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Distorted-Wave Analysis of Quasifree Proton Inelastic Scattering*

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Using a complex square-well distorting potential, the quasifree inelastic scattering of protons has been calculated in the impulse approximation. Calculations were performed for the nucleus C^{12} at both 160 and 1014 MeV to compare with experiment. The calculation was done with parameters determined from other data and was therefore essentially parameter-free. The agreement with experiment is quite satisfactory and relatively independent of the parameters. While the magnitude of the cross section is well determined, the location of the quasifree peak is in disagreement with experiment and only slightly shifted from the no-distortion case.

I. INTRODUCTION

At energies greater than about 100 MeV, a number of nuclear processes behave in a particularly simple manner. Specifically, it appears that one can evaluate the cross section for (p, p') reactions to discrete states and reactions such as $(p, 2p)$ by assuming that the incident and outgoing particles are essentially plane waves which undergo attenuation when traveling through the nuclear medium. That is to say, they are of the form

$$u = \exp(ikz),$$

where $k = \alpha + i\beta$. The quantity α is a measure of the depth of the real optical potential in which the particle finds itself; β is a measure of the imaginary part of the potential, or the mean free path in nuclear matter.

One of the simplest processes involves the quasifree scattering from an incident nucleon by the nucleons present in the target. This process manifests itself by a large peak in the inelastically scattered particle spectrum at an energy (in the laboratory) corresponding to the free two-body kinematics. This peak is broadened by the internal motion of the target nucleons and therefore a study of this peak can be used to study the momentum distribution of the target nucleons. Some time ago Wall and Roos[1] applied a plane-wave analysis of the quasifree scattering process to their data at 160 MeV. They basically used an impulse approximation analysis due to Wolff,[2] modifying it to include a momentum distribution consistent with the shell model and known single-particle binding energies. In this analysis they used an unattenuated plane wave, i.e., $\beta = 0$ and $\alpha =$ wave number of the incident particle. Another key assumption was the use of closure over those states in the residual nucleus connected with the initial nucleus by nucleon-nucleon scattering. Essentially, these two assumptions permitted the differential cross section $d^2\sigma/d\Omega dE$, where $d\Omega$ and dE refer to the solid angle and energy of the outgoing particle, to be written in terms of the free nucleon-nucleon cross section and the momentum distribution of the single-particle states in the target nucleus.

While this analysis fit the data qualitatively there were two serious disagreements between the calculations and the experimental results. The magnitude of the cross section predicted by these calculations was too large, and the location of the peak was systematically lower than the observed peak position.

The present paper is an attempt to improve upon these calculations by taking into account the optical distortion of the incident and outgoing particles. In particular, we attempt to fit the earlier data of Wall and Roos[1] at 160 MeV and some data taken by Corley *et al.*[3] at 1014 MeV; both experiments were on C^{12}.

II. THEORY

A. Distorted-Wave Impulse-Approximation (DWIA) Theory of Quasifree Scattering

For a $(p, 2p)$ process where the parameter of the incoming particle is denoted by subscript 0, the two outgoing particles by 1 and 2, the particular nucleon struck in the nucleus by α, and the initial and final nuclei by A and $A-1$, respectively, Jacob and Maris[4] find for the cross section that

$$\frac{d^9\sigma}{d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_{A-1}} = \sum_{\text{final states } A-1} \sum_{\alpha=1}^{A} \frac{(\hbar c)^2 \bar{E}^2}{k_0 E_1 E_2 E_\alpha} \times \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_{A-1} - \mathbf{k}_0)\,\delta(E_1 + E_2 + E_{A-1} - E_0 - E_A) \times (d\sigma/d\Omega)^{\text{free}}(\bar{0}\bar{\alpha}\bar{1}\bar{2})\,|\,g_{A,A-1}{}^{\alpha}(\mathbf{k}_\alpha)\,|^2, \quad (1)$$

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[1] N. S. Wall and P. G. Roos, Phys. Rev. **150**, 811 (1969).

[2] P. A. Wolff, Phys. Rev. **87**, 434 (1958); N. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), Chap. 11, pp. 603–701.

[3] Daniel M. Corley, thesis, University of Maryland, 1969 (unpublished); D. M. Corley *et al.* (to be published). These data will be included in a comprehensive paper by H. Palevsky and the Brookhaven Group and collaborators.

[4] G. Jacob and T. Maris, Rev. Mod. Phys. **38**, 121 (1966).

where

$$g_{A,A-1}{}^{\alpha}(\mathbf{k}_\alpha)=[1/(2\pi)^{3/2}]\int d\mathbf{X}_\alpha d\mathbf{X}\exp(-i\mathbf{k}_\alpha\cdot\mathbf{X}_\alpha)$$
$$\times D_2{}^*(\mathbf{X}_\alpha)D_1{}^*(\mathbf{X}_\alpha)D_0(\mathbf{X}_\alpha)\varphi_{A-1}{}^*(\mathbf{X})\varphi_{A,A-1}(\mathbf{X}_\alpha,\mathbf{X}),\quad(2)$$

where all spatial and all barred kinematical variables are evaluated in the c.m. system. $(d\sigma/d\Omega)^{\text{free}}$ is the free nucleon-nucleon cross section in the c.m. frame, and the D's represent the distortion factors modifying plane waves, defined as

$$\chi_i(\mathbf{r}_i)=[1/(2\pi)^{3/2}]D_i(\mathbf{r}_i-\mathbf{R}_{A-1})\exp(i\mathbf{k}_i\cdot\mathbf{r}_i),\quad(3)$$

where χ_i is a distorted wave function in the lab system.

