the ground state, it appears that the $g_{7/2}$ plus phonon component is overestimated by them for the 343-keV state.

In view of these facts, it is felt that it would be instructive to have the level scheme in cadmium-111 examined on the basis of the intermediate-coupling approach of the unified nuclear model with $3s_{1/2}$, $2d_{5/2}$, and $2d_{3/2}$ orbits available to the odd neutron. The expansion coefficients in these wave functions could then be used to obtain the theoretical transition probabilities, so that they could afford a means of comparison with the present experimental values.

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Method for Determining Spins of Neutron Resonances*

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The proposed method for measuring the spin of S-wave neutron resonances is based on the detailed properties of the low-energy capture γ-ray spectrum. Measurements on nuclei with resonances of known spin show that this method gives reliable spin assignments. It is also applied to unassigned resonances in ^{167}Er and ^{187,189}Os.

1. INTRODUCTION

The spin J of the state formed in resonance neutron reactions is one of the most difficult of the resonance parameters to measure—even though for S-wave resonances it takes on only one of two values, namely, $J=I\pm\frac{1}{2}$, where I is the spin of the target nucleus. A number of experimental methods exist[1]; these include (a) the use of polarized neutron beams and samples, (b) parameter analysis of resonance data (thin-sample scattering and capture yields and transmission areas for both thick and thin samples) to determine the spin weighting factor $g_J=\frac{1}{2}(2J+1)/(2I+1)$, and (c) the various spin-dependent properties of the γ-ray spectrum emitted as a result of resonance neutron capture. Group (c), the largest of the three classes, has been reviewed by Bollinger.[2] This review includes the method of γ-ray multiplicities, recently investigated by Coceva *et al.*,[3] and also a preliminary report of the present method in which the low-energy γ-ray spectrum following resonance neutron capture is used to determine the population of low-lying levels having known spin.

Early measurements by Draper *et al.*[4] have shown that the shape of the low-energy γ-ray spectrum from resonance capture in ^{115}In differ for several resonances, but they were limited by the resolution of the sodium iodide scintillation spectrometers. Subsequent measurements by Fenstermacher *et al.*[5] for thermal and resonance capture in ^{167}Er and ^{177}Hf gave intensities of low-energy transitions which were quantitatively interpreted by Huizenga and Vandenbosch[6] to make spin assignments for the resonances. In the case of ^{177}Hf, these spins proved to be correct, but because of the large experimental errors, the resulting spin for the 0.46-eV resonance in ^{167}Er disagrees with the currently accepted value.[7]

The present studies are, in principle, similar to those cited above, but are carried out with the use of a high-resolution Ge(Li) γ-ray spectrometer and are a systematic extension of a preliminary high-resolution measurement by Prestwich and Bollinger,[8] who investigated low-energy γ rays from resonant capture in Ru and

* Work performed under the auspices of the U.S. Atomic Energy Commission.
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[1] J. E. Lynn, *The Theory of Neutron Resonance Reactions* (Clarendon Press, Oxford, 1968).
[2] L. M. Bollinger, in *Slow Neutron Physics*, edited by J. A. Harvey (Academic Press Inc., New York, to be published).
[3] C. Coceva, F. Corvi, P. Giacobbe, and G. Carraro, Nucl. Phys. **A117**, 586 (1958).
[4] J. E. Draper, C. Fenstermacher, and H. L. Schultz, Phys. Rev. **111**, 906 (1968).
[5] C. A. Fenstermacher, J. E. Draper, and C. K. Bockelman, Nucl. Phys. **10**, 386 (1959).
[6] J. R. Huizenga and R. Vandenbosch, Phys. Rev. **120**, 1305 (1960).
[7] *Neutron Cross Sections* (U.S. Department of Commerce, Springfield, Va.), 2nd ed., Suppl. No. 2.
[8] W. E. Prestwich and L. M. Bollinger (private communication).

