tions. Even if the Coulomb-excitation cross section were raised by a factor of 2, this discrepancy would be serious.

The theoretical level schemes (S.M.) in Fig. 8 are from the effective-interaction shell-model calculations of Lips,[28] whose calculations provide a least-squares fit to nuclei with $20<Z<28$ and $N=28$. These spectra represent a substantial improvement over the older $(1f_{7/2})^n$ calculations of McCullen *et al.*,[2] since they include configurations of the form $(f_{7/2})^2p_{3/2}$ for ^{51}V and $(f_{7/2})^4p_{3/2}$ for ^{53}Mn. The results are somewhat similar to those of Auerbach.[29] The success of the model can be further tested by looking at γ-ray transition rates. These are calculated[28] with a free-particle $M1$ operator and $E2$ effective charge of $1.6e$, the value reported[26] to fit the ground-state quadrupole moment and $BE2$ values in ^{51}V. The calculated results are about the same for both nuclei. In both cases, the $E2$ transition rates are well represented, but the calculated $BM1$ strengths are too small by factors of 3–8. An extension of the model to include the $f_{5/2}$ configuration[28] does fit both $E2$ and $M1$ transition rates.

The presently available data may also be interpreted in the strong-coupling model of Scholz and Malik,[2] who use Coriolis coupling of deformed single-particle states to a deformed rotating core. The ground-state quadrupole moment and $E2$ transition rates are well represented by a small negative deformation, with the parameter[2] $\beta=0.15$. This would be quite consistent with the shell-model picture of the nucelus. The calculated excitation energies are given[2] for $\beta=-0.32$ and reproduced in Fig. 8. The calculated $M1$ transition rates are not in agreement with the measurements.

ACKNOWLEDGMENTS

The authors acknowledge the assistance of Dr. W. R. Harris, who provided absolute detection efficiencies of the 35-cc Ge(Li) detector and FORTRAN programs used in processing the γ-ray spectra. Two of us (D. M. S. and M. T. M.) are especially grateful for the opportunity afforded us as summer visitors at Brookhaven, and express our appreciation to Dr. D. E. Alburger, who made the facilities of the van de Graaff Laboratory available to us.

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[28] K. Lips and M. T. McEllistrem, Phys. Rev. C **1**, 1009 (1970).

[29] N. Auerbach, Phys. Letters **24B**, 260 (1967).

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Scattering of 14.5-MeV Protons from the Even Isotopes of Germanium*

T. H. Curtis, H. F. Lutz, and W. Bartolini
Lawrence Radiation Laboratory, University of California, Livermore, California 94550

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Protons from the 90-in. cyclotron at Lawrence Radiation Laboratory (LRL)–Livermore have been scattered from enriched targets of ^{70,72,74,76}Ge, with an over-all energy resolution (FWHM) of 70 keV. The incident proton energy was approximately 14.5 MeV, and angular distributions were obtained from 20° to 150°, in 10° intervals. The elastic-scattering results were analyzed using the optical model. Angular distributions were obtained for inelastic proton groups from seven levels in ^{70}Ge, nine levels in ^{72}Ge, eleven levels in ^{74}Ge, and ten levels in ^{76}Ge—all below 3.5-MeV excitation. These included scattering to the quadrupole and octupole one-phonon vibrational levels and scattering to the 0^+, 2^+, 4^+ two-quadrupole–phonon triplet. The data were analyzed using a coupled-channel computer code and a vibrational model of the nucleus. Strengths to excited states (β_l) and spins and parities of levels were determined and are compared with Coulomb-excitation decay studies and with other scattering experiments.

I. INTRODUCTION

PROTON scattering is an established technique for the study of nuclear structure. We are currently engaged in a series of proton-scattering experiments from the even-even isotopes of Cd,[1] Ti,[2] and Ge using a vibrational model and the coupled-channel analysis of Tamura[3] to interpret our data. The germanium isotopes have not been studied extensively by scattering techniques. The energy levels, spins, and parities of ^{70,72,74}Ge have been deduced mainly from the decay of the neighboring isotopes of As and Ga. The results of these decay measurements will be discussed and compared to our results later in this article.

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* This work was performed under the auspices of the U.S. Atomic Energy Commission.

[1] H. F. Lutz, W. Bartolini, and T. H. Curtis, Phys. Rev. **178**, 1911 (1969).

[2] H. F. Lutz, W. Bartolini, T. H. Curtis, and G. M. Klody, Phys. Rev. **187**, 1479 (1969).

[3] T. Tamura, Rev. Mod. Phys. **37**, 679 (1965); Oak Ridge National Laboratory Report No. ORNL-4152, 1967 (unpublished).

Dickens *et al.*,[4] using 10.5-MeV protons, have observed the quadrupole and octupole vibrational levels in each of the four isotopes, and have used a distorted-wave Born-approximation (DWBA) code to analyze their angular distributions. Their results for β_l agree with this work.

The existence of the strong 2^+ and 3^- levels in each of the germanium isotopes suggests that a vibrational model be used to interpret the germanium level structure. However, in a pure vibrational model, one predicts a two-phonon 0^+, 2^+, 4^+ triplet of levels at twice the energy of the one-quadrupole–phonon state. In each of the germanium isotopes, low-lying states with spins 0^+, 2^+, and 4^+ have been observed, but they are shifted rather far from twice the energy of the first 2^+ level. In particular, in ^{72}Ge, the 0^+ member of the triplet at 0.690 MeV is below the excitation energy of the one-quadrupole–phonon level at 0.835 MeV.

We have chosen to analyze our results using a coupled-channel calculation. Although the DWBA is sufficient for the interpretation of weakly excited levels, it is believed to be a poor approximation for strong collective levels and, particularly, is not well suited to handle two-step processes, such as are involved in the excitation of the two-quadrupole–phonon triplet. One could conceivably use a higher-order Born approximation, but this calculation would be more complex than a coupled-channel approach. The coupled-channel calculation has the capability of handling exactly multiphonon processes within the framework of a vibrational model. Thus, the germanium isotopes appeared to be a reasonable and interesting region to test some of the two-phonon vibration ideas. As will be seen, fitting using a two-phonon picture was only partly successful.

A second point of interest in these isotopes is a study of the fractionation of the octupole state. The use of the term fractionation implies more than the mere presence of several octupole states. By fractionation, we mean that a strong state in one isotope is apparently divided into several weaker states of the same multipole order in a nearby isotope. However, the sum of the total cross sections to the weaker states is approximately the same as the total cross section to the original state. In the germanium isotopes, protons fill the $2p_{3/2}$ subshell while neutrons in ^{70}Ge fill the $1f_{5/2}$ subshell, in ^{72}Ge fill the $2p_{1/2}$ subshell, and in ^{74,76}Ge are beginning to populate the $1g_{9/2}$ subshell. Veje[5] suggests that fractionation be associated with unfilled subshells, and indeed we find a tendency in ^{74}Ge and ^{76}Ge for the octupole strength to be split more or less evenly among several levels.

TABLE I. Isotopic composition of germanium targets used in this experiment.

Atomic percent	Target ^{70}Ge	^{72}Ge	^{74}Ge	^{76}Ge
70	91.4	2.70	0.69	0.82
72	3.73	90.9	1.14	0.52
73	1.01	1.27	1.56	0.55
74	2.70	4.23	95.8	4.56
76	1.18	0.93	0.82	93.55

II. EXPERIMENTAL METHOD

The LRL 90-in. variable-energy cyclotron produced a beam of approximately 14.5-MeV protons, which were momentum-analyzed using a 90° bending magnet with a 76.2-cm radius of curvature. For the several different cyclotron runs required to obtain the data, the incident proton energy varied from 14.35 to 14.6 MeV, but was constant for any one isotope. The beam was focused using a quadrupole triplet to a spot on the targets at the center of a 60.9-cm-diam scattering chamber. The beam was monitored with a Faraday cup and current integrator.

