PHYSICAL REVIEW C VOLUME 1, NUMBER 4 APRIL 1970

Effect of the Knockout-Exchange Mechanism in Nucleon-Nucleus Scattering*

JAY ATKINSON†

Lawrence Radiation Laboratory, Livermore, California 94550

AND

V. A. MADSEN

Department of Physics, Oregon State University, Corvallis, Oregon 97331

(Received 15 August 1969; revised manuscript received 12 January 1970)

We have studied the contribution of the knockout-exchange mechanism to inelastic proton scattering in the distorted-wave Born approximation (DWBA) as a function of two-body force range, orbital-angular-momentum transfer, incident-particle energy, and bound-particle energy and configuration. Several important features are revealed: (1) The exchange mechanism is found to be important for reasonable ranges (≈ 1 F) and for energies as high as 150 MeV; (2) the exchange-to-direct ratio is quite sensitive to the orbital-angular-momentum transfer L, and the exchange amplitude can exceed the direct for large L; (3) non-normal transfers $[(-1)^L \neq (-1)^{\Delta\pi}]$ are usually negligible but may become significant for high L, especially if no spin flip is involved; (4) both the magnitude and phase of the exchange amplitude relative to the direct are sensitive to the initial and final single-particle configurations, particularly to the radial quantum numbers; (5) in collective excitations constructive interference occurs among exchange amplitudes to a sufficient extent that the inclusion of exchange can double the cross section for 2^+ states. Expressions for direct and exchange amplitudes are obtained in the zero-range limit of the PWBA which qualitatively explain the behavior observed in the numerical results. A new method is presented for estimating collective enhancement due to core polarization by using empirical $B(E2)$ values. Calculation of the absolute cross section for the 2^+ excitation in ^{118}Sn(p, p'), including exchange and core polarization, yields a two-body effective interaction strength which agrees both with strengths derived from charge-exchange reactions and with those obtained from a semirealistic two-nucleon interaction.

I. INTRODUCTION

IN the past few years, there has been a growing interest in the microscopic model of inelastic nuclear scattering. Calculated angular distributions are usually in reasonable agreement with experimental results, but whether absolute cross sections can be obtained from a no-parameter model is not yet clear. Recent work by Amos *et al.*[1] and Petrovich *et al.*[2] on the problem of the interaction for inelastic scattering indicates that in some instances an effective interaction obtained from the two-nucleon data can reproduce experimental absolute cross sections if collective enhancements are included in some way. Agassi and Schaeffer[3] have been able to fit the 55-MeV data[4] for the 3^-, $T=0$ level in ^{40}Ca with an interaction obtained from the nuclear structure work of Gillet and Sanderson.[5]

* Work supported in part by the U.S. Atomic Energy Commission.

† Present address: Department of Physics, Oregon State University, Corvallis, Ore.

[1] K. A. Amos, V. A. Madsen, and I. E. McCarthy, Nucl. Phys. **A94**, 103 (1963); K. A. Amos, *ibid.* **A103**, 657 (1967).

[2] F. Petrovich, H. McManus, V. A. Madsen, and J. Atkinson, Phys. Rev. Letters **22**, 895 (1969).

[3] A. Agassi and R. Schaeffer, Phys. Letters **26B**, 703 (1968).

[4] K. Yagi *et al.*, Phys. Letters **10**, 186 (1964).

[5] V. Gillet and E. Sanderson, Nucl. Phys. **91**, 292 (1967).

The calculations in Refs. 1–3 did include exchange. However, until recently most inelastic scattering calculations have neglected the effects of particle exchange in the hope that exchange amplitudes are small because they involve matrix elements of bound with continuum single-particle wave functions. In addition to this theoretical argument, which in the region of 10–50-MeV projectiles is unconvincing, the following empirical evidence for the lack of importance of the exchange contribution can be offered: the success of the liquid-drop model for collective excitation by inelastic scattering, the smallness of the (p, n) cross sections compared to (p, p'), the success of the Lane model[6] for (p, n) analog transitions, and the success of the microscopic model excluding exchange in fitting (p, p') angular distributions. This evidence implies that, in many cases, particle-exchange amplitudes are either negligibly small or sufficiently similar to the direct amplitudes that their effect can be simulated by using an altered strength of interaction in a purely direct calculation.

The few calculations that have included exchange provide considerable evidence that exchange amplitudes are not negligible. We now review this evidence.

The first important calculation of inelastic scattering

[6] A. M. Lane, Phys. Rev. Letters **8**, 171 (1962).

in DWBA, by Levinson and Banerjee[7] for $^{12}C(p, p')^{12}C$, did include the knockout-exchange amplitude. However, although the authors state that the exchange amplitude is always smaller than the direct, there is no further indication of their relative importance.

Knockoutlike amplitudes arising from use of a space-exchange operator in an unsymmetrized formulation also were calculated by Agodi and Schiffrer.[8] Their results have been cited as evidence that the exchange cross section is small. While it is true that the exchange cross section is a factor of about 4 smaller than the direct for this case, the inclusion of the exchange amplitude could enhance the cross section by as much as a factor of 2 over that calculated with a Serber force but neglecting the exchange integral.

A very interesting result was obtained by Une *et al.*[9] in a study of the $^{13}C(p, n)$ reaction. They show that for orbital-angular-momentum transfer $L=0$, which dominates the $^{13}C(p, n)$ analog excitation, the exchange cross section is a factor of more than 10 lower than the direct. However, for $L=2$ transfers, the exchange cross sections they calculate are actually larger than the direct. This is the first indication of the sensitivity of the exchange amplitude to orbital-angular-momentum transfer.

Amos *et al.*[1] show that a local representation of the two-body t operator seems to give about the right magnitude for the $^{89}Y(p, p')$ cross section for the excitation of the $\frac{9}{2}^+$ state. This result is the first good evidence that the strength of the interaction for inelastic scattering is closely related to that in the two-nucleon problem. Aside from this important conclusion, Ref. 1 showed that the phases of the direct and exchange radial integrals are surprisingly close to each other. A further interesting result was that for many sets of parameters considered, the exchange cross section was larger than the direct.

The calculations discussed indicate that, for some cases at least, the knockout contribution to inelastic scattering and charge-exchange reactions is not negligibly small, and that inclusion of exchange will allow calculation of absolute cross sections in reasonable agreement with experiment. Many questions remain, however, concerning the nature of exchange amplitudes. For example, one can argue convincingly that the knockout amplitude must go to zero for high energies because it involves the matrix element of a rapidly oscillating continuum function with a bound one. One would like to know how high the energy must be before exchange becomes negligible. Another question is that of the two-body-force-range dependence of the exchange amplitude. In the zero-range limit the space-direct and space-exchange amplitudes are equal for a Serber force. It would be interesting to know how near zero range the interaction must be before the exchange and direct amplitudes are essentially the same and whether realistic nuclear force ranges are near zero range in this sense.

The successful description of inelastic proton scattering in terms of the liquid-drop collective model, in which no exchange can be included, may have to do with cancellation of exchange terms in a microscopic model.[10] It is well known that the large cross section for excitation of collective states is due to constructively coherent contributions from a large number of single-particle transitions. The possibility that the phases of exchange and direct amplitudes may not be the same for forces of realistic range means that, while the various direct single-particle amplitudes will be constructively coherent, the exchange amplitudes may tend to cancel each other. However, the nearness in phase of the direct and exchange in Ref. 1 leads one to question seriously the exchange cancellation argument. Further study of the phases is needed.

The result of Ref. 9, that relative sizes of direct and exchange amplitudes are dependent on orbital-angular-momentum transfer, is also of great importance because it will strongly affect the relative cross sections for different states of the same nucleus when they involve different transfers. Much more information on the angular-momentum-transfer dependence of exchange amplitudes is needed.

In view of these questions we have made a systematic study of DWBA exchange amplitudes as a function of various parameters of the theory. A preliminary report of some of our results has been published.[11]

II. FORMALISM

The present calculations are based on the formulation of Amos, Madsen, and McCarthy.[1] This section summarizes their results and gives expressions for the coefficients of direct and exchange amplitudes in the cross section for a number of cases of interest. As in Ref. 1, this formulation and the numerical results presented in this paper do not include spin-orbit distortion. For completeness we present a formulation including spin-orbit distortion in Appendix A. The treatment given in this section and Appendix A does not include contributions from the heavy-particle stripping mechanism. For a discussion of this and other aspects of the exchange problem see the forthcoming book by Austern.[12]

[7] C. A. Levinson and M. K. Banerjee, Ann. Phys. (N.Y.) **3**, 67 (1958).

[8] A. Agodi and G. Schiffrer, Nucl. Phys. **50**, 337 (1964).

[9] T. Une, S. Yamazi, and H. Yoshida, Progr. Theoret. Phys. (Kyoto) **35**, 1010 (1966).

[10] N. K. Glendenning and M. Veneroni, Phys. Rev. **144**, 839 (1966).

[11] J. Atkinson and V. A. Madsen, Phys. Rev. Letters **21**, 295 (1968).

[12] N. Austern, *Direct Nuclear Reaction Theories* [John Wiley & Sons, Inc., New York (to be published)].

The notation is nearly the same as that in Ref. 1. Isospin projections are β for the projectile and α for the target nucleons. The total and projection quantum numbers for nuclear spin and isospin are J_iM_i and T_iP_i in the initial state. Quantum numbers which are definite are labeled i and f for initial and final states and those which are summed over are labeled 1 and 2. The transfer quantum numbers are orbital angular momentum IN, spin $I'N'$, and isospin $\tau\rho$. The caret is used with coordinates to denote unit vectors $\hat{r}=\mathbf{r}/r$, and with angular momenta for the quantity $\hat{J}=(2J+1)^{1/2}$. The index j stands for all bound-state quantum numbers, j, l, α.

The effective interaction between the projectile and a target nucleon is assumed to be central and is given by the expression

$$t(0,1)=t(r_{01})\sum_{TS}A_{TS}\mathcal{P}_{TS}, \tag{1}$$

where A_{TS} is a numerical coefficient and $\mathcal{P}_{TS}$ is the projection operator onto two-particle states of isospin T and spin S.

The space-direct and space-exchange single-particle amplitudes for definite orbital-angular-momentum transfer are given by

$$F_{LM}^{j_1j_2}=(-1)^{l_1}\int\int\chi_f^{(-)}(\mathbf{r}_1)\chi_i^{(+)}(\mathbf{r}_0)t(r_{01})\times[\phi_{j_1l_1}(\mathbf{r}_1)\phi_{j_2l_2}(\mathbf{r}_0)]_L^M d^3r_0d^3r_1, \tag{2a}$$

$$G_{LM}^{j_1j_2}=(-1)^{l_1}\int\int\chi_f^{(-)}(\mathbf{r}_1)\chi_i^{(+)}(\mathbf{r}_0)t(r_{01})\times[\phi_{j_1l_1}(\mathbf{r}_1)\phi_{j_2l_2}(\mathbf{r}_0)]_L^M d^3r_0d^3r_1. \tag{2b}$$

It is clear that in the limit of zero-range effective interaction $t(r_{01})$ these two amplitudes are identical.

When the space part of the effective nucleon-nucleon interaction is expanded in spherical harmonics

$$t(r_{01})=\sum_{\lambda}v_\lambda(r_0,r_1)Y_\lambda^\mu(\hat{r}_0)Y_\lambda^{\mu*}(\hat{r}_1), \tag{3}$$

these amplitudes may be written

$$F_{LM}^{j_1j_2}=\langle l_2\|Y_L\|l_1\rangle\hat{L}^{-1}\sum_{L_1L_2}(4\pi)^2i^{L_1-L_2}Y_{L_2}^M(\hat{k}_f)Y_{L_1}^0(0)\hat{L}^{-1}\langle L_2\|Y_L\|L_1\rangle(-1)^{L_1}C(L_1L_2L;0MM)\times\int R_{L_2}(r_0)g_L^{j_1j_2}(r_0)R_{L_1}(r_0)r_0^2dr_0, \tag{4a}$$

$$G_{LM}^{j_1j_2}=(-1)^{l_1}\sum_{\lambda L_1L_2}(4\pi)^2i^{L_1-L_2}C(L_1L_2L;0MM)Y_{L_2}^M(\hat{k}_f)Y_{L_1}^0(0)\langle L_2\|Y_\lambda\|l_1\rangle\langle l_2\|Y_\lambda\|L_1\rangle W(L_2l_1L_1l_2;\lambda L)\times\int\int R_{L_2}(r_0)R_{l_2}(r_1)v_\lambda(r_0,r_1)R_{L_1}(r_1)R_{l_1}(r_0)r_0^2r_1^2dr_0dr_1, \tag{4b}$$

where

$$g_L^{j_1j_2}(r_0)=\int R_{l_2}(r_1)v_L(r_0,r_1)R_{l_1}(r_1)r_1^2dr_1, \tag{5}$$

and R_L and R_l are continuum and bound radial wave functions.

In terms of $F_{LM}^{j_1j_2}$ and $G_{LM}^{j_1j_2}$, the differential cross section is given by the expression

$$\frac{d\sigma}{d\Omega}=\left(\frac{2m}{4\pi\hbar^2}\right)^2\frac{k_f}{k_i}[2(2J_i+1)]^{-1}\sum_{II'LM}(2I+1)(2I'+1)\Big|\sum_{j_1j_2}d_{j_1j_2}(II'L)\times[D_{j_1j_2}^+(II')(F_{LM}^{j_1j_2}+G_{LM}^{j_1j_2})+D_{j_1j_2}^-(II')(F_{LM}^{j_1j_2}-G_{LM}^{j_1j_2})]\Big|^2, \tag{6}$$

where

$$d_{j_1j_2}(II'L)=\hat{j}_1\hat{j}_2\begin{pmatrix} j_1 & \frac{1}{2} & l_1\\ j_2 & \frac{1}{2} & l_2\\ I & I' & L\end{pmatrix}\begin{cases} S(J_iJ_fI;j_1j_2\alpha_1\alpha_2) & \text{nuclear isospin not assumed pure}\\ 1 & \text{pure nuclear isospin,}\end{cases} \tag{7}$$

$$D_{j_1j_2}^+(II')=t_{01}+t_{10}, \tag{8}$$

$$D_{j_1j_2}^-(II')=t_{00}+t_{11} \tag{9}$$

with

$$t_{TS}=A_{TS}\epsilon_S(I')\begin{cases}\gamma_T(\alpha_1\alpha_2\beta_1\beta_2) & \text{nuclear isospin not assumed pure}\\ \sum_\tau S(J_iJ_fI;T_iT_f\tau;j_1j_2)\theta_T(\tau) & \text{pure nuclear isospin,}\end{cases} \tag{10}$$

$$\epsilon_S(I')=(2S+1)(\tfrac{1}{2}-\tfrac{2}{3}\delta_{I'1}\delta_{S1}), \tag{11}$$

$$\theta_T(\tau)=C(\tfrac{1}{2}\tfrac{1}{2}\tau;\beta_i,-\beta_f,\beta_i-\beta_f)C(T_iT_f\tau;P_i,-P_f,P_i-P_f)\epsilon_T(\tau), \tag{12}$$

$$\gamma_T(\alpha_1\alpha_2\beta_i\beta_f)=\tfrac{1}{2}[\delta_{\alpha_1\alpha_2}\delta_{\beta_i\beta_f}-(-1)^T\delta_{\alpha_1\beta_f}\delta_{\alpha_2\beta_i}], \tag{13}$$

TABLE I. Inelastic scattering coefficients for the case in which the nuclear isospin is not assumed to be conserved. For this case they do not depend on the total angular momentum transfer I or any single-particle quantum numbers besides the charge.

