Shell-Model Analysis of the Excited States of He^4

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Using a theory previously developed for the O^{16} nucleus, the energy levels of He^4 were calculated using the nuclear shell model and the random-phase approximation (RPA). Though no attempt was made to optimize the parameters of the calculation, the predicted energies of the excited states compare favorably with those determined from experimental analyses. Qualitative agreement with experiment is obtained for the calculated electron inelastic-scattering form factor to the first 0^+ excited state and for the integrated photoabsorption cross section. However, the prediction of the majority of the $E1$ strength in the higher of the $1-$, $T=1$ levels does not conform to most experiments which indicate the lower level to have the stronger $^1P_1$ component. The RPA produces some improvement over an ordinary shell-model calculation by decreasing the energies of the lower excited states by several hundred keV and by increasing the relative $E1$ strength of the lower $1-$, $T=1$ level.

I. INTRODUCTION

SEVERAL shell-model calculations[1–3] have been performed to determine theoretically the odd-parity excited states of He^4. These calculations used the particle-hole model and the bound-state wave functions of the harmonic oscillator as a basis. By considering the excitation of more than one particle-hole pair, Vashakidze and Mamasakhlisov[4] determined the energies of certain even-parity states of He^4 in addition to the odd-parity states formed by single-particle excitations from the $1s$ to the $1p$ shell. Moshinsky and his co-workers[5,6] have determined both the even- and odd-parity states for a system of four nucleons in a harmonic-oscillator well. The spacing of the oscillator levels was derived from the rms charge radius of He^4 determined from electron elastic-scattering experiments. Their initial calculations utilized central forces for the residual nucleon-nucleon interaction. The calculation was later repeated using the more realistic Brueckner-Gammel-Thaler potential. The results of these shell-model calculations will be reviewed later in the context of comparisons between experiment and results obtained in the present calculation.

The cluster model or resonating-group approximation is another theoretical model which has been used to determine levels in He^4 by searching for resonances in the $t+p$, He^3+n, or $d+d$ systems. A calculation assuming charge invariance, using a central nucleon-nucleon interaction and considering only the first two channels found a virtual state with the quantum numbers $J=0+$, $T=0$ at approximately the same energy as measured experimentally.[7] (A $1-$, $T=0$ state predicted by the calculation was later determined to be spurious.[8]) Prior to this, Laskar[9] had formulated the complete 3×3 scattering matrix using the same model but performed no calculations which considered cross-channel coupling.

The present analysis of the He^4 nucleus also utilizes the nuclear shell model with harmonic-oscillator basis functions and the random-phase approximation (RPA) as developed for O^{16} in a previous paper[10] [afterwards referred to as (I)]. This theory includes all the matrix elements of the Tamm-Dancoff approximation (TDA) for one-hole–one-particle (1h-1p) and 2h-2p excitations from an $A=4$ nucleus closed in the $1s$ shell, plus the matrix elements corresponding to those "backward-going diagrams" which occur in the first RPA. When the harmonic-oscillator shell model is applied to He^4, a truncated set of bound-state wave functions is used to represent resonances in the continuum. As Barrett[2] has indicated, for the excitation of particles into states of nonzero orbital angular momentum, the centrifugal barrier for He^4 is sufficiently high ($\approx$40 MeV above the zero of the nuclear potential for p states) to increase the lifetime of the excited configurations and lend credence to their description by bound-state shell-model wave functions within the nuclear confines. For a particle in an s state there is no such barrier. The first $0+$, $T=0$ excited state is largely the result of excitation of a particle from the $1s$ to the $2s$ shell. However, since this $0+$ state is only slightly above the $t+p$ threshold, its description as a bound state should still be valid. A brief report of some preliminary shell-model results for He^4 has already been presented by the author.[11] The current study will determine all the energy levels of He^4 obtainable within the limits of the present model with additional concentration on determining the electromagnetic transition strengths between the ground

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state and the excited states. Since the excited states of He^4 are very high in energy (20 MeV or above), these energies determined from the RPA are not expected to be materially different from those determined in an ordinary shell-model calculation. However, Gillet and Melkanoff[12] have shown that the ground-state correlations introduced by the RPA did affect significantly the electron-scattering form factor of the strongest giant-dipole states of O^{16} (both of which are above 20-MeV excitation). The RPA could influence the $1-$, $T=1$ states of He^4 in a similar fashion. Our results for the lowest $0+$, $T=0$ state will be compared with those of Blomqvist[13] who used the RPA in a description of monopole vibrations in closed-shell nuclei including He^4. Several deficiencies in his model (to be enumerated later) were corrected by the model used in the present investigation.

In summary, the hole-particle shell model has been reasonably successful in reproducing the general features of the He^4 spectrum, particularly the odd-parity levels. The extension of the model to further study the even-parity excited states and to determine other properties of the He^4 structure, notably the electromagnetic transitions, hopefully would enjoy the same degree of success.

II. THEORETICAL CONSIDERATIONS

A. Eigenvalues and Eigenvectors in the Random-Phase Approximation

The method for obtaining the energies and wave functions for excited states of nuclei according to the random-phase formalism was developed in detail in (I). Only the essential features of that method will be reproduced here for the sake of continuity and to define operators used later in obtaining expressions for the matrix elements of one-body operators.

The excited states $|k\rangle$ are generated from the ground state $|g\rangle$ by the operators $\Gamma_k^\dagger$ which satisfy the commutation relation

$$[\mathcal{H}, \Gamma_k^\dagger]=\omega_k\Gamma_k^\dagger, \tag{1}$$

which means

$$\Gamma_k^\dagger|g\rangle=\omega_k|k\rangle \tag{2}$$

and

$$\Gamma_k|g\rangle=0 \qquad \text{for all } k. \tag{3}$$

Here $\mathcal{H}$ is the nuclear Hamiltonian consisting of one-particle and two-particle operators, and the ω_k are the energy eigenvalues. In this theory, the ground state $|g\rangle$ contains hole-particle pairs in addition to a component of the closed-shell "vacuum," $|0\rangle$.