Attempts to use 1-mg/cm^2 self-supporting Ge targets proved unsuccessful. Exposure to the 20–200-nA proton beam for several hours caused the extremely brittle targets to fracture. When examined, the targets showed signs of thermal stress. To prevent further deterioration of the targets, they were dipped in a dilute solution (2%) of polystyrene (CH) dissolved in benzene. Thus coated, the targets were preserved, but contaminant peaks due to carbon and hydrogen in oxygen (which formed as GeO on the surface of the targets) were present in our spectrum. The deterioration of the targets prevented a reliable measure of their areal density, and the absolute values of the cross section were determined by normalizing the forward-angle elastic scattering to a calculated curve. The details of this normalization are given in Sec. III.

The isotopic constitution of each of the targets is given in Table I. The targets were supplied by the isotopes division of Oak Ridge National Laboratory.

A single lithium-drifted silicon detector 2000 μ thick, operated at 300 V reverse bias and subtending 2.5×10^{-4} sr, was used to count the scattered particles. It was not necessary to use a counter telescope to identify protons due to the high negative Q values and the low yields of other reactions. The detector was cooled to -75°C, using a dry-ice and acetone mixture, to reduce leakage current. Toward the end of the Ge runs, thermoelectric coolers were used to

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[4] J. K. Dickens, F. G. Perey, and R. J. Silva, Oak Ridge National Laboratory Report No. ORNL-3714, 1964 (unpublished), p. 1.

[5] C. J. Veje, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **35**, No. 1 (1966).

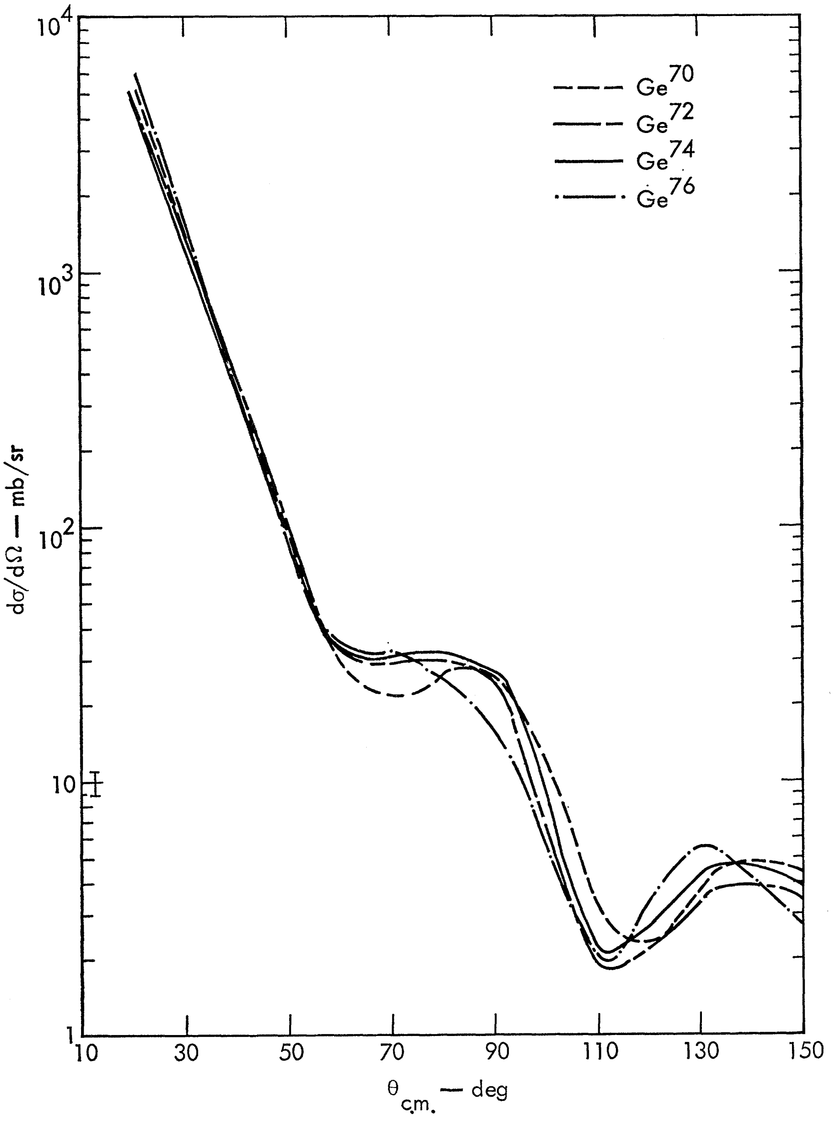

FIG. 1. Elastic-scattering cross sections for the germanium isotopes. Absolute values of the cross sections were determined using the normalization procedure described in the text.

cool the detectors to −25°C. This was sufficient to reduce leakage currents, and thermoelectric coolers were considerably more straightforward to use. Pulses from the detector were analyzed and stored in an 800-channel analyzer.

Punched paper tape from the analyzer was converted to cards, which were input for several programs written for the CDC-3600 computer. The spectra were plotted, and peak positions were determined visually and used as input to a program that fit a Gaussian to each peak. In this experiment, we could resolve peaks separated by 70 keV.

III. ELASTIC SCATTERING

The elastic-scattering cross sections were used to normalize the data and to provide optical-model parameters to use as input data for the coupled-channel analysis of the inelastic scattering.

A. Normalization

To assign an absolute scale to our cross-section measurements, the forward-angle elastic scattering was normalized to a calculated cross section. This was done first for ^{70}Ge. Using values of the optical-model parameters obtained from the formulas given by Perey[6] ($V_0=50.7$ MeV, $W_D=13.7$ MeV, $V_{SO}=-7.5$ MeV, $r=1.25$ F, $a=0.65$ F, $\bar{a}=0.47$ F), the elastic cross section from 20° to 50° was calculated from the optical model. This cross section was relatively independent (<2% for a 5-MeV change in the real

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[6] F. G. Perey, Phys. Rev. **131**, 745 (1963).

part of the central potential) at forward angles. The 20°, 30°, 40°, and 50° ^{70}Ge points were normalized to this curve. The forward-angle points of the other three elastic cross sections were then normalized to the ^{70}Ge, to provide the normalizations for the rest of the data. This provided only one normalization to a calculated curve instead of four, with the reasonable assumption that at forward angles the four elastic cross sections would have the same magnitude.

Because of the uncertainty in the normalization procedure, we have assigned 10% error bars to most of our cross sections, which is larger than statistical error in most cases. For some of the very small inelastic cross sections, the statistical error plus the error due to background subtraction is of the order of, or larger than, 10%. This occurs for a cross section of about 0.15 mb/sr. Hence, for cross sections smaller than this value, we have assigned an absolute error of 0.015 mb/sr. It should be kept in mind in viewing all angular distributions that the error bars assigned are absolute, not relative, errors.

The elastic cross sections for all four isotopes have been plotted on one graph in Fig. 1 for easy comparison. The graph is qualitatively similar to a graph given by Dickens *et al.*[7] for 10.5-MeV protons scattered by the four germanium isotopes. The ^{70}Ge has the most pronounced minimum, at about 70°, with the minimum in each of the other isotopes becoming progressively less pronounced. This qualitative agreement supports our normalization procedure.