I	Colliding nucleons	$D_{j_1j_2}^{+}(II')$	$D_{j_1j_2}^{-}(II')$
0	Alike	$\frac{1}{2}A_{10}$	$\frac{3}{2}A_{11}$
0	Unlike	$\frac{1}{4}(A_{10}+3A_{01})$	$\frac{1}{4}(A_{00}+3A_{11})$
1	Alike	$\frac{1}{2}A_{10}$	$-\frac{1}{2}A_{11}$
1	Unlike	$\frac{1}{4}(A_{10}-A_{01})$	$\frac{1}{4}(A_{00}-A_{11})$

and $S(J_iJ_fI;j_1j_2\alpha_1\alpha_2)$ and $S(J_iJ_fI;T_iT_f\tau;j_1j_2)$ are spectroscopic amplitudes[13] for the nondefinite and definite nuclear isospin, respectively.

The coefficients $D_{j_1j_2}^{\pm}(II')$ appearing in the expression for the cross section Eq. (6) are given in Tables I–III for most cases of interest for both spin-flip ($I'=1$) and non-spin-flip ($I'=0$) transitions.

III. PLANE-WAVE AMPLITUDES

It is interesting to try to obtain simple plane-wave amplitudes for a qualitative comparison with our numerical DWBA results. It is possible to calculate plane-wave exchange amplitudes exactly for a Gaussian interaction with harmonic-oscillator bound states. However, the results are complicated and not particularly enlightening.

TABLE II. Inelastic scattering coefficients for charge transfer for the case in which nuclear isospin is assumed to be definite. For the (p,n) or (n,p) reactions the coefficients $D_{j_1j_2}^{\pm}(II')$ are the table entries multiplied by $-C(T_iT_f1;P_i,-P_f,P_i-P_f)$ $S(J_iJ_fI;T_iT_f1;j_1j_2)$. For the (p,p') and (n,n') reactions there is an additional factor of $1/\sqrt{2}$. For the latter two reactions there are, in addition to the $\tau=1$ transfer term obtained from this Table, $\tau=0$ transfer terms which must be added to get the $D_{j_1j_2}^{\pm}(I\,I')$ coefficients. These are given in Table III.

I'	Even coefficient (+)	Odd coefficient (−)
0	$1/4(3A_{01}-A_{10})$	$1/4(A_{00}-3A_{11})$
1	$-1/4(A_{10}+A_{01})$	$+1/4(A_{00}+A_{11})$

[13] V. A. Madsen, Nucl. Phys. **80**, 177 (1966).

Recognizing that the space-exchange and space-direct amplitudes are exactly the same in the zero-range limit, we have made an expansion about zero range[14,15] keeping only the leading order terms. The leading term beyond zero range should contain differences between direct and exchange amplitudes and enable us to identify some of the features to be seen in our DWBA amplitudes.

With a spherically symmetric short-range interaction

$$V=V_0g(|\mathbf{r}_0-\mathbf{r}_1|), \tag{14}$$

where g is normalized to have a unit integral over

TABLE III. Inelastic scattering coefficients for no charge transfer for the case in which nuclear isospin is assumed to be definite. For (p,p') and (n,n') reactions the $\tau=0$ term in the coefficients $D_{j_1j_2}^{\pm}(II')$ are obtained by multiplying the table entry by $(-1)^{1/2-p_i+T_i-P_i}\{\delta_{T_iT_f}/[2(2T_i+1)]^{1/2}\}S(J_iJ_fI;\,T_iT_f0;\,j_1j_2)$. For $T_i=T_f=0$ transitions this is the only contribution, but for other cases one must add the $\tau=1$ contribution from Table II.

I'	Even coefficient (+)	Odd coefficient (−)
0	$3/4(A_{10}+A_{01})$	$1/4(A_{00}+9A_{11})$
1	$1/4(3A_{10}-A_{01})$	$1/4(A_{00}-3A_{11})$

d^3r_0 or d^3r_1, the integral of V with some function $f(\mathbf{r}_1)$ is

$$V_0\int f(\mathbf{r}_1)g(|\mathbf{r}_0-\mathbf{r}_1|)d^3r_1=V_0[1+c_2\nabla_0^2]f(\mathbf{r}_0) \tag{15}$$

to second order in the range parameter in $g(r)$. In Eq. (15), the coefficient c_2 is

$$c_2=\int\tfrac{1}{6}r^2g(r)d^3r.$$

Of course, $f(\mathbf{r}_0)$ must be a reasonably smooth function, so the derivatives involved in the Taylor-series expansion all exist. This integration formula can be applied to calculate the space-direct and space-exchange amplitudes F and G, Eqs. (2). This calculation is made for plane-wave continuum functions in Appendix B. The resulting amplitudes in the cutoff approximation for an even interaction with a short-range

[14] An expansion about zero range has been used recently (Refs. 2, 15) to approximate the effects of exchange in DWBA.

[15] F. Petrovich and H. McManus (unpublished).

Yukawa form $e^{-\alpha r}/\alpha r$ are

$$F_{LM}^{j_1j_2}=(1-q^2/\alpha^2)\tilde{F}_{LM}^{j_1j_2}, \qquad (16a)$$

$$G_{LM}^{j_1j_2}=[1+(1/2\alpha^2)(\kappa_1^2+\kappa_2^2-k_i^2-k_f^2+q^2)] \times\tilde{F}_{LM}^{j_1j_2}+\tilde{G}_{LM}^{j_1j_2}, \qquad (16b)$$

where $\tilde{F}_{LM}^{j_1j_2}$ is the zero-range plane-wave amplitude, q is the momentum transfer $\kappa^2=2m\epsilon/\hbar^2$, and $k^2=2mE/\hbar^2$, ϵ being the single-particle binding energy. The term $\tilde{G}_{LM}^{j_1j_2}$, discussed below for the non-normal transfers, vanishes for normal transfers $[(-1)^L=(-1)^{l_2-l_1}]$ for the special case of identical initial and final single-particle states, as shown in Appendix B. Restricting the discussion to this case, we have for Eq. (16b)

$$G_{LM}^{j_1j_2}=\{1+(1/\alpha^2)[\kappa^2-\tfrac{1}{2}(k_i^2+k_f^2)+\tfrac{1}{2}q^2]\}\tilde{F}_{LM}^{j_1j_2}. \qquad (16b')$$

For the validity of this expression the energy should be low. For $\alpha=1.0$, we require $k\ll 1$ or $E\ll 20$ MeV. Two differences in Eqs. (16a) and (16b′) due to the finite-range correction are immediately apparent. The first is that, compared to $F_{LM}^{j_1j_2}$, $G_{LM}^{j_1j_2}$ decreases with increasing projectile energy E_i. This property of the DWBA amplitudes will be seen later in the numerical results given in Sec. IV. The second feature of Eqs. (16) is that $F_{LM}^{j_1j_2}$ falls off more rapidly with increasing angle than does $G_{LM}^{j_1j_2}$. This qualitative characteristic of exchange angular distributions, which is also present in the DWBA,[11] gives rise in the plane-wave approximation to an L dependence of exchange-to-direct cross-section ratios. The reason is as follows: The zero-range amplitude is roughly proportional to $j_L(qr)$. The higher the L value, the larger the value of q (and θ) where the Bessel function has its main peak. With the finite-range correction reducing the direct amplitude and increasing the exchange as q is increased, the latter amplitude becomes more important for higher L values. Such an L dependence is seen also in DWBA; this will be discussed further in Sec. IV.

It is also suggested by Eq. (16b′) that the exchange amplitude should be of increasing importance relative to the direct for higher particle binding energy $\hbar^2\kappa^2/2m$. As will be discussed in Sec. IV, this dependence was tested for DWBA and found to be (weakly) present there also.

In the case of a non-normal transfer [for which $(-1)^{L+1}=(-1)^{l_2-l_1}$] the direct amplitude, in particular the zero-range direct amplitude $\tilde{F}_{LM}^{j_1j_2}$, is identically zero. The only contribution to inelastic scattering comes from exchange, and the only nonzero part of Eq. (16b) is $\tilde{G}_{LM}^{j_1j_2}$, Eq. (B9). Carrying out angular integrations we obtain, for the special case $j_1l_1=j_2l_2$,

$$G_{LM}^{j_1j_2}=\tilde{G}_{LM}^{j_1j_2}=8\pi V_0 i^{1-L}QT_{L(L1)}^M(\hat{q},\hat{Q}) \times\sum_\lambda(-1)^\lambda\langle l_1\|Y_L\|\lambda\rangle\hat{l}_1W(L\lambda Ll_1;l_11) \times\int j_L(qr)R_{l_1}(r)U_{l_1,\lambda}(r)r^2dr, \qquad (17)$$

where

$$T_{L(L1)}^M(\hat{q},\hat{Q})=(4\pi/3)^{1/2}[Y_L(\hat{q})Y_1(\hat{Q})]_L^M \qquad (18)$$

with

$$\mathbf{q}=\mathbf{k}_f-\mathbf{k}_i,$$
$$\mathbf{Q}=\tfrac{1}{2}(\mathbf{k}_f+\mathbf{k}_i), \qquad (19)$$

and

$$U_{l,\lambda}(r)=-\left(\frac{l+1}{2l+1}\right)^{1/2}\left(\frac{dR_l}{dr}-\frac{l}{r}R_l\right), \qquad \lambda=l+1$$
$$=\left(\frac{l}{2l+1}\right)^{1/2}\left(\frac{dR_l}{dr}+\frac{l+1}{r}R_l\right), \qquad \lambda=l-1$$
$$=0, \qquad \text{otherwise.} \qquad (20)$$

The tensor $T_{L(L1)}^M$ is zero at 0° and 180° scattering angle, and it vanishes everywhere for $M=0$ when $\mathbf{k}_i$ is along the z axis. This is shown in Sec. V to be a general property of the DWBA non-normal amplitude. The quantity $\sum_M(T_{L(L1)}^M)^2$, which appears as a factor in the cross section, is

$$\sum_M|T_{L(L1)}^M(\hat{q},\hat{Q})|^2=\text{const} \times 4k_i^2k_f^2\sin^2\theta/[(k_f^2-k_i^2)^2+4k_i^2k_f^2\sin^2\theta], \qquad (21)$$

where θ is the scattering angle. For small-energy-transfer reactions this is a fairly flat function that peaks at 90° and goes to zero at $\theta=0°$ and 180°. The other factor in the cross section in the cutoff PWBA is approximately proportional to $j_L(qR)^2$, where R is the cutoff radius. This latter factor also gives the angular distribution for a normal transfer L in the cutoff PWBA. Thus we should expect the angular distribution for a non-normal L transfer to be similar to a normal angular distribution multiplied by the right-hand side of Eq. (21), which makes it go to zero at $\theta=0°$ and 180°. In the special case of no energy transfer, the function Eq. (21) is simply a constant in angle. In that case, however, $j_L(qr)$ is zero at $\theta=0°$ and $Q=0$ at 180°.

IV. PROPERTIES OF EXCHANGE AMPLITUDES

In Ref. 11, some of the important properties of the exchange amplitudes were noted. It was shown that the

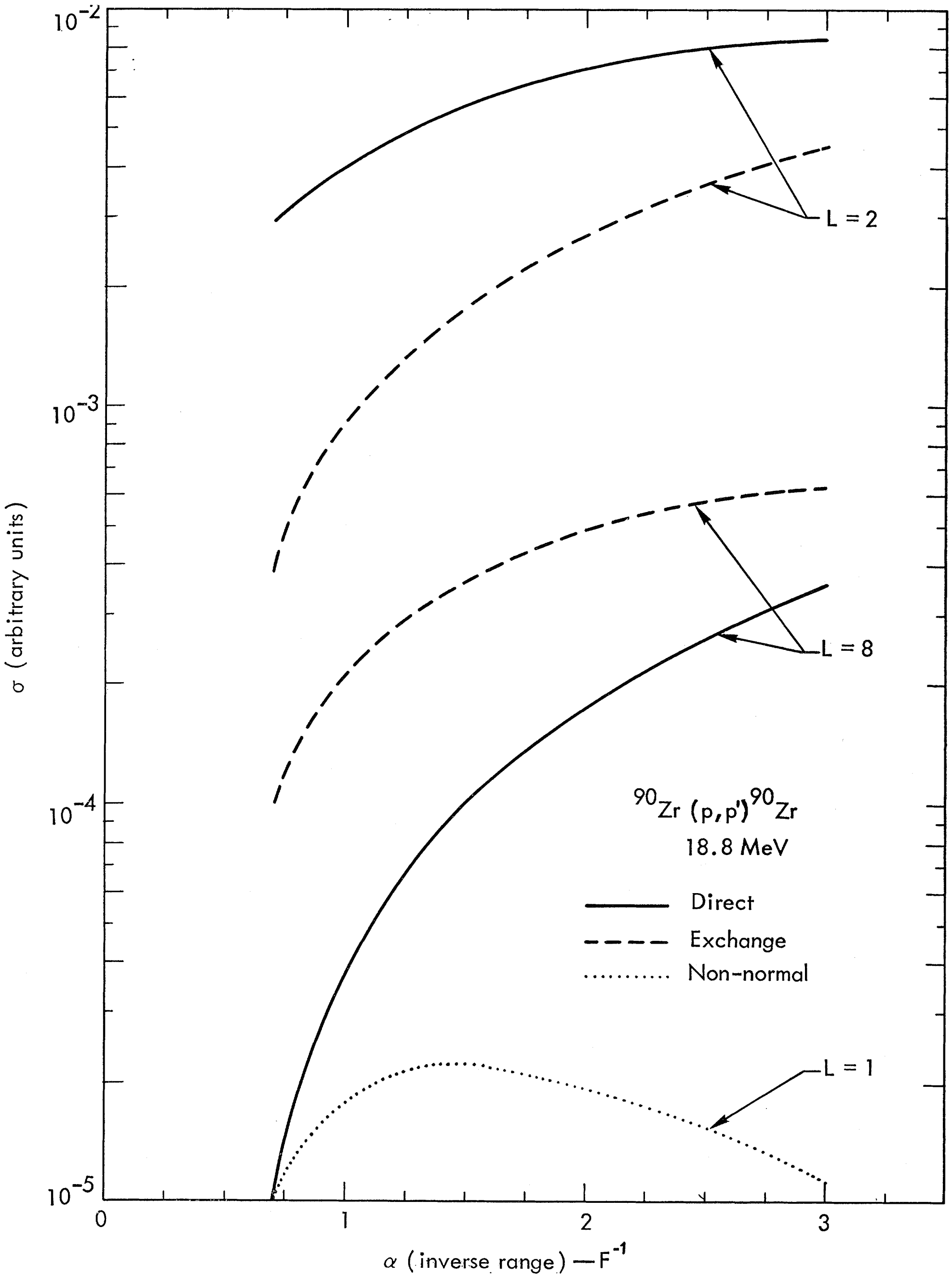


FIG. 1. Dependence of direct and exchange cross sections in $^{90}Zr(p, p')$ on the inverse range α of the Yukawa force, $V_0(e^{-\alpha r}/\alpha r)$. V_0 is adjusted so that $V_0 \propto \alpha^3$. L is the orbital-angular-momentum transfer. $L=8$ excites the 8^+ state while $L=1$ and $L=2$ both contribute to the 2^+ excitation. The non-normal $L=1$ transfer is pure exchange (see Sec. V).

ratio of exchange-to-direct cross sections increased rapidly with increasing orbital-angular-momentum transfer for a Yukawa-force range of the order of one Fermi. Previously, it had been assumed that exchange was important only for forces of short range, in which limit the space-direct and space-exchange amplitudes, Eqs. (2), approach each other. However, for high transfers we find that the exchange amplitude actually exceeds the direct and becomes increasingly important at longer ranges. The increase in importance of exchange is not because of any absolute increase in the exchange amplitude with increasing range, but rather because the direct amplitude decreases more rapidly than the exchange as the range increases. This relatively faster decrease is due to the fact that, for transfer of orbital angular momentum L, only the L multipole of the two-body interaction contributes to the direct amplitude, whereas for exchange all multipoles of the interaction contribute. For a long-range force, the magnitude of successive multipoles falls off rapidly with multipole order, so that direct amplitudes for high L are expected to be weak. This behavior is illustrated in Fig. 1, where the dependence of direct and exchange amplitudes on inverse range is shown for $L=2$ and 8 with a constant volume interaction ($V_0\alpha^{-3}=$ const). For $L=8$, the direct cross section falls off more rapidly than the exchange as the range gets longer (α gets smaller); for $L=2$ the situation is reversed.

