about 10%. The general effects of the RPA, however, are not pronounced.

The present results dictate that a more reliastic force be used as well as lower 2*s* and 1*d* energies. Calculations using the Tabakin potential are currently in progress and a brief report of the results will be given in the near future.

ACKNOWLEDGMENTS

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Scattering of 19–30-MeV Alpha Particles from C^{12} †

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Elastic and inelastic excitation functions and differential cross sections of α particles scattered from C^{12} have been measured. Excitation functions were taken from 18.90 to 30.06 MeV in 20-keV steps. Differential cross sections were measured at the energies of anomalies in the excitation functions. Optical-model and phase-shift fits to the elastic angular distributions were attempted without success. A statistical analysis of the excitation function indicates that the reaction is proceeding mainly through the compound nucleus with a coherence width Γ of 360 keV (c.m.).

INTRODUCTION

THE scattering of α particles from light nuclei has recently been of interest to several workers.[1] This interest has increased since the report by Singh *et al.*[2] of evidence for intermediate structure in these reactions. In a series of experiments on *s-d* shell nuclei, structure of two to three times the compound nuclear width was observed in the 22–27-MeV region of excitation in Si^{28}, Si^{30}, and P^{32}. Singh and others[3,4] have interpreted this as evidence for simple structure, possibly 4p-4h states.

An investigation of this same region in other light nuclei seemed worthwhile and feasible with the expanded energy range of the newer tandem accelerators. We began with an investigation of the α elastic and inelastic scattering from C^{12}.

α scattering from C^{12} has been studied as a function of energy and angle up to 19 MeV (for a list of references see Ref. 1). Above this energy, only scattered cyclotron data are available. For this reason we began our investigation at 18.9 MeV. Excitation functions were measured in 20-keV steps to 30.06 MeV. Angular distributions were then measured at the energies of the anomalies in the excitation functions.

EXPERIMENTAL TECHNIQUE

The doubly charged helium beam from the Williams Laboratory tandem Van de Graaff accelerator struck self-supporting targets in an ORTEC 17-in. scattering chamber. For the excitation functions, the data for elastic scattering from C^{12} and O^{16} were measured simultaneously, using LiOH evaporated on 20-μg/cm² carbon foils.[5,6] The differential cross sections were measured using 100-μg/cm² C^{12} foils. Monitor runs indicated that the carbon thickness increased linearly with time due to hydrocarbon contamination, increasing by about 18% in 6 days of exposure to the beam. The data were suitably corrected.

The accelerator energy was originally calibrated using

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[1] E. B. Carter, G. E. Mitchel, and R. H. Davis, Phys. Rev. **133**, B1421 (1964).

[2] P. P. Singh, B. A. Watson, S. S. Kroepfl, and T. P. Marvin, Phys. Rev. Letters **17**, 968 (1966).

[3] P. P. Singh (private communication).

[4] J. P. Schiffer, J. Phys. Soc. Japan Suppl. **24**, 319 (1968).

[5] J. F. Morgan, Ph.D. thesis, University of Minnesota, 1968 (unpublished).

[6] Clark Bergman, Ph.D. thesis, University of Minnesota, 1968 (unpublished).

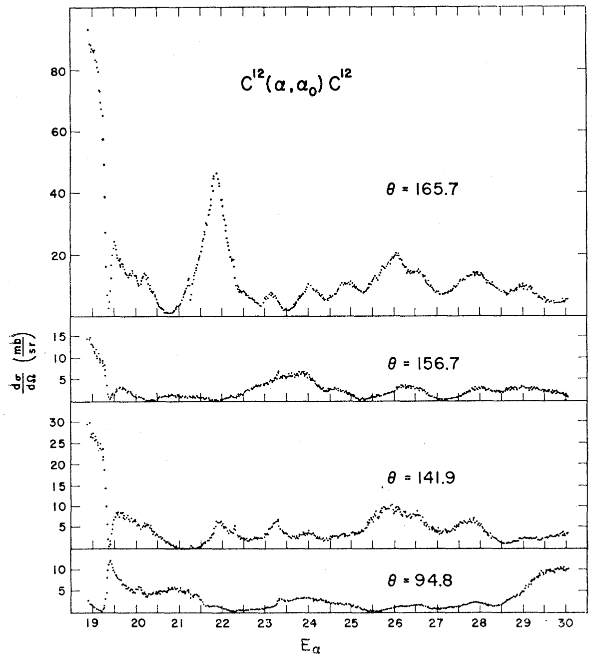

FIG. 1. Elastic excitation functions from $E_\alpha = 18.9$ to 30.06 MeV. The c.m. scattering angle is shown.

several (p, n) thresholds and subsequently checked to an accuracy of ±15 keV, using the $C^{12}(p, p')C^{12*}$ reaction at the 14.22-MeV resonance. The analyzing magnet geometry was such that the maximum energy dispersion of the beam was less than 20 keV. The peak-to-valley ratio of the $C^{12}(p, p')C^{12}$ resonance indicated that the actual energy spread was always much less than this.

For the excitation functions, eight ORTEC surface-barrier detectors were mounted on a 35-cm-diam ring which fit into the scattering chamber. This allowed each angle to be set independently to ±0.1°. Solid angles for the rectangular apertures, determined by measurement with a traveling microscope to 1% accuracy, were typically 0.500 msr; the angular acceptance was 0.5°.

For most of the differential cross-section measurements, three surface-barrier detectors were used, mounted 30° apart. Circular apertures gave an angular acceptance of 1.2° and a typical solid angle of 3.50± 0.04 msr.

The electronic equipment was conventional. Since only 4 ADC's were available, more than four detectors were accommodated by routing the ADC output.[6] All of the digitized signals were fed a CDC 3100 computer, in which a block of memory was assigned to each detector.[7] At the end of each run, the computer wrote the data on magnetic tape, automatically advanced the machine energy,[6] and the experimenter started the next run. Data were then read back from tape into another section of memory, converted to absolute cross sections, and written on another tape for future analysis.