It was shown in (I) that a suitable $\Gamma_k^\dagger$ is[14]

$$\Gamma_k^\dagger=\sum_q c^k{}_q S_q^\dagger+\sum_q \bar{c}^k{}_q \bar{S}_q, \tag{4}$$

where the $S_q^\dagger$ and $\bar{S}_q$ are, respectively, the hole-particle state creation and annihilation operators, the $c^k{}_q$ are the components of the eigenvectors occurring in the usual shell-model approximation and the $\bar{c}^k{}_q$ are the coefficients of those components of the excited states resulting from the destruction of particle-hole pairs in the ground state referred to as ground-state correlations.

If we write

$$S_q^\dagger=S^\dagger(qJM), \tag{5}$$

where the individual angular momenta are coupled to J and M, and q refers to state quantum numbers other than J and M, then

$$\bar{S}_q=(-)^{J+M}S(qJ-M), \tag{6}$$

where the phase and coupling to $-M$ give this destruction operator the same rotational properties as $S_q^\dagger$. As an example, $S_q^\dagger$ for a 1h-1p state is

$$S^\dagger(\bar{j}_1 j_2 JM)=\sum_{m_1m_2}(j_1m_1j_2m_2\,|\,JM)(-)^{j_1+m_1}a_{j_2m_2}^\dagger a_{j_1-m_1}, \tag{7}$$

where the $(j_1m_1j_2m_2\,|\,JM)$ is a Clebsch-Gordan coefficient and the $a^\dagger$ and a are the fermion creation and destruction operators. The isospin can easily be included on the same footing as angular momentum.

In (I) the $\Gamma_k^\dagger$ were defined with an additional constant term c^k. It was shown there that c^k did not affect the eigenvalue problem (even for states of $J=0+$, $T=0$). Since c^k is just a constant it commutes with $S_q^\dagger$ and $\bar{S}_q$. Whether c^k is included in the definition of $\Gamma_k^\dagger$ or not, the usual RPA orthonormality condition

$$\langle k\,|\,k'\rangle=\langle g\,|\,[\Gamma_k,\Gamma_{k'}^\dagger]\,|\,g\rangle=\delta_{kk'}=\sum_q(c^{k'}{}_q c^k{}_q-\bar{c}^{k'}{}_q\bar{c}^k{}_q) \tag{8}$$

(where the $c^k{}_q$, etc., are taken to be real) is obtained, since

$$[S_q,\bar{S}_{q'}]=[\bar{S}_q^\dagger, S_{q'}^\dagger]=0 \tag{9}$$

and the usual appeal is made to the quasiboson approximation

$$\langle g\,|\,[S_q,S_{q'}^\dagger]\,|\,g\rangle=-\langle g\,|\,[\bar{S}_q^\dagger,\bar{S}_{q'}]\,|\,g\rangle\approx\delta_{qq'}. \tag{10}$$

The basis used to describe the odd-parity states consisted of 1h-1p states with $1\hbar\omega$ excitation. The even-parity states were constructed from all the possible 1h-1p and 2h-2p configurations with oscillator energy $2\hbar\omega$ above the $A=4$ closed shell. The eigenvalue calculation included in addition to all the matrix elements of an ordinary 1h-1p and 2h-2p shell-model calculation

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[14] Very often $\Gamma_k^\dagger$ is defined with the second term negative.

those matrix elements which occur in the first-order RPA. The derivation of the eigenvalue problem is described in detail in (I) and will not be repeated here. Formulas for obtaining the required matrix elements of the nucleon-nucleon interaction are contained in Appendix A of (I). There the hole-particle matrix elements are given in terms of the particle -particle matrix elements and the appropriate angular-momentum coupling coefficients.

For a nucleus as light as He^4, a valid description of the energies and wave functions of the excited states requires the elimination of the spurious states of center-of-mass motion. Since the present calculation considered all possible configurations of a given oscillator excitation, this elimination was accomplished merely by adding to the nuclear Hamiltonian an inordinately large component of the c.m. Hamiltonian. Upon diagonalization of the interaction matrix the spurious states occur at very high energy, become decoupled and no longer affect the composition of the low-energy eigenstates.

B. Matrix Elements of One-Body Operators in the Random-Phase Formalism

In order to determine the strength of electromagnetic transitions, the quantities $\langle J_f M_f \mid T \mid J\, M \rangle$, the matrix elements of appropriate tensor operators between the initial and final states, must be evaluated. The operator T is defined by

$$T=\sum_{\alpha,\beta;J,M}\langle\beta \mid T_{JM} \mid \alpha\rangle a_\beta^\dagger a_\alpha, \tag{11}$$

where the T_{JM} will be the various multipole operators and $\langle\beta \mid$ and $\mid \alpha\rangle$ are single-particle states. The reduced matrix elements are defined by the Wigner-Eckart theorem

$$\langle J_f M_f \mid T_{JM} \mid J_i M_i\rangle=(-)^{J_f-M_f}\times\begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i\end{pmatrix}(J_f \,||\, T_J \,||\, J_i). \tag{12}$$

Observables (such as cross sections) are functions of the squares of matrix elements. Taking the matrix element of T between initial and final states, squaring, averaging over initial states, summing over final states, and using Eq. (12), the result is

$$(2J_i+1)^{-1}\sum_{M_i,M_f}\mid\langle J_f M_f \mid T \mid J_i M_i\rangle\mid^2 = (2J_i+1)^{-1}\sum_J (J_f \,||\, T_J \,||\, J_i)\mid^2. \tag{13}$$