For a finite-range force, the exchange amplitudes are expected to decrease relative to the direct with increasing energy due to poor overlaps in matrix elements of the interaction taken between the slowly varying bound and rapidly varying continuum wave functions for projectile and target nucleon. This expectation is borne out by our calculations. Figure 2 shows the energy dependence[16,17] for $L=2$ and 8 of the exchange-to-direct cross-section ratio. At 150 MeV, this ratio is 0.04 for $L=2$. This small ratio does not mean that exchange is negligible at 150 MeV since the corresponding amplitude ratio is 0.2. Assuming com-

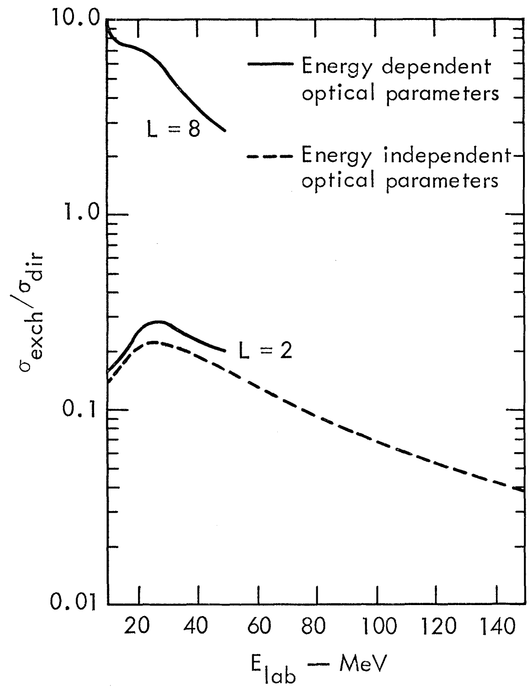


FIG. 2. Energy dependence of the ratio of exchange-to-direct cross sections. The solid curves are calculated with the energy-dependent optical parameters of Becchetti and Greenlees (Ref. 16) and the dashed curve is calculated with the potentials of Gray *et al.* (Ref. 17) for all energies.

pletely constructive interference the cross section with exchange would be almost 50% greater than the direct alone.

The form of the plane-wave exchange amplitude [Eq. (16b′)] suggests that the importance of exchange amplitudes relative to direct increases with binding energy. This is verified to some extent in DWBA, as shown in Table IV, where the exchange-to-direct cross-section ratio for a number of cases is given as a function of the single-particle binding energy. The $L=2$ and $L=4$ ratios reproduce the expected behavior quite well. The $L=0$ ratios are much less sensitive to the binding energy and are not in agreement with Eq. (16b′). It should be noted that the effect considered here is small since one must go to highly unrealistic binding energies to produce a significant change in $\sigma_{\rm exch}/\sigma_{\rm dir}$.

The irregular behavior of the $L=0$ ratios can be understood by recognizing that lower-orbital-angular-momentum transfers take place generally at smaller radii. Since the plane-wave expressions [Eqs. (16a) and (16b′)] were derived in the cutoff approximation, they do not give an accurate account of processes that receive important contributions from inside the cutoff radius. Also the principal effect of the binding energy is to determine the form of the bound-state wave

TABLE IV. Dependence of the exchange-to-direct cross-section ratio on the single-particle binding energy in a Woods-Saxon potential. This particle is a $1p_{1/2}$ neutron (proton) in the case of $^{13}C(^{13}N)$ and a $1g_{9/2}$ proton in ^{90}Zr. L is the orbital-angular-momentum transfer.

$^{13}C(p,n)^{13}N$ Binding energy (MeV)	$\sigma_{\rm exch}/\sigma_{\rm dir}$ $L=0$	$L=2$	$^{90}Zr(p,p')^{90}Zr$ Binding energy (MeV)	$\sigma_{\rm exch}/\sigma_{\rm dir}$ $L=0$	$L=4$
5	0.215	0.88	1.0	0.0747	0.532
10	0.208	1.07	3.0	0.0736	0.549
20	0.208	1.37	5.7	0.0735	0.570
30	0.212	1.63	9.0	0.0728	0.595
50	0.250	1.98			

[16] F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. **182**, 1190 (1969).
[17] W. S. Gray, R. A. Kenefick, J. J. Kraushaar, and G. R. Satchler, Phys. Rev. **142**, 735 (1966).

TABLE V. Dependence of exchange-to-direct cross-section ratios on single-particle quantum numbers.

n_1	n_2	l_1	l_2	$\sigma_{\text{exch}}/\sigma_{\text{dir}}$
1	1	5	7	0.097
1	1	4	6	0.097
1	1	3	5	0.107
1	1	5	5	0.141
1	1	4	4	0.158
1	1	2	4	0.135
1	2	5	3	0.425
1	2	2	4	0.430
2	2	2	4	0.272
2	2	1	3	0.474
2	2	2	2	0.337
2	3	2	0	0.500

function in the asymptotic region. This region contributes more importantly to high-L than to low-L processes and hence one expects the exchange-to-direct ratios for higher L to be more sensitive to the binding energy, as indicated in Table IV.

The above argument suggests that a similar effect should be produced by the use of harmonic-oscillator wave functions, whose rapid asymptotic falloff is simulated by a tightly bound Woods-Saxon wave function. Une *et al.*[9] used harmonic-oscillator wave functions for $^{13}C(p, n)$. For $L=2$, these wave functions yield a ratio $\sigma_{\text{exch}}/\sigma_{\text{dir}} \approx 2$, whereas a calculation with Saxon well and a realistic binding energy of 5 MeV gives a value 0.88 for this ratio. Thus it appears that the relative importance of exchange may be seriously overestimated when calculated with harmonic-oscillator wave functions.

The properties of exchange amplitudes that we have discussed thus far have been calculated for transitions in which the bound nucleon is in the same single-particle state initially and finally. It is important to know also how the exchange amplitudes vary with single-particle quantum numbers. In Table V, exchange-to-direct cross-section ratios are given for various single-particle transitions involved in the $^{118}Sn(p, p')$ reaction discussed in Secs. VI and VII C. These results show that exchange is relatively more important for higher radial quantum numbers and that exchange is particularly important when $n_1 \neq n_2$. The dependence on orbital angular momentum is not as strong, but

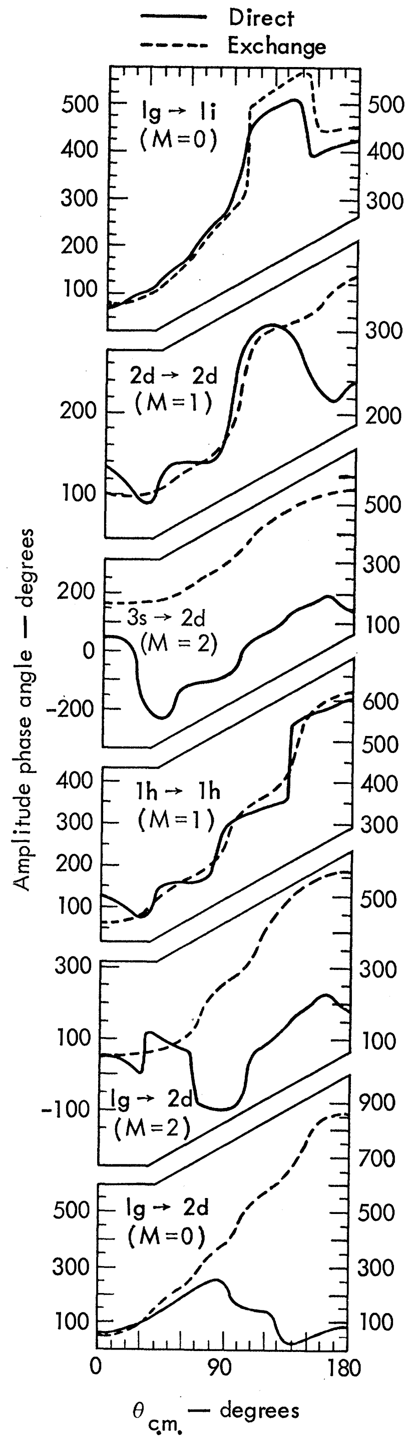


FIG. 3. Comparison of the direct and exchange phase angles of the complex scattering amplitude as a function of scattering angle for several important single-particle transitions in $^{118}Sn(p, p')^{118}Sn(2^+)$. M is the z projection of the orbital-angular-momentum transfer L. ($L=2$ for this excitation.)

exchange is favored by low l-values and by the condition $l_1=l_2$. Exchange could be particularly important for $0^+ \to 0^+$ transitions where a small calculated direct cross section often results from near-perfect cancellation of several direct single-particle amplitudes with different single-particle quantum numbers. The cancellation would be less complete for the single-particle exchange amplitudes. Such a transition occurs in $^{90}Zr(p, p')$ and is discussed in Sec. VII B.

Another important property demonstrated in Ref. 11 was the remarkable similarity of the phases of direct and exchange amplitudes as a function of scattering angle even for long-range forces. Figure 3 shows these phases for several strong transitions in $^{118}Sn(p, p')$ involving a variety of single-particle configurations. The amount of correlation between direct and exchange phases is dependent primarily on the radial quantum numbers of the initial and final states. Generally, transitions with $n_1=n_2$ exhibit stronger phase correlation than those with $n_1 \neq n_2$, the greatest similarity occurring for transitions between states having nodeless wave functions. For the proper interpretation of the phase plots in Fig. 3, it is important to note that the vertical scale has been extended beyond 360° to avoid the appearance of discontinuities. Thus in the case of the $3s \to 2d$ transition the two curves lie about 360° apart and the direct and exchange amplitudes are actually in phase over most of the angular range. Keeping this in mind, we see that there is reasonably good agreement between direct and exchange phases in the majority of cases and that, except for the $1g \to 2d(M=0)$ transition, the disparities that do occur extend over only a small fraction of the total angular range. The importance of these phase differences in collective transitions will be considered further in Sec. VI.

V. NON-NORMAL TRANSFERS

From Eq. (4a), we see that for the direct amplitude the orbital-angular-momentum transfer is restricted to even or odd values depending on the parity on the transition. The reduced matrix element $\langle l_2 \| Y_L \| l_1 \rangle$ is nonvanishing only when $(-1)^L = (-)^{l_2-l_1}$, and the latter phase factor has the sign of the change in parity of the transition. On the other hand, there is no such restriction on L in the exchange amplitude, Eq. (4b). Let us label orbital transfers for which $(-1)^L \neq (-)^{l_2-l_1}$ as non-normal transfers. These can occur, and do so only through the space-exchange amplitude.

Consider the exchange amplitude [Eq. (4b)] at 0° or 180°, for which $Y_{L_2}{}^M(\hat{k}_f) = Y_{L_2}{}^0(0)\delta_{M0}$. With $M=0$, the Clebsch-Gordan coefficient vanishes for $(-1)^L \neq (-1)^{L_1+L_2} = (-1)^{l_1+l_2}$; that is, the exchange amplitude goes to zero at 0° and 180° for non-normal transfers, as in the simple plane-wave result, Eq. (21). We know that, in the limit of short-range forces, the non-normal amplitudes must vanish at all angles, because they approach the direct, which are zero. It is important to determine the contribution of non-normal transfers to the cross section for typical interaction ranges and the dependence of this contribution on range.

Figure 4 shows a comparison of differential cross section for the various possible transfers in the $^{13}C(p, n)$ reaction assuming a pure $p_{1/2} \to p_{1/2}$ transition and a 1.1-F Yukawa force with a Serber mixture. (This Yukawa range is equivalent to the 1.8-F Gaussian range used by Une *et al.*[9] and was chosen to facilitate the comparison made in Sec. IV with their results.) In this case, the $L=0$ transfer dominates, the $L=2$ cross section is lower by a factor of about 2, and the non-normal $L=1$ contribution is down by a factor of about 100 compared to the $L=0$.

With a Serber force of range 1.4 F (the OPEP range) for the $^{14}C(p, n)^{14}N$ ground-state transition, which has an inhibited $L=0$ contribution related to the β decay, we find the non-normal $L=1$ cross section to be a factor of about 100 smaller than the dominant $L=2$.

The range dependence of a typical non-normal transfer was shown in Fig. 1 for a constant volume interaction ($V_0\alpha^{-3}$ = const). In contrast to the behavior shown for normal transfers, the non-normal cross section reaches a maximum at some point and then falls off rapidly as the range becomes small.

It is interesting to note for the case of $^{90}Zr(p, p')$ $^{90}Zr(2^+)$, that, although the non-normal ($L=1$) amplitude is only 0.034 as large as the total normal ($L=2$), it is relatively much larger with respect to the $L=2$ exchange component alone. Thus if the direct component is not large, one would expect that the non-normal contribution might be significant. Consider the 8^+ excitation in ^{90}Zr for which the exchange cross section is six times the direct. The normal ($L=8$), non-normal ($L=7$), and total differential cross sections for this case are shown in Fig. 5. The non-normal part is quite significant here, contributing one-fourth of the total cross section. Also apparent is the characteristic shape of the non-normal angular distribution mentioned at the beginning of this section and seen earlier in analytical form [Eq. (21)] in the plane-wave limit.

It should be pointed out that, while the non-normal $L=7$ transfer contributes significantly to the excitation of the 8^+ state on the basis of the $(1g_{9/2})^2$ wave function assumed in the calculation, it may actually be much less important. The $L=8$ normal amplitude is fairly strongly enhanced by collective effects (see Sec. VII B), but the non-normal $L=7$, being a spin-flip amplitude, is probably not. This suggests the interesting possibility that in the opposite case, where the normal transfer is coupled to $I'=1$ (spin flip) and the non-normal has $I'=0$, the non-normal cross section might be considerably enhanced relative to the normal.[18]

[18] F. Petrovich (private communication).