Absolute normalizations were obtained separately by scattering α particles from methane in a 5-cm gas cell.[8] Care was taken to normalize the excitation function at energies and angles where the cross section does not vary rapidly with energy, since the energy spread in the gas cell was 50 keV. The normalization factor was calculated at three points and the results were consistent within 3%. The effect of multiple scattering in the exit foil of the gas cell[9] introduces a 5% uncertainty. The statistical error in the normalization was usually 3%, giving an over-all normalization error of 6%. The differential cross sections were normalized to the excitation functions.

The statistical accuracy of the final data varied due to the rapid fluctuation of yield with energy and angle; a cross section of 10 mb was measured to a statistical accuracy of approximately 3%. The accuracy of the beam charge measurement was better than 2%.

RESULTS

The excitation functions are shown in Figs. 1–4. The elastic excitation functions are dominated by the tail of the resonance at 18.7 MeV and broad structure at higher energies. This behavior is in direct contrast

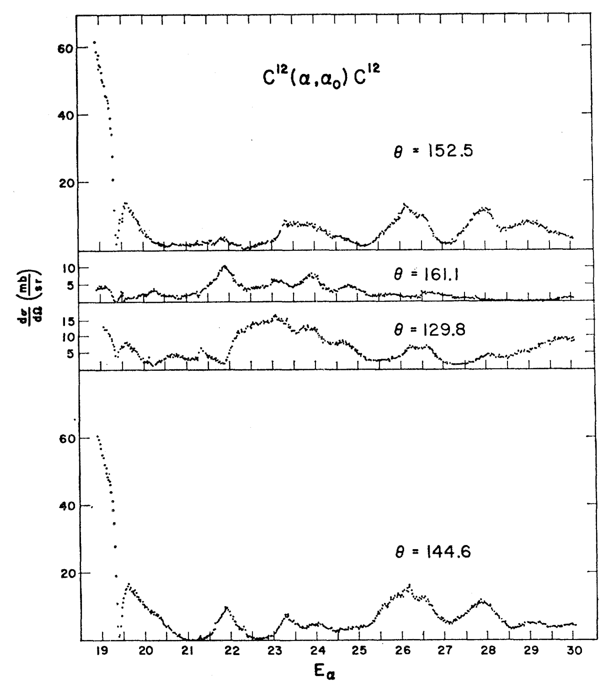

FIG. 2. Elastic excitation functions from $E_\alpha = 18.9$ to 30.06 MeV. The c.m. scattering angle is shown.

[7] R. K. Hobbie and R. W. Goodwin, Nucl. Instr. Methods **52**, 119 (1967); **58**, 342 (1968).

[8] Clarence Jacobs, MS thesis, University of Minnesota, 1967 (unpublished).

[9] T. C. Kan, MS thesis, University of Minnesota, 1969 (unpublished).

with the resonance structure seen in the same excitation region (22–27 MeV) of other light nuclei excited by elastic α scattering.[2,3,6] The inelastic data for scattering to the 4.43-MeV state show even less structure than the elastic data, but in general have a larger cross section.

In the region around 23.5 MeV of excitation, there is evidence for some narrow isolated structure superimposed on the slowly varying background. In an attempt to determine the nature of this, excitation functions over this region were taken with a 30-μg/cm^2 C^{12} target. The results are shown in Figs. 5 and 6. Each angle was chosen to be a zero of one of the first eight Legendre Polynomials. Two narrow resonances are evident, at α energies of 21.22 and 22.33 MeV, in both the elastic and inelastic channels. From the angles at which these disappear, they are determined to have spins and parities of 5^- and 6^+, respectively.

The differential cross sections are presented in Figs. 7–14. The elastic cross sections show the familiar diffraction structure, and are remarkably similar, especially above 23 MeV. The inelastic cross sections exhibit much less structure, with a gradual trend toward an increasing number of oscillations as the energy increases.

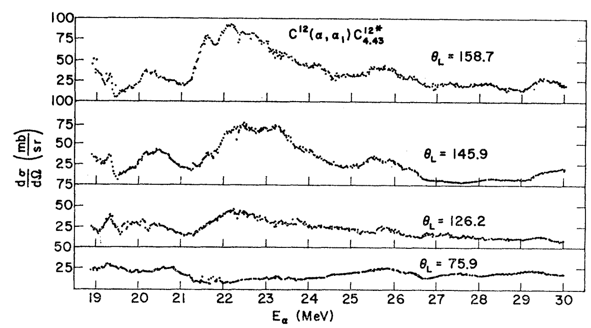

FIG. 3. Inelastic excitation functions from $E_\alpha=18.9$ to 30.06 MeV. The c.m. scattering angle is shown.

ANALYSIS

First, attempts were made to fit the angular distributions using the optical model. A standard optical model with Woods-Saxon form factors and a surface imaginary term was used. No fits to the entire range of data could be found despite several extensive grid searches on the geometry parameters, both "on" and "off resonance." The data was then truncated at 90° and attempts were made to fit the forward scattering, which should be dominated by potential scattering. In this manner, we were able to generate qualitative (i.e., in phase) fits to the data. The parameters are given in Table I. No further attempts were made to obtain optical-model fits to the data.

