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# Inclusion of Nuclear-Structure Calculations in Nucleon-Nucleus Scattering

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It is shown how one can use the results of a nuclear-structure calculation in the theory of nucleon-nucleus scattering. We restrict ourselves to the simple case of nucleon scattering on a hole nucleus. The correlations of the ground state have been included.

## INTRODUCTION

In a previous work[1] we have developed the general theory of nucleon-nucleus scattering on a hole nucleus in the framework of linear-response theory extending Migdal's approach[2] to the scattering problem. With a similar goal, the unrenormalized random-phase approximation (RPA) has been applied to the scattering problem, using a schematic model.[3] The details and restrictions of these methods can be found in Refs. 1 and 3, as well as in further references. It turns out that a calculation of the scattering process using an effective particle-hole interaction would be rather complicated, since one has to solve a complicated Fredholm problem.[1] Therefore, one has so far studied the problem only in the framework of a schematic model,[1,3] where the corresponding Fredholm determinant degenerates. But it is well known that the schematic model is only a poor approximation to the real situation (see for instance, Mikeska.[4]) For this reason we think one can obtain an improvement of the present status of the theory by including the results of the nuclear-structure calculation obtained with a normal effective particle-hole interaction. The deviations from the nuclear-structure calculation – caused by the matrix elements of the interaction between continuum-bound and continuum-continuum single-particle states – will be treated in this work by a schematic approach. This implies that these special matrix elements can be approximately represented by a separable particle-hole force with the help of a fitting procedure to the real particle-hole force. One may get further improvement using perturbation theory for the difference of the particle-hole force and the separable force as a final step. In the first section, we give a short summary of the nucleon-nucleus scattering theory on a hole nucleus in terms of Migdal's renormalized quantities. The explicit treatment of the model will then be given in the second section.

## I. GENERAL FORMALISM

It has been shown in Ref. 1, that the scattering state in the shell-model representation is given by:

$$\langle \phi_{k,i}{}^{(-)} | \psi_{p,j}{}^{(+)} \rangle = (z_i z_k z_p z_j)^{-1/2} \times \left\langle 0 \right| \psi_i^\dagger \psi_k \frac{i\eta}{E - H + i\eta} \psi_p^\dagger \psi_j \left| 0 \right\rangle e^{i(\delta_p + \delta_k)}, \tag{I.1}$$

and

$$\langle \phi_{r,i} | \psi_{p,j}{}^{(+)} \rangle = (z_i z_r z_p z_j)^{-1/2} \times \left\langle 0 \right| \psi_i^\dagger \psi_r \frac{i\eta}{E - H + i\eta} \psi_p^\dagger \psi_j \left| 0 \right\rangle e^{i\delta_p}. \tag{I.2}$$

Here, $|0\rangle$ denotes the normalized ground state of the compound system, $[H|0\rangle = E_0(A)|0\rangle, E_0(A) = 0]$; $E$ is by definition the energy of the scattering system $(E := \epsilon_p - \epsilon_j)$. $\psi_\mu^\dagger$ and $\psi_\mu$ are the Schrödinger creation and annihilation operators, respectively, of a nucleon with the quantum-number set $\mu$ fixed by an independent-particle Hamiltonian $H_S$. Asymptotically they behave as standing waves. With $z_\mu$ we denote Migdal's renormalization constant. If it is necessary, we will label the continuum states by $p$ and $k$, the hole states by $i$ and $j$, and the bound-particle states by $r$ and $s$ $(\delta_r = \delta_s = 0)$. The independent-particle-model states in (I.1) and (I.2) are defined by:

$$|\phi_{p,i}{}^{(+)}\rangle = e^{i\delta_p} (z_i z_p)^{-1/2} \psi_p^\dagger \psi_i |0\rangle, \tag{I.3}$$

and

$$|\phi_{r,i}\rangle = (z_i z_r)^{-1/2} \psi_r^\dagger \psi_i |0\rangle. \tag{I.4}$$