B. Optical-Model Fit to Elastic Scattering

The elastic angular distributions, properly normalized, were fit using an optical-model least-square search code. A central potential, using a real part V_0 with a Woods-Saxon form factor, an absorptive imaginary part W_D with a derivative Woods-Saxon form factor, and a spin-orbit potential V_{S0} with a Thomas form, were used in the analysis. The exact forms of the potential are given in an article by Tamura.[3]

It was not our purpose to find an individual set of parameters for each isotope, but rather to determine a constant or slowly varying set of parameters that fit the sequence of germanium isotopes. Thus, our best-fit set of parameters for each isotope was adjusted slightly to give the set of parameters listed in Table II. The modification consisted of changing the radius and diffuseness parameters by at most 2% to the values indicated, and the fits to the elastic scattering were not noticeably affected, at least not to the eye. Again, compared to the 10.5-MeV proton scattering of Dickens *et al.*,[4] we find our values of V_0 are the same to 5%, while our values of W_D are much different. In our case, W_D changes from 11.4 to 11.7 MeV in going from ^{70}Ge to ^{76}Ge, while the Oak Ridge value varies from 12.8 to 17.9 MeV over the same range. The values of $V_{S0}=-7.2$ MeV is an average of the four determinations. The elastic-scattering data are not particularly sensitive to this parameter, and the value has an uncertainty of 2 MeV.

TABLE II. Optical-model parameters determined from elastic scattering.

	This work		(p,p') [a]	
Isotope	V (MeV)	W_D (MeV)	V (MeV)	W_D (MeV)
^{70}Ge	50.3	11.4	52.8	12.8
^{72}Ge	50.4	11.5	52.3	14.7
^{74}Ge	50.3	11.6	51.6	17.9
^{76}Ge	50.8	11.7	52.4	16.4

For this work:

$E_0=14.5$ MeV $r_0=1.24$ F $r_c=1.25$ F

$W=0.0$ MeV $a=0.65$ F

$V_{S,O}=-7.2$ MeV $\bar{a}=0.57$ F

[a] Reference 4—no geometric parameters are quoted in this reference.

While V_0 is practically constant except for ^{76}Ge, our values for W_D have an $(N-Z)/A$ dependence given by

$$W_D=11.0+4[(N-Z)/A].$$

The constant 4 in this equation is smaller than the usual value of 16 or 17.

IV. INELASTIC SCATTERING

The levels observed in this experiment for which we have obtained angular distributions, with the spins we have assigned (in some cases previously known) and total cross sections, are listed in Table III. The energy-level determinations are determined from this work, using the known positions[8] of the first-quadrupole state in each isotope as a calibration. The uncertainties of these energies range from 5 to 20 keV, depending on how well the peak was defined. The uncertainties were determined by comparing the excitation energy for each level determined from the spectrum at each angle and seeing how well the results agreed with each other. As can be seen from Table III, our energy assignments are in quite reasonable agreement with previous measurements.

The inelastic angular distributions were analyzed using a coupled-channel code written by Tamura.[3] In this calculation, one assumes that the states of the target nucleus are restricted to a finite set cor-

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[7] J. K. Dickens, F. G. Perey, and R. J. Silva, Oak Ridge National Laboratory Report No. ORNL-3499, 1963 (unpublished), p. 20.

[8] R. L. Robinson, P. H. Stelson, F. K. McGowan, J. L. C. Ford, Jr., and W. T. Milner, Nucl. Phys. **74**, 281 (1965).

Table III. Energies, spins, and parities of levels observed in this work. A comparison is made to the previous most complete experiment for each isotope.

^{70}Ge					^{72}Ge					^{74}Ge					^{76}Ge				
This work			(p,p'), $(p,p'\gamma)$ [a]		This work			^{72}As, ^{72}Ga decay [b]		This work			^{74}As, ^{74}Ga decay [c]		This work			(p,p') [d]	
E_x (MeV)	I^π	σ_t (mb)	E_x (MeV)	I^π	E_x (MeV)	I^π	σ_t (mb)	E_x (MeV)	I^π	E_x (MeV)	I^π	σ_t (mb)	E_x (MeV)	I^π	E_x (MeV)	I^π	σ_t (mb)	E_x (MeV)	I^π
0	0^+		0	0^+	0	0^+		0	0^+	0	0^+		0	0^+	0	0^+		0	0^+
1.040	2^+	40	1.040	2^+	0.694	0^+	7	0.6913	0^+	0.596	2^+	43	0.598	2^+	0.562	2^+	47	0.562	2^+
1.212	0^+	3.5	1.216	0^+	0.835	2^+	30	0.8339	2^+	1.210	2^+	7	1.20	2^+	1.102	2^+	8	1.109	2^+
1.709	2^+	2.5	1.708	2^+	1.466	2^+	4	1.464	2^+	1.473	4^+	4	1.47	4^+	1.414	4^+	5.5	1.412	
2.162	4^+	3.3	2.153	4^+	1.727	4^+	3.6	1.728	4^+	1.726	0^+	5	1.71		1.902	0^+	7	1.908	
			2.157	2^+	2.043	2^+	1.3	2.065	3^+(2^+[e])	2.210	1^-	1.5	2.18	2^+[f,g]	2.03	4^+	4		
			2.307	0^+	2.4	2^+, 4^+	1.0	2.402	(2^+)	2.546	3^-	7	2.53		2.286	3^-	2		
			2.452	(3)				2.464	(4^+)	2.956	3^-	7	2.95	3^-				2.517	
			2.536	$(1^+, 2^+)$	2.522	3^-	16	2.515	3^-	3.077	3^-	2	3.05					2.567	
2.565	3^-	20	2.562	3^-				2.940	(1^-)	3.160	4^+	3.5	3.16		2.710	3^-	7	2.687	3^-
			2.807	(4^+)	2.956	3^-	4	2.943	(3^-)	3.385	4^+	1.7	3.33					2.727	
			2.887	0				3.036	2^-	3.512	4^+	1.5	3.41					2.763	
			2.945	(2, 1)	3.092	4^+	3						3.57					2.838	
			3.047	3^+				3.094	2^+						2.966	3^-	3.8		
3.063	4^+	6	3.059	4^+											3.196	3^-	3		
			3.107	0											3.498	2^+	3		
3.429	5	3.2																	

[a] Reference 11.
[b] Reference 16.
[c] Reference 22.
[d] Reference 21.
[e] Reference 20.
[f] Reference 23.
[g] Reference 24.

responding to the different reaction channels being considered. One also assumes that the channels are distinct, so that overlap between them may be ignored, and that each individual channel may be described by the optical model (generalized to include collective motion), so that the different channels are coupled together. With these assumptions, the Schrödinger equation for the nuclear wave function becomes a finite set of coupled differential equations, which are then solved numerically. The code then calculates the differential cross sections for both the elastic and inelastic reactions. The code includes spin-orbit distortions, complex form factors, and Coulomb excitation. As we used the code for this analysis, the excited states of the nuclei are described using a vibrational model with the strength to the excited state as a variable parameter. The code allows various coupling schemes, including a mixture of one- and two-phonon contributions to a single state. The discussions of the experimental results are organized in accord with the coupling schemes used to analyze the data.

The first coupling scheme to be considered was the coupling of the one-phonon quadrupole and octupole vibrational states to the ground state $(0^+\text{-}2_1^+\text{-}3_1^-)$. (The subscript refers to the number of vibrational phonons.) Next, we consider the results of coupling the ground state, the one-quadrupole–phonon state and the so-called two-phonon triplet. In this scheme, we allow the possibility of one-phonon contributions to the levels in the triplet. Hence, the coupling scheme becomes $(0^+\text{-}2_1^+\text{-}0_{1,2}^+\text{-}2_{1,2}^+\text{-}4_{1,2}^+)$. Figure 2 shows a schematic representation of the various coupling parameters that go, into the description of these coupling schemes. The β_{02} parameter is the coupling of the ground state to one-phonon 2^+ state, and the parameters β_{20}, β_{22}, β_{24} are the coupling strengths from the one-phonon state to each of the two-phonon states. These coupling strengths all arise from operators that are linear in α, where α is the coefficient of an expansion of the nuclear radius in spherical harmonics. In addition, there are transitions from the ground state directly to the two-phonon state that are quadratic in α and are denoted by β_{00}', β_{02}', β_{04}'. The cross sections we calculated turned out to be rather insensitive to the last three parameters, and most often they were set equal to β_{20}, β_{22}, and β_{24}, respectively. The direct excitations of the two-phonon state by an operator quadratic in α is to be kept distinct from the excitation of a one-phonon contribution to each of the members of the triplet. This possibility was included in the coupling scheme by the parameters β_{00}'', β_{02}'', and β_{04}''. It was found necessary in optimizing our fits to the two-phonon states to include some one-phonon contributions.