Another phenomenological method of analyzing α-particle elastic scattering is the Blair-McIntyre cutoff model.[10,11] The scattering amplitude is given by

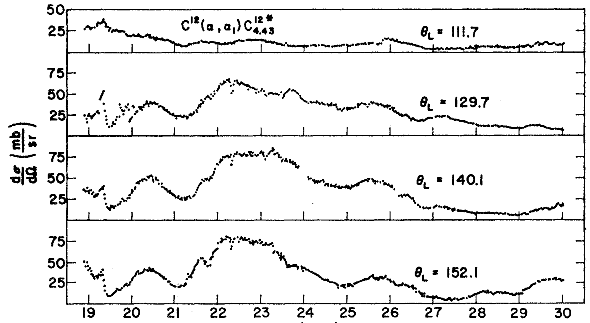

FIG. 4. Inelastic excitation functions from $E_\alpha=18.9$ to 30.06 MeV. The c.m. scattering angle is shown.

$$f(\theta)=f_C(\theta)+(i/2k)\sum_l(2l+1)\times\exp(2i\omega_l)[1-A_l\exp(2i\delta_l)]P_l\cos\theta,$$

where f_C is the Coulomb amplitude, and ω_l is the Coulomb phase shift. In this model, A_l and δ_l are given by

$$A_l=\{1+\exp[(l_A-l)/\Delta_A]\}^{-1},$$

$$\delta_l=\delta_0\{1+\exp[(l-l_d)/\Delta_d]\}^{-1}.$$

Grid searches were carried out on the five parameters l_A, Δ_A, δ_0, l_d, and Δ_d. When the deepest minimum was found, the parameters were allowed to vary to find a converged minimum. The results were quite ambiguous. None of the angular distributions were well fit. Even when the model gave the correct number of oscillations, the predicted cross sections were low, especially at back angles. More important, the parameters varied in a random fashion with energy, in contrast to qualitative theoretical predictions.

Because of these difficulties it was decided to attempt a complete phase-shift analysis of the data by taking A_l and δ_l as the parameters. A starting point for the phase shifts was obtained by using the smooth cutoff phases for the angular distribution best fit by the Blair-McIntyre model.[2,3,12] The choice of starting parameters may have had a significant effect on the results (see below). For 27-MeV α particles $kR\sim 8$, which implies that at least nine partial waves are needed in the cal-

TABLE I. Optical-model parameters for the data forward of 90° c.m. The fits had χ^2 per degree of freedom of about 3.

$V=140$ MeV	$W_{\text{surface}}=5.8$ MeV
$R_r=1.30$ F	$R_i=1.75$ F
$a_r=0.65$ F	$a_i=0.75$ F

[10] J. S. Blair, Phys. Rev. **95**, 1218 (1964).
[11] J. A. McIntyre, K. H. Wong, and L. C. Becher, Phys. Rev. **107**, 1337 (1960).
[12] E. B. Carter, Phys. Letters **278**, 202 (1968).

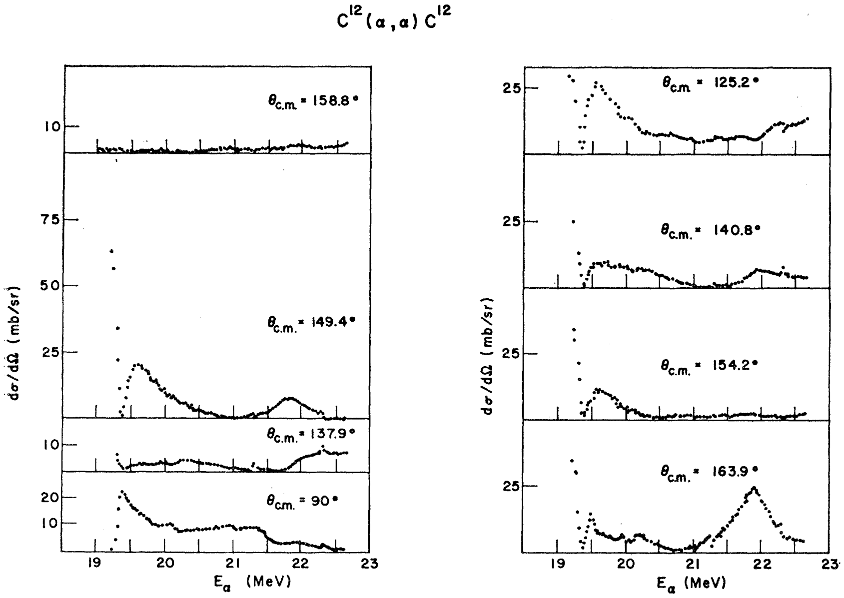

FIG. 5. Elastic excitation functions from $E_\alpha = 19.0$ to 22.64 MeV.

culation. Stringent requirements were placed on the parameters in an attempt to insure a consistent set of phase shifts.[5,6] The assumption was made that one or at most two partial waves were responsible for the deviation from potential scattering. Defining

$$\psi_l = |A_l \exp(2i\delta_l)|$$

for each partial wave, $d\sigma_l/d\psi_l$ was calculated. The parameters of the partial wave with the largest derivative were allowed to vary, the derivatives again examined, and the process repeated until the best fit, in the least-square sense, was obtained. The final parameters obtained by this method were then used as the starting parameters for the next higher energy. In this manner, phase shifts were obtained over the entire energy range.

Using the above procedure, it was not possible to obtain a consistent set of phase shifts. The individual angular distributions were well fit (χ^2 per degree of freedom was in the range 2–4), and a smooth variation of the phase shifts with energy was obtained. However, working down in energy from the highest-energy set of phase shifts inevitably produced a different set of phases. (Allowing more than one partial wave to vary at a time or picking the partial waves to be varied in a random order had no effect on the above results.) We attribute this to two causes. Since no angular distribution was well fit by the cutoff model, the calcula-

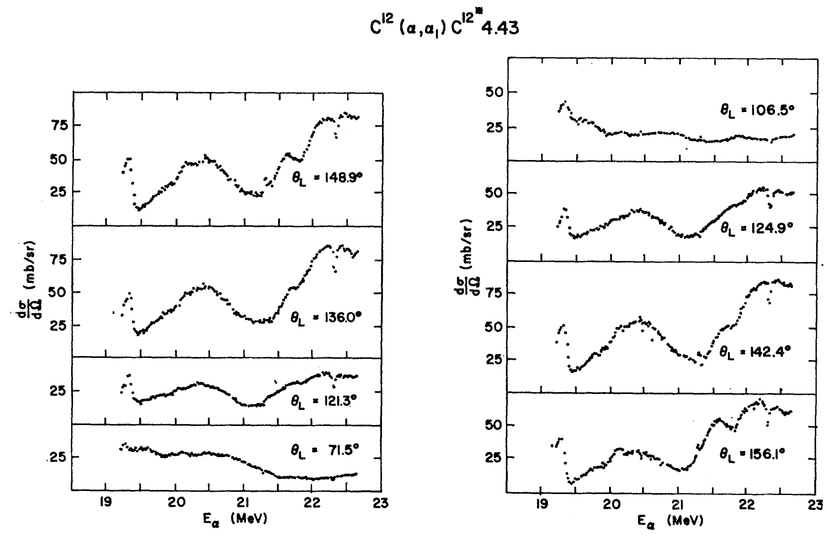

FIG. 6. Inelastic excitation functions from $E_\alpha = 19.0$ to 22.64 MeV.