By comparison with the definition of the renormalized response function,[5]

$$\tilde{L}_{\alpha\mu\lambda\nu}(\omega) := -(z_\alpha z_\mu z_\lambda z_\nu)^{-1/2} \times \langle 0 | [\psi_\lambda^\dagger \psi_\alpha (\omega - H + i\eta)^{-1} \psi_\nu^\dagger \psi_\mu - \psi_\nu^\dagger \psi_\mu (\omega + H - i\eta)^{-1} \psi_\lambda^\dagger \psi_\alpha] | 0 \rangle, \tag{I.5}$$

we obtained for (I.1) the following expression for the scattering states:

$$\langle S_0 | S \rangle = -i\eta \, \tilde{L}_{kjip}(E), \tag{I.6}$$

$$\langle B_0 | S \rangle = -i\eta \, \tilde{L}_{rjip}(E). \tag{I.7}$$

Here, we have introduced the abbreviations

$$|S\rangle := e^{-i\delta_p} |\psi_{p,j}{}^{(+)}\rangle,$$

$$|S_0\rangle := e^{-i\delta_k} |\phi_{k,i}{}^{(+)}\rangle,$$

and

$$|B_0\rangle := |\phi_{r,i}\rangle. \tag{I.8}$$

Assuming that the zero-order response can be represented by the quasiparticle shell-model response and the effective particle-hole interaction ("irreducible vertex part") to be weakly energy-dependent in the considered energy region, we have derived the following equations[1] for $\tilde{L}$:

$$\tilde{L}_{\mu\alpha\beta\nu}(\omega) = \tilde{L}^s_{\mu\beta}(\omega)[\delta_{\mu\nu}\delta_{\alpha\beta} - 2\pi \sum_{\rho\sigma} \tilde{I}_{\mu\rho\beta\sigma}(\omega) \tilde{L}_{\sigma\alpha\rho\nu}(\omega)], \tag{I.9}$$

or

$$\tilde{L}_{\mu\alpha\beta\nu}(\omega) = \tilde{L}^s_{\nu\alpha}(\omega)[\delta_{\mu\nu}\delta_{\alpha\beta} - 2\pi \sum_{\rho\sigma} \tilde{L}_{\mu\rho\beta\sigma} \tilde{I}_{\sigma\alpha\rho\nu}(\omega)].$$

Here, $\tilde{I}$ is the effective particle-hole interaction (irreducible vertex in the particle-hole channel). The shell-model response is defined by

$$\tilde{L}^s_{\nu\mu}(\omega) := (n_\nu - n_\mu) \frac{n_\nu + n_\mu}{\omega + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)}, \tag{I.11}$$

where $n_\nu, n_\mu$ are the quasiparticle occupation numbers. $\epsilon_\nu$ denotes the single-particle energy in the quantum state $\nu$. From the Eqs. (I.6), (I.7), (I.9), and (I.11) we can now deduce the equations for the scattering states:

$$\tilde{\rho}_{\nu\mu,S} = (n_\nu - n_\mu)[E_S + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)]^{-1} \times [\delta_{p\nu}\delta_{j\mu}(-i\eta) - 2\pi \sum_{\sigma\lambda} \tilde{I}_{\nu\sigma\mu\lambda}(\omega) \tilde{\rho}_{\lambda\sigma,S}], \tag{I.12}$$

where $\tilde{\rho}_{\nu\mu,M}$ is the matrix element of the quasiparticle density matrix

$$\tilde{\rho}_{\nu\mu,M} = (z_\nu z_\mu)^{-1/2} \langle 0 | \psi_\mu^\dagger \psi_\nu | M \rangle. \tag{I.13}$$