Because of the two-particle nature of the interaction (impulse approximation) the final states contributing to the cross section are clustered in a narrow energy band.[2] Therefore, the energy of the final nucleus can be replaced with an average value, $E_{A-1}{}^{\text{av}}=T_{A-1}+M_{A-1}{}^{\text{av}}$ and closure of $\varphi_{A-1}(\mathbf{X})$ can be employed to eliminate the wave functions of the recoil nucleus.

Therefore, the sum over the final states of the recoil nucleus $(A-1)$ yields

$$\frac{d^9\sigma}{d\mathbf{k}_1d\mathbf{k}_2d\mathbf{k}_{A-1}}=\sum_{\alpha=1}^{A}\frac{(\hbar c)^2\bar{E}^2}{k_0E_1E_2E_\alpha}\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_{A-1}-\mathbf{k}_0)$$
$$\times\delta(E_1+E_2+E_{A-1}{}^{\text{av}}-E_0-E_A)(d\sigma/d\Omega)^{\text{free}}(\bar{0}\bar{\alpha}\bar{1}\bar{2})$$
$$\times\int d\mathbf{X}\,|\,\varphi_A{}^{\alpha}(\mathbf{k}_\alpha,\mathbf{X})\,|^2,\quad(4)$$

where

$$\varphi_A{}^{\alpha}(\mathbf{k}_\alpha,\mathbf{X})=\int d\mathbf{X}_\alpha\exp(-i\mathbf{k}_\alpha\cdot\mathbf{X}_\alpha)D_2{}^*D_1{}^*D_0\varphi(\mathbf{X}_\alpha,\mathbf{X}).\quad(5)$$

The integral $\int d\mathbf{X}\,|\,\varphi_A{}^{\alpha}(\mathbf{k}_\alpha,\mathbf{X})\,|^2$ can be further simplified if we assume that the target nucleus can be represented by a simple product-type wave function of the form

$$\varphi_A(\mathbf{X}_\alpha,\mathbf{X})=\varphi_\alpha(\mathbf{X}_\alpha)\varphi_{A-1}(\mathbf{X}),\quad(6)$$

where $\varphi_\alpha(\mathbf{X}_\alpha)$ is the single-particle shell-model wave function and $\varphi_{A-1}(\mathbf{X})$ is the wave function representing all the particles not involved in the collision. Using the orthonormality of $\varphi_{A-1}(\mathbf{X})$ yields

$$\frac{d^9\sigma}{d^3k_1d^3k_2d^3k_{A-1}}=\sum_{\alpha=1}^{A}\frac{(\hbar c)^2\bar{E}^2}{k_0E_1E_2E_\alpha}\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_{A-1}-\mathbf{k}_0)$$
$$\times\delta(E_1+E_2+E_{A-1}{}^{\text{av}}-E_0-E_A)$$
$$\times(d\sigma/d\Omega)^{\text{free}}(\bar{0}\bar{\alpha}\bar{1}\bar{2})\,|\,g^{\alpha}(\mathbf{k}_\alpha)\,|^2,\quad(7)$$

where Eq. (2) becomes

$$g^{\alpha}(\mathbf{k}_\alpha)=[1/(2\pi)^{3/2}]\int d\mathbf{X}_\alpha$$
$$\times\exp(-i\mathbf{k}_\alpha\cdot\mathbf{X}_\alpha)D_2{}^*D_1{}^*D_0\varphi_\alpha(\mathbf{X}_\alpha).\quad(2')$$

Equation (2′) is the distorted momentum distribution of the struck particle (α).

Since we are describing a (p,p') experiment, we must integrate over the recoil nucleus and the unobserved particle. Using the momentum δ function to integrate over $\mathbf{k}_2$ leaves only the integral over $\mathbf{k}_{A-1}$ to be evaluated. Evaluation of this integral becomes simpler if (following Azhgirey *et al.*[5]) one does the integral in $\mathbf{k}_{A-1}$ space with the set of coordinates (k_{A-1},θ,Φ), where θ is the angle between $\mathbf{k}_{A-1}$ and $\mathbf{x}=\mathbf{k}_0-\mathbf{k}_1$, and Φ is the angle of revolution around x measured from the plane formed by $\mathbf{x}$ and $\mathbf{k}_0$. This yields

$$\frac{d^3\sigma}{dE_1d\Omega_1}=\frac{1}{(\hbar c)^2}\sum_{\alpha=1}^{A}\int d\varphi\int dk_{A-1}$$
$$\times\frac{k_{A-1}\bar{E}^2k_1}{k_0E_\alpha x}\frac{d\sigma^{\text{free}}}{d\Omega}(\bar{0}\bar{\alpha}\bar{1}\bar{2})\,|\,g^{\alpha}(\mathbf{k}_\alpha)\,|^2,\quad(8)$$

where the limits of the integral over k_{A-1} are given by the condition $|\cos\theta_0|\leq1$, where

$$\cos\theta_0=\frac{\hbar^2c^2x^2+\hbar^2c^2k_{A-1}{}^2-m^2c^4-(E_1+E_{A-1}{}^{\text{av}}-E_0-E_A)}{2\hbar^2c^2xk_{A-1}};$$

these limits coming from the integration over the energy-conserving δ function. If we use a bound-state wave function of the type

$$\varphi_{njlm_j}(r)=[u_{njl}(r)/r]\{[1/2l+1)][(l-m+\tfrac{1}{2})^{1/2}$$
$$\times Y_{l,m+(1/2)}(\theta,\varphi)\alpha+(l+m+\tfrac{1}{2})^{1/2}Y_{l,m-(1/2)}\beta]\},\quad(9)$$

we must average $|\,g(k)\,|^2$ over the $2j+1$ values of m_j, since the value of m_j of the struck nucleon is unknown.