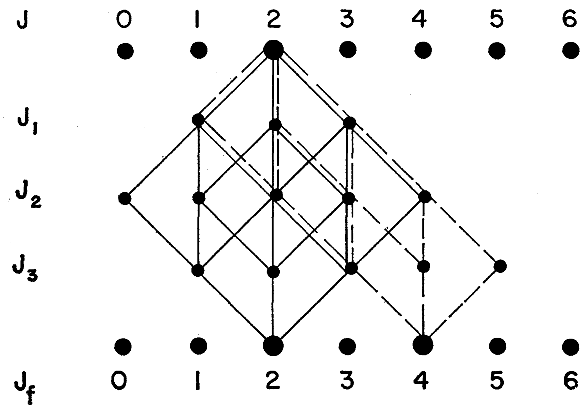

FIG. 1. Schematic representation of a four-step dipole cascade from a spin-2 initial state to both a spin-2 (solid lines) and a spin-4 (dashed lines) final state. The relative populations of the spin-4 and spin-2 final states are 10 and 19, respectively, so that $R_{42}\approx 0.5$.

found that the γ-ray intensities were correlated with the resonance spin.

2. THEORY

We consider resonant S-wave neutron capture in medium-weight even-Z odd-N target nuclei to form even-even final nuclei. The state thus formed has an excitation energy $E_x \approx B_n + E_n$ relative to the $J^\pi = 0^+$ ground state of the $A+1$ product nucleus. [Here B_n is the neutron binding energy ($\sim$5–8 MeV) and E_n is the neutron energy.] Transitions to the ground state will proceed largely by γ-ray cascades through the intermediate and low-lying levels, the latter typically including the well-spaced sequence of 2^+, 4^+, 6^+ rotational or 2^+, 0^+, 2^+, 4^+ vibrational states. These nuclei provide an opportunity to study how the capturing-state spin affects the relative population of discrete low-lying states with known spin. The existence of these few well-spaced levels through which the cascade must proceed also ensures that an appreciable intensity of γ rays will originate at each level. This cascade process has been treated by Huizenga and Vandenbosch[6] in an extension of their study of the ratios of cross sections for exciting isomers in the (n,γ) and (γ,n) reactions; it has also been recently discussed by Pönitz.[9]

In view of these existing theoretical discussions and because the emphasis of the present work is to empirically investigate the usefulness of this method, these population ratios will not be calculated in detail. It suffices to mention that similar results which point out the essential characteristics of the γ-ray cascade process can be derived in a simple way by making the reasonable assumptions that (i) the intermediate levels can be treated statistically (i.e., we consider a density of levels rather than a succession of discrete levels),

[9] W. P. Pönitz, Z. Physik **197**, 262 (1966); Bull. Am. Phys. Soc. **13**, 1389 (1968).

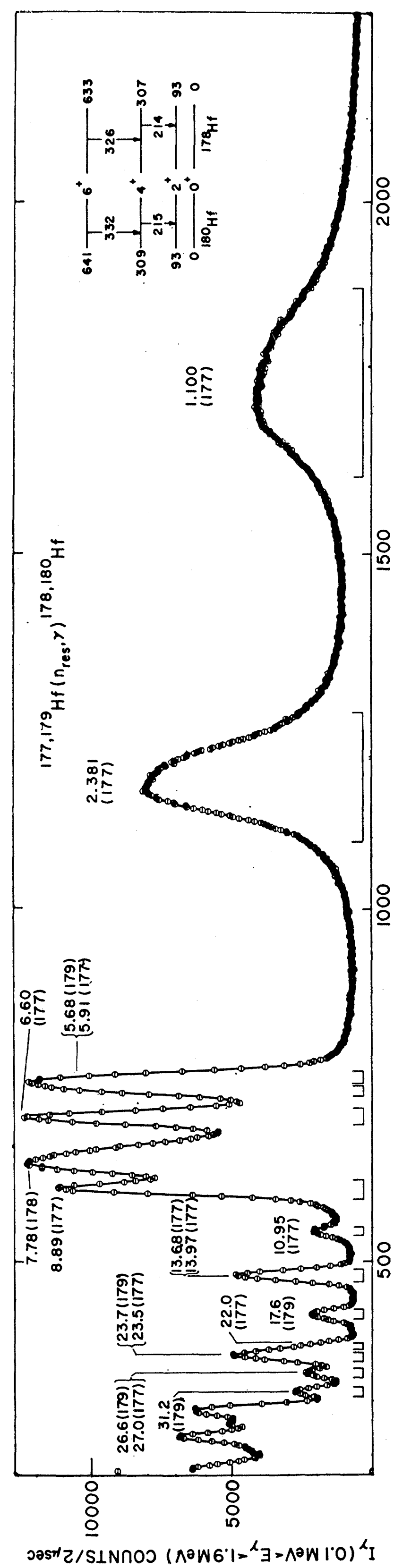

FIG. 2. Time-of-flight spectrum of γ rays (0.1 MeV$<E_\gamma<$1.8 MeV) from resonance neutron capture in a target of natural hafnium. Neutron energies (in eV) and isotopic assignment are given for the resonances. The energy-level diagrams for low-lying levels of ^{178,180}Hf are shown in the inset. The time-of-flight regions used in the determination of R_{ab} are indicated below the corresponding resonances.