We concluded the analyses of our data by considering all the other levels to be a simple coupling to the ground state, connected by a single one-phonon parameter β_{0I}. This coupling scheme is denoted by $(0^+\text{-}I^\pi)$.

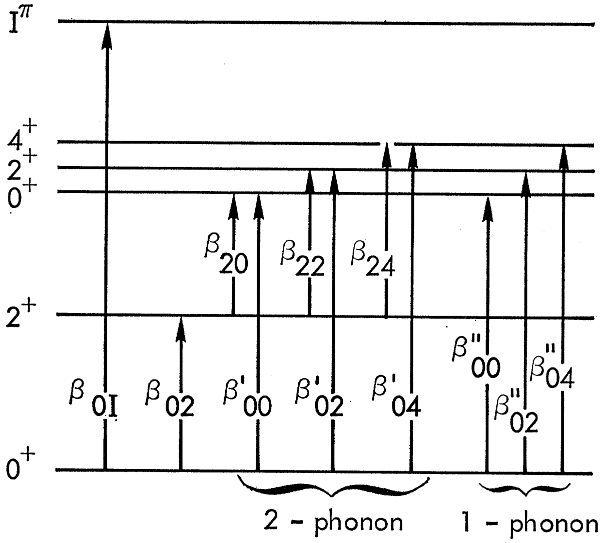

FIG. 2. Schematic representation of the deformation parameters (β) used in the coupled-channel analysis of Tamura (Ref. 3). The unprimed and double-primed quantities are linear in α, while the single-primed quantities are quadratic in α.

The spin assignments for the one-phonon–quadrupole and octupole states and for the triplet were already known. For the other levels, we tried various L values to obtain the best fits, and in some cases we differ from previous measurements. Further comments will be made when we discuss the particular levels.

A. Results for $0^+\text{-}2_1^+\text{-}3_1^-$ Coupling Scheme

The 2^+ and 3^- one-phonon states were the strongest inelastic levels seen in the experiment. The positions of the 2^+ levels were used as the calibration energy, and the positions of the 3^- states were identified by their strength and systematic appearance at about 2.6 MeV.

The intent of the fitting procedure was to have as few variable parameters as possible. We did not vary the potential well depths, therefore, or the geometric parameters (r or a) determined from the elastic-scattering analysis, except for W_D. In the optical-model analysis of the elastic scattering W_D, the strength of the absorptive part of the central potential accounts for all the cross section to inelastic and reaction channels. In the coupled-channel analysis, we explicitly calculate some of these nonelastic channels, and thus W_D must be reduced by some amount. Since the strength of the cross section to the inelastic channel is determined by the deformation parameter β, there is a $W_D \times \beta$ ambiguity—the more W_D is reduced, the smaller β must be.

To resolve this ambiguity, we used the accepted values[4,9] of β_2 and β_3 for the 2^+ and 3^- states of ^{70}Ge.

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[9] P. H. Stelson and L. Grodzins, Nucl. Data **1**, 21 (1965).

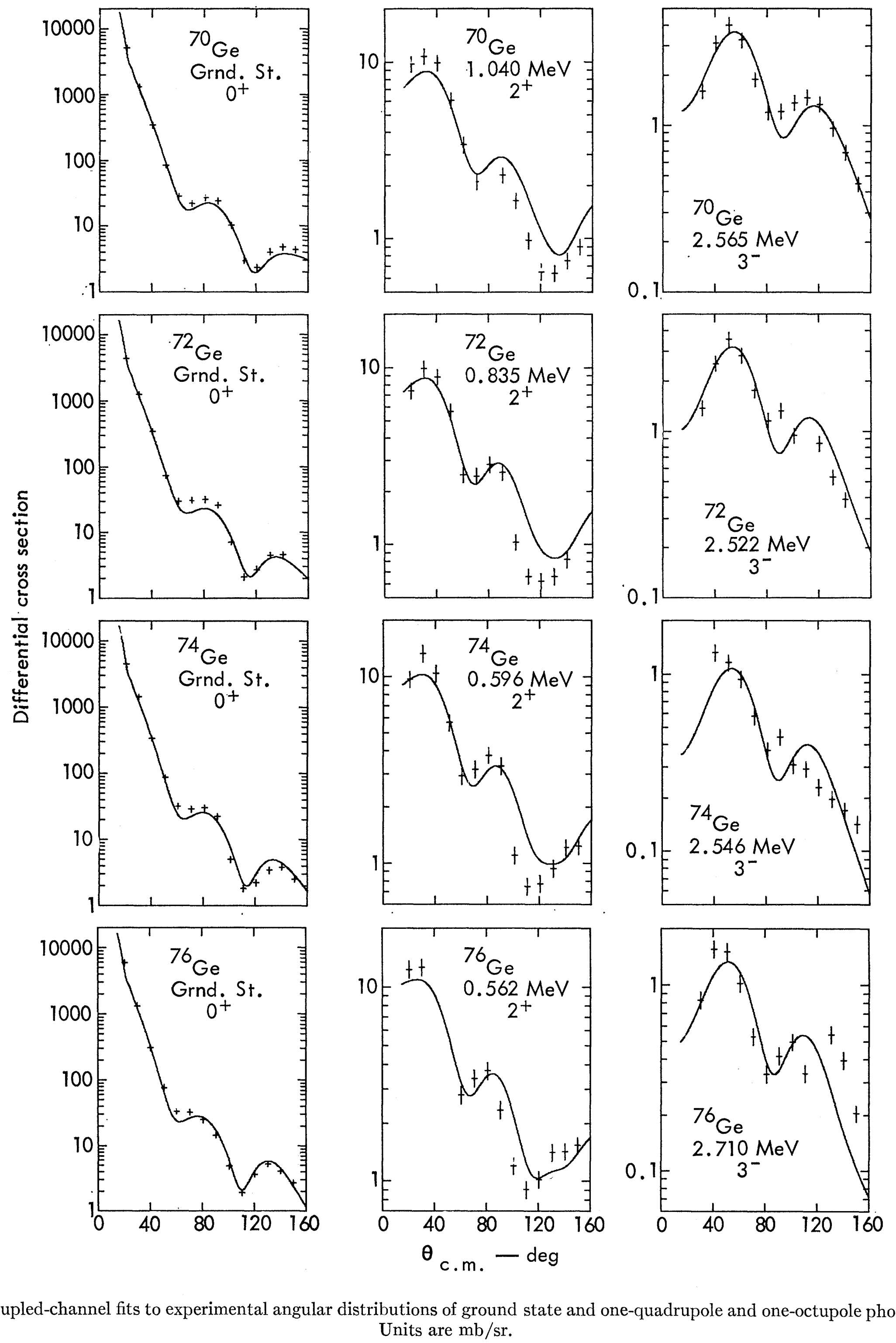

FIG. 3. Coupled-channel fits to experimental angular distributions of ground state and one-quadrupole and one-octupole phonon states. Units are mb/sr.