When Eq. (9) is inserted in Eq. (2′), and use is made of the relationship

$$[1/(2j+1)]\sum_{m_j}|\,g_{njlm_jm_l}(k_\alpha)\,|^2$$
$$=[1/(2l+1)]\sum_{m_l}g_{njlm_l}(k_\alpha)\,|^2,$$

where

$$g_{njlm_l}(k_\alpha)=[1/(2\pi)^{3/2}]\int d\mathbf{X}_\alpha\exp(-i\mathbf{k}_\alpha\cdot\mathbf{X}_\alpha)D_2{}^*D_1{}^*D_0$$
$$\times[u_{njl}(X_\alpha)/X_\alpha]Y_{l,m_l}(\theta_\alpha,\Phi_\alpha),\quad(2'')$$

so the expression for the cross section becomes

$$d^2\sigma/dE_1d\Omega_1=[1/(\hbar c)^2]\sum_{\alpha=1}^{A}\int d\varphi\int dk_{A-1}k_{A-1}$$
$$\times(\bar{E}^2k_1/k_\alpha E_\alpha x)(d\sigma/d\Omega)^{\text{free}}(\bar{0}\bar{\alpha}\bar{1}\bar{2})$$
$$\times[1/(2l+1)]\sum_{m_l}|\,g_{nlm_l}(k_\alpha)\,|^2.\quad(8')$$

For the case where there is no distortion

$$[1/(2l+1)]\sum_{m_l}|\,g_{njlm_l}\,|^2$$

[5] L. S. Azhgirey *et al.*, Nucl. Phys. **79**, 609 (1966).

has no Ω_{k_α} dependence, i.e., it depends only on the magnitude of k_α. Although strictly true only for the case where there is no distortion, we assume this quantity is independent of the angle Φ for the case where distortion is present. Since $d(\sigma/d\Omega)^{\text{free}}(\bar{0}\bar{\alpha}\bar{1}\bar{2})$ has no Φ dependence, $\bar{E}^2$ is the only variable depending on Φ and therefore,

$$d^3\sigma/dE_1d\Omega_1=[1/(\hbar c)^2]\sum_{\alpha=1}^{A}(4\pi k_1/k_0x)\int(dk_{A-1}k_{A-1}/E_\alpha)$$
$$\times[m_0^2+E_0E_\alpha-p_0p_\alpha\cos(x,k_\alpha)\cos(x,k_0)]$$
$$\times(d\sigma/d\Omega)^{\text{free}}(\bar{0}\bar{\alpha}\bar{1}\bar{2})[1/(2l+1)]\sum_{m_l}|g_{njlm_l}(k_\alpha)|^2. \quad (8'')$$

Unfortunately, to calculate $d^2\sigma/d\Omega_1dE_1$ as written here for 20 values of E_1 would require 16 h of computer time on a machine with the capabilities of a UNIVAC 1108. Therefore,

$$[1/(2l+1)]\sum_{ml}|g|^2$$

was calculated utilizing several additional approximations. Specifically, the approximation used in doing the Φ integral in k space was also utilized in doing the angular integrations in $g(k)$.

Since for the no-distortion case

$$[1/(2l+1)]\sum_{ml}|g|^2$$

is independent of θ_k, doing the angular integrations in $g(k)$ assuming $\theta_{k_\alpha}=0$ will give the correct result even though only the $m=0$, Y_{lm_l} will contribute. Since for the no distortion case this simplification has no affect on the numerical result, we make the same simplification here. Taking $\theta_{k_\alpha}=0$ and utilizing only the $m=0$, Y_{lm_l} gives

$$[1/(2l+1)]|g_{njlm_l}(k_\alpha)|^2=[1/(2l+1)]\sum_{ml}|$$
$$\times\int X_\alpha dX_\alpha u_{njl}(X_\alpha)D_2^*D_1^*D_0\exp(-ik_\alpha X_\alpha\cos\theta_{X_\alpha})$$
$$\times Y_{l,0}(\theta_{X_\alpha},\Phi_{X_\alpha})d(\cos\theta_{X_\alpha})d\Phi_{X_\alpha}|^2. \quad (10)$$

If the Φ dependence in the distortion factors is ignored, we have

$$[1/(2l+1)]|g_{njlm_l}(k_\alpha)|^2=[1/(2l+1)]\sum_{ml}|2\pi$$
$$\times\int X_\alpha dX_\alpha u_{njl}(X_\alpha)D_2^*(X_\alpha)D_1^*(X_\alpha)D_0(X_\alpha)$$
$$\times\exp(-ik_\alpha X_\alpha\cos\theta_{X_\alpha})_{l,0}(\theta_{X_\alpha},0)d(\cos\theta_{X_\alpha})|^2. \quad (11)$$