The present analysis will consider transitions only from a $J=0+$ ground state. Therefore, $J_i=0$ and one has $J=J_f$ as the order of the only multipole contributing to the transition. The problem reduces to the evaluation of

$$\langle g \mid \Gamma_{JM} T \mid g\rangle=\langle g \mid [\Gamma_{JM}, T] \mid g\rangle, \tag{14}$$

where T is given by Eq. (11) and $\langle JM \mid=\langle g \mid \Gamma_{JM}$. The matrix element (14) can be evaluated in two equivalent fashions. The $a^\dagger a$ can be converted into a coupled hole-particle creation or annihilation operator (i.e., $S^\dagger$ or $\bar{S}$ depending on which of the single-particle states is initially occupied) with an appropriate phase and Clebsch-Gordan sum. Equation (14) is then evaluated using the commutation relations (9) and (10). Alternately the $\bar{S}^\dagger$ and S in Γ_{JM} can be decomposed into single-particle creation and annihilation operators. The commutator with T results in sums of products of these creation and annihilation operators which are then contracted using the anticommutation relations for such operators. Certain terms must still be discarded resulting in what is tantamount to the quasiboson approximation. The Wigner-Eckart theorem is invoked and the expression for the reduced matrix element of the transition is found to be

$$(J_f=J \,||\, T_J \,||\, J_i=0) = \sum_{\alpha,\beta}(-)^s[c^J{}_{\alpha\beta}(\beta \,||\, T_J \,||\, \alpha)-\bar{c}^J{}_{\alpha\beta}(\alpha \,||\, T_J \,||\, \beta)]. \tag{15}$$

In the single-particle reduced matrix elements on the right-hand side, α refers to the hole quantum numbers and β to particle quantum numbers. The phase $(-)^s$, where $s=j_\alpha+j_\beta-J$, results from having coupled holes to particles in our $S_q^\dagger$ and S_q.

In the isospin formalism, s has the additional terms $\frac{1}{2}+\frac{1}{2}-T$, where T is the isospin of the final state. The reduced matrix elements in isospin space of the charge operator

$$e=\tfrac{1}{2}(1+\tau_3) \tag{16}$$

and the magnetic moment operator

$$\mu=\tfrac{1}{2}(\mu_p+\mu_n)+\tfrac{1}{2}(\mu_p-\mu_n)\tau_3 \tag{17}$$

must then be included in (15) in a manner specified by the electromagnetic operator being considered. In both expressions (16) and (17), the first term, essentially the unity operator, is an isoscalar contributing to transitions to $T=0$ states. The second term is an isovector contributing to transitions to $T=1$ states. The eigenvalues of τ_3 are the isospin projections ($+\frac{1}{2}$ or $-\frac{1}{2}$) of the nucleon isospinor. The μ_p and μ_n are, respectively, the proton and neutron magnetic moments in units of the nuclear magneton.

C. Electromagnetic Transitions

Using the wave functions determined from the energy level calculations, the strengths of electromagnetic transitions from the ground state to the excited states will be calculated. These strengths will be determined by evaluating the electron inelastic-scattering form factor for the first excited $0+$, $T=0$ state and the total integrated photoabsorption cross section for those states which can be connected to the ground state

by the electric dipole, electric quadrupole, and magnetic dipole operators. However, transitions to the $1+$, $T=0$ states will be reduced from those to $1+$, $T=1$ states by a factor of approximately

$$(\mu_p+\mu_n)^2/(\mu_p-\mu_n)^2\approx 0.04$$

and will therefore be neglected.

Frosch *et al.*[15] have measured the differential cross section for inelastic electron scattering from He^4 and have found that it proceeds predominantly through a longitudinal (or Coulomb) monopole transition to the first $0+$, $T=0$ excited state. They have determined the inelastic form factor from the relation

$$F_{\text{inel}}=\left[\frac{(d\sigma/d\Omega)_{\text{inel}}}{\sigma_{\text{point}}{}^{I}}\right]^{1/2}, \tag{18}$$

where

$$\sigma_{\text{point}}{}^{I}=\frac{e}{2E_0}\frac{\cos^2(\tfrac{1}{2}\theta)}{\sin^4(\tfrac{1}{2}\theta)}\frac{1}{1+(2E_0/Mc^2)\sin^2(\tfrac{1}{2}\theta)} \tag{19}$$

is the cross section for electron inelastic scattering from a point nucleus, E_0 is the energy of the incident electron, θ is the scattering angle of the electron, and M is the mass of the target nucleus.

In the plane-wave Born approximation, the form factor for a longitudinal inelastic monopole transition is

$$F_{\text{inel}}=(4\pi)^{1/2}(0\,||\,\hat{M}_0{}^{\text{Coul}}\,||\,0), \tag{20}$$

where $J_i=J_f=0$, and the atomic number of the target nucleus is not included in the definition, consistent with the definition of Frosch *et al.*[15] with whose results our calculation will be compared. The carat above the Coulomb operator $M_0{}^{\text{Coul}}$ indicates that the operations in isospin space are to be included. The multipole operators and their reduced matrix elements used here are as given in the comprehensive analysis of the electron-scattering problem by deForest and Walecka.[16]

The total integrated photoabsorption cross section for a transition to a *discrete* excited level of a nucleus is[17]

$$\int_{\text{one level}}\sigma_{\text{abs}}(E)\,dE=(2\pi)^3\alpha[(\hbar c)^2/E_{fi}](2J_i+1)^{-1}\times\sum_J\{|\,(J_f\,||\,\hat{T}_J{}^{\text{el}}(q)\,||\,J_i)\,|^2+|\,(J_f\,||\,\hat{T}_J{}^{\text{mag}}(q)\,||\,J_i)\,|^2\}, \tag{21}$$

where E_{fi} is the energy of the excited state; $q=E_{fi}/\hbar c$, and α is the fine structure constant. For the cases considered here $J_i=0$, $J_f=J$ so the $T_J{}^{\text{el}}$, the transverse electric multipole operator, and $T_J{}^{\text{mag}}$, the transverse magnetic multipole operators, will not both appear in transitions to the same level because of the different parity selection rules for electric and magnetic multipole operators.