tion was being started far from the physical phase shifts and could not converge to them. Also the spacing of the angular distributions (300 keV in the compound nucleus) made continuation over the energy range difficult, despite the lack of narrow structure. These conclusions are reinforced by the fact that given one angular distribution well fit by the cutoff model and small energy intervals, Singh[3] has been able to obtain closed sets of phase shifts using essentially the same procedure.

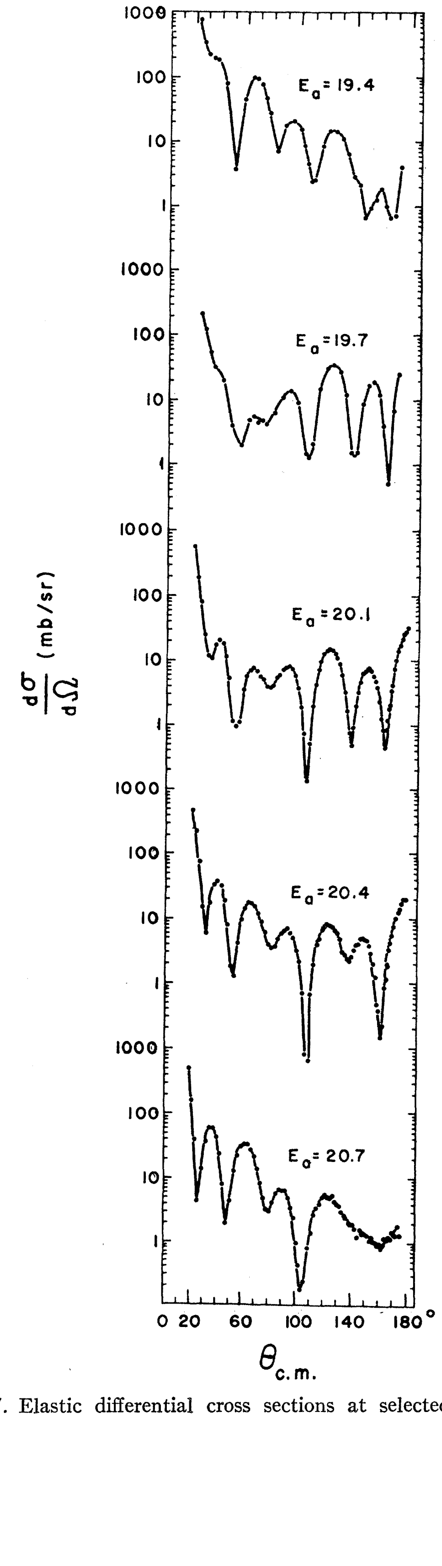

FIG. 7. Elastic differential cross sections at selected energies.

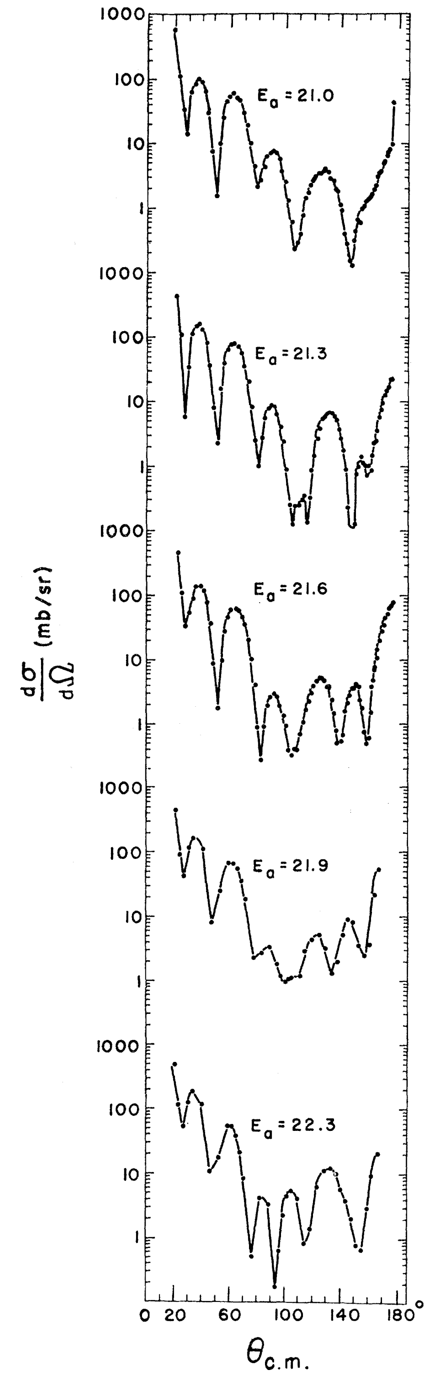

FIG. 8. Elastic differential cross sections at selected energies.