B. Distortion Factors

In Eq. (3), the distortion factor to be used in finding the distorted momentum distribution, Eq. (10), is defined. In a usual distorted-wave analysis these functions are formed by solving the Schrödinger equation for elastic scattering with an appropriate optical potential. Some time ago, it was observed by Amos[6] that at incident proton energies greater than about 50 MeV the solutions in a realistic optical potential could, in fact, be reasonably well represented by attenuated plane waves. The result is not surprising inasmuch as we are beginning to satisfy the criteria for the validity of the Glauber or high-energy approximation.[7]

We have therefore used this result to express our distortion factors as

$$D(\mathbf{r})=\exp\left(\frac{-iE}{(\hbar c)^2k}\int_{\mathbf{r}}^{\infty}V(x)dx\right),$$

where the coordinate, energy E, and the momentum k, and optical potential $V(x)$ are appropriate to the particular particle being considered. In the calculations reported here, we further simplified the calculation by using only a complex square well. The distorting factors then become

$$D=\exp\{[-iE/(\hbar c)^2k][V(E)+iW(E)]\Delta S\},$$

where ΔS is the path length in the potential (or nucleus). The energy dependence of the potential arises, in part, from the use of a local representation of the nucleon-nucleus potential.

III. PARAMETRIZATION

The calculations which we wish to report in no way represent an exhaustive multidimensional parameter search to attempt to fit the data precisely. Rather we prefer to use parameters which are, in fact, consistent with elastic nucleon-nucleon scattering, the shell-model, and known binding energies, and an energy-dependent optical model for scattering to determine the absorption properties. In other words, we wish to develop a calculation in which all parameters are essentially determined from different independent experiments. To this end we have used the following parametrization of the appropriate parameters in Eq. (8) of Sec. II.

A. Nucleon-Nucleon Scattering

(i) $(d\sigma/d\Omega)^{\text{free}}=(1.9+230./T+4850/T^2)\times(1+0.1\cos^2\theta)$,

where T is the c.m. kinetic energy and θ is the c.m. scattering angle. This formula[8] was used for the 160-MeV analysis, while for the 1014-MeV scattering we used

(ii) $(d\sigma/d\Omega)^{\text{free}}=Ae^{-Bx^2}$,

where $A=15.2\times10^{-26}\ \text{cm}^2$,

$B=-5.78\times10^{-6}\ (\text{MeV}/c)^2$,

[6] K. A. Amos, Nucl. Phys. **77**, 225 (1966).

[7] R. J. Glauber, in *Lectures in Theoretical Physics* (University of Colorado Press, Boulder, Colorado, 1958), Vol. 1, p. 374.

[8] K. F. Riley, Nucl. Phys. **13**, 407 (1959).

and

$$X=-(p_0-p_1)^2$$

with p_0 the incident–particle four-momentum and p_1 the outgoing–particle four-momentum.[3]

B. Single-Particle Wave Functions

The ABACUS II program of Auerbach[9] was used to generate the single-particle wave functions. The form of the nuclear potential was

$$V_{RE}(r)=-VRE/[1+\exp(r-R)/a].$$

In addition, a spin-orbit term was used with

$$V_{SO}=-VSO(1/r)[(d/dr)V_{RE}(r)]\mathbf{l}\cdot\boldsymbol{\sigma}.$$

Table I gives the parameters used in these equations.

C. Distorting Optical Potentials

As indicated in Sec. III B, the distortion was taken to be that of an attenuated plane wave in a complex square well. The potential parameters were taken to be

(i) at 160 MeV,

$$V=55 \text{ MeV}-0.3T,$$
$$W=10 \text{ MeV},$$
$$R=2.96\times10^{-13} \text{ cm},$$

where T is the kinetic energy of the particle undergoing distortion. These values are close, but not identical, to those of Perey, and Bohr and Mottelson.[10]

(ii) At 1014 MeV,

$$R=2.29\times10^{-13} \text{ cm},$$
$$V=-20 \text{ MeV},$$
$$W=100 \text{ MeV}.$$

These values are the same as those found by Palevsky *et al.*[11] in their optical-model analysis of the elastic scattering of 1014-MeV protons on C^{12}. Inasmuch as the outgoing proton energy is always greater than 700 MeV, we have not varied the magnitude of the distorting potential for this particle.

TABLE I. Parameters of the shell model used to generate the wave functions for the struck particles.

State nlj	Binding energy MeV	R 10^{13} cm	a 10^{13} cm	VRE MeV	VSO MeV
$1s_{1/2}$	34.0	2.96	0.55	59.5	0.0
$1p_{3/2}$	15.8	2.96	0.55	55.5	9.0

[9] E. Auerbach (unpublished). We would like to thank Elliot Auerbach for making this program available to us.

[10] F. G. Perey, Phys. Rev. **131**, 745 (1963); A. Bohr and B. R. Mottelson, in *Nuclear Structure* (W. A. Benjamin, Inc., New York, 1969), Vol. I, Eqs. (2)–(176).

[11] H. Palevsky *et al.*, Phys. Rev. Letters **18**, 1200 (1967).