For the transverse matrix elements to be consistent with the conservation of electromagnetic current, the wave functions used must be exact eigenstates of the total nuclear Hamiltonian. This requirement is only approximately satisfied by shell-model calculations using harmonic-oscillator wave functions. Corrections for this fact and for the effects of nuclear recoil can be made only in very simple cases and are beyond the scope of the present treatment. On the other hand, all the matrix elements were corrected for center-of-mass motion and the proton charge radius as outlined by Tassie and Barker,[18] and the long-wave approximation was not used.

III. PARAMETERS OF THE CALCULATION

No optimization of the parameters was attempted in the present study of the He^4 nucleus. The single-particle energies, nucleon-nucleon interaction, and harmonic-oscillator size parameter were all selected *a priori* as "reasonable values" of a particular quantity. While the effects of the variation of some of the parameters and their reliability will be discussed, the illustrated energy spectrum and the electromagnetic transitions were all determined from this initial parameter choice.

The single-particle energy of the $1s_{1/2}$ hole was determined from the difference in binding energies of the He^3 and He^4 nuclei. Energies of the $p_{3/2}$ and $p_{1/2}$ particles were obtained from the broad but well-known p-wave virtual states occurring in the n(or p) $+He^4$ system.[19] The splitting of the $1p_{3/2}$ and $1p_{1/2}$ states was taken as 2.6 MeV. There is as yet no evidence for s- or d-wave resonances. In fact, the s-wave phase shifts for the scattering of neutrons from He^4 appear to be almost entirely hard sphere.[20] The narrow resonance at about 16.7 MeV in the $A=5$ system is known to consist of $d+t$ (or $d+He^3$) clusters.[21] Even the broad level at 20 MeV is now believed to have the same cluster character and to differ from the 16.7-MeV state only in the value of the total angular momentum.[22] The uncertainty in the $2s$ energy is rather pointedly demonstrated in the variety of values used for this energy in shell-model calculations for the very light nuclei. Fraser and Spicer[23] in a study of mass-3 and mass-5 systems adopted a value of 45 MeV for the $1p_{3/2}$-$2s_{1/2}$ energy difference on the basis that the $n+He^4$ s-wave phase shift appeared

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TABLE I. Single-particle spectrum (MeV).

$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$	$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$
−20.6	0.95	3.5	22.2	24.2	27.2

to be increasing in that energy region. Consistent with this splitting, the energy of the $|(1s_{1/2})^{-1}2s_{1/2}\rangle$ configuration would be ≈65 MeV. This is the value adopted by Blomqvist[13] though no reason was given for this choice. Two recent papers on He^4 came to our attention after the present calculations were completed. Vashakidze *et al.*[24] determined the single-particle energies of He^4 from a Hartree-Fock calculation using the Tabakin potential. What they would consider their best results (for $\hbar\omega=20$ MeV) indicate ≈36 MeV for the $|1s^{-1}2s\rangle$ energy. Surprisingly, their $2s_{1/2}$ state is almost 2 MeV below the $d_{5/2}$ state. Kuo and McGrory[25] studied the effect on the binding energy of He^4 of the excitation of particles into other orbits and incidentally calculated the energies of some of the excited levels in He^4. Their single-particle energies were obtained from the harmonic-oscillator energy $\hbar\omega=13$ MeV which gave the best agreement with experiment for the excited levels. Thus their $|1s^{-1}2s\rangle$ energy would be only 26 MeV.

Since direct experimental evidence is lacking for the energies of $2s$ or $1d$ particles, the following procedure was used to estimate those energies. The position of an unsplit $1p$ state was determined assuming that the shifts of the $1p_{1/2}$ state and the $1p_{3/2}$ state from the unsplit level were in the ratio of $(l+1)/l$, as would be the case for an effective one-body spin-orbit potential. This gave a separation of 22.4 MeV for the $1s$ and $1p$ levels (which is close to the value $\hbar\omega=21.8$ MeV derived from the oscillator size parameter used in this calculation). A $2s$-$1d$ level was then set at twice this energy (though these states are degenerate only in an oscillator well) and the $1d$ levels were then somewhat arbitratily split by 5 MeV. This splitting, however, is reasonably consistent with the early work of Elliott and Lane[26] on the two-body spin-orbit force which indicates that for the oscillator size parameter and force range used in the present calculation, the $1d$ splitting is about 2–3 times the $1p$ splitting. The analysis of Vashadkize *et al.*[24] favors a value of ≈7 MeV for the $1d$ splitting. While a somewhat larger value for the separation of the $1d_{5/2}$ and $1d_{3/2}$ than 5 MeV is indicated, it will be found that the important features of the results were not sensitive to this splitting.

The single-particle spectrum finally adopted is given in Table I.

A simple two-body central potential[27] consistent with the low-energy two-nucleon data (viz., the triplet and singlet scattering lengths and the binding energy of the deuteron) and containing only triplet-even and singlet-even components was used for the nucleon-nucleon interaction. No parameters of the force were adjusted since it was expected

(1) that the use of a free nucleon-nucleon force should still be valid for a nucleus as light as He^4;

(2) that because of the large energy gaps between major shells, the neglected configurations should not affect the results of the calculation appreciably; therefore it would not be necessary to adjust the effective interaction to compensate for the truncation of the basis.

The potential used was of the form

$$V(r)=V_0\exp(-r^2/r_0^2)\,(W+MP^r+BP^\sigma+HP^rP^\sigma), \tag{22}$$

where P^r and P^σ are, respectively, the operators which exchange the space and spin coordinates of the nucleons.

The parameters of the potential used are given in Table II.

The size parameter $b=(\hbar/M\omega)^{1/2}$ adopted for the harmonic-oscillator wave functions $\psi_{ho}\sim\exp[-\frac{1}{2}(r/b)^2]$ was 1.38 fm determined from the electron-elastic-scattering form factor for the He^4 nucleus. This value has also been used in several previous shell-model investigations of the He^4 nucleus.[1–3,5]

IV. RESULTS

A. He^4 Energy Spectrum—Comparison with Experiment and Previous Calculations

The energy levels of He^4, up to about 50 MeV, predicted by the present calculation using the fixed-parameter set described in Sec. III are compared in Fig. 1 to the known levels in He^4. Experimental data are taken from Fig. 1 of the paper by Werntz and Meyerhof[28] (WM). Their level scheme was obtained through an analysis of $H^3(p,n)He^3$ cross-section and polarization

TABLE II. Force parameters.