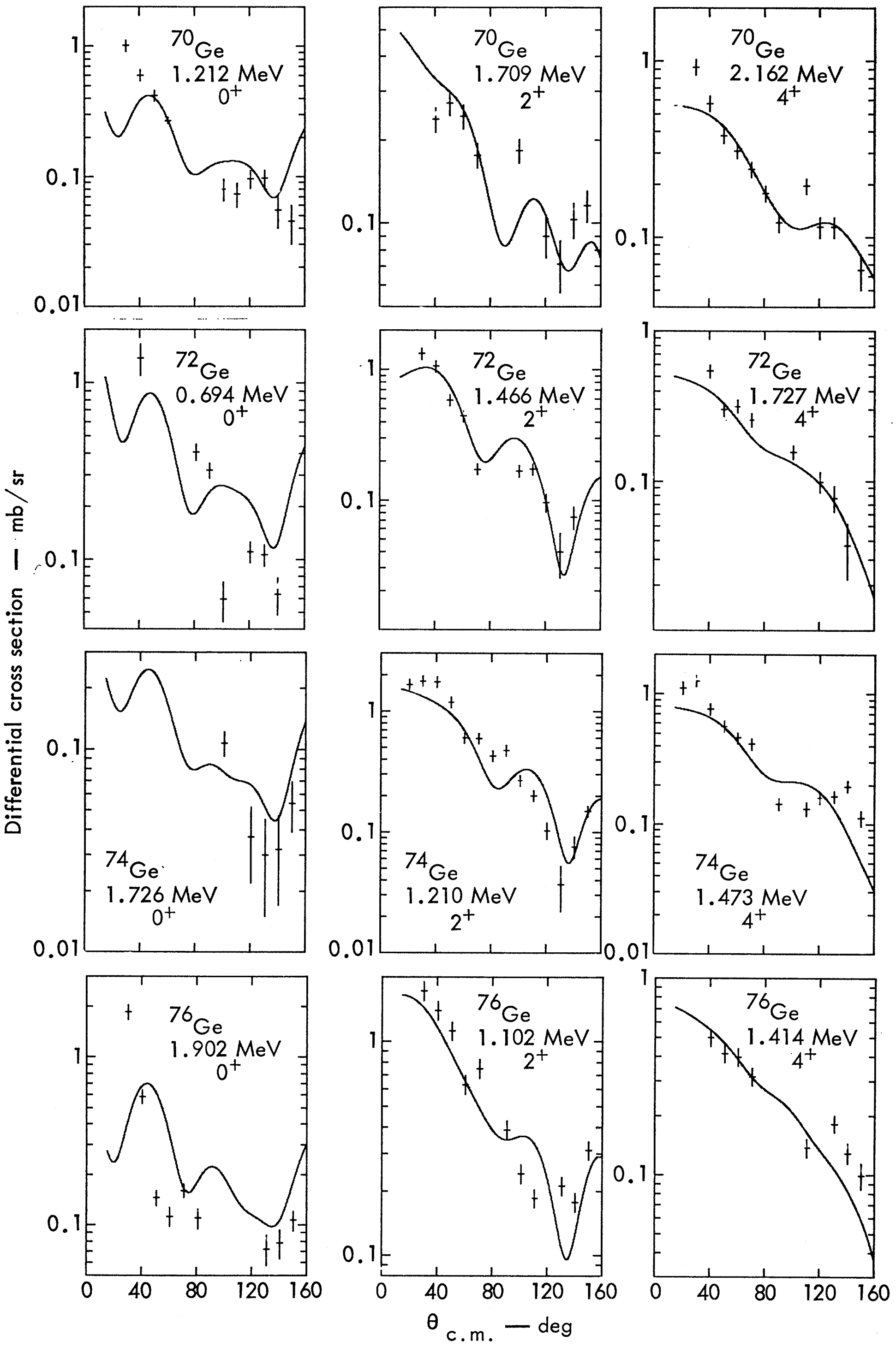

FIG. 4. Coupled-channel fits to experimental angular distributions of one- and two-quadrupole phonon states.

TABLE IV. Parameters of fit to quadrupole and octupole one phonon states.

Isotope	W_D (MeV)	Q (MeV)	J^π	This work	β [a]	(d, d') [b]
^{70}Ge	8.4	−1.040	2^+	(0.22)	0.22	0.20
		−2.565	3^-	(0.25)	0.25	0.20
^{72}Ge	9.1	−0.835	2^+	0.22	0.25	0.20
		−2.522	3^-	0.23	0.23	0.21
^{74}Ge	9.0	−0.596	2^+	0.23	0.29	
		−2.546	3^-	0.13		
^{76}Ge	8.8	−5.62	2^+	0.22	0.27	
		−2.710	3^-	0.14		

[a] β_{02} values are taken from Stelson and Grodzins, Ref. 9; β_{03} values are taken from Dickens *et al.*, Ref. 4.

[b] Reference 14.

Keeping these fixed, as well as the other parameters, we varied W_D in the coupled-channel code to obtain the fit shown in Fig. 3 for ^{70}Ge. This was obtained with a value of W_D equal to 8.4 MeV, or a reduction of 3 MeV from the 11.4 MeV determined from the optical-model analysis of the elastic scattering. The total cross section to the 2^+ and 3^- levels in ^{70}Ge is 60 mb. Hence, a 3-MeV change in the absorptive part of the potential accounts for 60 mb of inelastic cross sections, or 0.5 MeV/10 mb. This figure was used to calculate the amount W_D should be reduced for all the other coupled-channel fits made in this experiment.

The fits to the ^{72,74,76}Ge ground state and to the quadrupole and octupole one-phonon states are shown in Fig. 3, and the fitting parameters are summarized in Table IV. The values of β determined in this experiment for the latter three isotopes also are compared with previous measurements in Table IV. For ^{72}Ge, the results for β_{02} and β_{03} are quite good—within 10%. For ^{74,76}Ge, our values of β_{02} are somewhat smaller than the values quoted by Stelson and Grodzins,[9] but are quite close to our values of β_{02} for ^{70,72}Ge. No previous measure of the β_{03} to the octupole states for ^{74,76}Ge could be found.

B. Results for 0^+-2_1^+-$0_{1,2}^+$-$2_{1,2}^+$-$4_{1,2}^+$ Coupling Scheme

The fits to the triplet were not as satisfactory as the fits to the one-phonon states of Sec. IV A. The graphs of the calculated curves and experimental prints are shown in Fig. 4, and the results of the fitting are summarized in Table V. The β_{02} parameter, the strengths from the ground state to the one-phonon 2^+ level, was fixed by the results of Sec. IV A. The β_{2I} parameters were then adjusted to give the best fits that could be obtained. In general, the β_{0I}' parameters were set equal to β_{2I} parameters, as mentioned earlier. However, for ^{76}Ge, some improvement was found by allowing the β_{0I}' parameters to change for the 2^+ and 4^+ levels. In general, the β_{2I} parameters did not differ by more than 15% from their corresponding β_{02} parameter, which is consistent with a simple two-phonon picture. It was necessary in all cases to include some one-phonon contributions to each of the levels in the triplet. This procedure improved the fits by changing somewhat the structure of the calculated angular distributions. In Table V, we have listed the one-phonon β parameter, and the product of the β from the ground state to the first 2^+ times the β from the 2^+ to each of the members of the triplet. A comparison of these two quantities gives an indication of the relative amounts of one-phonon and two-phonon contribution to the triplet. The percent of one-phonon contributions $\beta_{0I}''^2/[(\beta_{02}\times\beta_{2I})^2+\beta_{0I}''^2]$ is listed in the last column of Table V.

In general, we were able to fit the 2^+ and 4^+ members of the triplet, while the 0^+ curves bear little resemblance to the data. The fit of the anomalous 0^+ (below the first 2^+) in ^{72}Ge is no better nor no worse than the fits to other 0^+ levels, and, thus, we can not make any statement about that particular level.