At this time, the results of Carter's[12] phase-shift analysis of all the available $C^{12}(\alpha, \alpha)C^{12}$ data were published. Using his phase shifts as a starting point,

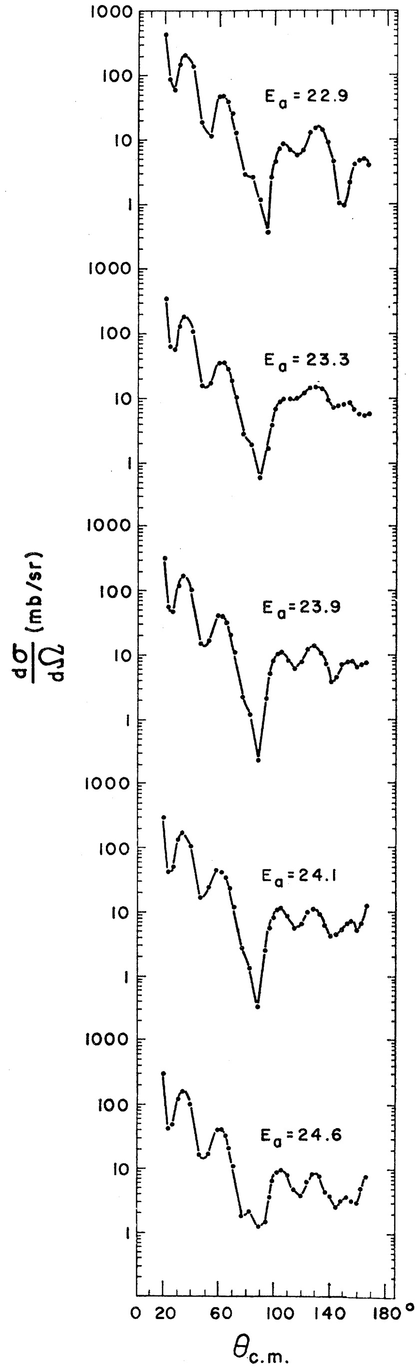

FIG. 9. Elastic differential cross sections at selected energies.

another attempt was made to fit the data. Interpolation from his graphs at 21.0 MeV provided a set of starting parameters for generating a fit. The final parameters at 21.0 MeV did not differ much from the initial values, and what changes there were are presumably do to errors in reading the graphs. Using these final phases, we proceeded as above to trace out the phase shifts as a function of energy. In this manner a smooth set of phase shifts was obtained giving equally good fits to the individual angular distributions but differing significantly from any previous set we had determined and from Carter's published values. As before, it was impossible to work backward from the end points and obtain the same values for the phase shifts.

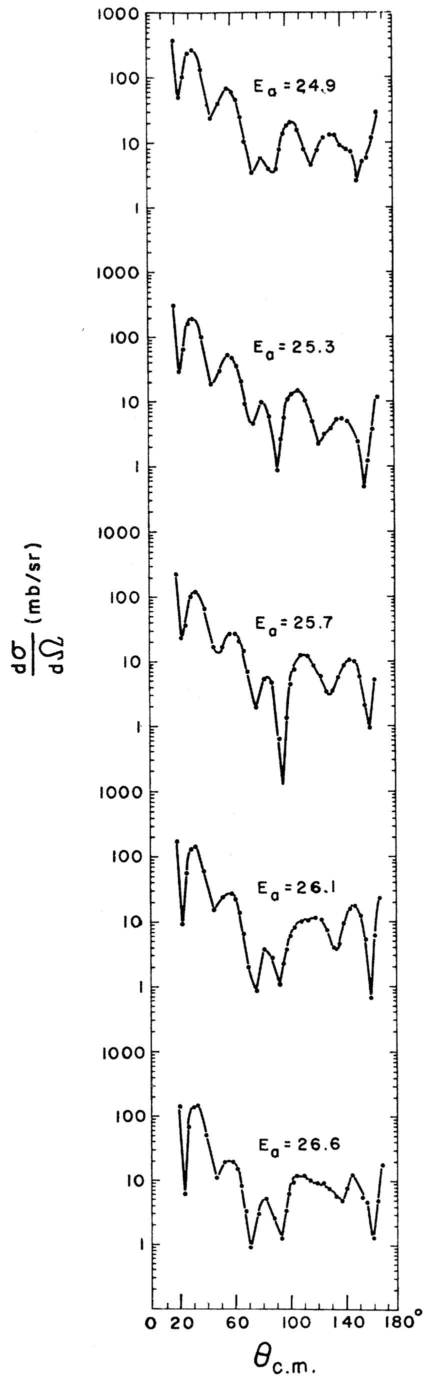

FIG. 10. Elastic differential cross sections at selected energies.

The excitation energy in the compound nucleus for this experiment was 22–29 MeV. At this energy one would expect to be in a region of many overlapping levels. Preliminary calculations, based on some estimates given by Ericson,[13] give a density of 250 levels per MeV at 25 MeV and a width, based on the neutron lifetime, of 30 keV. Since neither estimate takes into account angular momentum effects, they are known to badly underestimate the width and overestimate the density.[14] Also, in light nuclei the proton width is usually as large as the neutron width and the α width is larger than either. Assuming the width to be too small by three or four would boost the estimate to 120–150

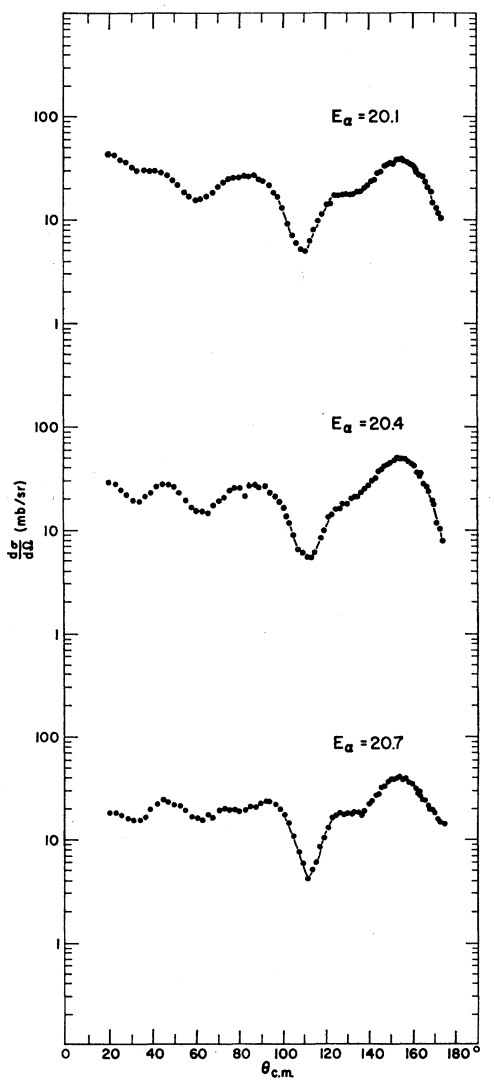

FIG. 11. Inelastic differential cross sections with the C^{12} left in its 4.43-MeV first excited state.