V_0 (MeV)	r_0 (fm)	W	M	B	H
−48.75	1.85	0.42	0.42	0.08	0.08

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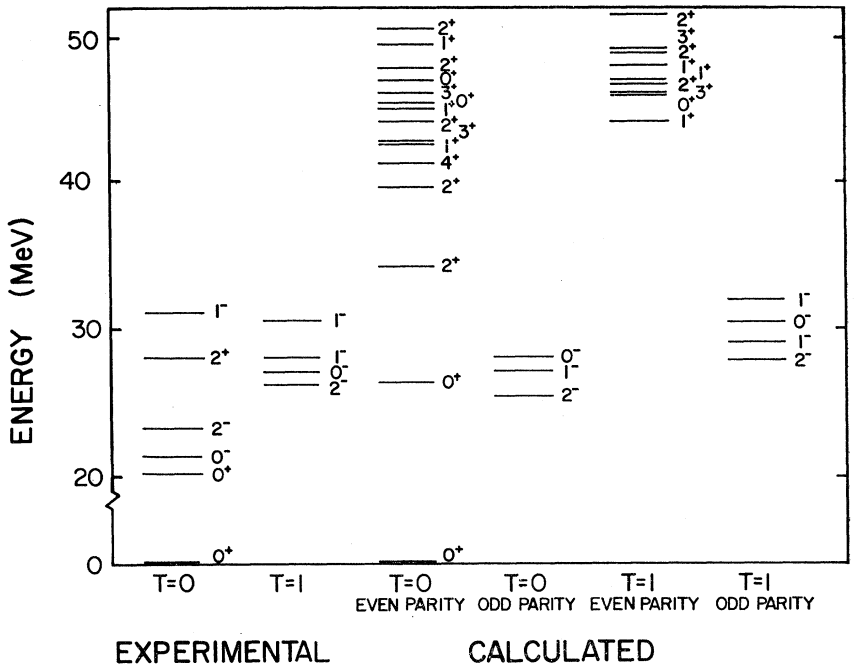

FIG. 1. Comparison of the observed energy levels of He^3 and those calculated from the present theory using the "reasonable" parameter set of Sec. III. Experimental data are taken from Fig. 1 of Ref. 28.

data and checked for consistency with $He^3(p, p)He^3$ results. They favor their solution "II" for the odd-parity $T=1$ states because of its compatibility with a particular set of phase shifts and p-wave channel spin mixing angle determined by Morrow and Haeberli[29] in their analysis of the $He^3(p, p)$ reaction. One very important result of this choice is that the lower $1-$, $T=1$ level will be predominantly ^1P and the upper level largely ^3P. The relationship of this interpretation to the present calculation of these same levels will be indicated shortly.

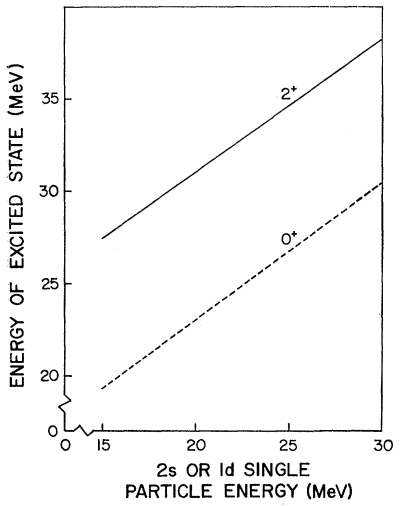

FIG. 2. Variation of the calculated energies of the first $0+$, $T=0$ and $2+$, $T=0$ excited states of He^4 as a function of the $2s$ and $1d$ single-particle energies, respectively.

Our model is successful in producing the first $0+$ excited state at a relatively low energy (26.3 MeV), although the single-particle energy of its dominant configuration $|(1s_{1/2})^{-1}2s_{1/2}\rangle$ was taken as 44.8 MeV. The particle-hole interaction has thus lowered the energy of this configuration by 19 MeV. Consistent with experiment, the next even-parity $T=0$ level is a $2+$ with the correct energy separation from the $0+$ level. While both the $0+$ and $2+$ levels are about 6 MeV too high, Fig. 2 demonstrates that their positions are strongly dependent on the position of the $2s$ and $1d$ single-particle energies, respectively, whose values cannot be obtained from experiment. Figure 2 shows that if the $2s$ and $1d$ energies are $\approx$16 MeV (a $1s$-$2s$ or $1s$-$1d$ separation of 36 MeV) the energies of both the lowest $0+$ and lowest $2+$ excited states will agree with the results of WM. It should be remarked that the position of the $2+$ level is only weakly dependent upon the $1d_{3/2}$–$1d_{5/2}$ splitting. Moreover, the wave functions for the lowest $0+$ and $2+$ excited states are essentially unaffected by such an energy variation. The $1+$ and $3+$ levels would of course be sensitive to the positions of the individual $1d_{3/2}$ and $1d_{5/2}$ states, respectively, not just their "center of gravity." WM have found that the deuteron widths of the $0+$ and $2+$ levels are large, comparable to those of neutron and proton channels. The effects of the $0+$ deuteron width are suppressed, of course, since this

[29] W. Haeberli and L. Morrow, in *Few Body Problems, Light Nuclei and Nuclear Interactions*, edited by Guy Paic and Ivo Slaus (Gordon and Breach Science Publishers, Inc., New York, 1969).

TABLE III. Shell-model prediction of $T=2$ states of He^4 (energies in MeV).