Hsu and French[10] have made a microscopic calculation of the vibrational states of the even nickel isotopes, using a variety of techniques including the Tamm-Dancoff and random-phase approximations. They conclude that the 2^+ and 4^+ members of the two-phonon triplet should appear where expected in the vibrational spectrum, but that the 0^+ member should appear at approximately 5-MeV excitation. If we take these results over to the germanium isotopes, they provide an explanation for our fitting of the 2^+ and 4^+ states while not fitting the 0^+. Attempts to fit the 0^+ states with a simple $L=0$ curve where not successful.

C. Results for 0-I^π Coupling Schemes

These results are grouped by isotope, and all shown in Figs. 5(a) and 5(b). The parameters of the fit to

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[10] L. S. Hsu and J. B. French, Phys. Letters **19**, 135 (1965).

TABLE V. Parameters of fit to states with one- and two-quadrupole phonons.

Isotope	W_D (MeV)	Q (MeV)	J^π	β_{02}	β_{0I}' two phonon	β_{2I} two phonon	β_{0I}'' one phonon	$\beta_{02}\times\beta_{2I}$	% of one-phonon contributions
^{70}Ge	8.9	−1.212	0^+	0.22	0.22	0.22	0.02	0.048	15
	8.9	−1.709	2^+	0.22	0.23	0.23	0.01	0.051	4
	8.9	−2.162	4^+	0.22	0.18	0.18	0.05	0.040	61
^{72}Ge	9.1	−0.694	0^+	0.22	0.22	0.22	0.04	0.048	41
	9.1	−1.466	2^+	0.22	0.18	0.18	0.06	0.040	70
	9.1	−1.727	4^+	0.22	0.20	0.20	0.01	0.044	5
^{74}Ge	8.5	−1.726	0^+	0.23	0.15	0.15	0.02	0.035	25
	8.5	−1.210	2^+	0.23	0.29	0.29	0.06	0.067	45
	8.5	−1.473	4^+	0.23	0.24	0.24	0.02	0.055	12
^{76}Ge	8.1	−1.902	0^+	0.22	0.23	0.23	0.02	0.051	13
	8.1	−1.102	2^+	0.22	0.30	0.20	0.03	0.066	17
	8.1	−1.414	4^+	0.22	0.23	0.20	0.00	0.051	0

each level are listed in Table VI. We shall discuss the results isotope by isotope.

^{70}*Ge*

This isotope has been studied following the decay of ^{70}As and by $(p, p'\gamma)$ on ^{70}Ge by Van Patter *et al.*,[11] by the decay of ^{70}As by Born *et al.*[12] and de Ruiter *et al.*,[13] and by (d, d') by Kregar and Elbek.[14] Brown *et al.*[15] see 22 levels below 3.5 MeV, using (p, p') and a spectrograph, but assign no spins or parities.

We observe two additional levels, at 3.063 and 3.429 MeV. Van Patter *et al.* observe the levels shown in Table III, including a 3^+ level at 3.047 MeV and a 4^+ level at 3.059 MeV. The fit of a 4^+ curve is quite good, except for the low-angle points, and our level at 3.063 MeV is probably the 3.059-MeV level of Van Patter *et al.* However, Brown *et al.* report only one level in this region, at 3.053 MeV. The 3.429-MeV level we see is reported by Brown *et al.* also. We were able to observe points only from 80° to 130°. A tentative assignment of 5^- is made on the basis of the coupled channel fit.

^{72}*Ge*

This isotope has been studied in great detail following the decay of ^{72}Ga and ^{72}As by Camp[16] and Brun *et al.*,[17] by the decay of ^{72}Ga by Bhattacherjee *et al.*,[18] by Rester *et al.*,[19] and by Tirsell and Bloom,[20] by (p, p') by Darcey,[21] and by (d, d') by Kregar and Elbek.[14]

We see four additional levels at 2.043, 2.4, 2.956, and 3.092 MeV. The 2.043 level is probably the same as the 2.065 level seen by Camp and identified as 3^+. Brun *et al.*, Bhattacherjee *et al.*, and Tirsell *et al.*

TABLE VI. Parameters of fits to states with 0^+-I^π coupling schemes.

Isotope	W_D (MeV)	Q (MeV)	J^π	β_{0I}
^{70}Ge	11.0	−3.063	4^+	0.12
	11.2	−3.429	5^-	0.14
^{72}Ge	11.2	−2.043	2^+	0.04
	11.5	−2.4	2^+	0.02
			4^+	0.04
	11.3	−2.956	3^-	0.11
	11.3	−3.092	4^+	0.11
^{74}Ge	11.5	−2.210	1^-	0.07
	11.3	−2.956	3^-	0.04
	11.5	−3.077	3^-	0.07
	11.2	−3.160	4^+	0.10
	11.5	−3.385	4^+	0.07
	11.5	−3.512	4^+	0.06
^{76}Ge	11.5	−2.030	4^+	0.07
	11.6	−2.286	3^-	0.11
	11.5	−2.966	3^-	0.10
	11.5	−3.196	3^-	0.09
	11.5	−3.498	2^+	0.08

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[11] D. M. Van Patter, P. F. Hinrichsen, and M. H. Shapiro, Bull. Am. Phys. Soc. **13,** 1427 (1968); P. F. Hinrichsen, D. M. Van Patter, and M. H. Shapiro. Nucl. Phys. **A123,** 250 (1969); D. M. Van Patter, R. Rikmenspoel, and P. N. Trehan, *ibid.* **27,** 467 (1961).