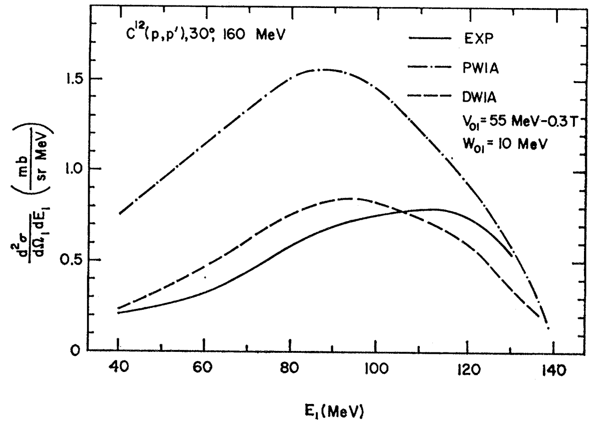

FIG. 1. The experimental results, the PWIA calculation, and DWIA calculation for the quasifree scattering of 160-MeV protons from C^{12} at 30°.

IV. RESULTS AND DISCUSSION

Using the above parameter sets, the main results of this analysis are given in Figs. 1–4, and these results are discussed in the following subsections.

A. Effect of Distortion on the Magnitude of the Cross Section

When examining Figs. 1–4 the effect of the distortion on the magnitude of the cross section is the most striking result. That the DWIA theory decreased the values of the cross section with respect to the plane-wave impulse-approximation (PWIA) theory is no great surprise since all cases calculated included the imaginary part of the optical potential which results in an e^{-a}-type factor in the distorted momentum distribution $g(k)$. However, the size of the decrease was of interest, and it was found that in all cases the distortion reduced the magnitude of the cross section to values more in line with the experimental data. In fact, when calculating the ratio of the experimental value of the cross section at the peak to the theoretical value of the cross section at the peak, one finds that for the four cases calculated (160 MeV at 30°, 160 MeV at 50°, 1014 MeV at 9.05°, 1014 MeV at 20.2°) the PWIA theory gives ratios of 0.50, 0.52, 0.32, and 0.53, whereas the DWIA theory gives ratios of 0.92, 1.02, 0.86, and 1.28, respectively. In all cases the improvement is substantial, and one can say that the inclusion of absorptive processes is necessary for any reasonable quantitative description of the cross section.

B. Effect of Distortion on the Peak Position

Next to changes in magnitude the second most obvious feature apparent from Figs. 1–4 is the fact

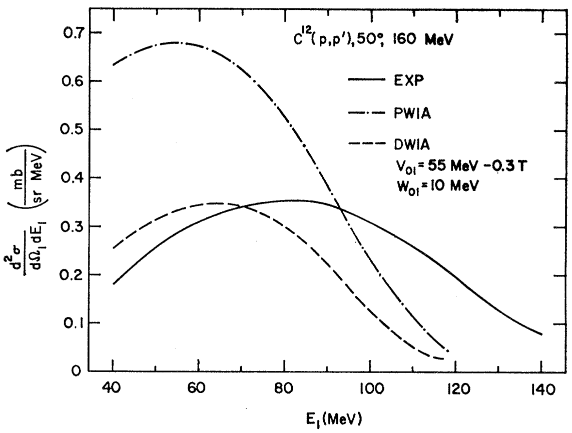

FIG. 2. The experimental results, the PWIA calculation, and the DWIA calculation for the quasifree scattering of 160-MeV protons from C^{12} at 50°.

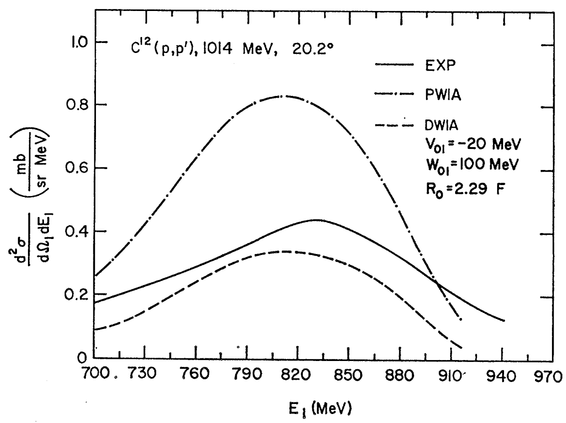

FIG. 4. The experimental results, the PWIA calculation, and the DWIA calculation for the quasifree scattering of 1014-MeV protons from C^{12} at 20.2°.

that the DWIA calculation gives a peak position which is, in all cases, as good as or slightly better than the PWIA results. As will be shown later, the real part of the optical potential is responsible for this shift.

C. Effect of Distortion on the Shape

Whether or not the DWIA theory gives a spectrum shape which is an improvement on the PWIA spectrum is not readily apparent by examining Figs. 1–4 as given. Therefore, to facilitate this examination, the experimental, DWIA, and PWIA results were all normalized to one and plotted in Figs. 5–8. In all four cases, the DWIA curve drops below the experimental curve on both the high- and low-energy sides of the peak more sharply than the PWIA curve does. That is, the DWIA spectrum is narrower than either the PWIA or experimental spectra.

We have undertaken another series of calculations to determine the sensitivity of the calculations to various parameters. These tests were as follows.

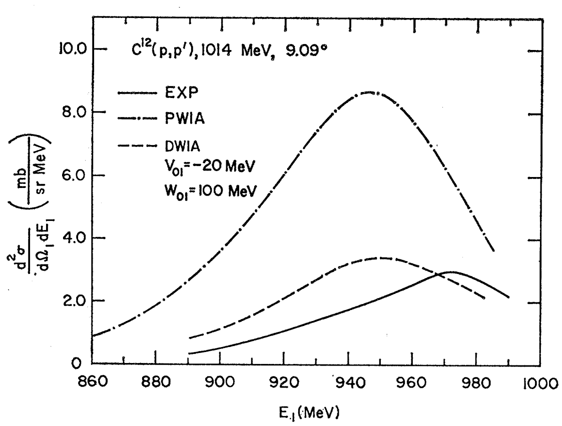

FIG. 3. The experimental results, the PWIA calculation, and the DWIA calculation for the quasifree scattering of 1014-MeV protons from C^{12} at 9.09°.