The knowledge of these matrix elements is equivalent to the knowledge of the states of the compound system. By $|M\rangle$ we denote a scattering state $|S\rangle$ as well as a bound state $|B\rangle$. Since we also want to calculate the effect of the continuum single-particle states on the bound states of the compound system, we have to give the system of equations for the matrix elements $\tilde{\rho}_{\nu\mu,B}$.[1,2] We derive these equations by inserting the spectral representation of the renormalized response function

$$\tilde{L}_{\nu\sigma\mu\beta}(\omega) = -\sum_M \left( \frac{\tilde{\rho}_{\nu\mu,M} \tilde{\rho}^*_{\beta\sigma,M}}{\omega - E_M + i\eta} - \frac{\tilde{\rho}_{\sigma\beta,M} \tilde{\rho}^*_{\mu\nu,M}}{\omega + E_M - i\eta} \right) \tag{I.14}$$

into Eq. (I.9) or (I.10) and taking the limit $\omega \to \pm E_B$. Then the pole terms give the following set of equations:

$$[E_B + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)] \tilde{\rho}_{\nu\mu,B} = -(n_\nu - n_\mu) 2\pi \sum_{\alpha\beta} \tilde{I}_{\nu\alpha\mu\beta} \tilde{\rho}_{\beta\alpha,B}, \tag{I.15}$$

$$[-E_B + \epsilon_\mu - \epsilon_\nu - i\eta)] \tilde{\rho}^*_{\mu\nu,B} = -(n_\nu - n_\mu) 2\pi \sum_{\alpha\beta} \tilde{I}_{\nu\alpha\mu\beta} \tilde{\rho}^*_{\alpha\beta,B}, \tag{I.16}$$

$$[E_B + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)] \tilde{\rho}^*_{\nu\mu,B} = -(n_\nu - n_\mu) 2\pi \sum_{\alpha\beta} \tilde{\rho}^*_{\beta\alpha,B} \tilde{I}_{\beta\mu\alpha\nu}, \tag{I.17}$$

$$[-E_B + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)] \tilde{\rho}_{\mu\nu,B} = -(n_\nu - n_\mu) 2\pi \sum_{\alpha\beta} \tilde{\rho}_{\alpha\beta,B} \tilde{I}_{\beta\mu\alpha\nu}, \tag{I.18}$$

which are equivalent to (I.12). $\sum_{\alpha\beta}$ means summation over the discrete states as well as integration over the continuum states. Insertion of the spectral representation into the equation of motion also gives the completeness relation:

$$\sum_M (\tilde{\rho}_{\nu\mu,M}\tilde{\rho}^*_{\lambda\sigma,M} - \tilde{\rho}_{\sigma\lambda,M}\tilde{\rho}^*_{\mu\nu,M}) = (n_\mu - n_\nu)\delta_{\nu\lambda}\delta_{\sigma\mu}, \tag{I.19}$$

which can be proved by complex integration.

## II. TREATMENT OF THE MODEL

In our approach we want to take into account the solutions of a nuclear-structure calculation. These solutions are known for many forms of the particle-hole interactions. See for instance Brown, Costillejo, and Evans,[6] also Haug and Weigel.[7] In addition see Gillet.[8] They are the result of the diagonalization of Eqs. (I.15)-(I.18) in a finite space, since all single-particle quantum numbers are restricted to bound states only. We denote the states resulting from this procedure by $|n\rangle$, and the corresponding density matrix elements are $\tilde{\rho}_{\nu\mu,n}$, where $\nu$ and $\mu$ belong to the discrete part of the spectrum of $H_S$. We assume in our further procedure that the $\tilde{\rho}_{\nu\mu,n}$ are known, since we can either obtain them by a standard procedure or we can use the results of a previous calculation. With the help of (I.19) we can express the unknown $\tilde{\rho}_{\nu\mu,M}$ equivalently by the matrix elements $\langle 0|C_n^\dagger|M\rangle$ and $\langle 0|C_n|M\rangle$, where $C_n^\dagger$ creates the state $|n\rangle$.[9] The connecting relations are:

$$\tilde{\rho}_{\nu\mu,M} = \sum_n (\langle 0|C_n^\dagger|M\rangle\tilde{\rho}^*_{\mu\nu,n} + \langle 0|C_n|M\rangle\tilde{\rho}_{\nu\mu,n}), \tag{II.1}$$

