states dominates many of the features of elastic nucleon-deuteron scattering, and has a range which is determined by the size of the deuteron wave function.[13] Clearly, similar but less well understood long-range effects should occur in the three-nucleon continuum states.[14]

The electric dipole photodisintegration of He^3 is the simplest process, that is closely related to experiment, in which long-range three-body scattering plays an essential role. In fact, very pronounced scattering effects must occur in the three-body break-up channel if the two-nucleon interactions can support a bound state and thereby imply the existence of the two-body channel.[15] Clearly, the coupling between the two-body and three-body channels is a long-range mechanism.

The dependence of the cross section for two-body breakup on final-state scattering is less pronounced. However, we shall show that significant rescattering effects are to be expected. Moreover, because of the importance of long-range nucleon-deuteron interactions, these effects are, to a large extent, unrelated to the on-the-energy-shell nucleon-deuteron scattering parameters: The ^2P n-d scattering is highly inelastic and the real part of the phase shift is close to zero, whereas in the photodisintegration to a ^2P n-d state the scattering effects are those which correspond to an attractive n-d interaction and almost zero inelasticity.

In Sec. 2, we describe the general formalism for the three-particle disintegration of a bound state. The three-particle equations are simplified using a separable approximation for the two-particle off-the-energy-shell scattering amplitudes (Sec. 3). This permits an exact treatment of the three-particle aspects of the problem and automatically obeys the three-particle unitarity relations connecting the two- and three-body photodisintegration channels. Employing separable interactions is mathematically advantageous in that it reduces the three-body problem to that of solving a set of one variable integral equations. However, the separable approximation used cannot accurately represent

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the physical nucleon-nucleon interaction, and hence our calculation rests on the assumption that the main dynamical features of the three-nucleon continuum states are insensitive to the details of the nuclear forces. We note that similar treatments of the low-energy *n-d* scattering problem have met with considerable success,[16,17] indicating that separable interactions are a reasonable approximation in the photodisintegration problem for photon energies at least up to 20 MeV.

It is straightforward to solve the three-nucleon bound-state problem with simple two-nucleon potentials. But the study of the dependence of the photodisintegration cross sections on the properties of the underlying potential would pointlessly illustrate the sensitivity to the bound-state rms radius, which is the parameter that determines the over-all magnitude of the cross sections. The fact that a particular simple potential yields a bound state with the correct radius is of course without physical significance, since the radius, like the binding energy, is a sensitive function of the details of a complex nuclear interaction. For these reasons we have chosen to compare the predictions of different phenomenological bound-state wave functions with the same radius. The wave functions are described in Sec. 4 and their different features are illustrated by reference to the charge form factors of the three-particle nuclei, which constitute by far the most reliable, but not particularly exacting, source of experimental information on the bound states.

The expressions for the various photodisintegration amplitudes and cross sections are given in Sec. 5. In Sec. 6, we interpret the results.

2. TRANSITION OPERATORS FOR BREAKUP OF A THREE-BODY BOUND STATE

Consider a system of three particles with Hamiltonian

$$H=H_0+\sum_{\alpha=1}^{3} V_\alpha, \tag{2.1}$$

where V_α is the potential between particles labelled β and γ. The bound state and scattering properties of the system can be obtained if the resolvent of the total Hamiltonian,

$$G(s)=[H-s1]^{-1}, \tag{2.2}$$

is known. The essence of the Faddeev formulation of the three-particle problem is to solve completely all the two-particle subsystems before attempting to tackle the three-particle system. Accordingly we introduce the resolvent and transition operators [$G_\alpha(s)$ and $T_\alpha(s)$, respectively] for the β-γ subsystem:

$$G_\alpha(s)=[H_0+V_\alpha-s1]^{-1} \tag{2.3}$$

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and

$$T_\alpha(s)G_0(s)=V_\alpha G_\alpha(s), \tag{2.4}$$

where $G_0(s)$ is the resolvent operator corresponding to H_0. From (2.2) and (2.3) it follows that

$$G(s)=G_\alpha(s)-\sum_{\beta\neq\alpha} G_\alpha(s)V_\beta G(s) \tag{2.5a}$$

$$=G_\alpha(s)-\sum_{\beta\neq\alpha} G(s)V_\beta G_\alpha(s). \tag{2.5b}$$

If we define the operator

$$\Omega_\alpha(s)\equiv 1-\sum_{\beta\neq\alpha} V_\beta G(s), \tag{2.6}$$

then

$$G(s)=G_\alpha(s)\Omega_\alpha(s). \tag{2.7}$$

Combining Eqs. (2.6) and (2.7) and using (2.4), we obtain

$$\Omega_\alpha(s)=1-\sum_{\beta\neq\alpha} V_\beta G_\beta(s)\Omega_\beta(s)$$

$$=1-\sum_{\beta\neq\alpha} T_\beta(s)G_0(s)\Omega_\beta(s). \tag{2.8}$$

Let $\Phi_{\alpha n\mathbf{q}}$ be the asymptotic state describing the free motion of a particle α with momentum $\mathbf{q}$ with respect to a bound state (n) of particles β and γ, and Ψ_B be the bound-state wave function of all three particles. To first order in the electromagnetic Hamiltonian H_e, the amplitude for the transition between these two states is

$$\langle\Phi_{\alpha n\mathbf{q}}\mid \mathcal{U}_\alpha(E+i\epsilon)\mid\Psi_B\rangle=\langle\Psi_{\alpha n\mathbf{q}}^{(-)}\mid H_e\mid\Psi_B\rangle. \tag{2.9}$$

Here $\Psi_{\alpha n\mathbf{q}}^{(\pm)}$ are the scattering states in the αn channel with energy E and are given by

$$\Psi_{\alpha n\mathbf{q}}^{(\pm)}=\Phi_{\alpha n\mathbf{q}}-G(E\pm i\epsilon)\sum_{\beta\neq\alpha} V_\beta\Phi_{\alpha n\mathbf{q}}. \tag{2.10}$$

Thus,

$$\mathcal{U}_\alpha(E\pm i\epsilon)=H_e-\sum_{\beta\neq\alpha} V_\beta G(E\pm i\epsilon)H_e, \tag{2.11}$$

and using (2.6) and (2.8) we obtain the Faddeev equations for the transition operator

$$\mathcal{U}_\alpha(E\pm i\epsilon)=\Omega_\alpha(E\pm i\epsilon)H_e$$

$$=H_e-\sum_{\beta\neq\alpha} T_\beta(E\pm i\epsilon)G_0(E\pm i\epsilon)\mathcal{U}_\beta(E\pm i\epsilon). \tag{2.12}$$

If α is taken to be zero, then Eqs. (2.11) and (2.12) define the operator for the transition to an asymptotic state of three free particles.

For the moment we drop the spin-isospin variables. Then in the center of mass the three-particle basis states may be characterized by any pair (α=1, 2, or 3) of the vectors $\mathbf{p}_\alpha$ and $\mathbf{q}_\alpha$, where $\mathbf{p}_\alpha$ is the relative momentum of particles β and γ and $\mathbf{q}_\alpha$ is the momentum of particle α with respect to the β-γ subsystem.

The coupling between the two-body and three-body channels is illustrated by Eq. (2.12); the amplitude for the transition to a state of three free particles is

$$\langle \mathbf{p}_\alpha \mathbf{q}_\alpha | \mathcal{U}_0(E) | \Psi_B \rangle = \langle \mathbf{p}_\alpha \mathbf{q}_\alpha | H_e | \Psi_B \rangle - \sum_{\beta=1}^{3} \int \frac{d^3\mathbf{p}_\beta' \hat{T}_\beta(\mathbf{p}_\beta, \mathbf{p}_\beta', E - q_\beta^2/2m_\beta) \langle \mathbf{p}_\beta' \mathbf{q}_\beta | \mathcal{U}_\beta(E) | \Psi_B \rangle}{p_\beta'^2/2\mu_\beta + q_\beta^2/2m_\beta - E}, \quad (2.13)$$

where $\hat{T}_\beta$ is the γ-α transition operator in the two-particle Hilbert space and μ_β and m_β are the appropriate reduced masses. If there is a two-particle bound state with wave function $\phi_{\beta n}$ and binding energy $\epsilon_{\beta n}$, then

$$\lim_{s \to -\epsilon_{\beta n}} (s + \epsilon_{\beta n}) \hat{T}_\beta(\mathbf{p}, \mathbf{p}', s) = g_{\beta n}(\mathbf{p}) g_{\beta n}(\mathbf{p}'), \quad (2.14)$$

where

$$g_{\beta n}(\mathbf{p}) = \langle \mathbf{p} | \hat{V}_\beta | \phi_{\beta n} \rangle. \quad (2.15)$$

Hence for every bound state of the two-particle subsystems the amplitude for the transition to three free particles contains a pole, the residue and position of which are given by

$$\lim_{E \to q_\beta^2/2m_\beta - \epsilon_{\beta n}} (E - q_\beta^2/2m_\beta + \epsilon_{\beta n}) \langle \mathbf{p}_\alpha \mathbf{q}_\alpha | \mathcal{U}_0(E) | \Psi_B \rangle = g_{\beta n}(\mathbf{p}_\beta) \langle \Phi_{\beta n \mathbf{q}_\beta} | \mathcal{U}_\beta(E) | \Psi_B \rangle. \quad (2.16)$$

A property which is closely related to (2.16) is three-particle unitarity. Taking the discontinuity across the set of overlapping bound-state scattering and three-particle scattering cuts,[18] one obtains

$$\begin{aligned} \mathcal{U}_\alpha(E+i\epsilon) - \mathcal{U}_\alpha(E-i\epsilon) &= -2\pi i \sum_{\alpha\gamma} \int W_{\alpha\gamma}(E+i\epsilon) | \Phi_{\gamma n \mathbf{q}_\gamma} \rangle \\ &\times d^3\mathbf{q}_\gamma \delta(q_\gamma^2/2m_\gamma - \epsilon_{\gamma n} - E) \langle \Phi_{\gamma n \mathbf{q}_\gamma} | \mathcal{U}_\gamma(E-i\epsilon) \\ &- 2\pi i \int\int W_{\alpha 0}(E+i\epsilon) | \mathbf{p}_\alpha \mathbf{q}_\alpha \rangle \\ &\times d^3\mathbf{q}_\alpha d^3\mathbf{p}_\alpha \delta(q_\alpha^2/2m_\alpha + p_\alpha^2/2\mu_\alpha - E) \langle \mathbf{p}_\alpha \mathbf{q}_\alpha | \mathcal{U}_0(E-i\epsilon). \end{aligned} \quad (2.17)$$

The operators $W_{\alpha\gamma}$ are given by

$$W_{\alpha\gamma} = \sum_{\beta \neq \alpha} V_\beta - \sum_{\beta \neq \alpha} \sum_{\delta \neq \gamma} V_\beta G(s) V_\delta, \quad (2.18)$$

and hence represent the scattering transitions from a three-particle state, in which β and γ are bound, to a state in which β and α are bound. Equation (2.18) also defines the operators $W_{\alpha 0}$. These represent the transitions from a state with β and γ bound to a state of three particles.