1. *Sensitivity to Changes in R_0*

The spectrum's sensitivity to changes in the value of the optical-potential radius, R_0, was tested by doing 1014-MeV calculations (at 9.05° and 20.2°) for two sets of R_0. Calculations were made with $R_0 = 2.29$ F (the value derived by fits to elastic proton scattering at 1 GeV) and with $R_0 = 2.96$ F (the value used for the 160-MeV calculations). The results are shown in Fig. 9. The 29% increase in the radius of the potential results in a 20% decrease in the magnitude of the cross section at 9.05° and a 22% decrease at 20.2°. In the 9.05° case, the experimental curve fell between the two curves given by the different radii, but in the 20.2° case the experimental curve was much higher than the curve calculated with either radius, although the curve calculated with $R_0 = 2.29$ F came closer to matching the experimental results.

2. *Sensitivity to Changes in V*

The sensitivity of the spectrum to changes in the real part of the potential was tested by doing the

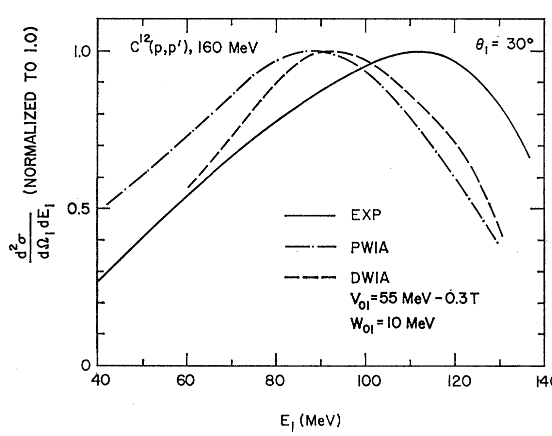

FIG. 5. The same curves as in Fig. 1 except normalized to accentuate the differences.

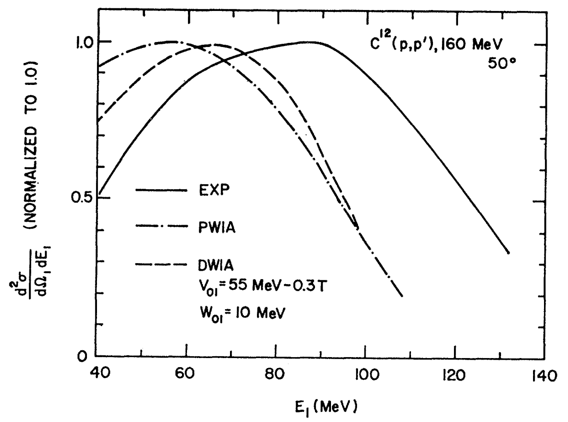

FIG. 6. The same curves as in Fig. 2 except normalized to accentuate the differences.

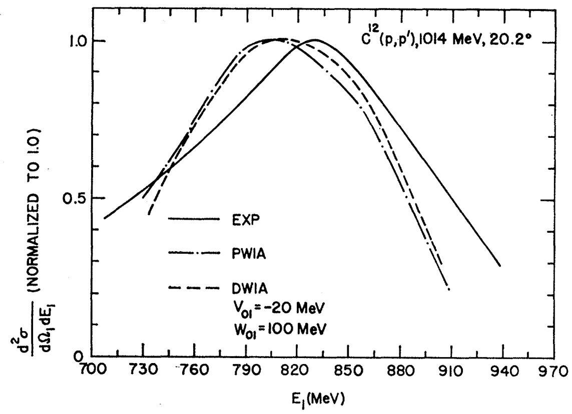

FIG. 8. The same curves as in Fig. 4 except normalized to accentuate the differences.

160-MeV calculations with the real part of the potential given by 60 MeV$-0.3T$, 55 MeV$-0.3T$, 50 MeV$-0.3T$, and 0 while keeping the imaginary part of the potential fixed at 10 MeV. The results of these calculations are shown in Figs. 10 and 11. Increasing the real part of the potential by 5 MeV results in decreases in the magnitude of the cross section of 4–10% as well as slight shifts (of 3–4 MeV) in the position of the peak to higher energies. Turning the real part of the potential off completely caused the peak to fall at the same energy as in the plane-wave case.

3. Sensitivity to Changes in W

The sensitivity of the spectrum to changes in the imaginary part of the potential was tested next by doing calculations (again at 160 MeV at 30° and 50°) where the imaginary part of the potential was given as 0, 5, 10, and 15 MeV while keeping the real part fixed at 55 MeV$-0.3T$. The plotted results are shown in Figs. 12 and 13. Unlike changes in V, changes in W do not produce noticeable shifts in the position of the curves' peaks, but introduced only changes in magnitude. In fact, increases of 5 MeV in W caused decreases of 17–23% in the magnitude of the cross section.