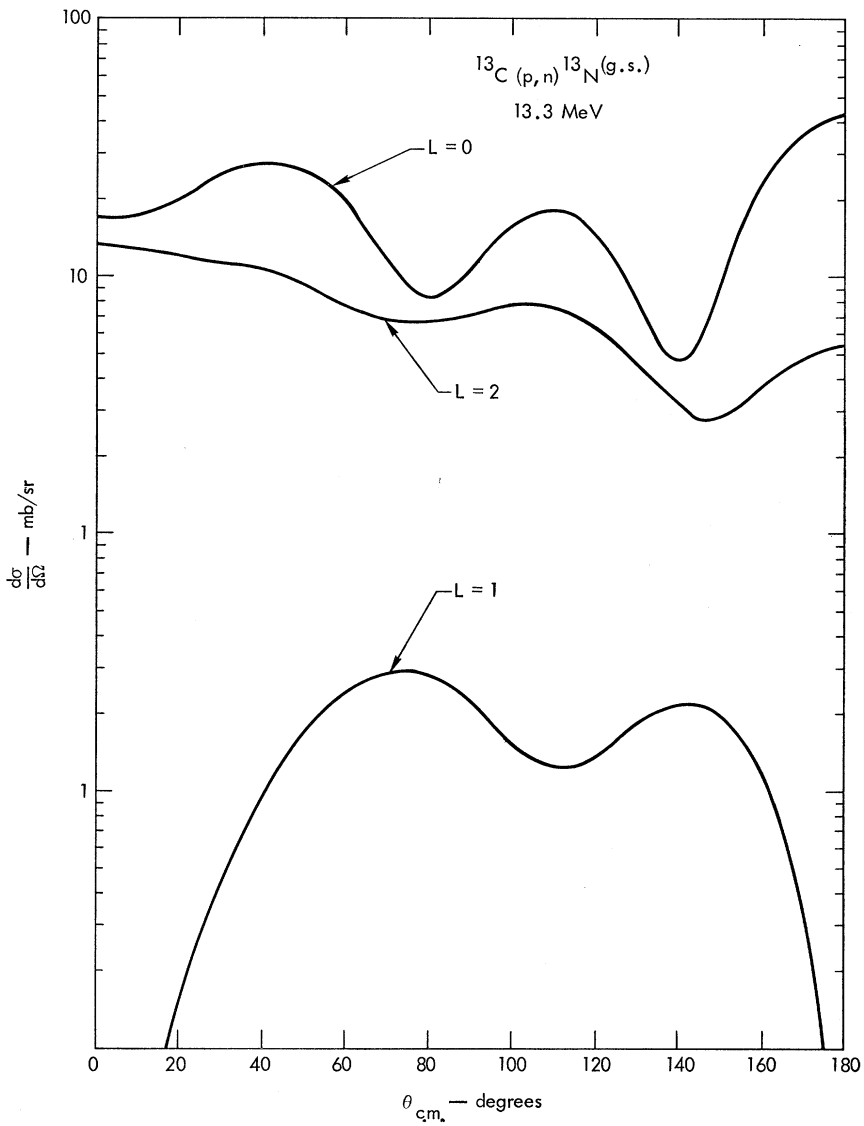


FIG. 4. Contributions to the $^{13}C(p, n)^{13}N$ angular distribution from all possible orbital-angular-momentum transfers L. The $L=0$ and $L=2$ curves include both direct and exchange contributions. $L=1$ is a non-normal transfer and hence pure exchange.

VI. EXCHANGE IN COLLECTIVE EXCITATIONS

In a microscopic description of a collective excitation, the importance of exchange depends not only on the magnitude of the various single-particle exchange amplitudes but also on the degree of constructive interference present in their coherent sum. Glendenning and Veneroni have suggested[10] that the phases of the exchange amplitudes will be essentially random. Thus one would expect that the constructive interference characterizing the direct amplitudes would not obtain in the exchange case. At variance with this prediction, however, is the remarkable similarity between direct and exchange phases observed by Amos *et al.*[1] for $^{19}F(p, p')$ and by us[11] for $^{13}C(p, p')$.

It is therefore of some interest to carry out a microscopic calculation of the cross section and angular distribution for a collective excitation. We have chosen for this purpose the (p, p') reaction leading to the 2^+ state in ^{118}Sn since microscopic wave functions exist[19] for this nucleus. In addition, the recent availability of the $^{118}Sn(p, p')^{118}Sn(2^+)$ angular distribution[20] provides a useful check for our calculation as well as the possibility of obtaining force constants from the absolute cross section (see Sec. VII).

Yoshida's collective RPA wave functions[19] include approximately 100 two-quasiparticle configurations. For computational convenience we have chosen the

[19] S. Yoshida, Nucl. Phys. **38**, 380 (1962).

[20] W. Makofske, W. Savin, H. Ogata, and T. H. Kruse, Phys. Rev. **174**, 1429 (1968).

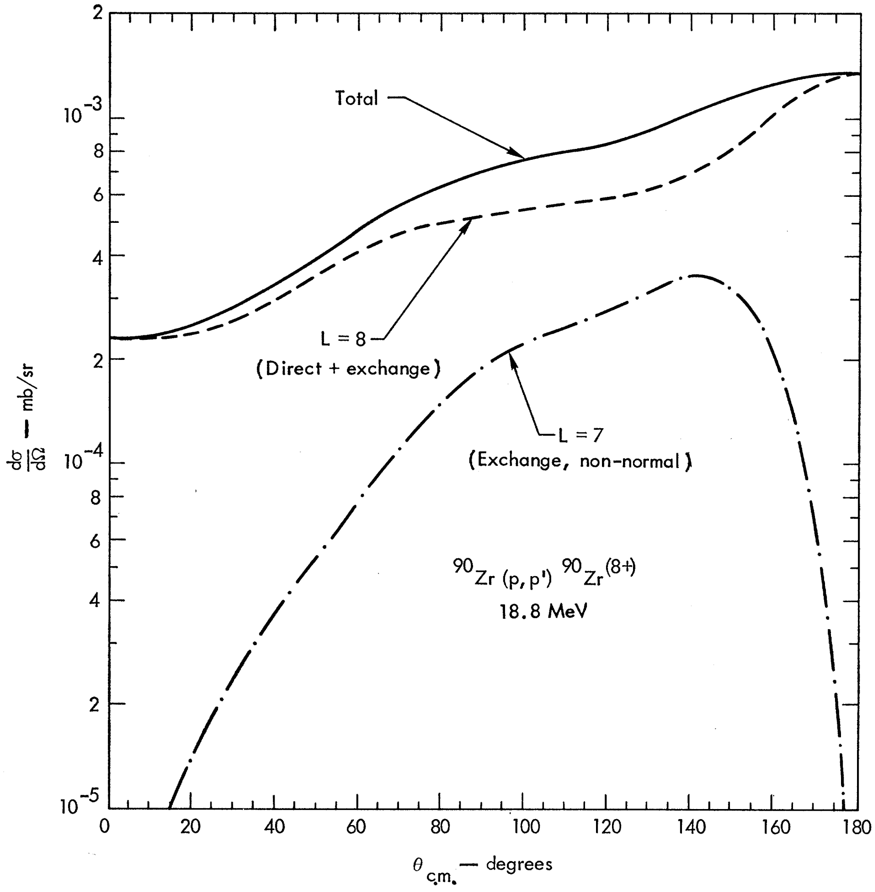


FIG. 5. Normal, non-normal, and total differential cross sections for $^{90}Zr(p, p')^{90}Zr(8^+)$. The two L transfers contribute incoherently so the total curve is just the sum of the other two.

46 strongest terms, which give about 70% of the total $E2$ transition strength. Optical parameters were obtained in the usual way from elastic data. A two-body Serber interaction was used with a Yukawa shape of range 0.7 F, and a Coulomb term was included for the proton transitions. (Angular distributions calculated with longer Yukawa ranges were generally in slightly poorer agreement with the data.) The results are shown in Fig. 6 with the Serber strength adjusted so that the direct-plus-exchange curve gives the best visual fit to the data. It is clear that exchange makes an important contribution here, its effect being to raise the total cross section by a factor of more than 2. This implies that the cancellation argument mentioned above is not entirely valid, and it is therefore of interest to examine the question of interference more closely.

To do this, we have made separate calculations of direct and exchange cross sections for each of the single-particle transitions in the 2^+ excitation. If we consider the ratio of exchange-to-direct cross section, σ_E^i/σ_D^i, for each transition i, and compute a properly weighted average of this quantity over all the transitions, the result $(\sigma_E^i/\sigma_D^i)_{\text{av}}=0.395$ is obtained. This is to be compared with the calculated ratio for the total exchange and direct cross sections, $\sigma_E^{\text{total}}/\sigma_D^{\text{total}}=0.224$. The observed difference is qualitatively what one would expect if there were relatively more destructive interference among the exchange amplitudes than among the direct. To see that this is indeed the case and to obtain a more quantitative idea of the importance of such interference, we define a quantity

$$\phi_D \equiv 1-\sigma_D/\sigma_D', \qquad (22)$$

which is a measure of the amount of cancellation occurring among the various single-particle direct amplitudes. Here σ_D is the actual direct total cross section and σ_D' is the cross section that one would get if all the direct single-particle amplitudes were exactly in phase, i.e.,

$$\sigma_D' = \left[\sum_i (\sigma_D^i)^{1/2}\right]^2. \qquad (23)$$

An analogous quantity ϕ_E is defined for the exchange cross sections. It is clear from Eqs. (22) and (23) that, if all single-particle amplitudes are in phase, ϕ will be zero. For the present case the results $\phi_D=0.096$ and

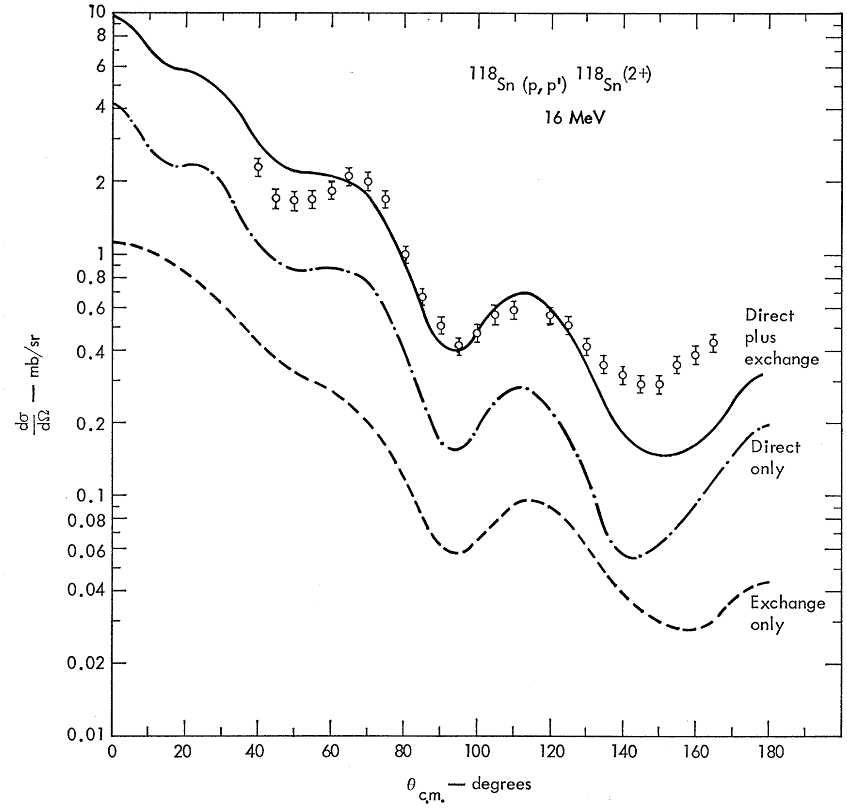
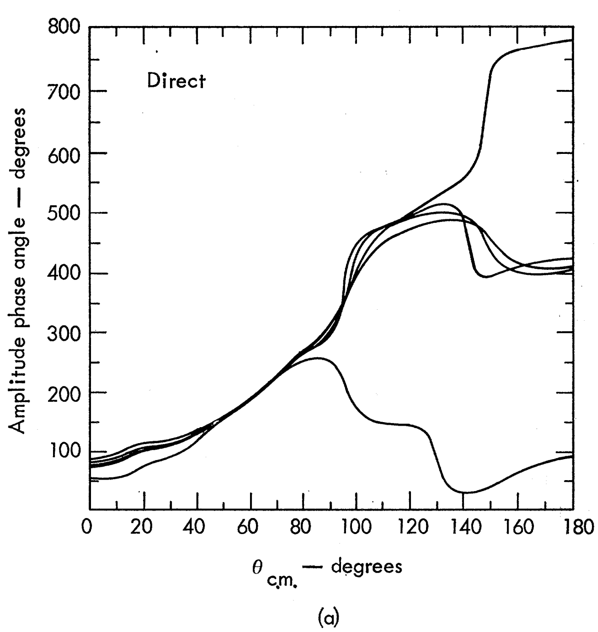


FIG. 6. Differential cross sections calculated with a 0.7-F Yukawa interaction plus Coulomb excitation but without collective enhancement (see Sec. VII). The same Serber strength V_S=224 MeV was used for all three curves. The data are taken from Ref. 20.



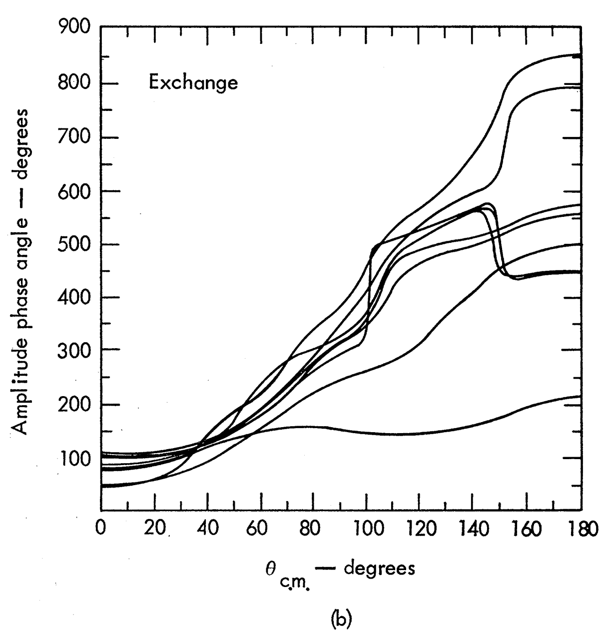


FIG. 7. Phase angles of direct and exchange complex scattering amplitudes as a function of scattering angle for typical single-particle transitions in ^{118}Sn(p, p'). To make the curves continuous, multiples of 360° are added to the phase angle where appropriate. Thus points which are 360° apart on the graph are actually in phase. There appear to be fewer curves in (a) than in (b) because of the symmetry of the direct form factor [Eq. (5)] with respect to interchange of initial and final bound states. No such symmetry occurs in the exchange. Thus, for example, $2d\to1g$ and $1g\to2d$ have identical direct amplitudes whose phase curves fall on top of each other.

$\phi_E=0.366$ are obtained. Thus, whereas the interference among the direct amplitudes is mostly constructive ($\approx$10% reduction), the cancellation among the various exchange amplitudes results in nearly a 40% reduction in the total exchange cross section. Qualitatively, therefore, randomness of the type anticipated by Glendenning and Veneroni[10] is actually present in our calculated single-particle exchange amplitudes but the resulting cancellation is not sufficient to make the total exchange amplitude negligible. It is interesting to note that, in spite of this significant cancellation, the resultant total direct and total exchange amplitudes are almost exactly in phase, there being only 1.5% cancellation between them.