3. APPROXIMATION SCHEME FOR CONTINUUM STATES

In this section, we apply the low-energy separable approximation scheme of Lovelace[19] to the photodisintegration of three-particle nuclei. The basic assumption is that the kernel of the three-particle Faddeev equation is mainly sensitive to the structure of the two-nucleon amplitudes in the vicinity of the singularities associated with the deuteron (denoted by d) and the singlet antibound state (denoted by s). This assumption, together with the analytic properties implied by (2.14) and the requirement of two-particle unitarity, leads to the use of a nonlocal separable two-nucleon interaction.[20] Assuming charge independence and using units where the nucleon mass is one, the transition operator in the three-particle Hilbert space is written as

$$\langle \mathbf{p}_\alpha \mathbf{q}_\alpha | T_\alpha(E) | \mathbf{p}_\alpha' \mathbf{q}_\alpha' \rangle = \sum_{n=s,d} \langle \mathbf{p}_\alpha | \alpha n \rangle t_{\alpha n}(E - \tfrac{3}{4} q_\alpha^2) \times \langle \alpha n | \mathbf{p}_\alpha' \rangle \delta_3(\mathbf{q}_\alpha - \mathbf{q}_\alpha'). \quad (3.1)$$

We take, in the subspace with isosopin I and spin S,

$$\langle \mathbf{p}_\alpha | \alpha n, IS \rangle = g_n(\mathbf{p}_\alpha) | I_{\beta\gamma} = I_n, I_\alpha, II_z \rangle \times | S_{\beta\gamma} = S_n, S_\alpha, SS_z \rangle, \quad (3.2a)$$

$$g_n(\mathbf{p}) = m_n / (p^2 + \mu_n^2), \quad (3.2b)$$

$$t_n(E) = [\lambda_n^{-1} + \int (d^3\mathbf{p} g_n^2(\mathbf{p}) / (\mathbf{p}^2 - E - i\epsilon))]^{-1}. \quad (3.3)$$

The parameters μ_n, m_n, and λ_n are adjusted to give a scattering length of -20.34 F and an effective range of 2.7 F for the $^1S_0$ state and a scattering length of 5.397 F and deuteron binding energy of 2.225 MeV for the $^3S_1$ state.

Suppressing for the time being the spin-isospin labels, the amplitude for disintegration into a nucleon-deuteron or a nucleon-singlet antibound state may be defined as

$$\langle \mathbf{q} | X_n(E) | \Psi_B \rangle = -\sqrt{\tfrac{1}{3}} \sum_{\alpha=1}^{3} \langle \alpha n, \mathbf{q} | G_0(E) \mathcal{U}_\alpha(E) | \Psi_B \rangle. \quad (3.4)$$

This definition takes into account the identity of the three nucleons. The off-the-energy-shell dependence has been chosen so as to simplify the Faddeev integral equation. Using (2.12), (3.4), and (3.1) we have

$$\langle \mathbf{q} | X_n(E) | \Psi_B \rangle = \langle \mathbf{q} | B_n(E) | \Psi_B \rangle + \sum_{m=s,d} \int d^3\mathbf{q}' \langle \mathbf{q} | Z_{nm}(E) | \mathbf{q}' \rangle t_m(E - \tfrac{3}{4} q'^2) \times \langle \mathbf{q}' | X_m(E) | \Psi_B \rangle, \quad (3.5)$$

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[20] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).

where

$$\langle \mathbf{q} \mid B_n(E) \mid \Psi_B\rangle = -\sqrt{\tfrac{1}{3}} \sum_{\alpha=1}^{3} \langle \alpha n, \mathbf{q} \mid G_0(E) H_e \mid \Psi_B\rangle \tag{3.6}$$

and

$$Z_{nm}(E) = -\tfrac{1}{3} \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} (1-\delta_{\alpha\beta}) \langle \alpha n \mid G_0(E) \mid \beta m\rangle. \tag{3.7}$$

The functions Z_{nm} correspond to effective nucleon-deuteron and nucleon-singlet potentials. They are nonlocal, energy-dependent, and correspond to the exchange of a nucleon from one three-body configuration to another.[19] The functions B_n correspond to the amplitudes for disintegration when the nucleon-deuteron or nucleon-singlet relative motion is described by a plane wave.

The amplitude for disintegration into three free nucleons is given by (2.12), (3.1), and (3.4). We have

$$\langle \mathbf{p}_\alpha \mathbf{q}_\alpha \mid X_0(E) \mid \Psi_B\rangle = \langle \mathbf{p}_\alpha \mathbf{q}_\alpha \mid B_0(E) \mid \Psi_B\rangle + \sum_{\beta=1}^{3} \sum_{m=s,d} \langle \mathbf{p}_\beta \mathbf{q}_\beta \mid \beta m\rangle t_m(E-\tfrac{3}{4}q_\beta^2) \langle \mathbf{q}_\beta \mid X_m(E) \mid \Psi_B\rangle, \tag{3.8}$$

where

$$\langle \mathbf{p}_\alpha \mathbf{q}_\alpha \mid B_0(E) \mid \Psi_B\rangle = \sqrt{\tfrac{1}{3}} \sum_{\beta=1}^{3} \langle \mathbf{p}_\beta \mathbf{q}_\beta \mid H_e \mid \Psi_B\rangle.$$

The first term is the plane-wave approximation for the photodisintegration amplitude. The second term corresponds to rescattering corrections, corresponding to the production, propagation, and subsequent decay of nucleon-deuteron and nucleon-singlet configurations. The result of evaluating X_0 when X_m is put equal to B_m only corresponds to the summation of the disconnected graphs. This will be referred to as the first rescattering approximation.

Our equations for two-body (3.5) and three-body (3.8) photodisintegration amplitudes correspond to the exact solutions of the three-nucleon–continuum-state wave functions for separable potentials. Hence provided these potentials are real, the amplitudes satisfy the unitarity relation (2.17). Further, since the separable approximation for the two-body T matrix has the correct behavior in the neighborhood of the deuteron bound state, the relation (2.16) is obeyed.

4. THREE-NUCLEON BOUND-STATE WAVE FUNCTIONS

In this section, we describe briefly the phenomenological bound-state wave functions used in the photodisintegration problem. There is one particular property of the bound-state wave function, its asymptotic behavior in configuration space, which may be important. It is well known that in the low-energy photodisintegration of the deuteron, the deuteron wave function can be approximately described by the asymptotic form, $A\exp(-\alpha_d r)/r$, if the normalization constant A includes an effective range correction. To determine the asymptotic behavior of a three-particle wave function one must consider all possible virtual disintegrations of the system into two and three separated particles. In this context the Faddeev equations are useful, since the kernels of these equations depend on the two-particle T matrix, and hence contain information regarding the two-particle bound states. If E_B is the three-particle binding energy, then

$$\Psi_B = \sum_{\alpha=1}^{3} \Psi_B^{(\alpha)},$$
$$\Psi_B^{(\alpha)} = -G_0(-E_B) T_\alpha(-E_B) \sum_{\beta\neq\alpha} \Psi_B^{(\beta)}. \tag{4.1}$$

Using the definition of the free resolvent operator G_0, the property (2.14) of the two-particle transition amplitudes and the Faddeev equation (4.1), it is possible to separate the parameters which characterize the asymptotic behavior of the three-particle bound-state wave function:

$$\Psi_B(\mathbf{p}_\alpha, \mathbf{q}_\alpha) = (p_\alpha^2/2\mu_\alpha + q_\alpha^2/2m_\alpha + E_B)^{-1} \times \left\{ \sum_{\beta n} \frac{N_{\beta n}(\mathbf{p}_\beta, \mathbf{q}_\beta)}{q_\beta^2 + (E_B - \epsilon_{\beta n}) 2m_\beta} + R(\mathbf{p}_\alpha, \mathbf{q}_\alpha) \right\}. \tag{4.2}$$

The internal structure of the wave function is determined by the functions N and R.

For the three-nucleon state we construct a wave function which has approximately the correct asymptotic behavior by considering the singlet antibound state as a zero-energy bound state ($\epsilon_s = 0$). Assuming isospin and spin to be $\frac{1}{2}$, we take

$$\Psi_B(\mathbf{p}_\alpha, \mathbf{q}_\alpha) = (p_\alpha^2 + \tfrac{3}{4}q_\alpha^2 + E_B)^{-1} \times \sum_{\beta=1}^{3} \sum_{n=d,s} \{N_n(\mathbf{p}_\beta, \mathbf{q}_\beta)/[q_\beta^2 + \tfrac{4}{3}(E_B - \epsilon_n)]\} \times | I_{\gamma\alpha} = I_n, I_\beta, I = \tfrac{1}{2}, I_z\rangle \mid S_{\gamma\alpha} = S_n, S_\beta, S = \tfrac{1}{2}, S_z\rangle. \tag{4.3}$$

In terms of the usual classification, this wave function contains a principal S state given by

$$u(\mathbf{p}_1, \mathbf{q}_1) = (\sqrt{\tfrac{1}{2}}) \sum_{\alpha=1}^{3} \{ f_d(\mathbf{p}_\alpha, \mathbf{q}_\alpha) - f_s(\mathbf{p}_\alpha, \mathbf{q}_\alpha) \}, \tag{4.3a}$$

where

$$f_n(\mathbf{p}_\alpha, \mathbf{q}_\alpha) = \frac{N_n(\mathbf{p}_\alpha, \mathbf{q}_\alpha)}{(p_\alpha^2 + \frac{3}{4}q_\alpha^2 + E_B)(q_\alpha^2 + \frac{4}{3}(E_B - \epsilon_n))} \tag{4.3b}$$

In addition, there is an S-wave state of mixed permutation symmetry, the S' state; the two components are

$$v''(\mathbf{p}_1, \mathbf{q}_1) = T''\{-(\sqrt{\tfrac{1}{2}})(f_d(\mathbf{p}_1, \mathbf{q}_1) + f_s(\mathbf{p}_1, \mathbf{q}_1))\}, \tag{4.3c}$$

$$v'(\mathbf{p}_1, \mathbf{q}_1) = T'\{-(\sqrt{\tfrac{1}{2}})(f_d(\mathbf{p}_1, \mathbf{q}_1) + f_s(\mathbf{p}_1, \mathbf{q}_1))\}, \tag{4.3d}$$

where T'' and T' are symmetry operators defined as

TABLE I. Parameters of bound-state wave functions with and without short-range repulsion (srr). In calculating the charge radii, proton and neutron mean-square radii of 0.7185 and −0.1258 F^2 have been assumed.