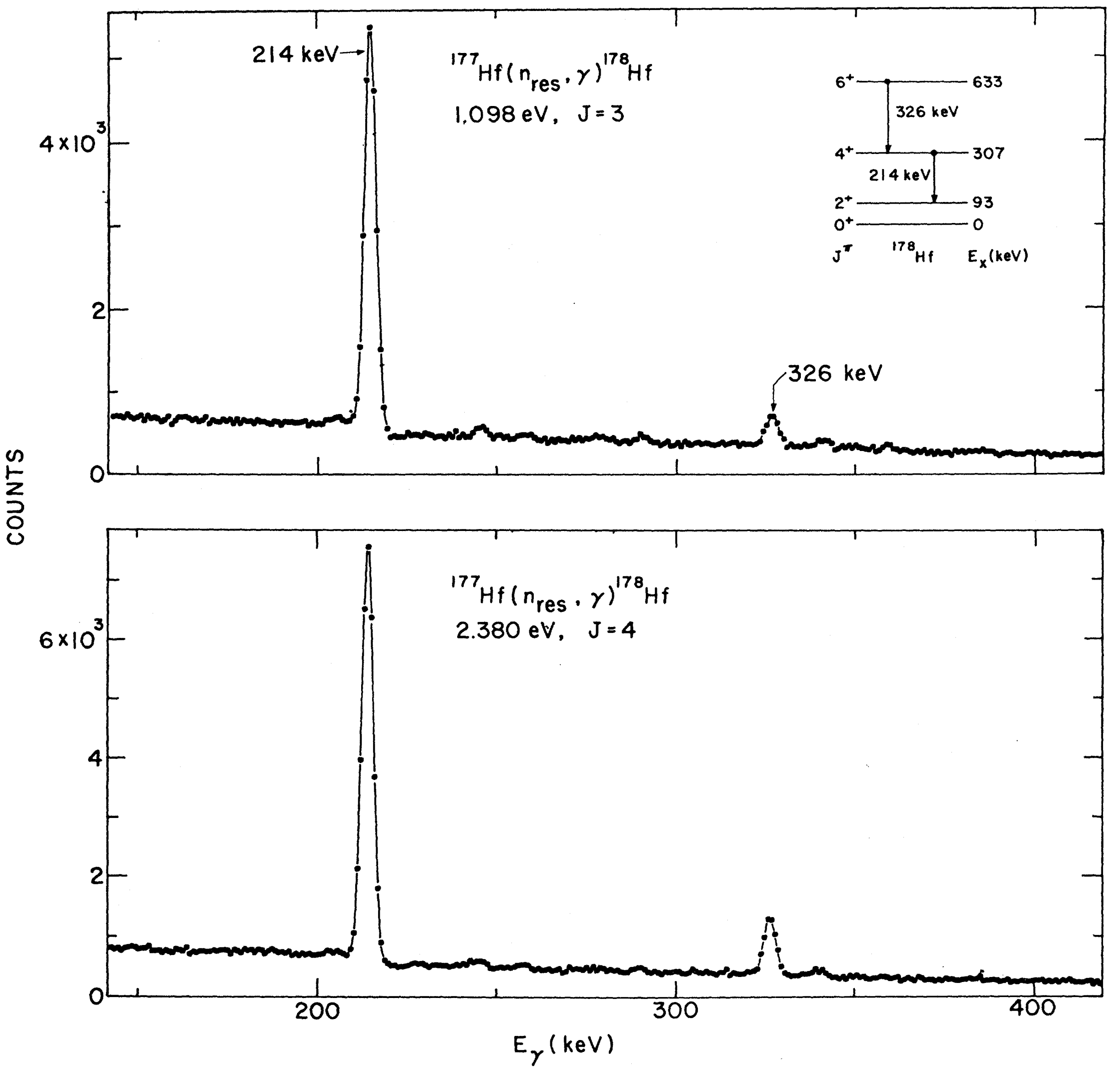

FIG. 3. A portion of the low-energy γ-ray spectrum from capture in the two lowest-energy resonances in ^{177}Hf. These spectra include the transitions deexciting both the lowest 6^+ and 4^+ levels as shown in the inset.