Another important question to consider is whether the destructive interference present among exchange amplitudes shows any dependence on scattering angle. In Fig. 7, the phases of the complex direct and exchange scattering amplitudes of the nine strongest transitions contributing to the ^{118}Sn$(p, p')^{118}$Sn(2^+) excitation are plotted as a function of the scattering angle. The direct phases are quite close together throughout the angular range, but the exchange phases tend to diverge at backward angles. This should tend to make the total exchange angular distribution fall off more rapidly than the angular distributions of individual single-particle exchange amplitudes. This effect can be seen quantitatively by taking the ratio

$$f^i=\left(\frac{d\sigma}{d\Omega}\right)^i_{\min}\bigg/\left(\frac{d\sigma}{d\Omega}\right)^i_{\max} \tag{24}$$

as a measure of the falloff for the ith single-particle exchange angular distribution. A weighted average of this falloff parameter over the nine strongest transitions in ^{118}Sn(p, p') gives $\bar{f}=0.054$, whereas the corresponding value for the total exchange angular distribution is $f=0.029$. We have previously pointed out[11] that the tendency of the exchange angular distribution to fall off more slowly than the direct will increase the back-angle scattering when exchange is included. The result just obtained indicates that this effect should be stronger for transitions between pure shell-model configurations than for collective excitations. In the latter, the cancellation among many exchange amplitudes at back angles will cause the total exchange angular distribution to fall off more rapidly, like the direct.

There is an additional interesting feature that arises from single-particle interference. We observe large qualitative differences among the single-particle exchange angular distributions and to a lesser extent among the direct. In summing the direct and exchange amplitudes separately however, these differences somehow tend to cancel and one obtains total direct and exchange angular distributions that exhibit considerable qualitative similarity. Again this implies that inclusion of exchange may have a much smaller effect on the shape of the angular distribution in a collective excitation than in a relatively pure single-particle excitation.

As stated in the Introduction, the liquid-drop model has been remarkably successful in describing collective nuclear transitions. Such calculations are inherently direct since they include no nucleon other than the projectile. It is clear from the results of this section that the reason for this success is not that exchange contribution is negligible, but rather that, when all the individual single-particle transition amplitudes are added together, the shape of the resulting total exchange angular distribution is remarkably close to the direct.

VII. ABSOLUTE CROSS-SECTION CALCULATIONS

A. Inclusion of Collective Enhancement

Love and Satchler[21] have shown that it is necessary to include collective enhancement to predict inelastic scattering cross sections. Even though the shell-model wave function including major configurations accounts for a large fraction of the wave function, the contribution from minor configurations can increase the cross section by a large factor. Yoshida[19] has examined the core contributions in the quasiparticle RPA. It follows from his results that in ^{118}Sn, for example, while the 51–82 major shell contributes 87% of the 2^+ wave function, inclusion of the 13% of the wave function due to minor configurations will increase the inelastic α-scattering cross section by a factor of about 6.

Thus, one is faced with the necessity of obtaining detailed information about the contribution of minor configurations. Attempts in this direction have been made by Zamick[22] and by Petrovich and McManus,[15] who carried out perturbation calculations based on realistic forces, which treat the core as a system of noninteracting particles in a Hartree-Fock well. They have not been entirely successful because of the failure to take into account the strong neutron-proton interaction for the core nucleons, which is expected to produce primarily $\tau=0$ enhancements.[18] However, better calculations of this type will be capable of predicting both inelastic scattering and electromagnetic transition rates.

An alternative approach, which does not require calculation of detailed wave functions, utilizes the knowledge that electromagnetic transitions and inelastic scattering are closely related. This fact has been ex-

[21] W. G. Love and G. R. Satchler, Nucl. Phys. **A101**, 424 (1967).
[22] L. Zamick (unpublished).

ploited by Love and Satchler,[21] who use the collective model to account for core contributions. Here we use the same idea but in an approximate microscopic picture.

The inelastic enhancement factor, which is calculated in Appendix C, is based on two assumptions. The first is that in both inelastic scattering and electromagnetic transitions the collective enhancement comes entirely from $\tau=0$ isospin transfers. The second assumption is that the ratios of inelastic single-particle amplitudes, both direct and exchange, to the corresponding electromagnetic amplitudes are independent of the single-particle quantum numbers.

The first assumption is a statement that the effective charge of the nucleons in the shell model is due to core motion in which the neutron and proton fluids are moving together following the motion of the extra-core nucleons. This is probably a good assumption since the low-lying strongly enhanced $E2$ transitions are those in which there is no isospin change. Further evidence is the near equality of the deformation parameter β determined from different kinds of reactions.[23] For example, (p, p') and (n, n') reactions would lead to very different values of β if the nuclear protons and neutrons were involved differently in the core excitations. For the quasiparticle RPA wave functions of Ref. 19, the core enhancement is within 2% of being due entirely to the $\tau=0$ transfer part.

The second assumption is rigorously true[24,19] for the direct amplitudes in the plane-wave approximation for forward scattering in the limit of zero Q-value scattering. It has been tested[25] in the direct distorted wave approximation for the $1f$, $2p$ shell, where variations of $\approx 30\%$ were found in the ratios. However, the phase difference between the direct inelastic and electromagnetic single-particle amplitude was always very nearly independent of the quantum numbers. It is clear from the results of Sec. IV that the exchange-to-direct amplitude ratios have fairly wide variations from one single-particle transition to another, so the second assumption cannot be very accurate for exchange. However, the phase will be given accurately for the large single-particle amplitudes because of the similarity in phase of direct and exchange amplitudes discussed in Sec. IV. When many single-particle amplitudes are added together, the random variations in amplitude will tend to average to zero.

As shown in Appendix C, it follows from these two assumptions that, for a spatially even force and a nucleus in which the shell-model wave function consists either entirely of proton or entirely of neutron configurations, the enhancement factor for the inelastic amplitude is

$$\epsilon(L)=[V_0e_0(L)\pm V_\tau]/(V_0\pm V_\tau), \tag{25}$$

the upper sign being for scattering of the projectile neutron or proton from like nucleons in the target. In Eq. (25), the parameter $e_0(L)$ is the $\tau=0$ electromagnetic effective charge. It is related to the neutron and proton *effective* charges (taken in units of the true proton charge) by the expression

$$e_0(L)=e_p(L)+e_n(L) \tag{26}$$

with

$$e_p(L)-e_n(L)=1. \tag{27}$$

Thus, if we know empirically the proton effective charge for a particular electromagnetic transition, we can calculate the inelastic enhancement. If there were a $\tau=1$ enhancement, then the right-hand side of Eq. (27) would be replaced by a $\tau=1$ effective charge $e_1(L)$, and this factor would then multiply the V_τ in the numerator of Eq. (25).

As a test of Eq. (25), we have calculated the inelastic enhancement for ^{118}Sn from Yoshida's quasiparticle-RPA wave functions[19] by explicitly including essentially all configurations and taking the ratio of the cross section to that calculated with the inclusion of only the cloud neutrons—those in the 51–82 major shell. In the notation of Ref. 19, we define the effective charges as

$$e_n=\frac{S_p[S^{(1)\prime}/S']^{1/2}-S_p^{(1)}}{S^{(1)}}, \tag{28a}$$

$$e_p=\frac{S_p[S^{(1)\prime}/S']^{1/2}+S_n^{(1)}}{S^{(1)}}. \tag{28b}$$

With these effective charges the cloud wave function gives the total electromagnetic transition rate. This differs from Yoshida's definition of effective charge, which would be appropriate only if there were protons filling the major shell. Equations (28) hold with neutrons, protons, or both, and they reduce to Yoshida's definition if there are only protons filling the major shell.

The neutron effective charge calculated from Eq. (28a) is $e_n=0.67$ from which, according to Eqs. (26) and (27), $e_0=2.35$. From Eq. (25), we obtain $\epsilon=2.01$. The inelastic cross section, when calculated using only the cloud configurations, should then be multiplied by a factor of 4.04. By detailed calculation including all configurations we find an enhancement of a factor of 3.34. If only direct terms are included the factor is 3.69, which shows that the second assumption involved in obtaining Eq. (25) is more accurate for direct than for exchange amplitudes. The angular distribution including only major-shell configurations

[23] P. H. Stelson, R. L. Robinson, J. H. Kim, J. Rapaport, and G. R. Satchler, Nucl. Phys. **68**, 97 (1965).
[24] W. T. Pinkston and G. R. Satchler, Nucl. Phys. **27**, 270 (1961).
[25] F. A. Schmittroth (private communication).

TABLE VI. Cross-section ratios for states in ^{90}Zr including collective enhancement with and without exchange. The enhancement factors used in the cross section comparison are those from Eq. (25). The cross section ratios are based on the value of $d\sigma/d\Omega$ at 40° for the 2^+ and 4^+ states, and at 60° for the 6^+ and 8^+ states.

		Enhancement factors[a]		$\frac{d\sigma}{d\Omega}(J^\pi) \Big/ \frac{d\sigma}{d\Omega}(2^+)$		
J^π	$e_p(L)$[b]	From Eq. (25)	From Ref. c	No exchange	Including exchange	Experimental
2^+	1.79	16.9	19.1	1.0	1.0	1.0
4^+	1.65	12.7	12.6	0.14	0.18	0.16±0.02
6^+	1.51	9.0	8.9	0.011	0.026	0.029±0.006
8^+	1.34	5.8	6.6	0.001	0.005	0.016±0.007

[a] Comparison between inelastic cross-section enhancement factors calculated with Eq. (25) and those from the detailed microscopic calculation of Petrovich and McManus (Ref. 15).
[b] Effective proton charge from Ref. 15.
[c] Reference 15.

and that including all configurations are essentially identical within 1% for $\theta<60°$ and never differ by more than 12% at any angle. Thus, for this example the assumptions involved in the derivation of Eqs. (25)–(27) appear to be valid to a reasonably good approximation. A further test is provided by the example of ^{90}Zr discussed below.

B. Inelastic Scattering in ^{90}Zr

The $J^\pi=0^+$, 2^+, 4^+, 6^+, 8^+ states in ^{90}Zr provide an especially interesting study for any theory of inelastic scattering because of their simple shell-model description. Structure calculations[26] indicate that (except for the 0^+) these states are well described by the $(1g_{9/2})^2$ proton configuration. In the 0^+ state (1.75 MeV) as well as the 0^+ ground state the $(1g_{9/2})^2$ mixes strongly with $(2p_{1/2})^2$; the wave functions are believed to be approximately $|\,\text{ground}, 0^+\rangle=0.8\,|\,p_{1/2}{}^2\rangle-0.6\,|\,g_{9/2}{}^2\rangle$ and $|\,1.75\text{ MeV}, 0^+\rangle=0.6\,|\,p_{1/2}{}^2\rangle+0.8\,|\,g_{9/2}{}^2\rangle$.

The population of these states by (p, p') was first studied by Gray *et al.*[17] at 18.8 MeV. They showed that the direct DWBA with a Yukawa force of range 1 F and strength 205 MeV gave a fair description of the 2^+, 4^+, and 6^+ states if one included a 40% cross-section enhancement for the 2^+ state. This enhancement was qualitatively expected on the basis of the empirical quadrupole enhancement. However, for the 8^+ state, the theory fell short of the data by a factor of more than 3 and also badly described the angular distribution. On the other hand, the calculated cross section for the 0^+ state was a factor of 5 larger than the upper limits that could be determined from the rather meager data at a few angles. In view of the very large exchange effect for higher L values, it appears that the inadequancy of DWBA, at least for the 8^+ state, may be overcome by the inclusion of exchange.

For comparison of various cross sections, we have used the value of the differential cross section for each state at a particular angle where the theoretical shape is in reasonable agreement with experiment.[17] These angles are 40° for the 2^+ and 4^+ states and 60° for the 6^+ and 8^+ states. The choice of this angle for the 8^+ state is somewhat arbitrary because of the large experimental errors and the poor agreement. The 0^+ state at 1.75 MeV will be considered separately.

If we now calculate the cross-section ratios for the 8^+ to the 2^+ state using the wave functions given above for the excited and ground states and using the interaction of Ref. 17, we obtain 3.1×10^{-3} without exchange and 1.5×10^{-2} when exchange is included. The experimental ratio[17] is $1.6\pm0.6\times10^{-2}$. However, in spite of the essentially perfect agreement, this result is not realistic because of the neglect of collective enhancement.[21] To remedy this deficiency in our calculation we have included collective enhancement using the approximation developed in part A of this section. As in Ref. 21, experimental effective charges for electromagnetic transitions are required. Since these are available only for the 2^+ state, we first use theoretical estimates made by Petrovich and McManus.[15] These are listed in Table VI along with theoretical estimates of the inelastic cross-section enhancement factors $\epsilon(L)^2$ calculated from Eq. (25). The exchange mixture was taken from the effective interaction used in Ref. 15 to represent approximately the G matrix calculated from the Kallio-Kolltveit force by cutting

[26] B. F. Bayman, A. S. Reiner, and R. K. Sheline, Phys. Rev **115,** 1627 (1959); I. Talmi and I. Unna, Nucl. Phys. **19,** 225 (1960).

it off below the separation distance. (This interaction, which will be referred to hereafter as the KK effective interaction, is an even 1-F Yukawa potential with an exchange mixture given below.) For comparison with our estimates, we show the enhancement factor obtained by Petrovich and McManus[15] as the ratio of the cross section including a microscopic description of the core to the cross section calculated without including the core. Also shown are ratios of differential cross sections including our estimate of the enhancement factor both with and without exchange. Except for 8^+ state, the calculated ratios are in good agreement with experiment. For the 8^+ state the inclusion of exchange improves the ratio by a factor of 5, but it still falls short of the data by a factor of 3.[27,28]

The theoretical estimate of effective proton charge $e_p(2)$ is somewhat smaller than the value 2.4 ± 0.5 determined by Love and Satchler[18] from the experimental $B(E2)$.[29] If we use the latter in Eqs. (26) and (27) to determine the effective charge, $e_0(2)$, then the enhancement factor, Eq. (25), for the Serber force is $\epsilon(2)=5.2\pm1.6$ and for the KK effective force of Ref. 15, $\epsilon(2)=6.6\pm2.1$. These errors are based on an error of 40% in the experimental $B(E2)$[29] and an estimated 10% error in the approximate method for calculating the collective enhancement factor. They do not include any of the error resulting from the departure of the isovector effective charge from the assumed value of 1. Using the Serber result, the $0^+\rightarrow2^+$ cross section in ^{90}Zr requires a strength of $V_S=-101\pm31$ MeV which has $V_0=-38\pm12$ MeV, $V_\tau=13\pm4$ MeV. The value of V_τ obtained in analysis of the (p,n) reaction in light nuclei[30] was about 24 MeV. Since no exchange was included, this latter number accounts for some exchange effect, whereas the one determined above does not. Using the results of Ref. 11 we obtain a corrected value $V_\tau=17$ MeV, which is somewhat higher than our result.