Asymptotic parameters: $E_B(\mathrm{He}^3)=0.18635$, $E_B(\mathrm{H}^3)=0.20472$, $\epsilon_d=0.05364$, $\epsilon_s=0.0$ (F^{-2})
Two-nucleon parameters: $\beta_d=1.193$, $\beta_s=1.173$ or $\gamma_d=2.34$, $\gamma_s=2.00$ (F^{-1})

Wave function	srr	ν_d^2 (F^{-2})	ν_s^2 (F^{-2})		h_d	h_s	Charge radius (F)	$P(S')$ (%)
I	no	4.0	7.0	He^3	1.490	−1.983	1.87	1.9
				H^3	1.646	−2.115	1.63	1.9
II	no	2.0	3.5	He^3	0.3677	−0.6762	1.95	0.97
				H^3	0.4094	−0.7265	1.74	1.0
III	yes	4.0	7.0	He^3	0.6195	−2.119	1.92	1.0
				H^3	0.6885	−2.272	1.72	1.0

follows:

$$T''=-(23)+\tfrac{1}{2}[(31)+(12)], \qquad (4.3e)$$

$$T'=\tfrac{1}{2}\sqrt{3}[(12)-(31)]. \qquad (4.3f)$$

The permutation of the coordinates of the particle α with those of β is denoted by $(\alpha\beta)$.

The expression (4.3) has the same form as the bound-state wave function obtained by applying the Faddeev equations and the separable approximation to the bound-state problem. In this case, the internal functions $N_n(\mathbf{p},\mathbf{q})$ are known; the $\mathbf{p}$ dependence is given by the two-particle form factor [e.g., the Yamaguchi form factor (3.2b)], and the $\mathbf{q}$ dependence is known numerically. Taking these results as a guide we consider two simple analytic forms for N. The first corresponds to the Yamaguchi two-nucleon interaction in which there is no short-range repulsion,

$$N_n(\mathbf{p},\mathbf{q})=h_n/(p^2+\mu_n^2)(q^2+\nu_n^2)^2. \qquad (4.4)$$

For the triplet two-nucleon state the Yamaguchi form factor corresponds to the Hulthen deuteron wave function

$$\tilde{\phi}_d(r)=A\exp(-\alpha_d r)[1-\exp(-\beta_d r)]/r \qquad (4.5)$$

with $\alpha_d=0.2316\ F^{-1}$ and $\mu_d=\beta_d+\alpha_d$. A second form for N is obtained by modifying the Yamaguchi form factors so as to simulate short-range repulsion in the two-nucleon subsystems, and adjusting the parameters ν_n and h_n so that the radius of the three-nucleon bound state and the S'-state probability density is unchanged. In the triplet two-nucleon state we take a form factor which corresponds to the deuteron wave function

$$\tilde{\phi}_d(r)=A\exp(-\alpha_d r)[1-\exp(-\gamma_d r)]^4/r, \qquad (4.6)$$

which, as $r\to 0$, goes to zero as r^3. The effective ranges characterized by the wave functions (4.5) and (4.6) are adjusted to be equal. An analogous procedure is adopted for the case of the singlet two-nucleon form factor, but we relate the size parameter γ_s to the singlet effective range by approximating the antibound state as a zero-energy bound state, $\alpha_s=0$. These considerations give

$$N_n(\mathbf{p},\mathbf{q})=\{\sum_{i=1}^{4}[m_{ni}/(p^2+\mu_{ni}^2)]\}[h_n/(q^2+\nu_n^2)^2]. \qquad (4.7)$$

Here

$$\mu_{ni}=\alpha_n+(i)\times(\gamma_n), \qquad i=1\text{–}4$$

and

$$m_{n1}=4(\mu_{n1}^2-\alpha_n^2),$$

$$m_{n2}=-6(\mu_{n2}^2-\alpha_n^2),$$

$$m_{n3}=4(\mu_{n3}^2-\alpha_n^2),$$

and

$$m_{n4}=-(\mu_{n4}^2-\alpha_n^2).$$

The "effective hard-core radius" is of the order of 0.4 F.

Thus in summary, the parameters of the constructed He^3 and H^3 wave functions fall into three classes: the known asymptotic parameters E_B, ϵ_d, and ϵ_s, the parameters which are determined by the properties of the $^3S_1$ and $^1S_0$ two-nucleon subsystems, i.e., β_n for the Hulthen-Yamaguchi case and γ_n for the situation with short-range repulsion, and finally the four parameters h_n, ν_n which we determine by the normalization, the S'-state probability, and the charge radius of the bound state, the ratio ν_d/ν_s being taken from three-nucleon calculations with separable potentials. We list the properties of the wave functions in Table I, and illustrate the effect of the short-range repulsion on the charge form factors in Figs. 1 (A) and 1 (B). As expected, the repulsion reduces the form factors at high-momentum transfer. The reduction is not sufficient to obtain agreement with experiment[21] and this may in-

[21] H. Collard, R. Hofstadter, E. R. Hughes, A. Johansson, M. K. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. 138, B802 (1964).

dicate that the combined repulsion between all three nucleons has not been taken into account. Note that, for the neutron charge form factor used,[22] the 1% S' state and the parameters characterizing the asymptotic behavior can account for the difference between the He^3 and H^3 form factors.

5. PHOTODISINTEGRATION AMPLITUDES AND CROSS SECTIONS

If $\mathbf{x}_\alpha$ is the position operator of nucleon α, and $\boldsymbol{\rho}_\alpha$ and $\mathbf{r}_\alpha$ the coordinate operators conjugate to $\mathbf{q}_\alpha$ and $\mathbf{p}_\alpha$, then

$$-i\hbar(\partial/\partial\mathbf{q}_1)=\boldsymbol{\rho}_1=\mathbf{x}_1-\tfrac{1}{2}(\mathbf{x}_2+\mathbf{x}_3)\equiv-\mathbf{r}'',$$
$$-i\hbar(\partial/\partial\mathbf{p}_1)=\mathbf{r}_1=\mathbf{x}_2-\mathbf{x}_3\equiv 2\mathbf{r}'/\sqrt{3},$$

where the prime and double prime indicate the symmetry properties (4.3f) and (4.3e), respectively. The dipole operator, if $\mathbf{R}$ is the c.m. coordinate and $\boldsymbol{\varepsilon}$ the photon polarization, has the form

$$\sum_{\alpha=1}^{3}\tfrac{1}{2}(1+\tau_z(\alpha))\boldsymbol{\varepsilon}\cdot\{\mathbf{x}_\alpha-\mathbf{R}\}=\boldsymbol{\varepsilon}\cdot\tfrac{1}{3}\{\tau_z'\mathbf{r}'+\tau_z''\mathbf{r}''\}.$$

This is an isovector, and hence the amplitudes for He^3 and H^3 disintegration are simply related by the Wigner-Eckart theorem if charge symmetry is assumed. The amplitudes for transitions to nucleon-deuteron or nucleon-singlet plane-wave states of isospin I and $I_z=-\frac{1}{2}$ are

$$\langle\mathbf{q}_1\,|\,B_{n=s,d}{}^{I=1/2}(E)\,|\,\Psi_B\rangle$$
$$=-(\surd\tfrac{1}{6})\int\frac{d^3\mathbf{p}_1 g_n(\mathbf{p}_1)\,\boldsymbol{\varepsilon}\cdot\{-\mathbf{r}''u\pm(\mathbf{r}''v''+\mathbf{r}'v')\}}{p_1{}^2+\frac{3}{4}q_1{}^2-E}, \tag{5.1}$$

$$\langle\mathbf{q}_1\,|\,B_s{}^{I=3/2}(E)\,|\,\Psi_B\rangle$$
$$=(\surd\tfrac{1}{3})\int\frac{d^3\mathbf{p}_1 g_s(\mathbf{p}_1)\,\boldsymbol{\varepsilon}\cdot\{-\mathbf{r}''u+(\mathbf{r}''v''-\mathbf{r}'v')\}}{p_1{}^2+\frac{3}{4}q_1{}^2-E}. \tag{5.2}$$

The dependence of B_n and the complete amplitudes X_n on the direction of the vector $\mathbf{q}_1$ is of the form

$$\langle\mathbf{q}_1\,|\,B_n{}^I(E)\,|\,\Psi_B\rangle=\boldsymbol{\varepsilon}\cdot\mathbf{q}_1 b_n{}^I(q;E), \tag{5.3}$$
$$\langle\mathbf{q}_1\,|\,X_n{}^I(E)\,|\,\Psi_B\rangle=\boldsymbol{\varepsilon}\cdot\mathbf{q}_1 x_n{}^I(q;E). \tag{5.4}$$

Substitution of (5.3) and (5.4) into the (3.5) gives an integral equation in one continuous variable; for the $I=\frac{3}{2}$ case there is just one channel corresponding to s-N scattering in ${}^2P_{1/2}$ state, but for the $I=\frac{1}{2}$ case there are two coupled channels corresponding to ${}^2P_{1/2}$ d-N and s-N scattering. The form of these integral equations and the method of solution are outlined in the Appendix.