4. Sensitivity to Distortion of the Unobserved Particle

Finally, we performed the DWIA calculation with and without a distorting factor for the unobserved particle. The basic argument is that while in a $(p, 2p)$ experiment calculated in a similar fashion the distortion of both protons is important, in this calculation we are not involved with the history of the unobserved particle after the collision and therefore need not

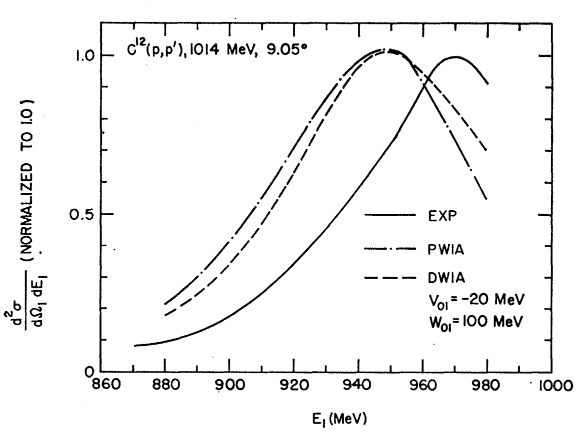

FIG. 7. The same curves as in Fig. 3 except normalized to accentuate the differences.

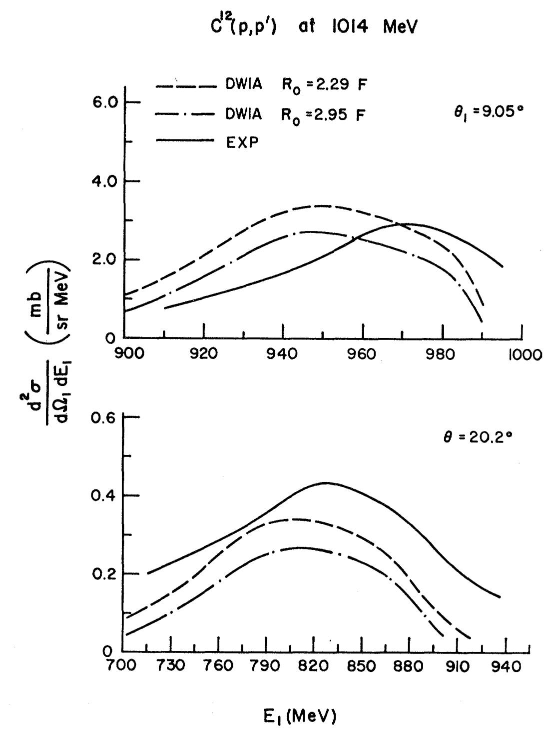

FIG. 9. DWIA calculations at 1014 MeV showing the effect of changing the radius of the square-well distorting potential.

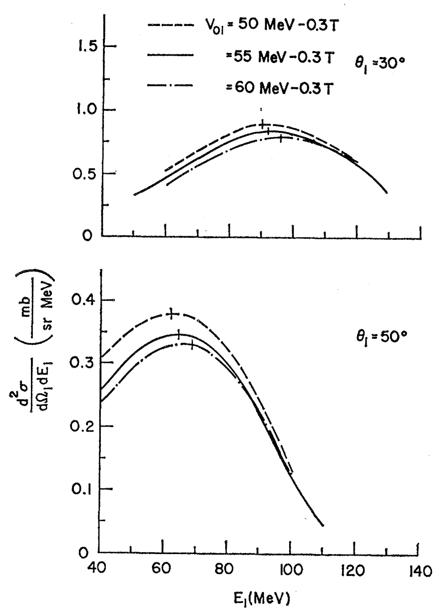

FIG. 10. DWIA calculations at 160 MeV showing the effect of small changes in the well depth of the real part of the distorting potential. The imaginary part of the potential was kept at 10 MeV.

consider it. The only property of the unobserved particle that plays any role in the present sort of formulation is its initial momentum distribution and the fact that we have a single nucleon-nucleon collision with it. We nevertheless formulated our program such that we could allow for a distortion on the observed particle, and in Fig. 14 illustrate the effect of including this distortion for the 160 MeV, 30° and 50°

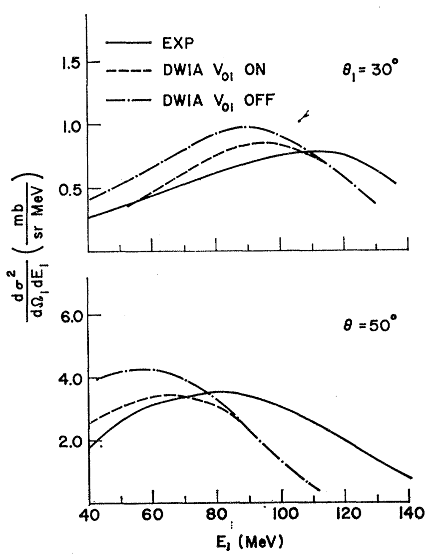

FIG. 11. DWIA calculations at 160 MeV showing the effect of the real part of the distorting potential keeping the imaginary part constant at 10 MeV but letting the real part be either 55 MeV$-0.3T$ (V_{01} on) or 0 (V_{01} off).