The KK effective interaction is[15] $t(0,1)=-73.4(e^{-r}/r)(1.64\mathcal{P}_{01}+\mathcal{P}_{10})$. Using the cross-section enhancement factor of $\epsilon^2=44\pm28$ given above, we obtain a differential cross section of 0.81 ± 0.52 mb/sr at 40° compared to the experimental value of 0.97 mb/sr. This is very good agreement. The values of V_τ and $V_{\sigma\tau}$ determined from this effective force are 18 and 12 MeV, also in excellent agreement with the value obtained from charge exchange reactions.[30]

These results are based on the experimental value,[29] $B(E2)=(4.2\pm1.5)\times10^{-50}$ cm^4, used by Love and Satchler[21] in their determination of the effective proton charge. Recently another measurement has come to our attention in which the result $B(E2)=(8.15\pm1.2)\times10^{-50}$ cm^4 was obtained.[31] With this new value, the KK effective interaction leads to an inelastic enhancement factor $\epsilon(2)=10.4\pm1.4$ and $d\sigma/d\Omega(40°)=2.01\pm0.54$ mb/sr, the latter number being a factor of 2 higher than experiment. Because of the complete disagreement between the two $B(E2)$ values we have not averaged them but have used each of them separately to calculate the interaction strength required to fit the data. These results will be summarized in Table VII. Experimental resolution of the $B(E2)$ discrepancy will be important in establishing the consistency of our results.

The angular distributions in ^{90}Zr are not greatly affected by inclusion of exchange. The best fit is obtained for the 2^+ state, shown in Fig. 8. The calculated curve does not include spin-orbit distortion. Its inclusion introduces more structure in the angular distributions and improves the agreement for the 2^+ state slightly.[17] The angular distributions for the 4^+ and 6^+ states are also in reasonable agreement with the experimental data, but for the 8^+ state the agreement is very poor, as shown by Gray *et al.*[17] The inclusion of exchange worsens the agreement slightly for the 8^+ state.

The ratio of the 8^+ cross section to that of other states could be improved somewhat by use of a longer-range force. However, such a force is probably unrealistic since the V_0 term is at least a two-pion-exchange interaction. The apparent disagreement of the calculated angular distribution with experiment probably indicates that there are other mechanisms involved. The weakness of the direct transition for the 8^+ state means that other mechanisms will be relatively more important in the excitation of the 8^+ than for other states. Better experimental data are needed to clarify the question of the mechanism for the 8^+ excitation.

As mentioned above, the data[17] for the 0^+ state at 1.75 MeV was insufficient to yield an angular distribution. More recently, this state has been seen via (p,p') at 12.7 MeV by Dickens *et al.*[32] who were unable to fit the data with a microscopic calculation. The inclusion of exchange greatly improves the angular distribution at angles beyond 100°. The magnitude

[27] Compared to the actual KK G matrix (Ref. 18) the 1-F Yukawa interaction overestimates the $L=8$ direct cross section by a factor of about 3 while affecting the exchange cross section very little. (Other multipoles are only slightly different for the two forces.) This is consistent with recent results (Ref. 28) which show that a Yukawa interaction of 2-F range gives a fairly good representation of the L dependence of a central G matrix obtained from the Hamada-Johnston potential. However since the direct amplitude is relatively small for the $L=8$ transfer the 8^+ to 2^+ cross-section ratio is overestimated by only 20% when the 1-F Yukawa interaction is used.

[28] W. G. Love, L. W. Owen, R. M. Drisko, G. R. Satchler, R. Stafford, R. J. Philpott, and W. T. Pinkston, Phys. Letters **29B**, 478 (1969).

[29] Yu. P. Gangrskii and I. Kh. Lemberg, Yadern. Fiz. **1**, 1025 (1965) [English transl.: Soviet J. Nucl. Phys. **1**, 731 (1965)].

[30] J. D. Anderson, S. D. Bloom, C. Wong, W. F. Hornyak, and V. A. Madsen, Phys. Rev. **177**, 1395 (1969).

[31] T. H. Curtis (private communication).

[32] J. K. Dickens, E. Eichler, and G. R. Satchler, Phys. Rev. **168**, 1355 (1968).

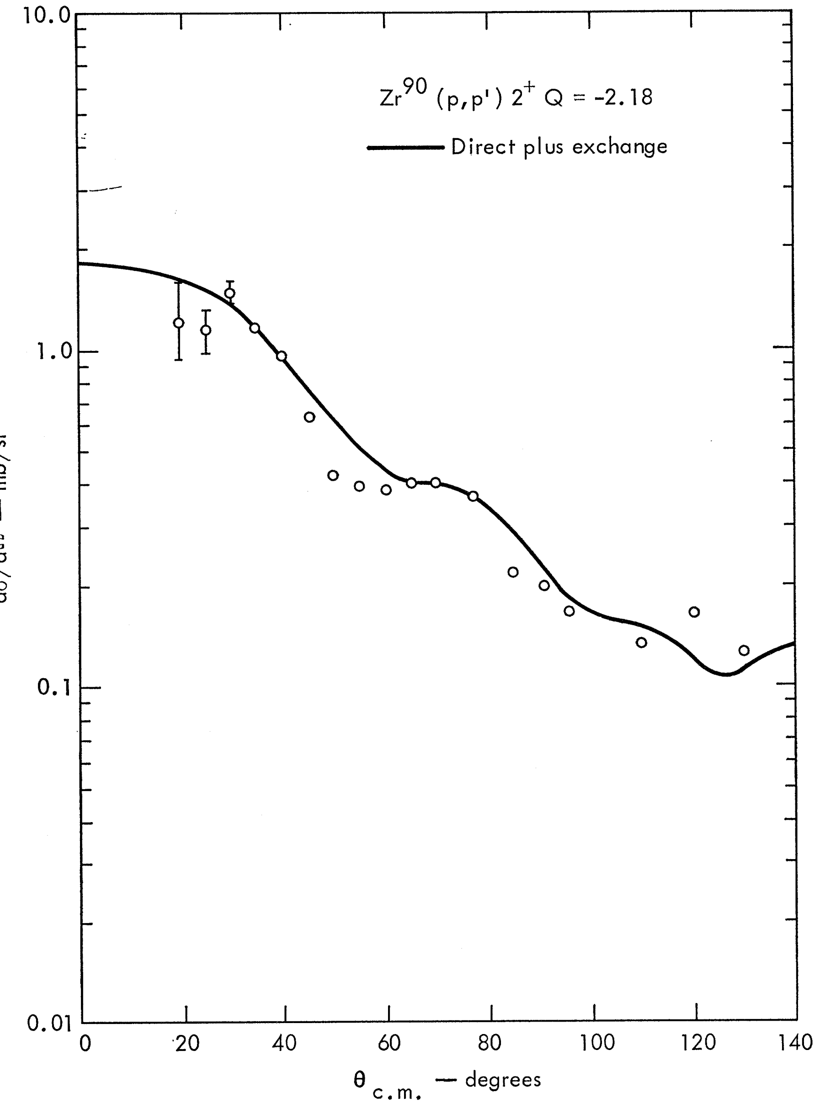


FIG. 8. A comparison of the experimental data (Ref. 17) with a calculation using a 1-F Yukawa force and the exchange mixture of the KK interaction. With the inclusion of exchange and collective enhancement a strength $V_{KK}=82$ MeV is required to fit the data. There are errors of about 50% associated with the cross-section enhancement factor (see text).

of the cross section is increased by a factor of 3 when exchange is included but is still a factor of 5 below the data. This large increase compared to that expected from the pure $(g_{9/2})^2 0^+ \rightarrow 0^+$ exchange-to-direct cross-section ratios[33] can be understood from Table V which shows that the exchange is relatively more important for larger numbers of radial nodes. The amplitude for this transition is proportional to the difference in single-particle amplitudes for the $2p_{1/2} \rightarrow 2p_{1/2}$ and the $1g_{9/2} \rightarrow 1g_{9/2}$ transition. These come close to cancelling each other for the direct terms,[17] but not for the exchange terms since the $2p_{1/2}$ exchange amplitude is expected from Table V to be somewhat larger than the $1g_{9/2}$ amplitude.

[33] This effect of the configuration mixing on the exchange-to-direct ratios was pointed out to us by F. Petrovich.

C. Inelastic Scattering in ^{118}Sn

In this section, we calculate the absolute cross section for excitation of the collective 2^+ state in ^{118}Sn(p, p') using the KK effective interaction and compare it to the experimental results of Ref. 20. We also calculate the Serber force strength required to fit the absolute cross section and compare it with that obtained for ^{90}Zr(p, p'). The radial dependence of both the Serber and KK effective forces is of the Yukawa form with a range of 1 F.

Although Yoshida's wave functions[19] for ^{118}Sn do include a large amount of collectivity, they still fall a little short of predicting the experimental $B(E2)$ for ^{118}Sn. Moreover, we do not use all configurations actually included by Yoshida, so our calculated value

TABLE VII. Summary of interaction strengths.

Reaction	Serber strength[a] (MeV)	KK strength[b] (MeV)
$^{90}Zr(p, p')2^+$ (2.18 MeV)[c]	101±31	81±26
$^{90}Zr(p, p')2^+$ (2.18 MeV)[d]	66±9	51±7
$^{118}Sn(p, p')2^+$ (1.22 MeV)	78±10	64±6
KK effective interaction		73[e]
(p, n) reactions[f]	150	73

[a] Coefficient of exchange operator: $(\mathcal{P}_{01}+\mathcal{P}_{10})e^{-r}/r$.
[b] Coefficient of exchange operator: $(1.64\ \mathcal{P}_{01}+\mathcal{P}_{10})e^{-r}/r$.
[c] Based on $B(E2)$ value of Ref. 29.
[d] Based on $B(E2)$ value of Ref. 31.
[e] Strength determined (Ref. 15) from KK interaction.
[f] Strength determined from average values obtained in Ref. 30 corrected to a 1-F range and for exchange. The strength given would be required to give the empirical value of V_τ but would be inconsistent with $V_{\sigma\tau}$.

for $B(E2)$ will fall short of experiment still further. To remedy this defect we again use the technique discussed in Sec. VII A for obtaining the inelastic enhancement factor. Whereas in Sec. VII A we used both Yoshida's cloud wave functions and his more complete collective wave functions to test the enhancement formula, we now regard the latter as the model wave function and calculate from it and the experimental $B(E2)$ the inelastic enhancement factor.

In this case, there are both neutron and proton configurations in the model state. The electromagnetic amplitude enhancement factor is

$$\frac{e_pS_p+e_nS_n}{S_p}=\frac{e_0(S_n+S_p)-(S_n-S_p)}{2S_p}=\left(\frac{B(E2)_{\text{expt}}}{B(E2)_{\text{theor}}}\right)^{1/2}, \quad (29)$$

where, again, S_n and S_p are as defined in Ref. 19. The quantities S_n+S_p and S_n-S_p are the primed sums in Eqs. (C3) and (C4). Our calculated $B(E2)$ is 1210 F^4 compared to the experimental value[34] of 1722 $F^4\pm 20\%$. From Eq. (29), we obtain $e_0=1.18\pm5\%$. The inelastic amplitude enhancement factor obtained from Eq. (C9) is

$$\epsilon=\frac{e_0V_0(S_n+S_p)-V_\tau(S_n-S_p)}{V_0(S_n+S_p)-V_\tau(S_n-S_p)}. \quad (30)$$

[34] T. H. Curtis, R. A. Eisenstein, D. W. Madsen, and C. K. Bockelman, Phys. Rev. **184**, 1162 (1969).

For the KK effective force $V_0/V_\tau\approx-2$ and for the Serber force $V_0/V_\tau=-3$. These ratios and the value of e_0 obtained above give $\epsilon=1.15\pm4\%$ (KK force) and $1.16\pm5\%$ (Serber force). With this value the KK effective force would require a strength of 64±6 MeV and the Serber force 78±10 MeV to fit the experimental data in Fig. 6. The error includes a 20% error in the measured $B(E2)^{34}$ from which the inelastic enhancement factor was calculated and a 15% error in fitting the theoretical differential cross section to experiment.

These results and the interaction strengths required for the $^{90}Zr(2^+)$ excitation are summarized in Table VII. The strength required for ^{118}Sn lies between the two strengths determined for ^{90}Zr on the basis of calculations from the two inconsistent experimental values of $B(E2)$, discussed in Sec. VII B.

VIII. DISCUSSION

It has been the hope in recent years that the microscopic model of nuclear inelastic scattering and charge-exchange reactions in the DWBA with some effective interaction would be adequate for the interpretation of experimental data. Once this has been established, these reactions can be used as a tool for nuclear spectroscopy. Inelastic scattering data are not as simple to interpret as one-particle transfer data, which directly give occupation numbers of nucleons in various single particle levels j. Because the j appears coherently in the inelastic scattering cross section, the interpretation is more complicated. On the other hand, the interference can yield important information about relative phase in configuration-mixed wave functions.

The primary application of inelastic scattering analysis probably will be for determining spins and parities of levels and for testing nuclear wave functions. For example, charge-exchange reactions give essentially the same information as β decay but do not have the severe limitation to low-lying states. In order for inelastic scattering and charge-exchange reactions to be useful for testing wave functions, the effective interaction must be one which does not vary much from nucleus to nucleus or from state to state in a given nucleus.

In general, it is possible to write the transition amplitude for inelastic nuclear scattering or charge exchange as a matrix element of the t operator

$$A=\langle\psi_f^{(-)}\Phi_f\,|\,t\,|\,\Phi_i\psi_i^{(+)}\rangle. \quad (31)$$

The t operator is complicated and generally nonlocal. From a practical point of view it is important to be able to approximate t sufficiently accurately with a simple interaction—one would hope even a local form. Unless exchange is included explicitly this will not be possible, because the knockout exchange is itself part of the nonlocality in Eq. (31). If nuclear forces were weak enough that a literal DWBA with real forces were accurate, the effect of antisymmetrization could be included as a particular nonlocality:

$$A=\langle\psi_f^{(-)}(0')\Phi_f(1',2,3\cdots N)\mid V(0,1)(\delta_{0,0'}\delta_{1,1'}-\delta_{0,1'}\delta_{1,0'})\mid\Phi_i(1,2,\cdots,N)\psi_i^{(+)}(0)\rangle,\quad(32)$$

where δ_{ij} is a Kronecker delta in the sum over spin and isospin coordinates and a Dirac delta in the integral over the space coordinates. The second term in the operator in Eq. (32) gives the exchange amplitude. In effect, what was being done in inelastic scattering calculations until recently was to try to replace the highly nonlocal operator in Eq. (32) by a local one. The procedure was surprisingly successful, except that it led to a strong L dependence, and therefore a state dependence, of the interaction. This L dependence is explained naturally by the explicit inclusion of exchange. The results presented in this paper show that in the 0–150-MeV range, no nucleon-nucleus scattering calculation with a simple local interaction which omits exchange will be accurate.

On the other hand, if exchange *is* included explicitly, there is considerable hope that realistic forces with the hard cores removed[15] by a Scott-Moszkowski separation[35,36] are capable of giving absolute cross sections in the DWBA. The results presented in Sec. VII with the KK effective interaction show that, if collective enhancement is taken into account, the DWBA including exchange is capable of giving correct ratios for various final states in ^{90}Zr(p,p'). The calculation still gives a small result for the 8^+ state, but we should reserve judgement on this transition until better experimental data are available. The question of how well the realistic force fits experimental absolute cross sections is somewhat obscured by the two inconsistent experimental values of $B(E2)$ for the $0^+\rightarrow2^+$ transition in ^{90}Zr. However, it appears that realistic forces have sufficient strength to explain experimental cross sections when both exchange and collective enhancement are included. The realistic forces have a proton-proton strength which is a factor of more than 10 weaker than that which was thought to be required before the importance of these effects was understood. At the same time, the KK effective interaction has almost exactly the isospin transfer strength which has been deduced from analysis of charge-exchange experiments[30] when the latter are corrected for the exchange effect. Thus, our results indicate that the same semirealistic central force is capable of explaining both inelastic scattering and charge-exchange reactions.