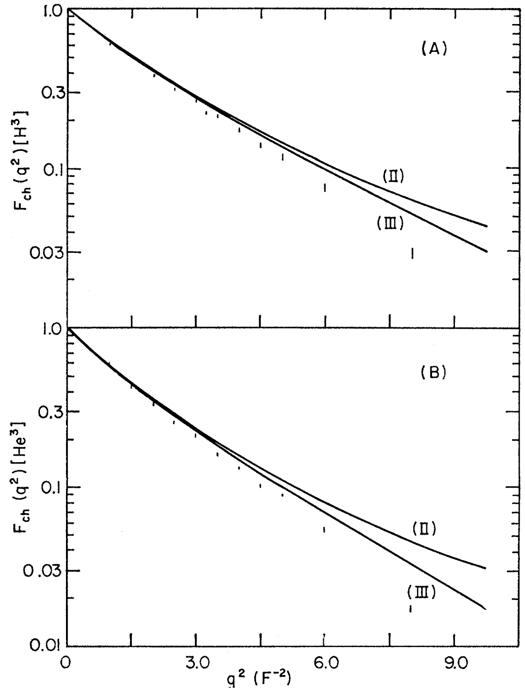

FIG. 1. Comparison of the charge form factors of wave functions II and III of Table I with the experimental data of Ref. 21. (A) corresponds to H^3 and (B) to He^3.

The differential cross section for photodisintegration into a nucleon-deuteron state is

$$d\sigma_2/d\Omega=\tfrac{1}{3}(2\pi)^2\alpha E_\gamma\sin^2\theta q^3\,|\,x_d{}^{I=1/2}(q;E=\tfrac{3}{4}q^2-\epsilon_d)\,|^2, \tag{5.5}$$

where we have summed over final spins and averaged over initial spins and polarization. In Eqs. (5.5), α is the fine structure constant, E_γ the photon energy, and q and θ are the magnitude and direction of the relative nucleon-deuteron momentum.

For the three-body break up, we assume that particle 1 is the unlike nucleon. Thus, I_{23} is equal to 1. There are four amplitudes corresponding to $I=\frac{1}{2}$ and $\frac{3}{2}$, and $S_{23}=0$ and 1. The plane-wave approximation terms in (3.8) when $I_z=-\frac{1}{2}$ are

$$\langle\mathbf{p}_1\mathbf{q}_1\,|\,B_0{}^{I=(1/2)S_{23}=0}(E)\,|\,\Psi_B\rangle$$
$$=(\surd\tfrac{1}{6})\,\boldsymbol{\varepsilon}\cdot\{-\mathbf{r}''u+(\mathbf{r}''v''+\mathbf{r}'v')\}\equiv b_{1/2}''+b_{1/2}{}^s, \tag{5.6}$$

$$\langle\mathbf{p}_1\mathbf{q}_1\,|\,B_0{}^{I=(1/2)S_{23}=1}(E)\,|\,\Psi_B\rangle$$
$$=-(\surd\tfrac{1}{6})\,\boldsymbol{\varepsilon}\cdot\{\mathbf{r}'u+(\mathbf{r}'v''-\mathbf{r}''v')\}\equiv b_{1/2}'+b_{1/2}{}^a, \tag{5.7}$$

$$\langle\mathbf{p}_1\mathbf{q}_1\,|\,B_0{}^{I=(3/2)S_{23}=0}(E)\,|\,\Psi_B\rangle$$
$$=-(\surd\tfrac{1}{3})\,\boldsymbol{\varepsilon}\cdot\{-\mathbf{r}''u+(\mathbf{r}''v''-\mathbf{r}'v')\}\equiv b_{3/2}'', \tag{5.8}$$

and

$$\langle\mathbf{p}_1\mathbf{q}_1\,|\,B_0{}^{I=(3/2)S_{23}=1}(E)\,|\,\Psi_B\rangle$$
$$=-(\surd\tfrac{1}{3})\,\boldsymbol{\varepsilon}\cdot\{\mathbf{r}'u+(\mathbf{r}'v''+\mathbf{r}''v')\}\equiv -b_{3/2}'. \tag{5.9}$$

[22] Nucleon-charge form factors used correspond to combination b' in C. de Vries, R. Hofstadter, and A. Johansson, Phys. Rev. **134**, B848 (1964). The corresponding proton and neutron mean-square radii are 0.7185 and −0.1258 F^2.

Here, the primes, double primes, a, and s denote the symmetry in relation to the permutation group of three variables. The scattering terms in (3.8) are given by

$$\langle \mathbf{p}_1\mathbf{q}_1 \mid C^{IS_{23}}(E) \mid \Psi_B\rangle = \sum_{\alpha=1}^{3} \sum_{m=s,d} \langle II_z, I_{23}, I_1 \mid I_{\beta\gamma}=I_m, I_\alpha, II_z\rangle \times\langle SS_z, S_{23}, S_1 \mid S_{\beta\gamma}=S_m, S_\alpha, SS_z\rangle R_m{}^I(\alpha), \quad (5.10)$$

where

$$R_m{}^I(\alpha)=g_m(\mathbf{p}_\alpha)t_m(E-\tfrac{3}{4}q_\alpha{}^2)\mathbf{q}_\alpha\cdot\boldsymbol{\varepsilon}x_m{}^I(q_\alpha;E).$$

Evaluation of the spin-isospin recoupling coefficients gives

$$\langle \mathbf{p}_1\mathbf{q}_1 \mid C^{I=(1/2)S_{23}=0}(E) \mid \Psi_B\rangle = T^S\{\tfrac{1}{2}(R_s{}^{1/2}(1)-R_d{}^{1/2}(1))\} +T''\{-\tfrac{1}{2}(R_s{}^{1/2}(1)+R_d{}^{1/2}(1))\}\equiv c_{1/2}{}^S+c_{1/2}'', \quad (5.11)$$

$$\langle \mathbf{p}_1\mathbf{q}_1 \mid C^{I=(1/2)S_{23}=1}(E) \mid \Psi_B\rangle = T'\{-\tfrac{1}{2}(R_s{}^{1/2}(1)+R_d{}^{1/2}(1))\}\equiv c_{1/2}', \quad (5.12)$$

$$\langle \mathbf{p}_1\mathbf{q}_1 \mid C^{I=(3/2)S_{23}=0}(E) \mid \Psi_B\rangle = T''\{-R_s{}^{3/2}(1)\} \equiv c_{3/2}'', \quad (5.13)$$

and

$$\langle \mathbf{p}_1\mathbf{q}_1 \mid C^{I=(3/2)S_{23}=1}(E) \mid \Psi_B\rangle = T'\{R_s{}^{3/2}(1)\}=-c_{3/2}'. \quad (5.14)$$

Summing over the final nucleon spins and averaging over the initial spin of the nucleus, the cross section for breakup into three nucleons is given by

$$d^5\sigma_3=(2\pi)^2\alpha E_\gamma \sum_{S_{23}=0,1} \mid \langle \mathbf{p}_1\mathbf{q}_1 \mid M^{S_{23}}(E) \mid \Psi_B\rangle \mid^2 \times d^3\mathbf{p}_1 d^3\mathbf{q}_1\delta(p_1{}^2+\tfrac{3}{4}q_1{}^2-E), \quad (5.15)$$

where

$$M^{S_{23}}(E)=-(\sqrt{\tfrac{2}{3}})X_0{}^{(1/2)S_{23}}(E)+(\sqrt{\tfrac{1}{3}})X_0{}^{(3/2)S_{23}}(E).$$

The average over photon polarizations can now be performed by using

$$\text{average over polarizations}\{\boldsymbol{\varepsilon}\cdot\mathbf{A}\boldsymbol{\varepsilon}\cdot\mathbf{B}\} =(1/2k^2)(\mathbf{k}\wedge\mathbf{A})\cdot(\mathbf{k}\wedge\mathbf{B}), \quad (5.16)$$

where $\mathbf{k}$ is the incident photon momenta.

The cross sections discussed below can then be obtained by suitable numerical integration of Eq. (5.15). In particular, there is no interference between amplitudes of different symmetry and isospin in the expression for the total cross section:

$$\sigma_3=(2\pi)^2\alpha E_\gamma\int d^3\mathbf{p}_1\int d^3\mathbf{q}_1\delta(p_1{}^2+\tfrac{3}{4}q_1{}^2-E) \times\{\tfrac{2}{3}\mid b_{1/2}''+c_{1/2}''\mid^2+\tfrac{2}{3}\mid b_{1/2}'+c_{1/2}'\mid^2+\tfrac{2}{3}\mid b_{1/2}{}^s+c_{1/2}{}^s\mid^2 +\tfrac{2}{3}\mid b_{1/2}{}^a\mid^2+\tfrac{1}{3}\mid b_{3/2}''+c_{3/2}''\mid^2+\tfrac{1}{3}\mid b_{3/2}'+c_{3/2}'\mid^2\}. \quad (5.17)$$

6. RESULTS

A. Dependence of Photodisintegration Cross Sections on Structure of Three-Nucleon Continuum States

The exact treatment of the three-particle aspects of the final states outlined in Sec. 3 represents by far the most complete approach to this aspect of the photodisintegration problem yet attempted.

For the two-body channel, the effects due to the distortion of the deuteron and the presence of the three-body channel are included. Further the theoretical $^2P_{1/2}$ phase shift and inelasticity are in reasonable agreement with the results of the phase-shift analyses of the differential nucleon-deuteron cross section.[23] This exact treatment of the final states shows that it is a poor approximation to neglect the nucleon-deuteron interaction in two-body photodisintegration.

In Fig. 2(A), the two-body photodisintegration cross sections are given in plane-wave approximation, where the nucleon-deuteron interaction is neglected [curve (b)] and in the case where the final-state interactions are taken into account [curve (a)]. It is

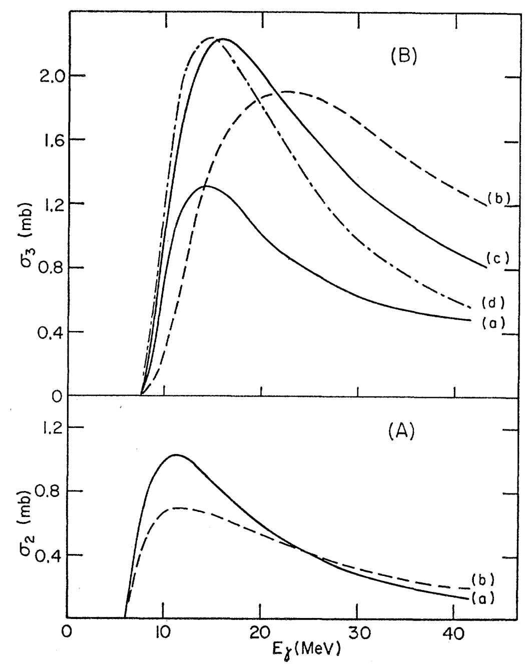

FIG. 2. Dependence of the photodisintegration cross section on the structure of the final states. (A) shows the two-body cross sections corresponding to the exact solution (a), and the plane-wave approximation (b). (B) shows the three-body cross sections corresponding to the exact solution (a), the plane-wave approximation (b), the exact solution with $^3S_1$ interaction equal to the $^1S_0$ interaction (c), and the first rescattering approximation (d). The bound state used is the He wave function I of Table I.