$$\langle 0|C_n|M\rangle = \sum_{\mu\nu}{}' (n_\mu - n_\nu)\tilde{\rho}^*_{\nu\mu,n}\,\tilde{\rho}_{\nu\mu,M}, \tag{II.2}$$

$$\langle 0|C_n^\dagger|M\rangle = \sum_{\mu\nu}{}' (n_\nu - n_\mu)\tilde{\rho}_{\mu\nu,n}\tilde{\rho}_{\nu\mu,M}, \tag{II.3}$$

where we have defined restricted sums as follows ($f_{\nu\mu}$ is an arbitrary function of $\nu$ and $\mu$):

$$\sum_{\mu\lambda}{}' f_{\lambda\mu} := \sum_{r,i}(f_{ri} + f_{ir}), \tag{II.4}$$

$$\sum_{\mu\lambda}{}'' f_{\lambda\mu} := \sum_{i,p}(f_{ip} + f_{pi}). \tag{II.5}$$

From Eqs. (I.15), (I.18), or (I.15), (I.17), respectively, we obtain the following equations for the "deviation" matrix elements[3]:

$$(E_n + E_M)\langle 0|C_n^\dagger|M\rangle = -2\pi\sum_{\mu\nu}{}'\sum_{\alpha\beta}{}'' \tilde{\rho}_{\mu\nu,n}\tilde{I}_{\nu\alpha\mu\beta}\tilde{\rho}_{\beta\alpha,M}, \tag{II.6}$$

$$(E_n - E_M)\langle 0|C_n|M\rangle = -2\pi\sum_{\mu\nu}{}'\sum_{\alpha\beta}{}'' \tilde{\rho}^*_{\nu\mu,n}\tilde{I}_{\nu\alpha\mu\beta}\tilde{\rho}_{\beta\alpha,M}. \tag{II.7}$$

According to our assumption, we approximate all matrix elements with at least one quantum number belonging to the continuum of $H_S$ by a schematic force

$$2\pi\tilde{I}_{\nu\alpha\mu\beta} = \lambda w_{\nu\mu}w_{\alpha\beta}. \tag{II.8}$$

With this approximation the solutions of (II.6) and (II.7) are

$$\langle 0|C_n^\dagger|M\rangle = -\lambda F_n^0 F_M \frac{1}{E_n + E_M}, \tag{II.9}$$

and

$$\langle 0|C_n|M\rangle = -\lambda F_n^{0*} F_M \frac{1}{E_n - E_M}, \tag{II.10}$$

where:

$$F_n^0 := \sum_{\nu\mu}{}' w_{\nu\mu}\tilde{\rho}_{\nu\mu,n}, \tag{II.11}$$

and

$$F_M := \sum_{\nu\mu}{}'' w_{\nu\mu}\tilde{\rho}_{\mu\nu,M}. \tag{II.12}$$

Since in the equations for $\tilde{\rho}_{pi,B}$ and $\tilde{\rho}_{ip,B}$ only matrix elements of the form (II.8) are involved, we can further obtain, by using Eqs. (II.1) and (II.8-12), the following solutions for these amplitudes:

$$\tilde{\rho}_{ki,B} = \frac{\lambda w_{ki}}{E_B - \epsilon_k + \epsilon_i + i\eta} F_B\left(1 - \lambda\sum_n |F_n^0|^2 \frac{2E_n}{E_n^2 - E_B^2}\right), \tag{II.13}$$

$$\tilde{\rho}_{ik,B} = \frac{-\lambda w_{ik}}{E_B + \epsilon_k - \epsilon_i} F_B\left(1 - \lambda\sum_n |F_n^0|^2 \frac{2E_n}{E_n^2 - E_B^2}\right). \tag{II.14}$$

Insertion of these solutions in the definition of $F_B$ gives the wanted dispersion relation for $E_B$:

$$1 = \lambda\sum_{ki}|w_{ki}|^2\left(\frac{1}{E_B - \epsilon_k + \epsilon_i + i\eta} - \frac{1}{E_B + \epsilon_k - \epsilon_i}\right)\left(1 - \lambda\sum_n |F_n^0|^2 \frac{2E_n}{E_n^2 - E_B^2}\right). \tag{II.15}$$