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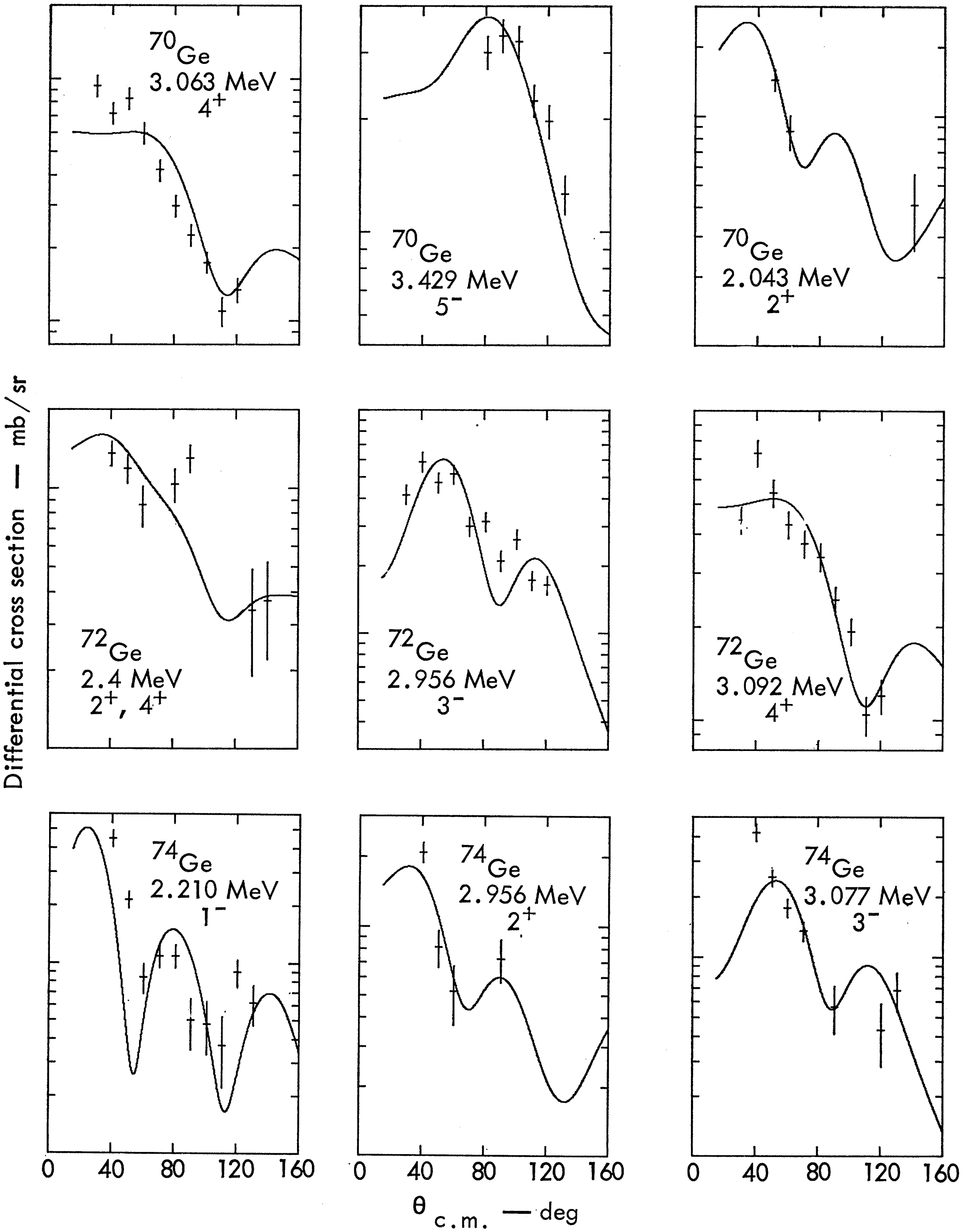
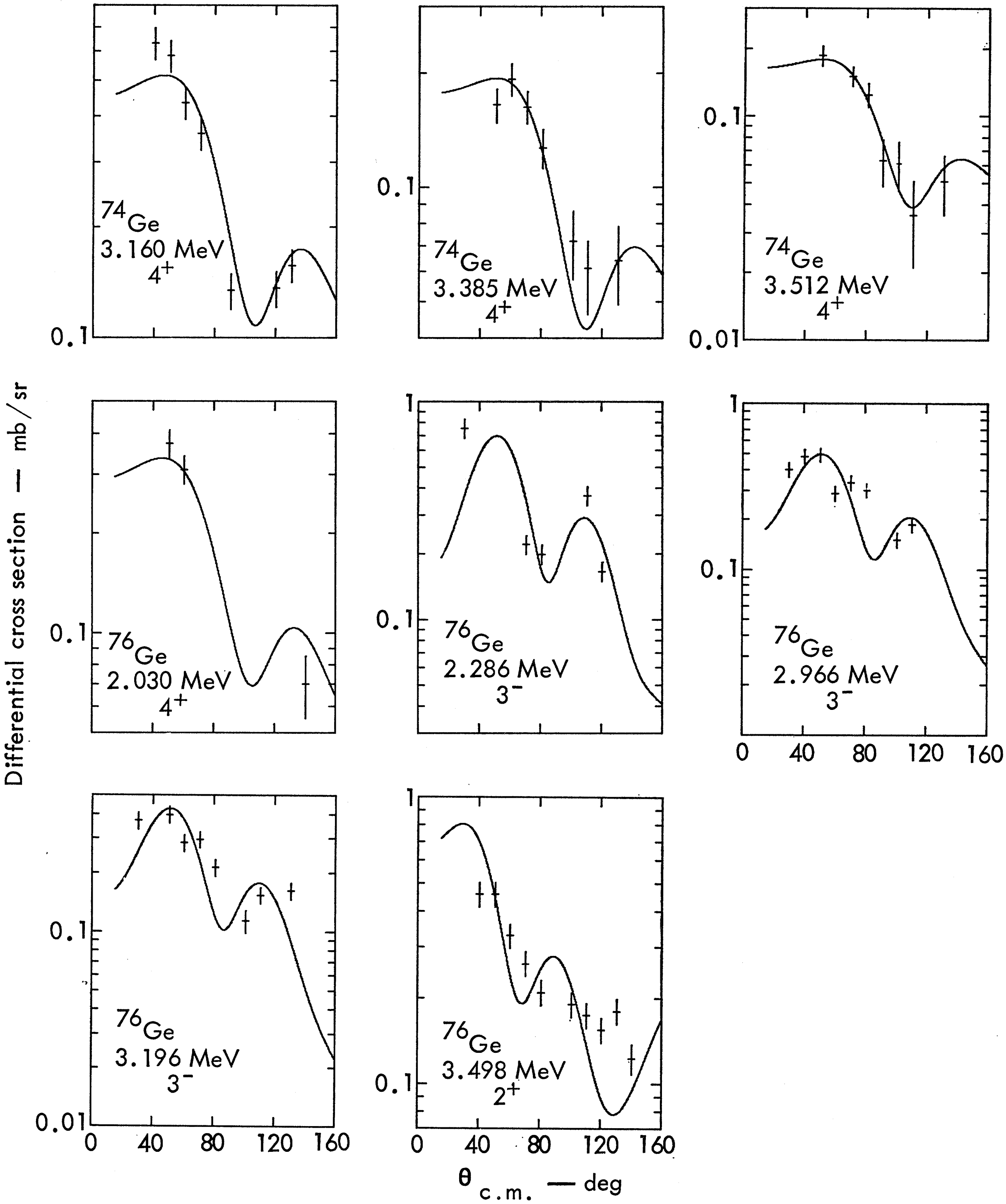


(a)

FIG. 5. Coupled-channel fits to experimental angular distributions to states analyzed with (0-I^π) coupling scheme.



(b)

FIG. 5. (*Continued*).

identify this level as a 2^+. Coupling to a 3^+ state of this magnitude is not possible with Tamura's coupled-channel code, so we have fit the angular distribution with a 2^+. With only three experimental points, we can make no definitive statement about the spin of this level. The group we observe at 2.4 MeV is an unresolved doublet reported by Camp, consisting of a 2^+ and a 4^+ level. Tirsell *et al.* identify it as a 2^+ and do not see the doublet. We were unable to fit it with a single L value, and have fit it with a sum of $L=2$ and $L=4$. The 2.956-MeV level has been fit with a 3^- assignment. A 1^- assignment is strongly forward-peaked and is not acceptable. Camp reports a 1^- level at 2.940 MeV and a 3^- level at 2.943 MeV. Apparently, we see the 3^- member of this doublet. Our 3.092-MeV level is best fit by a 4^+ curve. However, Camp reports a 2^+ level at 3.094, but can not exclude a 4^+. We therefore assign 4^+ on the basis of our fit. As a final note, Brun *et al.* and Bhattacherjee *et al.* see the 2.52-MeV level as a 2^+, and Tirsell *et al.* see it as either 2^+ or 3^-. This level is the strong octupole state that we positively assign as 3^-. Camp also reports the 2.52 as 3^-.