[23] W. T. H. van Oers and K. W. Brockman, Jr, Nucl Phys. A92, 561 (1967).

noteworthy that the magnitude of the rescattering corrections is similar to that of the simple two-body models of Eichmann[3] and Fetisov.[4] We shall see that the photodisintegration into three nucleons with the same isospin as the nucleon-deuteron state is very small. Accordingly it is possible that the models used by Eichmann and Fetisov are sufficient to account for the $^2P_{1/2}$ rescattering in the photodisintegration problem, while at the same time being completely inconsistent with the observed large inelasticity in $^2P_{1/2}$ nucleon-deuteron scattering on the energy shell.[16,23]

In the three-body break-up reaction, three fragments share the available energy and the nucleon pairs may be in relative S waves. Here, large rescattering corrections are probable. This is borne out by the results for the total cross section in Fig. 2(B). In this figure, curves (a), (b), and (d) represent the cross sections corresponding to the exact solution, the plane-wave approximation, and the first rescattering approximation[24] for the final states. It is evident that neither approximation (b) or (d) is particularly good.

In Fig. 2(B), curve (c) corresponds to the exact solution when the $^3S_1$ two-nucleon interaction has been weakened so as to be identical to the $^1S_0$ interaction, i.e., the deuteron is unbound. It is significant that this cross section and the plane-wave approximation (b) are much larger than the cross section (a). All these cross sections correspond exactly to final-state wave functions which are eigenstates of particular Hamiltonians. The essential difference is that the final states in (a) contain information about the deuteron, and hence the existence of the nucleon-deuteron channel, whereas the final states in (b) and (c) do not.

Barton and Gerasimov[15] have shown that the importance of final-state interactions in the three-body breakup is a necessary consequence of the form of the dipole operator, the dominant principal S state of He^3 and the near equality of the experimental two-body and three-body cross sections. This result was obtained by considering the bremsstrahlung-weighted sum rule for electric dipole photodisintegration.

$$J_T=\int_0^\infty \sigma_T(E_\gamma)(dE_\gamma/E_\gamma)=\tfrac{4}{3}\pi^2\alpha R_0^2, \qquad (6.1)$$

where α is the fine structure constant and R_0^2 is the mean-square radius of the odd nucleon in the bound state, i.e., the neutron in He^3 or the proton in H^3.

Clearly, the sum rule can be split into two-body and three-body break-up contributions

$$J_T=J_2+J_3. \qquad (6.2)$$

Alternatively, J_T can be decomposed according to the total isospin, $I=\frac{1}{2}$ or $\frac{3}{2}$, of the final state

$$J_T=J_{1/2}+J_{3/2}. \qquad (6.3)$$

This decomposition, which yields the nonrelativistic version of the Cabibbo-Radicati sum rule,[25] is most easily effected by considering the dipole operator as the zeroth component of an isovector

$$\mathbf{D}^i\cdot\boldsymbol{\varepsilon}=\sum_{\alpha=1}^{3}\tfrac{1}{2}\tau^i(\alpha)\mathbf{x}_\alpha\cdot\boldsymbol{\varepsilon}, \qquad (6.4)$$

where $i=+1$, 0, or -1, and forming the commutator

$$[\mathbf{D}^{+1}\cdot\boldsymbol{\varepsilon},\mathbf{D}^{-1}\cdot\boldsymbol{\varepsilon}]=-\sum_\alpha \tfrac{1}{2}\tau^0(\alpha)(\mathbf{x}_\alpha\cdot\boldsymbol{\varepsilon})^2. \qquad (6.5)$$

Assuming that He^3 and H^3 are the components of an isodoublet and noting the fact that $\mathbf{D}^i$ is an isotensor, the matrix elements of (6.5) give

$$2J_{1/2}-J_{3/2}=\tfrac{2}{3}\pi^2\alpha R_V^2. \qquad (6.6)$$

Here R_V^2 is the isovector radius of the three-nucleon ground state and is related to the mean-square radii for the odd and like nucleons by

$$R_V^2=2R_L^2-R_0^2. \qquad (6.7)$$

Assuming charge symmetry and no charge exchange effects,

$$R_L^2=r^2(He^3)-r^2(p)-\tfrac{1}{2}r^2(n) \qquad (6.8)$$

and

$$R_0^2=r^2(H^3)-r^2(p)-2r^2(n), \qquad (6.9)$$

where the quantities r^2 correspond to the He^3, H^3, proton, and neutron mean-square-charge radii.

To within 20% $R_L^2\simeq R_0^2$,[21] and hence

$$2J_{1/2}\simeq 2J_{3/2}\simeq J_T. \qquad (6.10)$$

Since experimentally $2J_2\simeq 2J_3\simeq J_T$,[(2)] Barton and Gerasimov[15] conclude that the two-body contribution must dominate the $I=\frac{1}{2}$ channel and that

$$J_3\simeq J_{3/2}=(2\pi^2\alpha/9)(5R_0^2-2R_L^2). \qquad (6.11)$$

But if one calculates the three-body break up using for the final states any complete set of eigenstates of a Hamiltonian which does not bind the deuteron, then

$$J_3=J_{1/2}+J_{3/2}\simeq 2J_{3/2}=(4\pi^2\alpha/9)(5R_0^2-2R_L^2). \qquad (6.12)$$

That is, J_3 is increased by a factor of 2 in accordance with the results of Fig. 2(B).

If the dipole operator $\mathbf{A}\cdot\mathbf{p}$ is used instead of $\mathbf{E}\cdot\mathbf{r}$, a substitution that would only be valid if the nuclear forces were local, then similar relations still apply. These relations now refer to the rms momentum in the ground state, and to the integrated cross sections weighted with E_γ^2 as opposed to E_γ^{-1}. In this case, the presence of two-body channel is reflected in the cross section for three-body break up at high E_γ. One can then reproduce the near equality of the low-energy two-body and three-body cross sections without in-

[24] By the first rescattering approximation we mean that the amplitudes for the production of the nucleon-deuteron and nucleon-singlet isobars are given by the plane-wave approximation.

[25] N. Cabibbo and L. A. Radicati, Phys. Letters **19**, 697 (1966). The isospin decomposition of the nonrelativistic Thomas-Reiche-Kuhn sum rule for the three-nucleon system has been given by J. S. O'Connell and F. Prats, Phys. Rev. **184**, 1007 (1969).

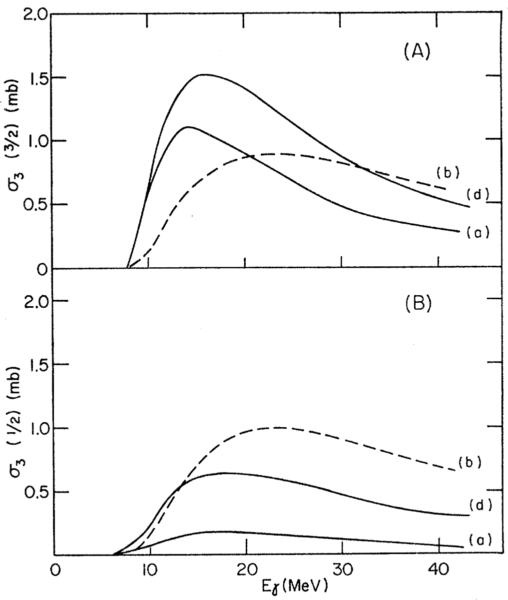

FIG. 3. Isospin decomposition of the three-body break-up cross section. (A) shows the $I=\frac{3}{2}$ contribution and (B) the $I=\frac{1}{2}$. Labels (a), (b), and (d) correspond to exact solution, plane-wave approximation, and first rescattering approximation, respectively. The bound state used is the He^3 wave function III of Table I.

cluding the final-state interactions. However for reasonable bound-state wave functions, the magnitudes of both cross sections are a factor of 2 too small.

Figure 3 illustrates the isospin decomposition for the cross section for three-body break up. We see that, for the bound-state wave function used, the exact solution, curves (a) and the plane wave approximation, curves (b), are consistent with (6.11) and (6.12) respectively. The first rescattering approximation (d) is poor for both the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ cross sections, but it does roughly reproduce the difference between these cross sections.

The mechanism giving rise to the suppression of the $I=\frac{1}{2}$ part of the three-body cross section seems to be rather complicated.[26] This cross section is given by (5.17),

$$\sigma_3(I=\tfrac{1}{2})=(2\pi)^2\alpha E_\gamma \int d^3\mathbf{p}_1 \int d^3\mathbf{q}_1 \delta(p_1^2+\tfrac{3}{4}q_1^2-E)$$
$$\times\tfrac{2}{3}\{|b_{1/2}''+c_{1/2}''|^2+|b_{1/2}'+c_{1/2}'|^2+|b_{1/2}^s+c_{1/2}^s|^2+|b_{1/2}^a|^2\}. \quad (6.13)$$

The dominant component in the ground state is the principal S state which depends on the spin-independent part of the nuclear interactions. This state alone with spin-independent final-state interactions gives rise to the amplitudes $b_{1/2}''$, $b_{1/2}'$, $c_{1/2}''$, and $c_{1/2}'$; and $\sigma_3(I=\frac{1}{2})=\sigma_3(I=\frac{3}{2})$ if the deuteron is unbound. The spin dependence of the interaction in the ground state generates a small S' admixture which in turn gives rise to the small amplitudes b^s and b^a. The spin dependence in the final state gives a nonzero value for $c_{1/2}^s$; but the necessary small value of $\sigma_3(I=\frac{1}{2})$ implies that $c_{1/2}^s$ is small, and that $c_{1/2}''$ and $c_{1/2}'$ interfere destructively with $b_{1/2}''$ and $b_{1/2}'$. Further, this interference reduces $\sigma_3(I=\frac{1}{2})$ at low energies when the dipole operator $\mathbf{E}\cdot\mathbf{r}$ is used, but for the operator $\mathbf{A}\cdot\mathbf{p}$, a less pronounced reduction at higher energies is produced. It would seem that the reduction in the low-energy cross section is mainly due to a modification of the long-range form, or equivalently the most rapidly varying part in momentum space, of the continuum wave function by the presence of the deuteron producing interaction. The analytic property of the amplitudes which is most likely to cause this is the pole in the three-body amplitude due to the existence of the two-body channel. This pole could characterize a long-range effective interaction [see Eq. (2.16)]. This conjecture is supported by the fact that the first rescattering approximation (d) which includes this property accounts quite well for the difference between $\sigma_3(I=\frac{1}{2})$ and $\sigma_3(I=\frac{3}{2})$.