The amplitudes for the scattering states are derived from (I.12) by the same procedure. One obtains:

$$\tilde{\rho}_{ki,S}=\delta_{ij}\delta_{kp}+\frac{\lambda F_S w_{ki}}{E_S+\epsilon_i-\epsilon_k+i\eta}\left(1-\lambda\sum_n |F_n^0|^2\frac{2E_n}{E_n^2-E_S^2}\right), \qquad \text{(II.16)}$$

$$\tilde{\rho}_{ik,S}=-\frac{\lambda F_S w_{ik}}{E_S+\epsilon_k-\epsilon_i}\left(1-\lambda\sum_n |F_n^0|^2\frac{2E_n}{E_n^2-E_S^2}\right), \qquad \text{(II.17)}$$

where

$$F_S=w_{pj}\left\{1+\left[\sum_{ki}\lambda|w_{ki}|^2\left(\frac{1}{E_S+\epsilon_k-\epsilon_i}-\frac{1}{E_S-\epsilon_k+\epsilon_i+i\eta}\right)\right]\left[1-\lambda\sum_n |F_n^0|^2\frac{2E_n}{E_n^2-E_S^2}\right]\right\}^{-1}. \qquad \text{(II.18)}$$

Use of the normalization condition for the state $|B\rangle$ gives $F_B$ as follows:

$$F_B^2=\lambda^{-2}\left\{\left(1-\lambda\sum_n |F_n^0|^2\frac{2E_n}{E_n^2-E_B^2}\right)^2\sum_{ki}|w_{ki}|^2\left[\left(\frac{1}{E_B+\epsilon_k+\epsilon_i}\right)^2-\left(\frac{1}{E_B+\epsilon_k-\epsilon_i}\right)^2\right]\right.$$
$$\left.+\sum_{r,l}\left[\left|\sum_n\left(\frac{F_n^0}{E_B+E_n}\tilde{\rho}^*_{lr,n}+\frac{F_n^{0*}}{E_B-E_n}\tilde{\rho}_{rl,n}\right)\right|^2-\left|\sum_n\left(\frac{F_n^0}{E_B+E_n}\tilde{\rho}^*_{rl,n}+\frac{F_n^{0*}}{E_B-E_n}\tilde{\rho}_{lr,n}\right)\right|^2\right]\right\}^{-1}. \qquad \text{(II.19)}$$

Now, all unknown quantities have been expressed in terms of the solutions of the nuclear-structure problem and the matrix elements between scattering-scattering states or scattering-bound states, respectively. The (complex) resonances are given by the solutions of Eq. (II.15). From (II.16) and (I.8) we can now immediately read off the wanted expressions for the $S$ matrix and the $T$ matrix obtaining:

$$S_{ki,pj}(E_{pj})=e^{2i\delta_p}\delta_{kp}\delta_{ij}-2\pi i\,\delta(E_{pj}-E_{ki})T_{ki,pj}(E_{pj}) \qquad \text{(II.20)}$$

with

$$T_{ki,pj}(E)=e^{i(\delta_p+\delta_k)}\lambda w_{pj}w_{ki}\left(1-\lambda\sum_n |F_n^0|^2\frac{2E_n}{E_n^2-E^2}\right)$$
$$\times\left\{1+\left[\lambda\sum_{p'j'}|w_{p'j'}|^2\left(\frac{1}{\epsilon_{p'}-\epsilon_{j'}-E-i\eta}+\frac{1}{\epsilon_{p'}-\epsilon_{j'}+E}\right)\right]\left(1-\lambda\sum_n |F_n^0|^2\frac{2E_n}{E_n^2-E^2}\right)\right\}^{-1}. \qquad \text{(II.21)}$$

Our formulas reduce to the corresponding ones of Ref. 3, where a different method has been used, if we neglect Migdal's renormalization and use a schematic model for the finite RPA, too. We are going to perform some calculations with the described method in the oxygen- and calcium-region using density-dependent forces for the bound-state calculation.[7]

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