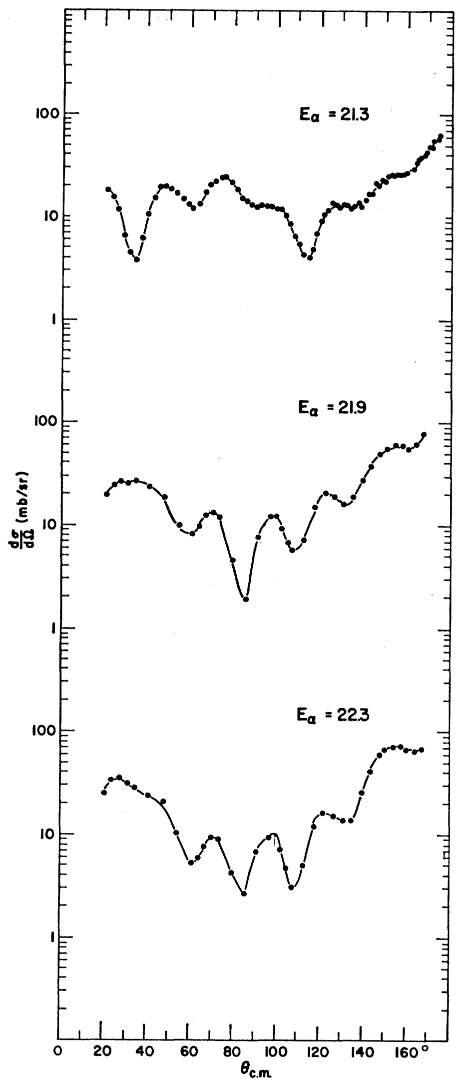

FIG. 12. Inelastic differential cross sections with the C^{12} left in its 4.43-MeV first excited state.

keV, compared to a value of 230 keV from the analysis of $N^{15}(p,\alpha)C^{12}$ at 16-MeV excitation.[15] Assuming the level spacing is underestimated by an order of magnitude for the higher angular momentum states, it is still reasonable to assume that Γ/D is greater than 1 and a statistical analysis will be meaningful.

The basis of the statistical analysis of the excitation function was the autocorrelation function[16]

$$C(\epsilon)=\frac{\langle[\sigma(E)-\langle\sigma\rangle][\sigma(E+\epsilon)-\langle\sigma\rangle]\rangle}{\langle\sigma(E)\rangle\langle\sigma(E+\epsilon)\rangle},$$

where $\sigma(E)$ is the differential cross section at some fixed angle as a function of excitation energy and the brackets indicate an energy average.

[13] T. Ericson, Phil. Mag. Suppl. **9**, 36 (1960).
[14] T. G. Dzubay, Phys. Rev. **158**, 977 (1967); Ph.D. thesis, University of Minnesota, 1966 (unpublished).
[15] G. M. Temmer, Phys. Rev. Letters **12**, 330 (1964).
[16] T. Ericson and T. Meyer-Kuchuk, Ann. Rev. Nucl. Sci. **16**, 183 (1966).

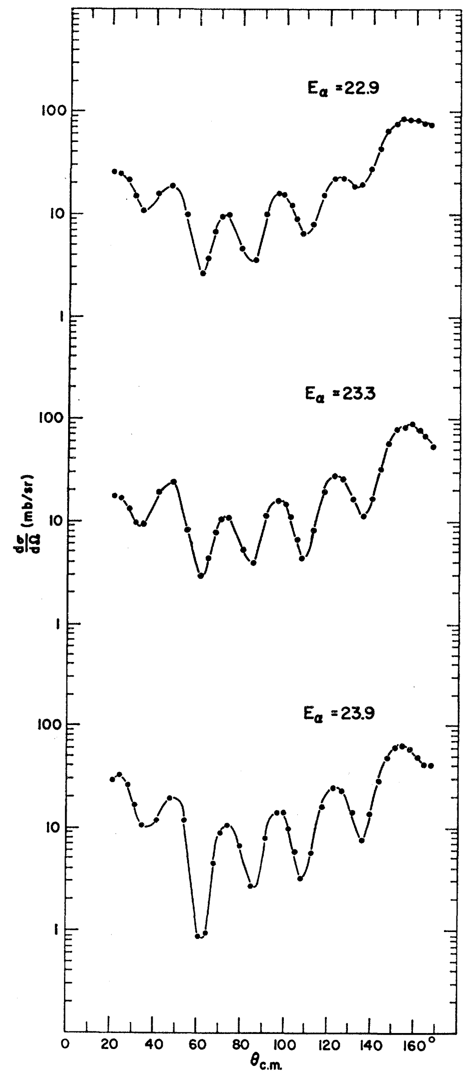

FIG. 13. Inelastic differential cross sections with the C^{12} left in its 4.43-MeV first excited state.