In the three-nucleon break-up reaction, the energy distribution of the odd nucleon shows several interesting features. Figure 4 illustrates several neutron spectra for the reaction $He^3(\gamma, n)2p$ with 12.5-MeV photons. The curves, which have been normalized to the same total cross section, correspond to different final-state wave functions: (a) the exact solution, (a′) the exact solution when only the final interaction between the two protons is included, (b) the plane-wave approximation, and (c) the exact solution when the $^3S_1$ two-nucleon interaction is equal to the $^1S_0$ interaction.

It is clear from Fig. 4 that the peak at high neutron energy is due to the final proton-proton $^1S_0$ interaction. We note that in marked contrast to the

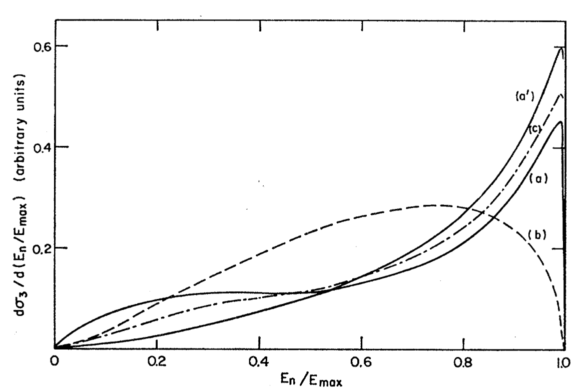

FIG. 4. Energy distributions of neutrons in $He^3(\gamma, n)2p$ for 12.5-MeV photons normalized to the same total cross section. Labels (a), (b), and (c) as in Fig. 2(B), and (a′) corresponds to the exact solution (a), retaining only the final proton-proton interaction.

[26] The assertion by Lehman and Prats that the suppression of $\sigma_3(I=\frac{1}{2})$ is a result of the near equality in strength of the singlet-even and triplet-even two-nucleon interactions is in error; D. R. Lehman and F. Prats, Report, 1969 (unpublished).

corresponding situation in nucleon-deuteron scattering,[15] the shape of this peak is not sensitive to the final neutron-proton interactions. This difference is probably attributable to the fact that in proton-deuteron scattering the mechanism for producing a $(pp)+n$ configuration is a pure rearrangement process comparable in strength to the $(np)+p$ production mechanism, whereas in the photodisintegration of He^3 the production of a $(pp)+n$ configuration is favored by the direct disintegration of the bound state.

The peak at medium neutron energies is a reflection of the final neutron-proton interactions. The singlet interaction would be expected to give rise to a bump in the energy distribution at approximately

$$E_n/E_{\max}=E_n/\tfrac{2}{3}(E_\gamma-E_B)=\tfrac{1}{4}, \qquad (6.14)$$

independent of the incident photon energy. On the other hand, if the neutron and proton come out preferentially in the $^3S_1$ state, this should give rise to some structure in the distribution at low neutron energy with position and strength more strongly dependent on the photon energy. In fact, this effect is difficult to detect because of the more prominent $^1S_0$ rescattering. However, if the $^3S_1$ interaction is put equal to the $^1S_0$, then the net enhancement at medium neutron energies is noticeably reduced; compare curves (a) and (c).

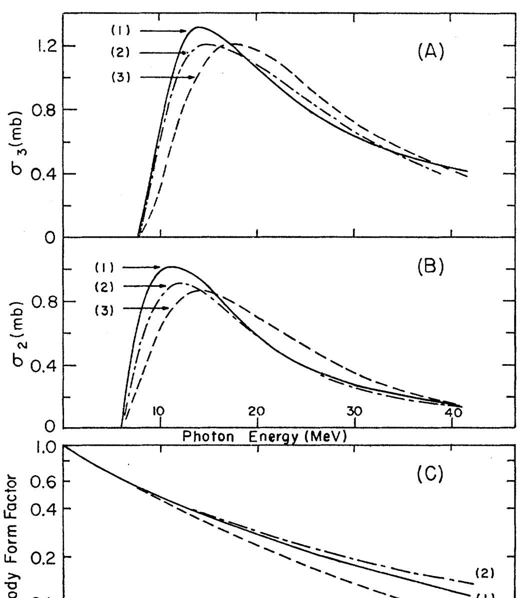

FIG. 5. Dependence of the three-body cross section (A), the two-body cross section (B), and the body form factor (C), on the shape of the principal S state of He^3. Curves (1), (2), and (3) correspond respectively to the principal S state of wave function I of Table I, the Gunn-Irving wave function (6.16), and the Irving wave function (6.17).

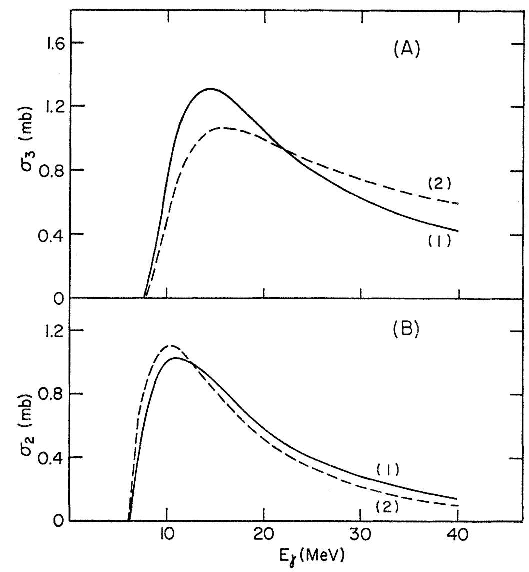

FIG. 6. Dependence of the three-body cross section (A), and the two-body cross section (B), on the S' admixture. The curve (2) corresponds to the complete He^3 wave function I of Table I, and curve (1) to the normalized S state of this wave function.

B. Dependence of Photodisintegration Cross Sections on Structure of Three-Nucleon Bound State

The bound-state properties considered are the analytic form of the principal S state, the asymptotic form, the S' state, and the two-nucleon short-range repulsion. All the results of this section correspond to the exact treatment of the final-state wave function.

The effects of the analytic form of the principal S state on the photodisintegration cross sections and on the body form factor,

$$F(q)=\int \exp(-i\tfrac{2}{3}\,\boldsymbol{\rho}_1\cdot\mathbf{q})\,\bar{u}(\mathbf{r}_1,\mathbf{p}_1)^2 d\tau, \qquad (6.15)$$

which enters into the evaluation of the charge form factors, are shown in Fig. 5. Curve (1) corresponds to the principal S state, normalized to one, of the He^3 wave function I, of Table I. The curves (2) and (3) correspond respectively to the Gunn-Irving

$$\bar{u}(\mathbf{r}_1,\boldsymbol{\rho}_1)=N\exp\{-\tfrac{1}{2}\mu(r_1^2+r_2^2+r_3^2)^{1/2}\}/(r_1^2+r_2^2+r_3^2)^{1/2},\qquad \mu=0.904\ \mathrm{F}^{-1} \qquad (6.16)$$

and the Irving

$$\bar{u}(\mathbf{r}_1,\boldsymbol{\rho}_1)=N\exp\{-\tfrac{1}{2}\mu(r_1^2+r_2^2+r_3^2)^{1/2}\},\qquad \mu=1.31\ \mathrm{F}^{-1} \qquad (6.17)$$

wave functions. Since all three wave functions have the same mean-square radius, $R^2=2.7\ \mathrm{F}^2$, the somewhat small differences in the photodisintegration cross sections in Fig. 5 reflect the differences in the analytic structure of the wave functions. It has been claimed that the use of a bound-state wave function with the

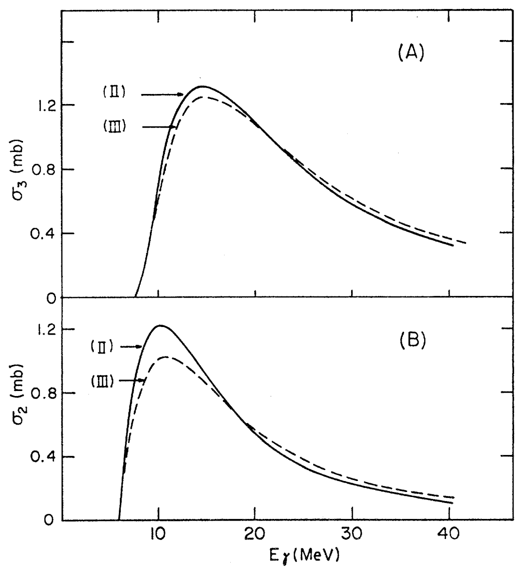

FIG. 7. Dependence of the three-body cross section (A), and the two-body cross section (B) on the two-nucleon short-range repulsion. Curves (II) and (III) correspond to He^3 wave functions II and III of Table I.

correct asymptotic form reduces the maximum value of the three-body cross section.[4,8] We found this to be true in the plane-wave approximation, but it is not true if final-state interactions are included: The maximum of curve (1) is greater than that of either curves (2) or (3) in Fig. 5(A).

The effects of the S' state can be seen by comparing curves (1) and (2) in Figs. 6(A) and 6(B). Curve (2) corresponds to the complete He^3 wave function I (Table I), which has a 1.9% S'-state probability density. Curve (1) corresponds to taking just the principal S state of the same wave function and renormalizing to one. If the phase of the S' amplitude is such that, like the experimental values,[21] the charge radii of He^3 is greater than H^3, then the bremsstrahlung-weighted sum rule (6.1) predicts that J_T decreases with the introduction of the S' state; the odd nucleon becomes more tightly bound and R_0 decreases. Furthermore, the results of the isospin decomposition, (6.3), (6.6), and (6.7), show that it is the isospin-$\frac{3}{2}$ contribution, and hence the three-body integrated cross section, which is reduced by the inclusion of the S' state. However, there is a tendency for the two-body cross section, particularly at low photon energies, to increase with the introduction of an S' state. This is because the S'-state admixture of the three-nucleon bound state results from the fact that the effective $^3S_1$ nuclear force is stronger than the $^1S_0$ force, and thus increases the nucleon-deuteron component of the wave function at the expense of the nucleon-singlet component. Hence the isospin-$\frac{1}{2}$ part of the three-body cross section decreases, and accordingly the total decrease in the three-body cross section is greater than it would be if the two-body channel were absent.