(ii) only radiative dipole transitions occur in the cascade, thereby restricting the intermediate levels to those allowed by $\Delta J=0, \pm 1$, (iii) the transition probability Γ_{ij} for any step $i\to j$ in the cascade is proportional to the level spacing $D(E_i, J_i) \propto \rho(E_i, J_i)^{-1}$ at the radiating state, and (iv) the energy dependence of the cascade is virtually the same for either capturing-state spin. In this simple model a number of factors are neglected which must be included in any detailed description of the γ-ray cascade process.[3,6,9] Among them are the dependence of the level density on the spin cutoff factor σ, the possible mixing of quadrupole transitions, the treatment of the low-lying levels, including the final state, in a nonstatistical manner, and the use of a distribution of values for the γ-ray multiplicity.

This simple description of the γ-ray cascade process gives the result that the probability of populating a low-lying level J_f after an n-step cascade is approximately proportional to the number of independent ways by which the capture state J can decay to this level under the restriction $\Delta J=0, \pm 1$ for each step in the cascade. [Here we omit any proportionality factors that cancel when the relative population R_{ab} of two final levels J_a, J_b formed by cascades from the same initial state J is computed.] This process is shown schematically in Fig. 1 for the example of a four-step dipole cascade from a spin-2 capture state to both a 2^+ (solid lines) and a 4^+ (dashed lines) final state. In this example, the relative population of these final states differs by a factor of 2 so that the ratio is $R_{42}\approx 0.5$. However, for a spin-3 capturing state this simplified model would clearly give equal population, $R_{42}\approx 1$, so that the ratio R_{42} is in this way sensitive to J. The corresponding relative population probabilities for final

TABLE I. Number of independent ways of populating a given level J_f from an initial state J by a four-step dipole cascade.

J_f \ J	0	1	2	3	4	5	6
0	3[a]	12[a]	6[a]	3[a]	1	0	0
2	6[a]	15[a]	19	16	10	4	1
4	1	4	10	16	19	16	10
6	0	0	1	4	10	16	19

[a] Includes the selection rule forbidding the $0\rightarrow0$ transition in the cascade.

states with $J_f=0$, 2, 4, or 6 formed by four-step cascades from capturing states with spin $J=0$–6 are summarized in Table I. A three-step dipole cascade would give similar results. Table I provides an adequate framework for the discussion of the results to be presented below; in fact, the quantitative agreement with these results is also generally quite good.

3. DATA ACQUISITION AND ANALYSIS

A. Experimental Details

In these measurements at the 25-m flight path of the CP-5 fast-chopper facility, a 40-cm^3 Ge(Li) spectrometer [full width at half-maximum (FWHM) = 3.98 keV at $E_\gamma=1.33$ MeV] detected γ rays from the target. Both the γ-ray pulse height (0.1 MeV$<E_\gamma<$1.9 MeV) and neutron time-of-flight data were recorded in a three-parameter magnetic-tape analyzer.

The time resolution (0.5–4.0 μsec/channel) of the 4096 time-of-flight channels, the delay (100–2000 μsec) prior to initiation of the timing sequence, and the rotor speed (2.8–16.3$\times10^3$ rpm) were appropriately selected for the desired time-of-flight region and neutron-energy resolution. The density of the target materials was in the range 0.8–3.9 g/cm^2 and for each element was selected with the criterion that attenuation in the target for the two γ rays of interest should differ by less than about 20%. The detector was shielded against neutrons scattered from the target by an absorber of borated paraffin, about 2 cm thick. For capture in low-energy resonances ($E_n<1$ eV), a 0.7-mm sheet of cadmium was inserted in the neutron beam at the chopper exit to prevent thermal overlap. Data were recorded on magnetic tape, and runs consisting of the 3×10^6 events needed to fill one tape required between 6 and 40 h, the latter time being the more typical.