The assumption made in connection with the calculation of the collective enhancement factor is probably close to being correct; that is, the core enhancement in nuclear transitions comes from the $\Delta T=0$ part of the interaction. The isovector effective charge cannot always have the assumed value of $e_1=1$; its value is, of course, model-dependent. The wave functions of Ref. 19 for which the cloud nucleons have almost exactly $e_1=1$ were calculated assuming equal neutron and proton orbits and using an isoscalar two-nucleon force. If the isospin dependence of the two-body force had been taken into account, it is expected that the value of e_1 would have been lower because of the stronger attraction of the valence neutrons for core protons and vice versa. Thus the difference e_1 between the neutron and proton effective charges is less than the value of unity that it would have if they each attracted core protons equally.[36a]

When a value of $e_1<1$ is used for estimating the cross section in ^{90}Zr, the resulting cross sections are increased. For example, for the rather extreme assumption of a purely isoscalar transition, $e_1=0$, the estimated enhancement factor would be ≈90 instead of the factor of ≈40 obtained from using $e_1=1$. In fact, the procedure could be reversed and used to calculate e_1 from proton or neutron inelastic scattering, when the $B(EL)$ is also known, if it can be established that a local effective-two-body interaction is adequate to describe inelastic scattering in DWBA.

It is probably worth pointing out here that the isovector effective charge could be obtained also from a comparison of the (p,p') and (n,n') results on the same nucleus. In ^{90}Zr, for example, if the simple $(g_{9/2})^2$ wave function were correct, the KK effective force would lead to an (n,n') to (p,p') cross-section ratio of $[(V_0-V_\tau)/(V_0+V_\tau)]^2\approx9$, neglecting Coulomb effects. The ratio of the deformation parameters β obtained with a collective-model analysis including the collective enhancement effects in the effective-charge approximation of Sec. VII A would be

$$\frac{\beta(n,n')}{\beta(p,p')}=\frac{[V_0e_0-V_\tau e_1]/[U_0-U_1(N-Z)/A]}{[V_0e_0+V_\tau e_1]/[U_0+U_1(N-Z)/A]},\quad(33)$$

where U_0 and U_1 are the strengths of the charge-independent and $\mathbf{T}\cdot\mathbf{t}$ terms in the nuclear optical potential. Taking the values $U_0=49$ MeV and $U_1=24$ MeV[37] (ignoring the isospin dependence of the imaginary part of the potential), one obtains from Eq. (33) for the $0^+\rightarrow2^+$ transition in ^{90}Zr, $\beta(n,n')/\beta(p,p')=1.46$ for $e_1=1$. If the transition were purely isoscalar, $e_1=0$, the β ratio would be 1.12.

In spite of the uncertainties connected with our knowl-

[35] S. A. Moszkowski and B. L. Scott, Ann. Phys. (N.Y.) **11**, 65 (1960).

[36] G. E. Brown, in *Unified Theory of Nuclear Models and Nucleon-Nucleon Forces* (North Holland Publishing Co., Amsterdam, 1967), 2nd ed.

[36a] We are grateful to Professor B. R. Mottelson for conversations clarifying this feature of isovector effective charge.

[37] G. R. Satchler, R. M. Drisko, and R. H. Bassel, Phys. Rev. **136**, B637 (1964).

edge of e_1, it is true that for any reasonable estimate of the isovector effective charge, large enhancement factors are obtained. This is particularly true when the model state consists entirely of the same kind of nucleon as the projectile, since the model-state cross section is then very small. In cases of inelastic scattering where $\Delta T=0$ transfer is possible, it seems to be true that no simple shell model is adequate to give absolute cross sections. This kind of difficulty is familiar from $E2$-transition calculations with shell-model wave functions, and the difficulty arises from precisely the same effect in the nucleus: the large contribution of a small fraction of the wave function containing a very large number of minor configurations involving the core.

On the other hand, charge-exchange reactions may be relatively free of these core effects. The agreement of the isospin-transfer parts of the KK interaction with phenomenological charge-exchange strength supports this possibility. The detailed microscopic treatment of collective enhancement in inelastic scattering is, of course, an interesting problem in itself. As a test for shell-model wave functions, however, the charge-exchange reaction is highly preferable to inelastic scattering just because it avoids the necessity of including the core enhancement.

IX. SUMMARY

We have reported in this paper the results of a study of the effect of knockout exchange on nuclear inelastic scattering. The following properties of the exchange mechanism have been demonstrated:

(1) There is a strong L dependence in the relative contribution of exchange and direct amplitudes for ranges $\gtrsim 0.5$ F due to a rapid decrease in the direct amplitude with increasing range when the orbital-angular-momentum transfer L is large.

(2) Exchange effects do decrease in importance with increasing projectile energy, but are not negligible in the 10–150-MeV range.

(3) The exchange contribution is relatively more important for transitions between single-particle states with radial nodes, particularly when the number of nodes is different in the initial and final single-particle wave function. There is a slight increase in the relative importance of exchange with increasing single-particle binding energy.

(4) Amplitudes for the direct-forbidden non-normal transfers, $(-1)^L \neq (-1)^{l_2-l_1}$, give a small contribution to the cross section in most cases, but they can be important for high-L transitions where exchange dominates.

(5) For single-particle transitions between states with the same radial quantum number the phases of the complex direct and exchange amplitudes are nearly the same. When $n_1 \neq n_2$ the phases can be very different.

(6) Exchange single-particle amplitudes contribute significantly to the cross section for collective states although there is some tendency for cancellation due to random phases. For a typical $0^+ \rightarrow 2^+$ transition the inclusion of exchange increases the cross section by a factor of about 2.

Simple plane-wave expressions have been found using an expansion of the interaction around zero range which demonstrate qualitatively the properties (1) and (2) above.

A simple method is presented and used for estimating the collective enhancement for inelastic scattering cross sections from the empirical nucleon effective charges. Application has been made to calculation of the relative cross sections for the 2^+, 4^+, 6^+, 8^+ states of ^{90}Zr including exchange and using effective charges obtained by Petrovich and McManus[15] from a detailed microscopic model. A 1-F Yukawa interaction normalized to reproduce the 2^+ cross section also gives the magnitudes of the 4^+ and 6^+ cross sections accurately. Although the inclusion of exchange increases 8^+ cross section by a factor of 5, the result still falls short of the data by a factor of 3.

Application has also been made to the calculation of absolute cross sections for the 2^+ excitations in ^{118}Sn(p, p') and ^{90}Zr(p, p'). The cross section was calculated with 1-F Yukawa interaction having an exchange mixture determined by Petrovich and McManus to give approximately the same scattering for the 2^+ excitation as G matrix for the KK interaction. With this interaction the absolute cross section is in agreement with experiment for ^{118}Sn(p, p'). There are experimental uncertainties connected with the estimate of collective effects in ^{90}Zr. The charge-exchange terms in the interaction are in agreement with those obtained by analysis of charge-exchange reactions.

ACKNOWLEDGMENTS

We wish to express appreciation to Dr. J. D. Anderson, Dr. H. McManus, and F. Petrovich for a number of valuable conversations. We are grateful to Dr. McManus and Mr. Petrovich for permission to use their collective enhancement factors and effective interaction before publication. We also thank Dr. S. D. Bloom, Dr. J. D. Anderson, and Dr. L. Schecter for critical reading of the manuscript.

APPENDIX A: ANTISYMMETRIZED FORMALISM WITH SPIN-ORBIT DISTORTION

We present in this Appendix a formulation of the antisymmetric nucleon-nucleus scattering modified to include the effects of spin-orbit distortions. The effect of spin-orbit distortions in direct reactions has been treated previously by Satchler[38] and by Tobocman.[39]

[38] G. R. Satchler, Nucl. Phys. **55**, 1 (1964).

[39] W. Tobocman, *Theory of Direct Nuclear Reactions* (Oxford University Press, New York, 1961).

The space-direct and space-exchange single-particle radial integrals for definite orbital-angular-momentum transfer are given by

$$\mathfrak{F}_L^{J_1J_2j_1j_2}=\frac{\langle l_2\|Y_L\|l_1\rangle}{\hat{L}}\frac{\langle L_2\|Y_L\|L_1\rangle}{\hat{L}}(-1)^{L_1}\times\int R_{J_2}^*(r_0)g_L^{j_1j_2}(r_0)R_{J_1}(r_0)r_0^2dr_0, \quad \text{(A1)}$$

$$\mathcal{G}_L^{J_1J_2j_1j_2}=(-1)^{l_1}\sum_\lambda W(L_2l_1L_1l_2;\lambda L)\langle L_2\|Y_\lambda\|l_1\rangle\times\langle l_2\|Y_\lambda\|L_1\rangle\iint R_{J_2}^*(r_0)R_{j_2}(r_1)v_\lambda(r_0,r_1)\times R_{J_1}(r_1)R_{j_1}(r_0)r_1^2r_0^2dr_1dr_0, \quad \text{(A2)}$$

where R_J and R_j are continuum and bound radial wave functions. We define a composite amplitude

$$\mathcal{A}(I,I',L)=\sum_{j_1j_2}d_{j_1j_2}(I,I',L)\times[D_{j_1j_2}^+(I,I')(\mathfrak{F}_L^{J_1J_2j_1j_2}+\mathcal{G}_L^{J_1J_2j_1j_2})+D_{j_1j_2}^-(I,I')(\mathfrak{F}_L^{J_1J_2j_1j_2}-\mathcal{G}_L^{J_1J_2j_1j_2})], \quad \text{(A3)}$$

where the coefficients are as defined in Sec. II.

In terms of these quantities the transition amplitude for particular initial and final projections of nuclear total angular momentum M_i, M_f and projectile spin μ_i, μ_f is

$$A\langle\chi_f^{(-)}(0)\Phi_f(1,2,\cdots A)\mid V_{01}\mid\chi_i^{(+)}(0)\Phi_i(1,\cdots A)-\chi_i^{(+)}(1)\Phi_i(0,2,\cdots A)\rangle=\sum_{IN}C(J_iJ_fI;M_i,-M_f,-N)(-1)^{J_i-M_i}\mathcal{A}(\mu_i\mu_fIN), \quad \text{(A4)}$$

where

$$\mathcal{A}(\mu_i\mu_fIN)=\sum_{LMI'J_1J_2L_1L_2}(4\pi)^2 i^{L_1-L_2-\pi}Y_{L_1}^0(0)Y_{L_2}^M(\hat{k}_f)\times\hat{J}_1\hat{J}_2\hat{L}^2\hat{I}'^2\begin{pmatrix}J_1 & \frac{1}{2} & L_1\\ J_2 & \frac{1}{2} & L_2\\ I & I' & L\end{pmatrix}C(J_1J_2I;\mu_i,-M-\mu_f,N)\times C(L_1\tfrac{1}{2}J_1;0\mu_i\mu_i)C(L_2\tfrac{1}{2}J_2;M,\mu_f,M+\mu_f)(-1)^{J_1-M-\mu_f-L_2-I}\mathcal{A}^{J_1J_2}(II'L), \quad \text{(A5)}$$

A is the number of target nucleons, χ is the projectile distorted wave, and Φ is the wave function for the target or residual nucleus. The cross section is

$$\frac{d\sigma}{d\Omega}=\left(\frac{2m}{4\pi\hbar^2}\right)^2\frac{k_f}{k_i}[2(2J_i+1)]^{-1}\sum_{IN\mu_i\mu_f}|\mathcal{A}(\mu_i\mu_fIN)|^2. \quad \text{(A6)}$$

APPENDIX B: ZERO-RANGE EXPANSION OF PWBA IN CUTOFF APPROXIMATION

We start by applying the expansion about zero range, Eq. (15), of the space-direct and space-exchange amplitudes, Eqs. (2). Carrying out the integration over r_1 gives us the following expressions

$$F=(-1)^{l_1}\sum_{m_1m_2}C(l_1l_2L;m_1m_2M)V_0\int\chi_f^*(\mathbf{r})\chi_i(\mathbf{r})\times(1+c_2\nabla^2)(\phi_{l_2}^{m_2}(\mathbf{r})\phi_{l_1}^{m_1}(\mathbf{r}))d^3r, \quad \text{(B1a)}$$

$$G=(-1)^{l_1}\sum_{m_1m_2}C(l_1l_2L;m_1m_2M)V_0\int\chi_f^*(\mathbf{r})\phi_{l_1}^{m_1}(\mathbf{r})\times(1+c_2\nabla^2)(\chi_i(\mathbf{r})\phi_{l_2}^{m_2}(r))d^3r, \quad \text{(B1b)}$$

where c_2 is the coefficient

$$c_2=\tfrac{1}{6}\int g(r)r^2d^3r. \quad \text{(B2)}$$

The first terms in Eqs. (B1) are the zero-range amplitudes, and they are identical. The second terms are the second-order corrections in the range parameter. Let us denote these integrals by

$$I_d^{(2)}=\int\chi_f^*\chi_i\nabla^2(\phi_{l_2}^{m_2}\phi_{l_1}^{m_1})d^3r, \quad \text{(B3a)}$$

$$I_e^{(2)}=\int\chi_f^*\phi_{l_1}^{m_1}\nabla^2(\chi_i\phi_{l_2}^{m_2})d^3r. \quad \text{(B3b)}$$

Using the Hermiticity property of the ∇^2 operator we may write these integrals alternatively as

$$I_d^{(2)}=\int\phi_{l_2}^{m_2}\phi_{l_1}^{m_1}\nabla^2(\chi_f^*\chi_i)d^3r, \quad \text{(B3a}')$$

$$I_e^{(2)}=\int\chi_i\phi_{l_2}^{m_2}\nabla^2(\chi_f^*\phi_{l_1}^{m_1})d^3r. \quad \text{(B3b}')$$

The form Eq. (B3a′) is very convenient if we are using plane waves since in that case the ∇^2 operation is simply

$$\nabla^2(\chi_f^*\chi_i)=-q^2\exp(-i\mathbf{q}\cdot\mathbf{r}), \quad \text{(B4)}$$

where $\mathbf{q}=\mathbf{k}_f-\mathbf{k}_i$ is the momentum transfer. Combining Eqs. (B1a) and (B4), we have simply

$$F_{LM}=(1-c_2q^2)\tilde{F}_{LM}, \quad \text{(B5)}$$

where $\tilde{F}_{LM}$ is the zero-range amplitude, the first term of Eq. (B1a). The finite-range effect expressed by the second term leads to a more rapidly falling angular distribution than the zero-range amplitude. This is a reasonable result, since the inclusion of the finite range makes the effective size of the diffracting object larger, leading to a narrower diffraction pattern.

In order to calculate the exchange integral, we take half the sum of the two forms Eqs. (B3b) and (B3b′) and carry out the ∇^2 operation, putting in plane waves for χ_f and χ_i, take the derivative of the plane waves, write the results in terms of $\mathbf{q}$ and $\mathbf{Q}=(\mathbf{k}_f+\mathbf{k}_i)/2$ and integrate by parts in the resulting $\mathbf{q}$ terms to get the result

$$I_e^{(2)}=\tfrac{1}{2}\int[-k_f^2-k_i^2+\kappa_1^2+\kappa_2^2+(2m/\hbar^2)(2U)+q^2]$$
$$\times\exp(-i\mathbf{q}\cdot\mathbf{r})\phi_{l_1}^{m_1}\phi_{l_2}^{m_2}d^3r+i\mathbf{Q}\cdot\int\exp(-i\mathbf{q}\cdot\mathbf{r})$$
$$\times(\phi_{l_1}^{m_1}\nabla\phi_{l_2}^{m_2}-\phi_{l_2}^{m_2}\nabla\phi_{l_1}^{m_1})d^3r, \quad \text{(B6)}$$

where $\hbar^2\kappa^2/2m$ is the single-particle binding energy and U is the binding potential.