For an infinite range of data, the autocorrelation function for a purely statistical process is given by[16]

$$C(\epsilon)=(1/N)[\Gamma^2/(\Gamma^2+\epsilon^2)],$$

where Γ is the correlation width and N the number of degrees of freedom open to the reaction. N is always less than or equal to $N_{\max}$, where $N_{\max_{g/2}}=\frac{1}{2}g$ (g even) or $\frac{1}{2}(g+1)$ (g odd), and g is the product of the spin factors for the initial and final states. For a wide range of angles around 90°, N may be taken equal to $N_{\max}$.[16] The finite range of the data effects the calculations in a well-studied manner.[17]

The experimental autocorrelation functions are shown in Fig. 15. The data at 129.8° and 161.1° show a marked departure from the Lorentz shape predicted above. Inspection of the data shows that these excitation functions are dominated by a group of structures between 22 and 25 MeV. It has been noted previously that when this occurs, the autocorrelation function will depart from the theoretical prediction.[18] For the six angles well

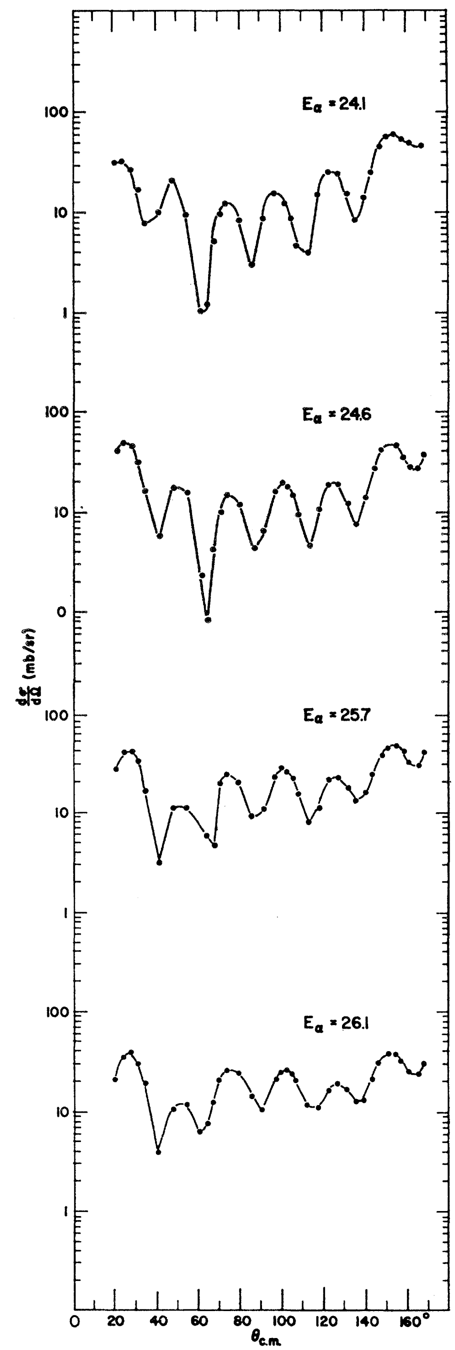

FIG. 14. Inelastic differential cross sections with the C^{12} left in its 4.43-MeV first excited state.

[17] P. J. Dallimore and I. Hall, Nucl. Phys. **88**, 193 (1966).

[18] P. J. Dallimore and B. W. Allardyce, Nucl. Phys. **A108**, 150 (1968).

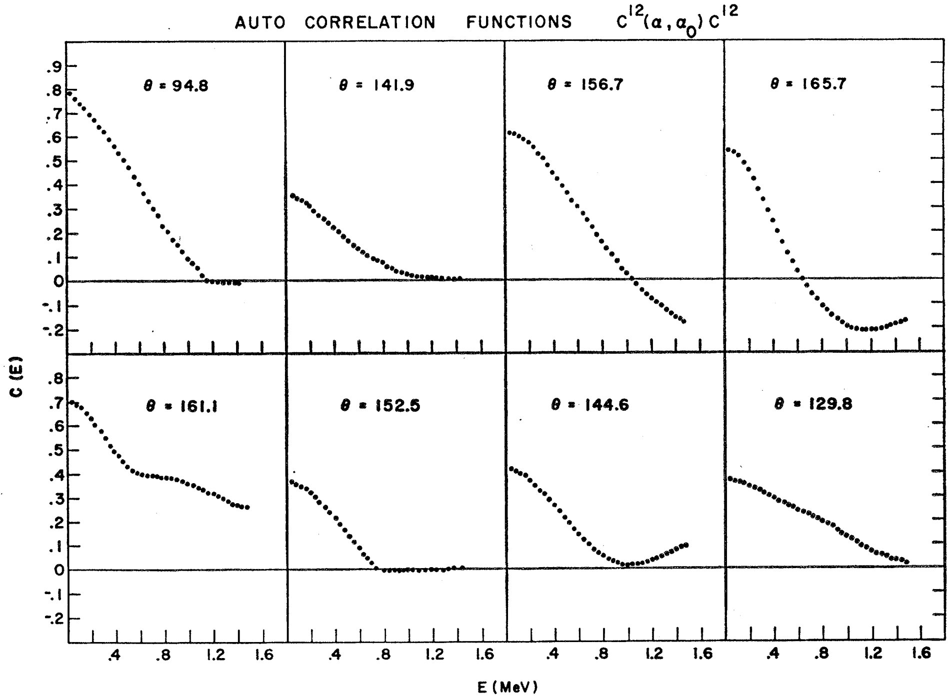

FIG. 15. Autocorrelation functions for various angles in the elastic channel.

represented by the Lorentz shape, the average Γ is 480 keV in the laboratory system or 360 keV in the c.m. system.

A further check on the statistical nature is the calculation of the cross-correlation function

$$C(\alpha,\alpha')=[\langle\sigma(\alpha)\sigma(\alpha')\rangle/\langle\sigma(\alpha)\rangle\langle\sigma(\alpha')\rangle]^{-1},$$

where $\sigma(\alpha)$ is the cross section at a given energy and angle in the reaction channel α, and the brackets indicate an average over energy. For a purely statistical reaction, $C(\alpha,\alpha')$ is zero. Taking into account the finite range of data effects[17] and the experimentally determined coherence width $\Gamma=360$ keV, the prediction is

$$C(\alpha,\alpha')=0.0\pm0.13.$$

Table II gives the experimentally determined values between the ground state and first excited state, for the various angles which were measured. In all but two cases, the numbers agree within the errors. The values are scattered on both sides of zero, as one would expect for errors introduced by finite range effects.