B. Data Analysis

The total time-of-flight spectrum [i.e., the spectrum (Fig. 2) with no restrictions placed on E_γ] is used to identify the neutron resonances in terms of their energy and their elemental or isotopic assignment[7] and to select the time-of-flight channels appropriate to each resonance. Similarly, the total γ-ray spectrum (no restriction on the time of occurrence of the γ-ray event) is scrutinized to identify the γ rays and to select regions (or channels) corresponding to the transitions deexciting the levels of interest. Then, for each well-resolved resonance (selected by the appropriate time-of-flight restrictions), a γ-ray spectrum (Fig. 3) is obtained by a scan of the magnetic tape, and the intensity of the γ rays which characterize the population probability of the selected levels is extracted from this spectrum. A resonance-to-resonance comparison of the *relative* intensity R_{ab} of these transitions is made to investigate the spin dependence of the relative population probability.

It should be emphasized that the relative population R_{ab} is considered since it not only suppresses any dependence of the individual populations $P_{a,b}$ on other variables, but also eliminates the need for normalization or absolute calibration of the individual γ-ray intensities $I_{a,b}$.

4. RESULTS

Several even-Z odd-N target nuclei with resonances of known spins were studied as test cases, namely, ^{183}W$(\frac{1}{2}^-)$, ^{135}Ba$(\frac{3}{2}^-)$, ^{105}Pd$(\frac{5}{2}^+)$, ^{95}Mo$(\frac{5}{2}^+)$, ^{167}Er$(\frac{7}{2}^+)$, and ^{177}Hf$(\frac{7}{2}^-)$. An example of the results is given in Fig. 3; for capture in each of the two lowest resonances in ^{177}Hf, namely, those at $E_n=1.10$ eV ($J=3$) and $E_n=2.38$ eV ($J=4$), these spectra show a γ ray from both the 6^+ level at 633 keV and the 4^+ level at 307 keV. The γ-ray intensities in the two spectra are normalized to the 215-keV ($4^+\rightarrow2^+$) line; they show that the 326-keV ($6^+\rightarrow4^+$) line is nearly twice as intense for the $J=4$ as for the $J=3$ resonance. This difference is about what is expected from Table I.

For each nuclide, the results for the well-resolved resonances for which the spins are known and for which the statistical error is $\Delta R_{ab}/R_{ab}\lesssim30\%$ are given in Table II. The lowest $2^+\rightarrow0^+$ transitions ($E_\gamma\approx100$ keV) from capture in W, Er, and Hf were not measured, and in Pd this γ ray ($E_\gamma=512$ keV) could not be resolved from the annihilation line. Table II gives the spins J_a, J_b of the emitting levels and the γ-ray energies in keV so that one can identify the relevant portion of the known low-lying level scheme for each product nuclide.[10]

5. DISCUSSION

Except for ^{95}Mo, the values of the ratios R_{ab} for nearly all resonances in each nucleus given in Table II cluster into two groups, consistent with the model described above and in good agreement with previous spin assignments. The only exceptions are the 101-eV resonance in ^{183}W and the 68- and 141-eV resonances in ^{105}Pd. For the latter two resonances, which have the previously assigned spins of 3 and 2, respectively, the error in R_{42} is such that in each case the value of this ratio falls within 2 standard deviations of the average

[10] C. M. Lederer, J. M. Hollander, and I. Perlman, *Tables of Isotopes* (John Wiley & Sons, Inc., New York, 1967), 6th ed.

TABLE II. Values of the relative population R_{ab} of the levels J_a and J_b for resonances with known spin. The energies (in keV) of the γ rays whose intensities are compared are given in columns 2 and 3. The previously assigned spins (column 5) and resonance energies are from Refs. 3 and 7.