0+	1+	2+
54.1	64.2	56.6
65.6		62.7

state is below the *d-d* threshold. A further important result in WM is that single-particle–hole states are found to have very large two-nucleon cluster widths. The lowest 0+ and 2+ states in our shell-model calculation are predominantly 1h-1p states in accord with the large deuteron widths found for those states in WM. Their analysis assumed only a ^1D wave contribution to the 2+ state. The combination of the two 1h-1p components in our 2+ wave function is largely ^1D. Interestingly, the next two 2+, $T=0$ levels predicted by our model consist almost entirely of 2h-2p states.

The present results for positions of the energy levels are only roughly comparable to those of Kramer and Moshinsky.[5] For the even-parity levels they made no estimates of the $1p$ or $1d$ splittings. It also appears that the two-body force they employed was weaker than that used here, since given excited states are at relatively higher excitation energies. Vashakidze and Mamasakhlisov[4] obtain results for the energy levels of He^4 for values of $\hbar\omega$ from 16 to 21.8 MeV. For $\hbar\omega=16$ and 21.8 MeV, considering both 1h-1p and 2h-2p excitations, their lowest 0+ state is at 20.7 and 28.1 MeV, respectively. Regardless of the value of $\hbar\omega$ they employ, their lowest 0+ and 2+ states are always separated by less than 2 MeV and they obtain a low-lying 4+ level ($\approx$25–30 MeV) because of incorrect treatment of the c.m. motion for that state, where up to 4p-4h excitations were allowed. The results published for other even-parity levels apparently considered only 1h-1p excitations and would have spurious c.m. components in $T=0$ states. Their odd-parity spectrum is comparable to ours, though for $\hbar\omega=21.8$ MeV their odd-parity levels are several MeV higher. The odd-parity results of the present calculation are similar to those obtained by de-Shalit and Walecka.[1] The ordering of the odd-parity levels follows the Landé interval rule, contrary to the spectrum of WM shown in Fig. 1. Furthermore our $T=0$ odd-parity spectrum is too compact. Barrett *et al.*[30] found that a tensor force depresses the 0−, $T=0$ state close to the 2−, T=0 level, in agreement with the analysis of $H^3(p,n)He^3$ polarization data. In Barrett's complete analysis[2] of the He^4 odd-parity levels, the use of a realistic nucleon-nucleon interaction, the Tabakin potential, is found also to force up the 1−, $T=0$ level, completing the agreement with the features of the experimental odd-parity $T=0$ spectrum.

The energies of the odd-parity $T=1$ states calculated here are close to the experimental values though slightly high and with a change in the order of 0− and 1− levels. More serious is the fact that this calculation predicts the upper 1−, $T=1$ level to be predominantly ^1P. This agrees with the result of de-Shalit and Walecka[1] (our calculations are very similar) and with Tombrello's phase-shift analysis[31] of $p+He^3$ and $n+H^3$ scattering data. Morrow and Haeberli[29] have reanalyzed the $p+He^3$ data and obtain another solution consistent with the lower states being predominantly ^1P. As previously mentioned, this is the result favored by the analysis of WM. Once more the calculation[2] with the Tabakin potential was found to produce the correct level ordering for $T=1$ odd-parity states and to increase substantially the ^1P component of the lower 1− state, though not sufficiently to obtain precise agreement with the latest experimental analyses. In view of the favorable results obtained with the Tabakin potential for the odd-parity levels of He^4, calculations by the author are in progress applying this potential to a reanalysis of the even-parity states.

There is, at this time, no strong experimental evidence for even-parity $T=1$ levels. Some of these levels obtained in our calculation will be considered in our discussion of the electromagnetic transitions.

Largely for the sake of completeness, the energies of the $T=2$ states (all of which are above 50 MeV) predicted by the present model are given in Table III.

As expected, the spectrum obtained using the RPA was substantially the same as that obtained in the usual shell-model approximation. The energies of a few of the states were reduced by small amounts. The largest effect of the RPA matrix elements was on the first 0+, $T=0$ excited state whose energy was reduced

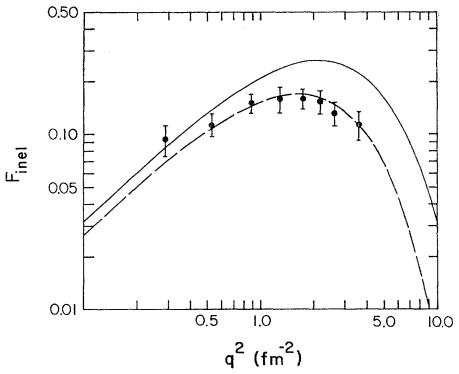

FIG. 3. A comparison of the calculated and observed $E0$ electron inelastic-scattering form factor versus the square of the momentum transfer. Experimental data are those of Ref. 15. The solid curve is obtained using the results of the energy-level calculation (the size parameter was 1.38 fm). The dashed curve is the form factor calculated using a size parameter of 1.6 fm and arbitrarily reducing the result by a factor of 0.6.

[30] B. R. Barrett, J. D. Walecka, and W. E. Meyerhof, Phys. Letters **22**, 450 (1966).

[31] T. A. Tombrello, Phys. Rev. **138**, B40 (1965); **143**, 772 (1966).

by slightly more than 0.5 MeV (compared to the 19 MeV produced by the particle-hole matrix elements). There were small effects on the odd-parity $T=1$ states as well. These were lowered by 0.2–0.4 MeV.