^{74}*Ge*

This isotope has been studied following the decay of ^{74}Ga and ^{74}As by Eichler *et al.*,[22] by the decay of ^{74}Ga by Ythier *et al.*,[23] by the decay of ^{74}As by Girgis and Van Lieshout,[24] Hamilton *et al.*,[25] and Kukoč *et al.*,[26] by neutron-capture γ rays on ^{73}Ge by Weitkamp *et al.*,[27] and by (p, p') by Brown *et al.*,[15] and Darcey.[21]

We see six additional levels, at 2.210, 2.956, 3.077, 3.160, 3.385, and 3.512 MeV. Ythier *et al.*, Girgis and Van Lieshout, Hamilton *et al.*, and Kukoč *et al.* report a 2^+ level at 2.20 MeV. We cannot fit a 2^+ to our angular distribution, and indeed the pronounced minimum in the angular distribution is quite characteristic of the 1^- assignment we have made on the basis of our best fit. A 1^- assignment can not be excluded by the decay data; however, all of the authors state that a 2^+ is most likely. Eichler *et al.* do not assign a spin to this level. A level at 2.95 is seen by both Eichler and Ythier and reported as 3^- by Ythier. We can fit either 2^+ or 3^-. Because of the 3^- assignment of Ythier and because we expect some fractionation of the octupole state, we assign 3^- also. We assign 3^- to the 3.077-MeV level because of our fit. It corresponds to the level at 3.05 MeV seen by Eichler *et al.* The other levels, at 3.160, 3.385, and 3.512 MeV are all given 4^+ assignments on the basis of the coupled-channel fits. They correspond to levels seen by Eichler *et al.* and Ythier *et al.*, but in their work no spin assignments are made.

^{76}*Ge*

This isotope has been studied by Darcey[21] using (p, p'), but no spin and parity assignments are made in this region of excitation. We see five additional levels, at 2.03, 2.286, 2.966, 3.196, and 3.498 MeV. The spin assignment of 4^+ to the 2.03-MeV level, based on a three-point angular distribution, is extremely tentative. The assignment of 3^- to the levels at 2.286, 2.966, and 3.196 MeV is justified by their coupled-channel fits, particularly for the latter two states. The 2^+ assignment for the 3.498 is also quite reasonable.

D. Electromagnetic Transition Rates Inferred from Deformabilities

From the deformation parameter β_{0l} deduced from the analysis of the inelastic-proton data, we may infer an electromagnetic transition rate. The usual formula, derived on the basis of a uniform charge

TABLE VII. Inferred electromagnetic transition roles expressed in single-particle units.

Isotope	E_x (MeV)	I^π	β	$G(L)$
^{70}Ge	1.040	2^+	0.22	19.8
	2.565	3^-	0.25	32.4
	3.063	4^+	0.12	10.4
	3.429	5^-	0.14	19.3
^{72}Ge	0.835	2^+	0.22	19.8
	2.043	2^+	0.04	0.7
	2.4	2^+	0.02	0.2
	2.4	4^+	0.04	1.2
	2.522	3^-	0.23	27.5
	2.956	3^-	0.11	6.3
	3.092	4^+	0.11	8.8
^{74}Ge	0.596	2^+	0.23	21.6
	2.210	1^-	0.07	2.7
	2.546	3^-	0.13	8.8
	2.956	3^-	0.04	0.8
	3.077	3^-	0.07	2.5
	3.160	4^+	0.10	7.2
	3.385	4^+	0.07	3.6
	3.512	4^+	0.06	2.6
^{76}Ge	0.562	2^+	0.22	19.8
	2.030	4^+	0.07	3.6
	2.286	3^-	0.11	6.3
	2.710	3^-	0.14	10.2
	2.966	3^-	0.10	5.2
	3.196	3^-	0.09	4.2
	3.498	2^+	0.08	2.6

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distribution, is

$$B(EL, 0^+\rightarrow L)=[(3/4\pi)ZeR^L\beta_{0I}]^2,$$
$$R=R_{\mathrm{EM}}\times A^{1/3}=1.2A^{1/3}\ \mathrm{F}, \qquad (1)$$

where R is the radius of the uniformly charged sphere. However, it is known that real charge distributions are not uniform, and in fact may be described more appropriately by a Fermi distribution. Owen and Satchler[28] have given graphs that allow one to calculate a correction factor $K(L)$, where L is the multipole order, which is the ratio of the inferred electromagnetic transition rate calculated using the Fermi-charge distribution to that calculated using a uniform charge distribution. For the germanium isotopes, assuming a Fermi distribution with parameters $r_0=1.10$ F and $a=0.6$ F, the factors are

$$K(1)=1.17,\quad K(2)=0.94,$$
$$K(3)=1.16,\quad K(4)=1.53,$$

and

$$K(5)=1.95. \qquad (2)$$

In addition, there is the well-known relation[29]

$$\beta_{\mathrm{EM}}=(R_{\mathrm{CP}}/R_{\mathrm{EM}})\beta_{\mathrm{CP}} \qquad (3)$$

between the β deduced in a charged particle (CP) interaction and the β deduced in a purely electromagnetic (EM) interaction. The radii used in this experiment are $R_{\mathrm{EM}}=1.2$ F and $R_{\mathrm{CP}}=1.24$ F.

Finally, we wish to express our values of $B(EL)$ in single-particle units, which are given by the formula

$$B(EL, 0^+\rightarrow L)_{\mathrm{SPU}}=[(2L+1)/4\pi](3/(3+L)]^2$$
$$\times(R)^{2L}e^2, \qquad R=1.2\ \mathrm{F}\times A^{1/3}. \qquad (4)$$

Combining Eqs. (1)–(4), we obtain

$$G(L)=\frac{K(L)(1.24/1.20)^2(3+L)^2}{4\pi(2L+1)}Z^2\beta_{0I}{}^2,$$

where $G(L)$ is the $B(EL)$ expressed in single-particle units, which is inferred from charged-particle data for a realistic charge distribution. The values obtained are listed in Table VII.

E. Fractionation of Octupole State

Consulting Table III, we see that the summed total cross sections for the octupole states seen in this experiment are as follows: ^{70}Ge–20 mb (1 state); ^{72}Ge–20 mb (2 states); ^{74}Ge–18 mb (3 states); ^{76}Ge–16 mb (4 states). In ^{74}Ge and ^{76}Ge, there are three and four octupole states, respectively, none of which are as strong as the strong 3^- state in ^{70}Ge or ^{72}Ge. This we identify as fractionation of the octupole state in ^{74}Ge and ^{76}Ge. In our study of the titanium isotopes,[2] several octupole states were observed in each isotope, but in that case the total octupole strength did not stay constant, and there was no strong level in one isotope that appeared to be split in the other isotopes. However, Yagi *et al.*[30] see six 3^- levels in ^{66}Zn(p,p'), each relatively weak $[B(EL), 0^+\rightarrow 3^-]=1$–2 single-particle units, instead of a single strong 3^- level, which one would expect in this region. The question of fractionation appears to be an open one and worthy of more investigation.

V. SUMMARY

We have scattered 14.5-MeV protons from enriched targets of ^{70,72,74,76}Ge, obtaining angular distributions for the ground state and seven–eleven excited states in each isotope. The elastic data were fit successfully, using reasonable optical-model parameters, except for the diffuseness of the imaginary part, for which we obtained a value of 0.57 F, about 0.1 F larger than normal. It was determined that the absorptive part of the potential W_D must be reduced by 0.5 MeV for each 10 mb of total cross section included as an inelastic channel in a coupled-channel calculation. Values of β_2 and β_3 for one-phonon–quadrupole and –octupole states of ^{72,74,76}Ge were determined that were in reasonable agreement with previous determinations. We were unable to fit the 0^+ member of the triplet, even if one-phonon and two-phonon contributions were mixed. This is supporting evidence for the view that the 0^+ member of the two-quadrupole–phonon triplet is at a much higher excitation energy than the 2^+ and 4^+ members. Spin and parity assignments were made on the basis of a coupled-channel fit for the other levels we observed, which are consistent in most cases with previous determinations. The fits we obtained, although not perfect in many cases, were better than those which would be obtained with a DWBA calculation. In particular, the simultaneous fitting of the ground state and one-phonon state allows the combined effect of β and W_D to be studied, while the fitting of the two-phonon triplet allows the investigation of a region in a way that would not be physically significant with the DWBA. Fractionation of the octupole state was observed in ^{74,76}Ge, with the total octupole strength for each of the isotopes approximately 18 mb.

ACKNOWLEDGMENT

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