For the bound-state wave functions of Table I, the He^3 and H^3 parameters differ by virtue of the difference in the asymptotic forms. We found, however, that the difference in the calculated cross sections for He^3 and H^3 break up is less than 5%, aside from a shift in the energy scale corresponding to the change in the threshold energy.

Finally, we examine in Figs. 7(A) and 7(B) the effect of using a three-nucleon bound-state wave function which includes short-range repulsion between nucleon pairs. Curves (II) and (III) correspond to the no repulsion II and short-range repulsion III wave functions of Table I. The difference in the charge form factors given by these wave functions is illustrated in Figs. 1(A) and 1(B). Note that aside from the two-nucleon correlations, these wave functions have similar properties; in particular, the charge radii and the asymptotic forms are the same.

Thus, in summary, the changes in the photodisintegration cross sections induced by modifications of the principal S state, the S' state, the asymptotic form, and the two-nucleon short-range repulsion properties of the three-nucleon bound state are all of the order of 10% or less and cannot be considered significant in the light of the present theoretical and experimental uncertainties of the photodisintegration process.

C. Comparison with Experiment

Previous studies of the two-body photodisintegration reaction have, in the main, concluded that there are no outstanding discrepancies between theory and experiment. In fact, the situation if far more complex. An important, and as yet unresolved, problem is to reconcile the experimental measurements on the total cross section[2] and the differential cross sections at an angle of 90°.[27,28] These measurements are related by

$$\sigma_2 = x(8\pi/3)(d\sigma_2/d\Omega)_{\theta=90°}$$

with

$$x \simeq 1.2$$

for photons with energies ranging from 6 to 40 MeV. This value for x is considerably larger than the value 1.05 ± 0.014, given by measurement of the angular distributions for an equivalent photon energy of 15.3 MeV [29] and is also larger than the values given by other less precise angular-distribution measurements.[2] Since the angular distributions are not very sensitive to x, the large value of 1.2 cannot be ruled out completely on this evidence alone. However, theoretical estimates of the electric quadrupole and magnetic dipole effects also give much smaller values for x, ranging from 1.03

[27] J. R. Stewart, R. C. Morrison, and J. S. O'Connell, Phys. Rev. 138, B372 (1965).

[28] B. L. Berman, L. J. Koester, and J. H. Smith, Phys. Rev. **133**, B117 (1964).

[29] B. D. Belt, C. R. Bingham, M. L. Halbert, and A. van der Woude, Ref. 14.

at 13 MeV to 1.09 at 40 MeV.[5,30] It seems most probable, therefore, that there is a systematic error in either the total cross-section data or the 90° data.

It is known that bound-state wave functions which are consistent with the experimental charge form factors give, in the absence of rescattering corrections, photodisintegration cross sections which are in agreement with the 90° data.[6,9] This agreement seems to be illusory in view of the enhancement of the low-energy two-body cross section due to final-state interaction effects [Fig. 2(A)]. This enhancement is typically of the order of 20% for all the ground-state wave functions used in this paper, and its existence is made plausible by the reduction of the $I=\frac{1}{2}$ part of the three-body cross section by the final-state interactions. It is not surprising therefore, that in Fig. 8 the theoretical values for $d\sigma/d\Omega$ at a proton angle of 90° are significantly larger than the experimental results.[31]

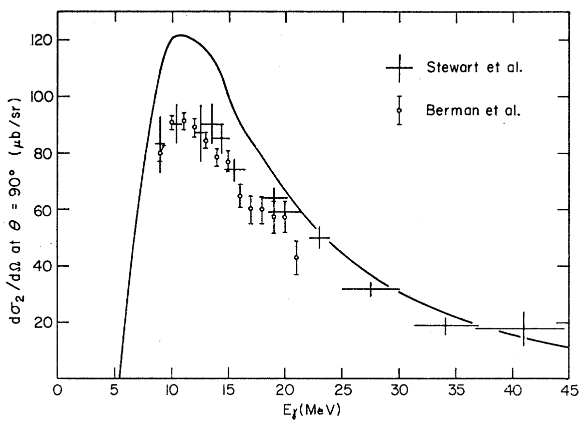

FIG. 8. Comparison of the differential cross sections for 90° for $He^3(\gamma, p)d$ reaction. The continuous curve corresponds to the He^3 wave function III. The experimental points have been taken from Refs. 27 and 28.

In Fig. 8, the theoretical results correspond to the He^3 wave function III of Table I, and it is clear from Figs. 5–7 that the discrepancy between theory and experiment could be reduced by modifying the bound-state wave function.[32] But within the present theoretical framework, it seems unlikely that any of the usual representations of the bound state could simultaneously agree with the 90° photodisintegration data and the charge form factors.

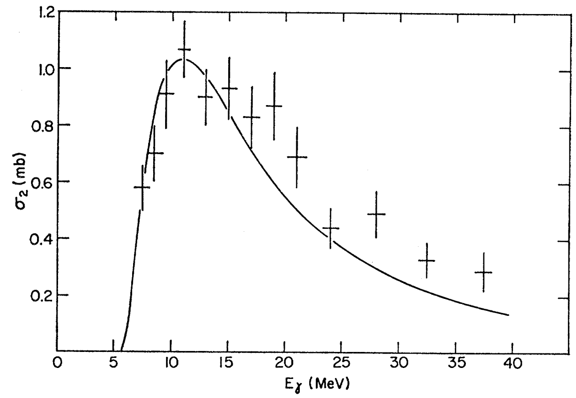

FIG. 9. Comparison of the experimental total cross sections for the reaction $He^3(\gamma, p)d$ of Ref. 2 with the theoretical results given by wave function III.

We investigated the effect of modifying the deuteron wave function. But since the correspondence between the deuteron wave function and the final-state interactions was not retained, the results are not definitive. Nevertheless, in the absence of rescattering, the use of the asymptotic form $\exp(-\alpha_d r)/r$ decreases the cross-section peak, and the use of the deuteron wave function with short-range repulsion [Eq. (4.6)] increases the cross-section peak. Clearly, the overlap between the deuteron and He^3 wave functions is important, but the most realistic deuteron wave function seems to give the greatest discrepancy with the 90° data.

It is possible, but unlikely, that the discrepancy is due to the neglect of noncentral forces, and the associated He^3 and deuteron D states. Since the He^3 D state is known to reduce the radius of the odd nucleon in the bound state, it must also, according to the dipole sum rule (6.1), reduce J_T; but a 9% D state gives only a 3% reduction.[33] Furthermore, the results of the isospin decomposition (6.3), (6.6), and (6.7), which hold separately for the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ photodisintegration

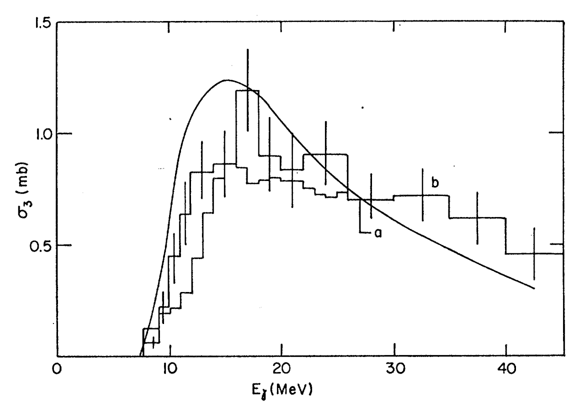

FIG. 10. Comparison of the total cross section for $He^3(\gamma, n)2p$ given by wave function III with the experimental results of Ref. 34, histogram (a), and the results of Ref. (2), histogram (b).

[30] The investigations, Refs. 3, 5, 6, and 10, of the role of the various multipole transitions in He^3 disintegration are far from complete. However, these values for x are of the same order of magnitude as those obtained in very extensive studies of deuteron photodisintegration, see F. Partovi, Ann. Phys. (N.Y.) **27**, 79 (1964).

[31] Experimental cross sections of Refs. 27 and 28 correspond to a laboratory angle of 90°, and accordingly should be smaller by about 1% than the cross sections at a 90° c.m. angle.

[32] Note that if He^3 wave functions used in Fig. 5 are modified to give smaller and more reasonable charge form factors, then the photodisintegration cross section increases.

[33] C. Lucas, Nucl. Phys. **A123**, 173 (1969).

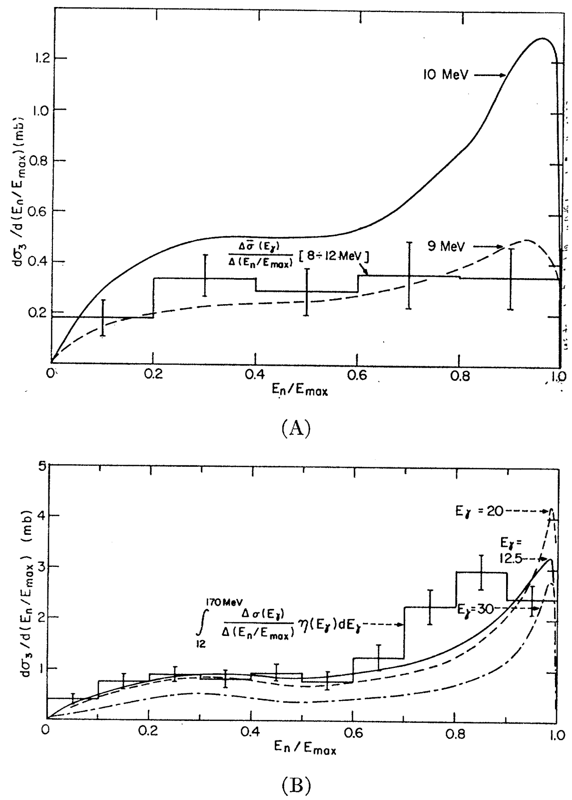

FIG. 11. The neutron energy distribution in $He^3(\gamma, n)2p$ at various photon energies given by wave function III. The experimental histograms are taken from Ref. 2.

channels, show that it is the spin-$\frac{3}{2}$ and isospin-$\frac{3}{2}$ contribution and hence the *three-body* integrated cross section which is reduced by the inclusion of a D state in He^3.