Target (I^π)	E_γ (J_a) (keV)	E_γ (J_b) (keV)	E_n (eV)	J	R_{ab}
$^{183}W(\frac{1}{2}^-)$	253 (4)	793 (2)	27.1	1	1.81±0.16
			40.6	1	1.65±0.42
			46.1	1	1.84±0.25
			47.8	0	0.91±0.19
			101	0	1.92±0.38
$^{135}Ba(\frac{3}{2}^-)$	1050 (4)	818 (2)	24.4	1	0.068±0.010
			82	2	0.113±0.013
			88	2	0.093±0.014
$^{105}Pd(\frac{5}{2}^+)$	713 (4)	613 (2)	11.8	3	1.72±0.24
			13.2	2	0.79±0.06
			25.3	3	1.66±0.14
			38.4	3	1.98±0.62
			55.2	3	1.99±0.24
			68.3	3	1.30±0.24
			77.7	2	0.85±0.12
			86.7	3	1.58±0.22
			126.3	3	1.57±0.47
			141.1	2	1.20±0.31
			202.6	2	0.78±0.24
			305.4, 313.6	2	0.90±0.29[a]
$^{95}Mo(\frac{5}{2}^+)$	852 (4)	776 (2)	45.1	3	0.34±0.01
			159.3	2	0.27±0.02
			358.4	3	0.31±0.03
			554.3	2	0.24±0.05
$^{167}Er(\frac{7}{2}^+)$	285 (6)	185 (4)	0.46	4	0.19±0.01
			0.58	3	0.13±0.01
			5.98	3	0.09±0.01
			9.38	3	0.09±0.01
$^{177}Hf(\frac{7}{2}^-)$	332 (6)	215 (4)	1.10	3	0.065±0.003
			2.38	4	0.123±0.003
			6.60	4	0.116±0.007
			8.89	4	0.111±0.006

[a] These two resonances are not resolved in the time-of-flight spectrum but, because both have $J=2$, they can be considered as a single resonance in our analysis.

for the relevant spin value when averaged over the remaining resonances in ^{105}Pd.

The ratio R_{42} for the 101-eV resonance in ^{183}W clearly falls into the $J=1$ group, contrary to the $J=0$ assignment in Ref. 7. The value in this comparison is based on measurements by several groups, two of which, however, have also reported a $J=1$ assignment from an area analysis of cross-section data. The most recent work included in this compilation is that of the Brookhaven group,[11] which reports $J=0$ on the basis of the results of a high-resolution Ge(Li) measurement in which they found no γ-ray intensity for a group of high-energy primary transitions ($E_\gamma > 5.5$ MeV) observed in the 1^+ resonances. In the range 25 eV $< E_n <$ 120 eV, the neutron-energy calibration of the present measurement yields resonance energies which agree within ±0.5 eV with those for ^{182,183}W given in Ref. 7. This eliminates the possibility that capture in the 103.8-eV resonance in ^{183}W may account for this discrepancy. From this discussion and the success of the method in giving resonance spins in agreement with previous assignments in nearly all other cases, one must accept this result as strong evidence for the $J=1$ assignment.

In general, this method gives reliable spin assignments for this class of nuclei when (i) the statistical error in R_{ab} is $\lesssim 25\%$, (ii) the resonances are well

[11] M. Beer, R. E. Chrien, O. A. Wasson, M. R. Bhat, M. A. Lone, and H. R. Muether, Bull. Am. Phys. Soc. 12, 105 (1967).

TABLE III. Values of the relative population R_{ab} of the levels J_a, J_b for unassigned resonances in ^{167}Er and ^{187,189}Os. The γ rays used to compute R_{ab} are given in columns 2 and 3. The spins in column 5 are based on the values of R_{ab} of this measurement, as given in column 6.

Target (I^π)	E_γ (J_a) (keV)	E_J (J_b) (keV)	E_n (eV)	J	R_{ab}
^{167}Er ($\frac{7}{2}^+$)	285 (6)	185 (4)	20.3	4	0.18±0.01
			22.0	3	0.12±0.01
			26.1	3	0.10±0.01
			27.4	4	0.18±0.01
^{187}Os ($\frac{1}{2}^-$)	323 (4)	155 (2)	9.5	1	0.29±0.08
			12.6	0	0.15±0.02
^{189}Os ($\frac{3}{2}^-$)	365 (4)	187 (2)	6.71	1	0.083±0.008
			8.96	2	0.158±0.006
			10.3	1	0.084±0.007
			18.7	2	0.146±0.009
			22.1	1	0.094±0.003
			27.4	2	0.153±0.012
			28.3	2	0.162±0.010
			30.8	2	0.143±0.030
			43.5	(2)	0.215±0.030
			50.5	2	0.153±0.010
			55.0	2	0.149±0.014
			60.7	2	0.159±0.014