B. Electron-Inelastic-Scattering Form Factor

The monopole electron-inelastic-scattering form factor calculated using Eq. (20) and the ordinary shell-model results is compared with the data of Frosch *et al.*[15] in Fig. 3. The calculated results are about 60% high and peak at too large a value of the momentum transfer. The RPA adds another 11% to the form factor, increasing the discrepancy. The effect of the RPA determined here is less than that obtained by Blomqvist.[13] [His $ZF(q)$ is to be identified with our F_{inel}, since $F(q)$ has a factor $1/Z$ in its definition.] In his wave functions, the amplitudes of RPA contributions are more than 40% of the usual shell-model amplitudes. The sum of these latter amplitudes is >1 in order to satisfy the RPA normalization condition (8). In our case, the presence of 2h-2p components tends to lower further the monopole strength. The primary cause of our differences with Blomqvist is not the fact that his states have spurious c.m. components which can be eliminated completely only by including 2h-2p components. (It is to be noted that Blomqvist's wave function also has a small $4\hbar\omega \mid 1s^{-1}3s\rangle$ component.) The present calculation was repeated neglecting 2h-2p components with the result that the RPA amplitude is still only 10% of the usual shell-model amplitude, but the 1h-1p components (and the form factor) are increased because of renormalization in the smaller basis. His results are apparently caused by using too large single-particle energies and compensating for this by more than doubling the usual strength of the Kallio-Kolltveit potential used as an effective interaction in order to obtain the monopole state at the correct experimental energy. Off-diagonal components of the interaction matrix (including the RPA terms) are then much larger than would be the case if agreement with experiment were obtained with smaller single-particle energies.

The dashed curve in Fig. 3 shows the effect on the form factor obtained from the shell model with a size parameter of 1.6 fm (spreading out the wave function) and arbitrarily multiplying the result by 0.6. (Of course, since the two parameters in the form-factor relation have been freely varied, there is no longer any connection with the results of the present energy-level calculation.) A good even-parity energy spectrum could be obtained using $b=1.6$ fm by adjusting the $2s$ and $1d$ single-particle energies. The 1h-1p component of the lowest 0+ state is insensitive to changes in the harmonic-oscillator parameter and single-particle energies. To obtain agreement with the experimental form factor the effects of a realistic two-body force would have to reduce the present 0.86 amplitude of the $\mid 1s^{-1}2s\rangle$ configuration to about 0.52. This effect is being investigated.

TABLE IV. Calculated photoabsorption cross sections to levels in He^4.

Level energy (MeV)	Level quantum numbers	Cross section (mb-MeV)
29.0	$J=1-, T=1$	6.7
31.9	$J=1-, T=1$	34.5
34.2	$J=2+, T=0$	0.76
47.9	$J=2+, T=0$	0.01
50.5	$J=2+, T=0$	0.03
46.8	$J=2+, T=1$	0.15
49.0	$J=2+, T=1$	0.52
51.6	$J=2+, T=1$	0.06

C. Integrated Photoabsorption Cross Sections

The integrated photoabsorption cross sections calculated from the present model for various levels in He^4 are given in Table IV. The sum of the listed terms is 42.7 mb MeV. The cross sections shown were evaluated at energies predicted by the theory. Evaluating the E1 cross sections at energies predicted by WM for the dipole states (placing our stronger state at 28.0 MeV and our weaker state at 30.5 MeV) would increase the results by another 3.6 mb MeV. The lowest quadrupole state would be somewhat weakened (by about 20%) because of the energy variation of the $E2$ transverse matrix elements.

The calculation indicates that 96% of the total cross section is due to electric dipole transitions. This supports the procedure used to obtain total photoabsorption cross sections wherein the differential cross section measured at 90° is merely multiplied by $8\pi/3$ to accomplish the integration over the $\sin^2\theta$ angular distribution characteristic of particles emitted following an $E1$ transition. It is significant that there are certain 2+ levels with observable quadrupole strengths but no 1+ levels with a measurable magnetic dipole strength. The quadrupole strengths of 2+, $T=0$ levels at 39.6 and 44.1 MeV are zero, consistent with the 2h-2p character of their wave functions.

Denisov and Kul'chitskii[32] have determined that the total integrated He^4 $(\gamma, p)H^3$ cross section between 20 and 50 MeV is 28.0 ± 2.0 mb MeV. If it is assumed that for all levels the neutron width equals the proton width and that the deuteron widths are zero, then the photoabsorption cross section from 20–50 MeV has a total value of 56 mb MeV. Thus our calculation produces almost 80% of this value. The assumption concerning the particle decay widths is compatible with the findings

[32] V. P. Denisov and L. A. Kul'chitskii, Yadern. Fiz. **6**, 437 (1967) [English transl.: Soviet J. Nucl. Phys. **6**, 318 (1968)].

of WM. Their results were obtained with zero deuteron width for all odd-parity levels including the dipole states. For the terms contributing to photoabsorption, only the lowest 2+ level has an appreciable deuteron width. However, our results show that its contribution is considerably smaller than the contribution from the dipole states.

The majority of our E1 strength is in the upper 1−, $T=1$ state contrary to an experiment[30] which indicates that the E1 strength of the upper state is only about half that of the lower state. This discrepancy, however, was expected in the light of our observation of the amplitude of the ^1P components of the dipole states.

The use of the RPA was not essential to the shell-model determination of the photoabsorption cross section. With the exception of the 1−, T=1 states the effects of the time-reversed matrix elements were quite negligible because of small RPA amplitudes in the eigenstates. In the lower electric dipole state considered in the usual shell model, the $E1$ contributions of the $|(1s_{1/2})^{-1}1p_{1/2}\rangle$ and $|(1s_{1/2})^{-1}1p_{3/2}\rangle$ configurations are of opposite sign, with the $p_{3/2}$ component having the larger magnitude. The $p_{1/2}$ RPA component interferes destructively with the $p_{1/2}$ shell-model contribution reducing the cancellation with the $p_{3/2}$ shell-model term and thus increasing the cross section to 9.0 mb MeV. In the upper state where the $p_{1/2}$ and $p_{3/2}$ shell-model contributions have the same sign, the RPA produces destructive interference lowering the cross section to 31.9 mb MeV. Percentagewise there is a large increase in the cross section to the lower state and the relative strength of the upper and lower states has been reduced from 5.1:1 down to 3.5:1. The essential fact remains that the present model predicts the upper 1−, $T=1$ level to have most of the $E1$ strength and the sum of the cross-section contributions from the two states remains virtually unchanged.