There is no difficulty in obtaining agreement between theory and experiment if, instead of the 90° data, the experimental two-body cross sections of Gorbunov and Varfolomeev[2] are taken. In this case, Fig. 9, the theoretical cross sections are slightly smaller than the experimental values at low photon energies and smaller still at higher energies, which is reasonable in view of the estimates of the effects due to transitions other than electric dipole.

Clearly, it is desirable to clarify the experimental situation regarding two-body break up. Confirmation of the results of Gorbunov[2] would imply that the present theoretical ideas on the reaction are substantially correct. If the 90° results of Berman[28] and Stewart[27] are correct, some way of reconciling this data with the charge-form-factor data is needed.

Turning to the three-body cross sections, we see in Fig. 10 that the theoretical cross sections are somewhat larger than the experimental values[2,34] at low energies. If, in the treatment of the ground state, a larger S' admixture or an Irving wave function is used (see Figs. 5 and 6), then this discrepancy could be reduced, but the agreement in Fig. 9 with the two-body cross section would then deteriorate. However, because of the large rescattering effects in the three-body channel, it is equally, or more, likely that the origin of this discrepancy lies in the treatment of the final continuum state.

There is limited experimental information on the differential cross sections for the three-body breakup. But the data on the neutron spectra in $He^3(\gamma, n)2p$ shows several interesting features. In Fig. 11, we compare the experimental and theoretical spectra at a number of different photon energies. In the light of the experimental errors the agreement is satisfactory with the exception of the absence of the peak at high neutron energy in the 8÷12-MeV experimental data. However, it should be emphasized that accurate neutron spectra and differential cross sections, in general, would be expected to be sensitive to the interference of electric dipole amplitudes with other multipole amplitudes, and hence are beyond the scope of this paper.

7. CONCLUSIONS

In the calculation of the low-energy photodisintegration of He^3, final-state interactions induce changes of the order of 20–25% in the two-body cross section and of the order of 100% in the three-body cross section. This implies that three-body scattering can produce significant modifications of the average value of the final-state wave functions taken over a large volume. This is particularly so in the three-body continuum state if the nuclear interactions bind the deuteron.

The bound-state properties, such as the shape of the principal S state, the asymptotic form, the S'-state probability, and the two-nucleon short-range repulsion are not directly essential to the understanding of the gross properties of the photodisintegration process.

In regard to the comparison of theory and experiment, inclusion of the final-state interactions has two effects. First, it permits a simultaneous, but rough, agreement with the experimental data on the total two-body and three-body photodisintegration cross sections[2] and the charge form factors of the three-particle nuclei.[21] Second, it suggests that, at least within the framework of simple two- and three-nucleon wave functions, central nuclear forces, and Siegert's theorem, the two-body differential cross sections at 90° [27,28] and the charge-form-factors data are inconsistent.

ACKNOWLEDGMENTS

It is a pleasure to thank Dr. G. Barton and Dr. J. S. O'Connell for useful discussions and correspondence.

[34] H. M. Gerstenberg and J. S. O'Connell, Phys. Rev. **144**, 834 (1966).

APPENDIX: METHOD OF SOLUTION OF INTEGRAL EQUATIONS FOR PHOTO-DISINTEGRATION AMPLITUDES

Substitution of Eqs. (5.3) and (5.4) into Eq. (3.5) gives an integral equation in one continuous variable:

$$x_m^I(q;E)=b_m^I(q;E)+(1/q^2)\sum_n \int d^3q'\,\mathbf{q}\cdot\mathbf{q}' \times\langle\mathbf{q}\mid Z_{mn}^I\mid\mathbf{q}'\rangle t_n(E-\tfrac{3}{4}q'^2)x_n^I(q';E) \quad \text{(A1)}$$

with

$$\langle\mathbf{q}\mid Z_{mn}^I\mid\mathbf{q}'\rangle=-2\langle II_z, I_{23}=I_m, I_1\mid I_{31}=I_n, I_2, II_z\rangle \times\langle SS_z, S_{23}=S_m, S_1\mid S_{31}=S_n, S_2, SS_z\rangle \times\frac{g_m(\frac{1}{2}\mathbf{q}+\mathbf{q}')g_n(\mathbf{q}+\frac{1}{2}\mathbf{q}')}{q^2+\mathbf{q}\cdot\mathbf{q}'+q'^2-E-i\epsilon}. \quad \text{(A2)}$$

We have to solve Eq. (A1) in two physical regions. For the three-nucleon final state

$$0\leq q^2\leq\tfrac{4}{3}E \quad \text{(A3)}$$

and for the nucleon-deuteron final state,

$$q^2=\tfrac{4}{3}(E+\epsilon_d). \quad \text{(A4)}$$

A method for solving the singular integral equation (A1) for the q^2 region given by (A3) has been outlined in Refs. (35) and (36). The integrations over the angular variables are performed analytically, and the variables q and q' then simultaneously rotated into the fourth quadrant of their complex plane

$$q=se^{-i\phi}, \qquad q'=te^{-i\phi}, \qquad s, t \text{ real and } \geq 0. \quad \text{(A5)}$$

This continuation of the function $x_n^I(q;E)$ is analytic as long as the contribution from the arc at infinity is zero and the integration contour $(0\leq t\leq\infty)$ does not cross a singularity of the integrand. In order to find $x_n^I(q;E)$ for q real, we now rotate the complex variable q back to the real axis, again making sure no singularities of the integrand are crossed.

For completeness we note here the final form of the integral equations following from this procedure. With

$$q=\alpha ue^{-i\phi}/(1-\mu), \qquad q'=\alpha ve^{-i\phi}/(1-v),$$

$$W_m^I(u,E;\alpha,\phi)=qx_m^I(q;E),$$

and

$$\bar{W}_m^I(u,E;\alpha,\phi)=qb_m^I(q;E).$$

We have [c.f. Eq. (A1)]

$$W_m^I(u,E;\alpha,\phi)=\bar{W}_m^I(u,E;\alpha,\phi) +\sum_n\int_0^1 dv\,\frac{\alpha e^{-i\phi}}{(1-v)^2}K_{mn}^I(u,v;\alpha,\phi)W_n^I(v,E;\alpha,\phi), \quad \text{(A6)}$$

where

$$K_{mn}^I(u,v;\alpha,\phi)=2\pi\int_{-1}^{+1}ydy \times q'^2\langle\mathbf{q}\mid Z_{mn}^I\mid\mathbf{q}'\rangle t_n(E-\tfrac{3}{4}q'^2). \quad \text{(A7)}$$

[35] J. H. Hetherington and L. M. Schick, Phys. Rev. **156**, 1647 (1967).

[36] I. M. Barbour and R. L. Schult, Phys. Rev. **155**, 1712 (1967).

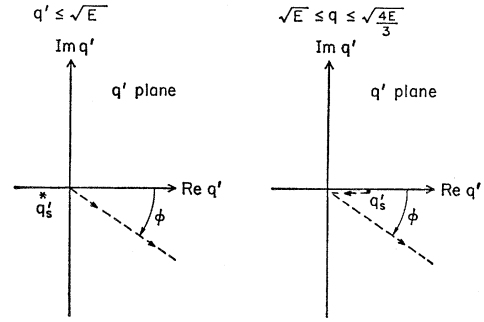

FIG. 12. Integration contours used for the solution of Eq. (A1).

The coupled integral equations as given by Eq. (A6) are valid analytic continuations of Eq. (A1) into the complex q and q' planes for $0<\phi<\frac{1}{2}\pi$ and are nonsingular in this region. Rotation of the complex variable q back to the real axis involves distorting the integration contour as shown in Fig. 12 in order to avoid the singularity of the kernel at $q'=\frac{1}{2}q-(E-\frac{3}{4}q^2+i\epsilon)^{1/2}$, which passes just below $q'=0$ when $q=\sqrt{E}$. This rotation gives

$$x_m^I(q;E)=b_m^I(q;E) +\sum_n q^{-1}\int_0^1 dv\,\frac{\alpha e^{-i\phi}}{(1-v)^2}K_{mn}^I(q,v;\alpha,\phi)W_n^I(v_1,E;\alpha,\phi) +\sum_n(2\pi^2i/q^3)g_m(E-\tfrac{3}{4}q^2)^{1/2} \times 2\langle II_z, I_{23}=I_m, I_1\mid I_{31}=I_n, I_2, II_z\rangle \times\langle SS_z, S_{23}=S_m, S_1\mid S_{31}=S_n, S_2, SS_z\rangle \times\int_0^{E-(1/2)q^2-q(E-3q^2/4)^{1/2}}d(q'^2)g_n[(E-\tfrac{3}{4}q'^2)^{1/2}] \times t_n(E-\tfrac{3}{4}q'^2)x_n^I(q';E)\theta(q^2-E). \quad \text{(A8)}$$

This rotation is valid for $0<\tan\phi\leq\text{Min}(\mu_s,\mu_d)/(\frac{4}{3}E)^{1/2}$ with q^2 in the region $0\leq q^2\leq\frac{4}{3}E$. For the parameters used, a suitable choice for ϕ was found to be

$$\phi=\tan^{-1}[\mu_s/\sqrt{(\tfrac{4}{3}E)^{1/2}}]. \quad \text{(A9)}$$

The solution of Eq. (A1) in the physical region for the nucleon-deuteron final state (A4) was obtained by solving the integral equation:

$$x_m^I(q;E)=b_m^I(q;E) +(1/q^2)\sum_n\int d^3q'[\theta(\tfrac{4}{3}E-q'^2)+\theta(q'^2-\tfrac{4}{3}E)] \times q\cdot q'\langle\mathbf{q}\mid Z_{mn}^I\mid\mathbf{q}'\rangle t_n(E-\tfrac{3}{4}q'^2)x_n^I(q'^2;E), \quad \text{(A10)}$$

where the first term in the square-brackets was taken to contribute to the inhomogeneous term of the integral equation and evaluated by using the solution obtained for $x_n^I(q;E)$ in the region (A3). The integration over dq' was approximated using the procedure described in Ref. (16). This procedure takes care of the singularity arising from the deuteron pole in t_d.