resolved so that intermediate values of R_{ab} are precluded, and (iii) at least one resonance of each value $J=I\pm\frac{1}{2}$ occurs so that each grouping of R_{ab} can be identified. This method has several advantages. Prime among them is that when not limited by statistical error the size of the effect used as the criterion for spin assignment is nearly 100%, and is thus much larger than that of most other methods. It should be noted that according to the elementary theory discussed above, the effect should be about a factor of 5 or 6 if the relative population of, say, the 6^+ and 2^+ levels were compared between the $I+\frac{1}{2}$ and the $I-\frac{1}{2}$ resonances. This was not feasible in any of the cases studied in these measurements, because, as mentioned above, the $2^+\rightarrow0^+$ transition was not observed or resolved or else the level structure was unfavorable to the observation of a strong $6^+\rightarrow4^+$ transition. Furthermore, since the nuclide in which capture takes place is identified by the specific γ rays of the product nucleus, it is not necessary that resonances in different isotopes or even in contaminant elements be resolved unless they produce capture γ rays that cannot be distinguished from those of interest. For example, the 38.4-eV resonance in ^{105}Pd appears on the tail of the strong resonance in ^{108}Pd at 33.2 eV, but capture in this latter resonance makes no contribution to the intensities of the 613-keV ($2^+\rightarrow2^+$) and 713-keV ($4^+\rightarrow2^+$) lines from ^{106}Pd. On the other hand, capture in ^{177}Hf and ^{179}Hf both have a $\sim$215-keV ($2^+\rightarrow0^+$) transition, but again the resonances in each isotope can be identified by their respective lowest $4^+\rightarrow2^+$ transitions which differ in energy by 6 keV. Thus in certain cases a completely unresolved resonance (i.e., one totally overlapping with another) could, in principle, be assigned a spin value by use of this method.

The method has also been used to obtain J for several unassigned resonances in ^{167}Er and ^{187,189}Os. These results are given in Table III for the resonances for which the statistical error in R_{ab} is less than 25%. These data provide the spins of four additional resonances in ^{167}Er. Of four resonances observed in ^{187}Os, two have a small statistical error and, in fact, have spins of opposite sign. The resonances in ^{189}Os given in Table III represent all the known resonances[7] in this nuclide, and for each a spin assignment is possible. Resonances with $J=2$ outnumber those with $J=1$, in accordance with the $(2J+1)$ factor in the usual Bethe-Bloch level-density formula.

The fluctuations in R_{ab} in Tables II and III can be accounted for by the statistical errors. This error usually is determined largely by that of the transition from J_a, the state of higher spin, which is typically higher in excitation energy, and has about one tenth the intensity of the transition from the level J_b. However, the $J_b=2$ levels used to determine R_{42} for ^{183}W and ^{105}Pd have excitation energies well above the respective lowest 2^+ levels. Consequently, the transition from J_b in these cases does not represent a large fraction of the full cascade intensity and its intensity is approximately equal to that of the transition from the level $J_a=4$, as is reflected in the ratios R_{42} of Table II. In an attempt to reduce the errors in R_{42} in these two cases, as well as to investigate a variation of this method, the following analysis was performed. The intensity I_b of the transition from J_b (which can be considered as a normalizing factor for the intensity I_a of the transition from J_a) was replaced by the summed intensity I_Σ in a band of γ-ray energies (typically 600 keV$<E_\gamma<$1100 keV). Since the statistical error in I_Σ is less than 1%, the errors $\Delta R_{4\Sigma}/R_{4\Sigma}$ (the ratio $R_{4\Sigma}$ being defined in analogy to R_{42}) were in the range 6–19% for ^{183}W resonances and 5–16% for ^{105}Pd resonances. This is an improvement over the ratio R_{42}, for which the errors are in the range 9–26% and 8–30% for ^{183}W and ^{105}Pd, respectively. For resonances in the other target nuclei, the differences between the errors in R_{ab} and in $R_{a\Sigma}$ were insignificant because of the small error in I_b.