Crone and Werntz[33] have studied theoretically the photodisintegration process in He^4. Their wave functions were members of $SU(4)$ supermultiplets. Shell-model wave functions inside the nucleus were coupled to wave functions in the continuum using boundary conditions determined from R-matrix theory. In their treatment, the resonances were not assumed to be isolated so that interference effects between multipoles could be evaluated. For the $He^4(\gamma, p)H^3$ process they obtain better agreement with experimental measurements of the total cross section by assuming that the dipole strength is mainly in the lower 1−, $T=1$ state. However, their peak is too sharp and the calculated results are lower than the experimental values in the high-energy tail. This they attribute to not varying the wave function with energy in this range.

Certain features of the present model are in general agreement with the results of Crone and Werntz. Their analysis of the He^4 $(\gamma, p)H^3$ and He^4 $(\gamma, n)He^3$ angular distributions indicates the presence of $E2$-$E1$ interference in the region of 29 MeV caused by a 2+, $T=0$ level. This agrees with the present results which indicate that a 2+, $T=0$ state at about this energy contains virtually all the $T=0$ quadrupole strength. Because of observed differences in the forward-backward asymmetries of the proton and neutron angular distributions they have added a $^1D_2$, 2+, $T=1$ state at an arbitrary energy of 35 MeV to account for this. In the present model, the quadrupole strength (and $^1D_2$ hole-particle component) is distributed among three 2+, $T=1$ states at 46.8, 49.0, and 51.6 MeV, with the second state dominating. Allowing for the fact that our model predicts energies several MeV high it would still seem that an energy somewhat in excess of 40 MeV is indicated for the strongest 2+, $T=1$ state.

The transverse *matrix elements* of Crone and Werntz were multiplied by a factor $E_{fi}/\hbar\omega$ for odd-parity states and $E_{fi}/2\hbar\omega$ for even-parity states (where E_{fi} is the experimental energy of a level) to make the matrix elements consistent with current conservation. Such a correction is strictly true if the nucleon-nucleon interaction has only a Wigner component. For a force with exchange terms, such as employed here, the corrections are more difficult to make. If these corrections in the present case were comparable to those made by Crone and Werntz, the calculated cross sections would increase, thereby improving the agreement with experiment. It would be surprising, however, to obtain exact agreement with experiment in view of the simplified approach used here in treating the photoabsorption process.

V. SUMMARY AND CONCLUSIONS

From the present analysis it is evident that the shell model can successfully describe the known excited states of He^4. The agreement with the experimental results is reasonably good in view of the simple nucleon-nucleon force assumed and the fact that there was no parameter search. Ground-state correlations of the type occurring in the first RPA are found to have a negligible effect on the positions of the energy levels. A more realistic force than that employed here is necessary to obtain a better quantitative description of the odd-parity spectrum.

The model does predict the over-all magnitude of the integrated photoabsorption cross section very well, though the $E1$ strength is concentrated in the wrong state. The monopole form factor to the lowest 0+ state is only qualitatively correct and the results indicate that the harmonic-oscillator size parameter should be increased. The RPA causes some redistribution of the $E1$ strength to the lower 1−, $T=1$ level and enhances the $E0$ electron inelastic-scattering form factor by

[33] L. Crone and C. Werntz, Nucl. Phys. **A134**, 161 (1969).

about 10%. The general effects of the RPA, however, are not pronounced.

The present results dictate that a more reliastic force be used as well as lower $2s$ and $1d$ energies. Calculations using the Tabakin potential are currently in progress and a brief report of the results will be given in the near future.

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Scattering of 19–30-MeV Alpha Particles from C^{12} †

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Elastic and inelastic excitation functions and differential cross sections of α particles scattered from C^{12} have been measured. Excitation functions were taken from 18.90 to 30.06 MeV in 20-keV steps. Differential cross sections were measured at the energies of anomalies in the excitation functions. Optical-model and phase-shift fits to the elastic angular distributions were attempted without success. A statistical analysis of the excitation function indicates that the reaction is proceeding mainly through the compound nucleus with a coherence width Γ of 360 keV (c.m.).

INTRODUCTION

THE scattering of α particles from light nuclei has recently been of interest to several workers.[1] This interest has increased since the report by Singh *et al.*[2] of evidence for intermediate structure in these reactions. In a series of experiments on *s-d* shell nuclei, structure of two to three times the compound nuclear width was observed in the 22–27-MeV region of excitation in Si^{28}, Si^{30}, and P^{32}. Singh and others[3,4] have interpreted this as evidence for simple structure, possibly 4p-4h states.

An investigation of this same region in other light nuclei seemed worthwhile and feasible with the expanded energy range of the newer tandem accelerators. We began with an investigation of the α elastic and inelastic scattering from C^{12}.

α scattering from C^{12} has been studied as a function of energy and angle up to 19 MeV (for a list of references see Ref. 1). Above this energy, only scattered cyclotron data are available. For this reason we began our investigation at 18.9 MeV. Excitation functions were measured in 20-keV steps to 30.06 MeV. Angular distributions were then measured at the energies of the anomalies in the excitation functions.

EXPERIMENTAL TECHNIQUE

The doubly charged helium beam from the Williams Laboratory tandem Van de Graaff accelerator struck self-supporting targets in an ORTEC 17-in. scattering chamber. For the excitation functions, the data for elastic scattering from C^{12} and O^{16} were measured simultaneously, using LiOH evaporated on 20-$\mu g/cm^2$ carbon foils.[5,6] The differential cross sections were measured using 100-$\mu g/cm^2$ C^{12} foils. Monitor runs indicated that the carbon thickness increased linearly with time due to hydrocarbon contamination, increasing by about 18% in 6 days of exposure to the beam. The data were suitably corrected.

The accelerator energy was originally calibrated using

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[2] P. P. Singh, B. A. Watson, S. S. Kroepfl, and T. P. Marvin, Phys. Rev. Letters **17**, 968 (1966).

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