TABLE II. Cross-correlation between the elastic and inelastic channels for various angles. The predicted value is 0.0, with an uncertainty of ±0.13 introduced by the finite range of the data.

θ lab	$C(\alpha,\alpha')$
75.9	0.07
126.6	0.01
145.9	0.16
158.7	0.18
140.1	−0.13
128.7	−0.06
111.7	0.03

DISCUSSION

The most striking feature of the data is the lack of structure above 19 MeV. For other light nuclei in this same excitation region, the excitation functions are qualitatively different. In α scattering on the Mg isotopes, structure with a width on the order of 150 keV is seen at these energies.[1] In O^{16}, one sees structure on the order of 100–500 keV [6] up to an energy of about 25 MeV, above which it gradually disappears. Recent measurements of C^{12} done below 16 MeV in 40–60-keV steps, reveal the same type of structure seen in the Mg isotopes.[3] At various isolated energies in this experiment there is evidence for 40-keV structure, but it is small and does not dominate the cross section as in the other reactions discussed above. It is probable that these few isolated resonances represent special states which are able to retain their identity in the background of the larger number of overlapping levels. Since recent experiments indicate that the isospin-forbidden excitation of analog states may take place in α scattering,[19] and since we are well above the energy (∼13 MeV) of the

[19] J. F. Morgan and A. R. Barnett, Williams Laboratory Progress Report No. AEC COO-1265-67, 1968, p. 18 (unpublished).

first $T_>$ states in O^{16}, the narrow isolated structure may be related to this effect.

The other striking feature of the data is that, in general, the magnitude of the cross section of the inelastic scattering to the 4.43-MeV state in C^{12} is larger than that to the ground state. This same effect has been noted in the scattering at lower energies.[20] Since it is now believed that the 4.43-MeV state is the second member of a rotational band built on the deformed ground state of C^{12}, this effect may be analogous to the "anomalous" proton scattering, observed by Cohen[21] and explained by Pinkston and Satchler[22] by the similarity between the inelastic nucleon matrix elements and the $BE(L)$ of the state.

Since neither the optical model nor the smooth cutoff model gives a good representation of the potential scattering in this region, most of the conclusions concerning the spectroscopy of O^{16} are negative. Carter[12] has recently published the results of a phase-shift analysis of α scattering from C^{12} between 20.0 and 37.0 MeV using isolated cyclotron differential cross sections. On the basis of this analysis, he suggests states at α energies of 20.4, 20.48, 21.1, and 25.2 MeV with widths between 0.3 and 0.6 MeV, and states at 21.0, 22.12, 28.8, and 29.0 MeV with widths greater than 1.0 MeV. Examination of the excitation functions reveals no evidence for these states, although the wider states might be difficult to observe. That these states are not actually seen is not surprising because of the difficulty in making an energy-dependent phase-shift analysis.

We also see no evidence for the 27.6-MeV state of O^{16} which has been observed in the $N^{14}(d, \alpha)C^{12}$, $C^{13}(He^3, \alpha)C^{12}$, and $C^{13}(He^3, He^3)C^{13}$ reactions.[23] This probably confirms the assignment of this state as the 1p-1h contribution to the total 3^- compound state of O^{16}, which would be difficult to excite in the C^{12}-α channel.

Finally, there is no experimental evidence for the existence of an 8^+ state which would be a member of the first rotational band in O^{16}. Various estimates have placed this state between 18 and 22 MeV of excitation depending on the amount of band kinking allowed in the calculation. We see no evidence in any of the data for structure which could be assigned a spin of 8, although anything of width greater than 40 keV should have been observed.

The results of the statistical analysis of the excitation functions are ambiguous. The shape of the autocorrelation function and the value of the 94.8° function at $\epsilon=0$ are consistent with a statistical interpretation of the reaction. Also the values of the cross correlations to the inelastic channel are not inconsistent with a statistical interpretation. However, the finite range of the data introduces such a large error in the estimate of these quantities that the agreement between theory and experiment may be meaningless.

The experimentally determined correlation width, $\Gamma = 360$ keV is not inconsistent with other values determined for O^{16} at lower excitation energies.[15] On the other hand, examination of the experimental data indicates the presence of angular cross correlations over a range of approximately 20°. One would expect for a statistical process an angular correlation given by $\delta \simeq 1/kR$.[18] For 27-MeV α particles kR is approximately 8, implying a correlation angle of $\sim 7.5°$. This same effect has recently been seen in other $\alpha + C^{12}$ reactions, where autocorrelations and cross-correlations have indicated a statistical process while the correlation angle is larger than the statistical estimate.[24]

In conclusion, the following facts may be drawn from the experiment. The data exhibit a significant change above 20.0 MeV. The same change seems to occur in the elastic scattering of α particles from O^{16} at a somewhat higher energy (25 MeV). Neither the optical model nor the smooth cutoff model give a good representation of the scattering in this energy region. The reaction seems to proceed mainly through a background of overlapping resonances in the compound nucleus with some narrow, isolated resonance superimposed on this background. The nature of these isolated resonances is not well understood.

ACKNOWLEDGMENTS

We wish to thank Professor Norton Hintz for suggesting this experiment and for many lengthy discussions of the results, and the entire Williams Laboratory crew for their assistance. Special thanks go to Dr. Clark Bergman for supplying the oxygen-carbon targets and for his aid in all phases of the experiment and its analysis.

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[21] B. L. Cohen and A. G. Rugin, Phys. Rev. **111**, 1568 (1958).

[22] W. T. Pinkston and G. R. Satchler, in *Proceedings of the International Conference on Nuclear Structure, Kingston, Canada, 1960*, edited by D. A. Bromley and E. W. Vogt (North-Holland Publishing Co., Amsterdam, 1960), p. 394.

[23] H. R. Weller, N. R. Roberson, and D. R. Tilley, Phys. Letters **25B**, 541 (1967).

[24] J. L. Black, H. M. Kuan, W. Gruhle, M. Suffert, and G. L. Latshaw, Nucl. Phys. **A115**, 683 (1968).