However, the most important result from this analysis using a summed γ-ray intensity I_Σ in place of the intensity I_b was that the resulting values of the ratio $R_{a\Sigma}$ (where $J_a=4$ or 6) also cluster into two groups corresponding to capture-state spins $J=I\pm\frac{1}{2}$. Insofar as they correlate with previously known spins or allow one to make new spin assignments, these results for $R_{a\Sigma}$ virtually duplicate those for R_{ab} in Tables II and III. This duplication includes the $J=1$ assignment for the 101-eV resonance in ^{183}W and the intermediate values of $R_{4\Sigma}$ for the 68- and 141-eV resonances in ^{105}Pd. Thus, this variant of the original method might prove extremely useful in certain cases—for example, in cases in which the transition from J_b cannot be resolved, or is too weak to give accurate values for R_{ab}, or must be excluded from the measured γ-ray spectrum.

The model developed in Sec. 2 can be investigated within limits by comparing the predicted and the measured variation in R_{ab} between the two resonance spins. In particular, the quantity $\mathcal{R}_{ab}\equiv\bar{R}_{ab}(J=I+\frac{1}{2})/\bar{R}_{ab}(J=I-\frac{1}{2})$ can be compared with the predictions of Table I or with those of a similar three-step dipole-cascade model. From Sec. 2 it is clear that R_{ab} should be sensitive to, among other things, both the average γ-ray multiplicity populating the levels J_a, J_b and to I itself. From Tables II and III the average values $\bar{R}_{ab}$ for each spin state are computed for each nuclide and from these the quantity $\mathcal{R}_{ab}$ is calculated. These experimental values of $\mathcal{R}_{ab}$ are given in column 4 of Table IV and are to be compared with the predicted values in columns 5 and 6. For ^{95}Mo, $\mathcal{R}_{42}$ is near unity, corresponding to the small spin-dependent effect noted above, while for the remaining nuclides the values of $\mathcal{R}_{ab}$ are consistent (within the framework of the model developed in Sec. 2) with an average γ-ray multiplicity (to J_a, J_b) of about 4. (The multiplicity of the ground-state cascade will be typically one or two steps more.) If one assumes both three-step and four-step cascades to the higher- and lower-lying levels of the pair J_a, J_b, respectively, then values for $\mathcal{R}_{ab}$ can also be calculated; these values are typically intermediate to the three-step and four-step case in Table IV. The extent of agreement between the experimental and predicted values of $\mathcal{R}_{ab}$ is noteworthy in view of the simple assumption and the elementary derivation of the predicted values. The data of Table IV can nonetheless be used as one criterion for the applicability of more sophisticated treatments of the cascade process itself.

TABLE IV. Comparison of the experimental ratios $\mathcal{R}_{ab}=\bar{R}_{ab}(I+\frac{1}{2})/\bar{R}_{ab}(I-\frac{1}{2})$ computed from Tables II and III with those predicted by assuming a three- or four-step version of the simple dipole cascade model. In column 2, J_+ and J_- correspond to $J=I+\frac{1}{2}$ and $J=I-\frac{1}{2}$, respectively.

Target	Res. spin J_+, J_-	Level spin J_a, J_b	Value of ratio $\mathcal{R}_{ab}$ Expt.	Theor. 4-step	Theor. 3-step
^{183}W	1, 0	4, 2	2.06±0.43	1.60	∞
^{135}Ba	2, 1	4, 2	1.52±0.24	1.97	2.57
^{105}Pd	3, 2	4, 2	2.06±0.09	1.90	2.33
^{95}Mo	3, 2	4, 2	1.28±0.08	1.90	2.33
^{167}Er	4, 3	6, 4	1.75±0.06	2.11	2.57
^{177}Hf	4, 3	6, 4	1.82±0.09	2.11	2.57
^{187}Os	1, 0	4, 2	1.93±0.60	1.60	∞
^{189}Os	2, 1	4, 2	1.73±0.04	1.97	2.57

Finally, in the case of ^{177}Hf the data of Table IV provide a comparison of the results of the present investigations with the relevant computations of Huizenga and Vandenbosch[9] for this nucleus. For values of the spin cutoff factor σ of 3, 5, and ∞, they obtain for the ratio $\mathcal{R}_{ab}$ the values 2.25, 2.54, and 2.33, respectively. In this instance one notes that while the relative population probabilities $R_{ab}(I+\frac{1}{2})$ and $R_{ab}(I-\frac{1}{2})$ are themselves sensitive to the parameter σ, the ratio $\mathcal{R}_{ab}$ of these is relatively insensitive to the value of σ. However, this aspect of the γ-ray cascade process must be investigated further before the effect of the spin cutoff factor σ can be neglected (or minimized) in all detailed calculations.

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