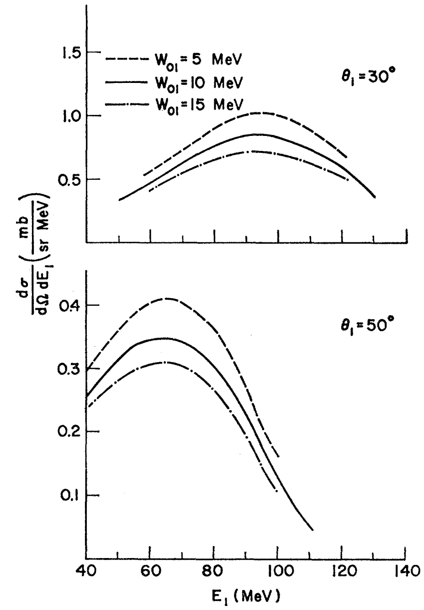

FIG. 12. DWIA calculations at 160 MeV showing the effect of changing the imaginary part of the distorting potential keeping the real part at 55 MeV$-0.3T$.

data. At both angles the addition of distortion lowered the cross section by a factor of 2 below the experimental values. It did not adversely affect the peak position, however, and in one of the two cases actually improved it. The fact that the spectra calculated without distortion on particle 2 gives better agreement with experiment than those calculated with dis-

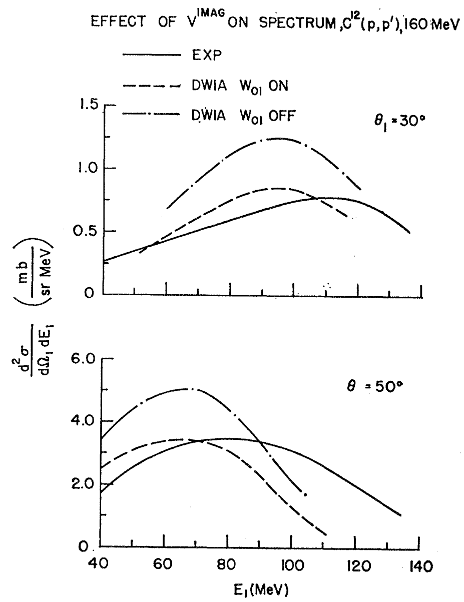

FIG. 13. DWIA calculations at 160 MeV showing the effect of having a real distorting potential of 55 MeV$-0.3T$ with an imaginary part of either 10 MeV or 0.

tortion included is consistent with our argument for neglecting this distortion.

Other calculations were undertaken to investigate the assumption that $\theta_{k\alpha}=0$ in the calculation of $|g(k)|^2$, and it was found that this approximation causes an overestimate of the peak cross section by about 12%, but there was no change of shape of the spectrum for the 1014 MeV 13.5° case. These and other calculations are present in a M.S. thesis[12] by one of the authors, copies of which are available.

V. CONCLUSIONS

We have presented a calculation which involves several nonessential simplifications such as the use of the square-well distorting factors, as well as several more basic assumptions such as the locality assumption of the optical potential, the use of on-energy-shell representation of the nucleon-nucleon scattering, and the impulse approximation. Despite this we find remarkably good agreement between experiment and the results of these calculations both with respect to the shape of the spectra as well as the over-all magnitude of the cross section.

There is, however, still one unexplained discrepancy between theory and experiment. Specifically, the location of the peak of the calculated spectrum falls consistently below that of the experiment. As was pointed out by Corley,[3] the location of the peak is as though the struck nucleons were bound with much less than the binding energies observed in $(p, 2p)$ experiments though the width of the peak is rather consistent with the momentum distributions found by fitting the observed binding energies in reasonable shell-model potentials. It is our feeling that the otherwise good agreement between theory and experiments indicates that a significant nuclear effect has been left out of our calculations.

As was pointed out in both Refs. 1 and 3, there is a discrepancy in the tails of the spectrum which correspond to struck particles with high internal momenta. That discrepancy is, in fact, better substantiated by the present calculations but until the peak-location discrepancy is resolved one cannot hope to get quantitative data on the higher-momentum components.

Note added in proof. A paper by T. de Forest, Jr., recently appeared [Nucl. Phys. **A132,** 305 (1969)] in which a similar analysis for quasifree inelastic electron scattering was performed. In both papers, binding energy effects are included. Our paper is similar to his plane-wave quasielastic model though our use of complex energy-dependent optical potential to generate the distorted waves is not obviously equivalent to the Perey effect. Our results indicate that the treatment we used incorporates enough of the nonlocal energy-dependent effects and that the renormalization of the wave functions is not necessary for agreement with experiment over our energy range.

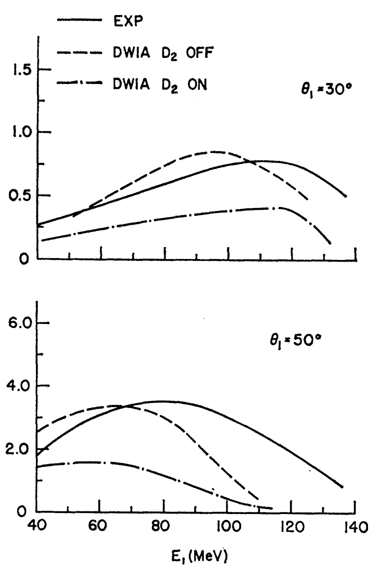

FIG. 14. DWIA calculations at 160 MeV showing the effect of including distortion on the unobserved particle.

ACKNOWLEDGMENTS

We would like to thank Dr. Daniel M. Corley for many useful discussions and much of his unpublished analysis and data. Much of the computing involved here was done at the University of Maryland Computer Science Center under a NASA Grant No. NsG 398.

[12] F. R. Kroll, M.S. thesis, University of Maryland, 1969 (unpublished).