The first term of Eq. (B6) is closely related to the direct term Eq. (B5). In particular, when a cutoff is taken beyond the range of the binding potential, the U term will be zero in the integrand. The rest of the terms in the bracket in Eq. (B6) are constant and can be factored out of the integral. In that case, we have for the space-exchange amplitude

$$G_{LM}=[1+\tfrac{1}{2}c_2(\kappa_1^2+\kappa_2^2-k_i^2-k_f^2+q^2)]\tilde{F}_{LM}+\tilde{G}_{LM}, \quad \text{(B7)}$$

where

$$\tilde{G}_{LM}=(-1)^{l_1}\sum_{m_1m_2}C(l_1l_2L;\,m_1m_2M)V_0c_2i\mathbf{Q}$$
$$\cdot\int\exp(-i\mathbf{q}\cdot\mathbf{r})(\phi_{l_1}^{m_1}\nabla\phi_{l_2}^{m_2}-\phi_{l_2}^{m_2}\nabla\phi_{l_1}^{m_1})d^3r. \quad \text{(B8)}$$

Using the gradient formula[40] we can write Eq. (B8) after some recoupling of angular momenta

$$\tilde{G}_{LM}=iV_0c_2\mathbf{Q}\cdot\int\exp(-i\mathbf{q}\cdot\mathbf{r})$$
$$\times\{R_{l_1}(r)\sum_\lambda(-1)^\lambda U_{l_2\lambda}(r)\sum_\Lambda\langle l_1\|Y_\Lambda\|\lambda\rangle\hat{l}_2$$
$$\times W(\Lambda\lambda Ll_2;\,l_11)-(-1)^{l_1+l_2-L}R_{l_2}(r)$$
$$\times\sum_\lambda U_{l_1\lambda}(r)(-1)^\lambda\sum_\Lambda\langle l_2\|Y_\Lambda\|\lambda\rangle$$
$$\times\hat{l}_1W(\Lambda\lambda Ll_1;\,l_21)\}\mathbf{T}_{L(\Lambda 1)}{}^M, \quad \text{(B9)}$$

where

$$U_{l,\lambda}(r)=-\left(\frac{l+1}{2l+1}\right)^{1/2}\left(\frac{dR_l}{dr}-\frac{lR_l}{r}\right), \qquad \lambda=l+1$$
$$=\left(\frac{l}{2l+1}\right)^{1/2}\left(\frac{dR_l}{dr}+\frac{l+1}{r}R_l\right), \qquad \lambda=l-1$$
$$=0, \qquad \text{otherwise} \quad (20)$$

and $\mathbf{T}_{L(\Lambda 1)}{}^M$ is the vector spherical harmonic. We note that for normal transfers, $(-1)^{l_1+l_2-L}=1$, so for the special case where the initial and final single-particle levels are the same, the two terms in Eq. (B9) will cancel each other and $\tilde{G}_{LM}{}^{l_1l_2}$ vanishes.

APPENDIX C: INCLUSION OF COLLECTIVE ENHANCEMENT

Collective enhancement is not expected to be important for spin-flip transitions ($I'=1$), so we consider here only $I'=0$, for which the cross section is

$$\frac{d\sigma}{d\Omega}=\left(\frac{2m}{4\pi\hbar^2}\right)^2\frac{k_f}{k_i}\frac{2J_f+1}{2(2J_i+1)}\left|\sum_\tau C(\tfrac{1}{2}\tfrac{1}{2}\tau;\,-\tfrac{1}{2}\tfrac{1}{2}0)\right.$$
$$\times(-1)^\tau\sum_{j_1j_2\alpha}\hat{j}_1\hat{j}_2\begin{pmatrix}j_1 & \tfrac{1}{2} & l_1\\ j_2 & \tfrac{1}{2} & l_2\\ L & 0 & L\end{pmatrix}S(J_iJ_fL;\,j_1j_2\alpha\alpha)$$
$$\left.\times C(\tfrac{1}{2}\tfrac{1}{2}\tau;\,\alpha-\alpha 0)(-1)^{1/2-\alpha}\sum_{TS}(2S+1)(2T+1)\tfrac{1}{2}(\tfrac{1}{2}-\tfrac{2}{3}\delta_{\tau 1}\delta_{T1})A_{TS}(F_{LM}{}^{j_1j_2\alpha\alpha}-(-1)^{T+S}G_{LM}{}^{j_1j_2\alpha\alpha})\right|^2, \quad \text{(C1)}$$

where τ is the isospin transfer and the charge index α is now written explicitly. Aside from the spectroscopic amplitude $S(J_iJ_fL;\,j_1j_2\alpha\alpha)$, the sign of the coefficient is different for neutrons and protons for the $\tau=1$ term but the same for the $\tau=0$ part. Thus for collective motions in which neutrons and protons are moving together as nuclear fluid, which will have

$$S(J_iJ_fL;\,j_1j_2\tfrac{1}{2}\tfrac{1}{2})=S(J_iJ_fL;\,j_1j_2-\tfrac{1}{2}-\tfrac{1}{2}),$$

the core contributions to the $\tau=1$ term are expected to be much less important than to the $\tau=0$.

[40] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), p. 124, Eq. (6.42).

In the microscopic picture, collective motions contributing to electromagnetic and inelastic enhancement of simple shell-model states are included by appropriate mixtures of very small percentages of configurations beyond the major shell. Let us assume that the part of the amplitude due to $\tau=1$ isospin transfers is not changed significantly by the inclusion of these minor configurations, but that the $\tau=0$ transfers are changed—increased significantly for collective states.

Corresponding to Eq. (C1) the reduced electric transition rate is

$$B(E,L)=\frac{1}{2J_i+1}\left|\sum_{j_1j_2\alpha}[\tfrac{1}{2}-\alpha]\langle\phi_{j_2\alpha}\|r^LY_L\|\phi_{j_1\alpha}\rangle S(J_iJ_fL;j_1j_2\alpha\alpha)\right|^2$$
$$=\frac{2}{2J_i+1}\left|\sum_{j_1j_2\alpha}S(J_iJ_fL;j_1j_2\alpha\alpha)[\tfrac{1}{2}-\alpha]\hat{L}\hat{j}_1\hat{j}_2\begin{pmatrix}j_1&\frac{1}{2}&l_1\\j_2&\frac{1}{2}&l_2\\L&0&L\end{pmatrix}\langle l_2\|r^LY_L(\hat{r})\|l_1\rangle\right|^2, \quad \text{(C2)}$$

where again α is the isospin projection of the nucleon. The first term in the brackets is the $\tau=0$ part and the second, the $\tau=1$. We define an isoscalar effective charge $e_0(L)$ in units of the proton charge for the $\tau=0$ part with the relation

$$e_0(L)\sum_{j_1j_2\alpha}{}' f_{j_1j_2\alpha}=\sum_{j_1j_2\alpha}f_{j_1j_2\alpha}, \quad \text{(C3)}$$

where $f_{j_1j_2\alpha}$ is the quantity in the sum in Eq. (C2) excluding the brackets. The primed sum extends only over major configurations, and the unprimed sum extends over all configurations. We also define an isovector effective charge with the relation

$$e_1(L)\sum_{j_1j_2\alpha}{}' f_{j_1j_2\alpha}\alpha=\sum_{j_1j_2\alpha}f_{j_1j_2\alpha}\alpha; \quad \text{(C4)}$$

The assumption that the collective enhancement comes almost entirely from the $\tau=0$ part of the amplitude is equivalent to setting $e_1\approx1$. With this assumption we can rewrite the effective electromagnetic transition operator in terms of neutron and proton effective charges, $e_p=\frac{1}{2}(e_0+1)$, $e_n=\frac{1}{2}(e_0-1)$. Thus we have $e_0=e_p+e_n$, $e_p-e_n=1$.

In order to use this information on effective charges in inelastic scattering we must make a further assumption. We assume that the electromagnetic single-particle matrix element and inelastic scattering single-particle amplitudes are approximately proportional:

$$F_{LM}{}^{j_1j_2\alpha\alpha}(\theta)=\langle l_2\alpha\|r^LY_L(\hat{r})\|l_1\alpha\rangle F_{LM}(\theta),$$
$$G_{LM}{}^{j_1j_2\alpha\alpha}(\theta)=\langle l_2\alpha\|r^LY_L(\hat{r})\|l_1\alpha\rangle G_{LM}(\theta), \quad \text{(C5)}$$

where $F_{LM}(\theta)$ and $G_{LM}(\theta)$ are proportionality factors independent of j_1j_2. With this assumption it follows from substitution of $F_{LM}{}^{j_1j_2\alpha\alpha}$ from Eq. (C5) into Eq. (C1) that the $\tau=0$ part of the inelastic scattering amplitude is enhanced by the factor e_0.

For electromagnetic transitions and for direct inelastic scattering, it is obvious what the $\tau=0$ and $\tau=1$ terms of the interaction are. In the latter case, they are respectively the terms in the amplitude coming from the V_0 and V_τ terms in the spin-independent term of the central effective two-body force

$$V_0+V_\tau\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2. \quad \text{(C6)}$$

These are precisely the combinations of A_{TS} that appear in Eq. (C1) for $\tau=0$ and $\tau=1$. For the case of exchange scattering the combinations of A_{TS} giving rise to pure charge-exchange or pure spin-exchange given in Eq. (C1) have in general no simple relationship to the spin-isospin dependence of the two-body force. However, for a spatially even force they are the same combinations as for the direct. We now restrict the derivation to the even force case, for which the cross-section equation (C1) becomes

$$\frac{d\sigma}{d\Omega}=\left(\frac{2m}{4\pi\hbar^2}\right)^2\frac{k_f}{k_i}\frac{2J_f+1}{2(2J_i+1)}\left|\sum_{j_1j_2\alpha}\hat{j}_1\hat{j}_2\begin{pmatrix}j_1&\frac{1}{2}&l_1\\j_2&\frac{1}{2}&l_2\\L&0&L\end{pmatrix}S(J_iJ_fL;j_1j_2\alpha\alpha)\right.$$
$$\left.\times2[V_0-4\alpha V_\tau](F_{LM}{}^{j_1j_2\alpha\alpha}+G_{LM}{}^{j_1j_2\alpha\alpha})\right|^2. \quad \text{(C7)}$$

Using Eqs. (C3)–(C5) and the assumption that $e_1=1$, we can write the cross-section equation (C7) including sums only over major configurations just by multiplying the V_0 term by the isoscalar effective charge e_0.

Analogous to the neutron and proton effective charges we define a neutron and proton inelastic enhancement factor which is to multiply the neutron and proton shell-model contributions to the inelastic amplitude in order to account for the core configurations:

$$\begin{Bmatrix}\epsilon_p(L)\\ \epsilon_n(L)\end{Bmatrix}=\frac{V_0e_0(L)\pm V_\tau}{V_0\pm V_\tau}. \qquad \text{(C8)}$$

Equation (C8) holds for (p, p') reactions; for (n, n') reactions the $\pm$ signs are reversed.

If the shell-model wave function consists only of protons or only of neutrons, the $\tau=0$ enhancement simply amounts to a multiplication of the cross section by a factor $\epsilon_p^2(L)$ or $\epsilon_n^2(L)$.

PHYSICAL REVIEW C VOLUME 1, NUMBER 4 APRIL 1970

Determination of the Quadrupole Moments of First Excited 2^+ States in Cd^{114} and Fe^{56} by Coulomb Excitation*

G. Schilling,† R. P. Scharenberg, and J. W. Tippie‡

Department of Physics, Purdue University, Lafayette, Indiana 47907

(Received 23 June 1969)

The differential Coulomb-excitation probability for the 0.558-MeV 2^+ excited state in Cd^{114} and for the 0.847-MeV 2^+ excited state in Fe^{56} have been measured by Coulomb exciting with O^{16} ions of 25- and 30-MeV incident energy. Inelastically scattered ions were separated from ions scattered elastically by requiring a coincidence with a deexcitation γ ray detected in a large scintillator placed close to the target. The differential excitation probability was interpreted in terms of the reorientation effect to yield the static electric quadrupole moments for the 0.558-MeV 2^+ state in Cd^{114}, $Q_{22}=-0.64\pm0.19\times10^{-24}$ cm^2, and for the 0.847-MeV 2^+ state in Fe^{56}, $Q_{22}=-0.345\pm0.054\times10^{-24}$ cm^2.

I. INTRODUCTION

Recent measurements of the static electric quadrupole moments of excited nuclear states[1–7] are providing insight into the character of the first excited 2^+ states which have been classified as vibrational states. The present paper is intended to describe in detail our determination of the static electric quadrupole moment of the 0.558-MeV 2^+ state in Cd^{114} and the 0.847-MeV 2^+ state in Fe^{56}. These experiments were carried out by measuring the differential Coulomb excitation cross section of 25- and 30-MeV O^{16} ions scattered off Cd^{114} and Fe^{56} nuclei. The scattered ions were detected in coincidence with the deexcitation γ radiation. The shape of the differential Coulomb excitation cross section can give conclusive evidence for the presence of the "reorientation effect."[8–10]

Consider a charged particle incident upon a target nucleus. In the language of the semiclassical Coulomb excitation theory[11] the differential excitation cross section can be written

$$d\sigma/d\Omega\ (\text{inelastic})=P(\xi,\theta)\,d\sigma/d\Omega\ (\text{Rutherford}), \qquad (1)$$

where $P(\xi,\theta)$ is the excitation probability. If $P(\xi,\theta)\ll 1$, then the elastic differential cross section $d\sigma/d\Omega$ (elastic) closely approximates the Rutherford cross section. Here ξ is the adiabaticity parameter associated

* Supported by the U. S. Atomic Energy Commission under Contract No. AT (11-1) 1746 (Chicago Operations Office).

† Present address: Department of Physics, University of Notre Dame, Notre Dame, Ind.

‡ Present address: Argonne National Laboratory, Argonne, Ill.

[1] J. de Boer *et al.*, Technical Report, California Instute of Technology, 1965 (unpublished).

[2] R. G. Stokstadt *et al.*, Nucl. Phys. **A91**, 319 (1967).

[3] J. J. Simpson *et al.*, Nucl. Phys. **A94**, 177 (1967).

[4] P. H. Stelson *et al.*, Bull. Am. Phys. Soc. **10**, 427 (1965).

[5] P. H. Stelson, in Proceedings of the Summer Study Group on the Physics of the Emperor Tandem Van de Graaff Region, Brookhaven National Laboratory, 1965, Vol. III, 1005 (unpublished).

[6] J. E. Glenn and J. X. Saladin, Phys. Rev. Letters **19**, 33 (1967).

[7] G. Schilling *et al.*, Phys. Rev. Letters **19**, 318 (1967).

[8] G. Breit and J. P. Lazarus, Phys. Rev. **100**, 942 (1955).

[9] G. Breit, R. L. Gluckstern and J. E. Russell, Phys. Rev. **103**, 727 (1956).

[10] G. Briet and R. L. Gluckstern, in *Encyclopedia* (Springer-Verlag, Berlin, 1959), Vol. XLI/I, p. 548.

[11] K. Alder *et al.*, Rev. Mod. Phys. **28**, (432 1956).