In this notation, the deformation considered by Gerace and Green[76] corresponds to $\epsilon=+0.2$. Filling the Cartesian harmonic-oscillator states for protons in Ca^{40}, we obtain

$$\rho_{\rm prot}=(2/\pi^{3/2})[(1+\tfrac{1}{3}\epsilon)(1+\tfrac{1}{3}\epsilon)(1-\tfrac{2}{3}\epsilon)]^{1/2}\times\exp[-(x^2/b_x^2+y^2/b_y^2+z^2/b_z^2)]\times[\tfrac{5}{2}+2(x^2/b_x^2+y^2/b_y^2+z^2/b_z^2)^2]. \quad (C2)$$

Substituting (C1) in (C2), changing to spherical coordinates, and angle averaging, we obtain

$$\bar{\rho}_{\rm prot}=(4\pi)^{-1}\int\rho_{\rm prot}d\Omega=2\pi^{-3/2}(1+\tfrac{1}{3}\epsilon)(1-\tfrac{2}{3}\epsilon)^{1/2}\times\int_0^1\exp[-r^2/b^2(1+\tfrac{1}{3}\epsilon-\epsilon\cos^2\theta)]\times[\tfrac{5}{2}+2(r^4/b^4)(1+\tfrac{1}{3}\epsilon-\epsilon\cos^2\theta)^2]d\cos\theta. \quad (C3)$$

Equation (C3) may be cast into a form convenient for comparison with spherical harmonic-oscillator densities by expanding the quantities of order ϵ out of the exponential and integrating. Retaining terms of order ϵ^2, we obtain

$$\bar{\rho}_{\rm prot}=2\pi^{-3/2}[1-\tfrac{1}{6}\epsilon^2]\exp(-r^2/b^2)\times[\tfrac{5}{2}+2r^4/b^4+\tfrac{18}{45}(r^4/b^4)\epsilon^2-\tfrac{16}{45}(r^6/b^6)\epsilon^2+\tfrac{4}{45}(r^8/b^8)\epsilon^2]. \quad (C4)$$

Recognizing $2\pi^{-3/2}(\frac{5}{2}+2r^4/b^4)$ as the Ca^{40} ground state in a spherical basis, it is evident that (C4) represents the removal of $\epsilon^2/6$ or 0.67% of the protons from the spherical ground state and redistribution of them in higher states. One may verify that the normalization of (C4) is correct to order ϵ^2.

To simplify the inclusion of the single proton in an excited Nilsson state, it is most convenient to rewrite (C4) in terms of spherical harmonic-oscillator radial functions $R_{nl}(r)$. Clearly, there is no unique way of doing so, but the following form has the advantage of a small number of radial functions with roughly comparable coefficients.

$$\bar{\rho}_{\rm prot}=\rho_s+(2\pi r^2)^{-1}[-\tfrac{1}{6}\epsilon^2R_{00}^2-\tfrac{1}{2}\epsilon^2R_{01}^2-\tfrac{25}{12}\epsilon^2R_{02}^2-\tfrac{1}{6}\epsilon^2R_{10}^2+\tfrac{2}{3}\epsilon^2R_{12}^2+\tfrac{9}{4}\epsilon^2R_{04}^2], \quad (C5)$$

where ρ_s is the density of the Ca^{40} ground state for a spherical harmonic oscillator. Using Nilsson's tabulated wave functions,[83] removing a single proton from orbit 8 and placing it in orbit 14 with $\epsilon=0.2$ corresponds to a change in density of

$$\delta_\rho=(2\pi r^2)^{-1}[-0.5R_{02}^2+0.3795R_{03}^2+0.1205R_{11}^2]. \quad (C6)$$

Thus, using (C5) and (C6) and including the fact that the probability for deformation is 20%, the total density correction from the deformed admixture is

$$\Delta\rho=(10\pi r^2)^{-1}[-0.0067R_{00}^2-0.0200R_{01}^2-0.5833R_{02}^2-0.0067R_{10}^2+0.3795R_{03}^2+0.1205R_{11}^2+0.0267R_{12}^2+0.0900R_{04}^2]. \quad (C7)$$

This correction is extremely small and is graphed in Fig. 24.

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Coulomb Distortion in Heavy-Ion Reactions

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The dynamics of nuclear distortion is studied for two spherical nuclei under the influence of their mutual Coulomb interactions. Both the quadrupole and the octupole degrees of freedom are considered. Realistic stiffness and effective-mass parameters are employed. It is found that the energy needed to bring the two nuclei into contact increases as a result of the distortions. However, in the most favorable case, the increase is only a few percent. This is much smaller than previous estimates using either the liquid-drop model or the adiabatic model.

I. INTRODUCTION

PREVIOUS estimates of the increase in the Coulomb-barrier height were obtained by studying the dynamics of distortion using the liquid-drop model,[1,2] or by using an adiabatic model with realistic stiffness parameters.[3] A very large increase in the Coulomb-barrier height was obtained. The use of liquid-drop parameters to describe the vibrational properties of a nucleus is inappropriate, however, as they do not reproduce the experimental excitation energies and the transition rates for the first-quadrupole vibrational state. On the other hand, the distortion estimated by considering the deformation potential alone, as done in Ref. 3, would be correct only if the

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collision process were infinitely slow, so that the nuclei adjusted themselves to the potential minimum during the whole process. However, as we shall see later, because the collision time is much smaller than the nuclear-quadrupole oscillation time, this assumption of an adiabatic process is questionable.

We have studied the dynamics of distortion using realistic stiffness and effective-mass parameters C and D for the quadrupole and the octupole degrees of freedom. Also, the nuclear diffuseness has been taken into account. As in previous studies we have limited ourselves to head-on collisons of two spherical nuclei.

II. THEORY

The two nuclei under consideration are assumed to have a Fermi-type charge distribution with a diffuseness a and half-density radii $R^{(i)}$:

$$\rho^{(i)}(r,\theta)=\rho_0^{(i)}/\{1+\exp[(r-R^{(i)})/a]\}, \quad (1)$$

where the half-density radii are given by

$$R^{(i)}=R_0^{(i)}[1+\sum_{\lambda=0}^{N}\beta_\lambda^{(i)}Y_{\lambda0}(\theta^{(i)})], \qquad i=1,2. \quad (2)$$

However, not all the deformation parameters are independent. In fact, from the conservation of volume and the constancy of the center of mass, we have

$$\beta_0=-(4\pi)^{-1/2}\sum_{\lambda=1}^{N}\beta_\lambda^2+O(\beta^3) \quad (3)$$

and

$$\beta_1=-\tfrac{3}{2}(3/\pi)^{1/2}\sum_{\lambda=2}^{N-1}\frac{(\lambda+1)\beta_\lambda\beta_{\lambda+1}}{[(2\lambda+1)(2\lambda+3)]^{1/2}}+O(\beta^3). \quad (4)$$

The Hamiltonian of the system can be written as follows:

$$H=\tfrac{1}{2}\mu\dot{r}^2+\tfrac{1}{2}\sum_{i,\lambda}D_\lambda^{(i)}\dot{\beta}_\lambda^{(i)2}+\tfrac{1}{2}\sum_{i,\lambda}C_\lambda^{(i)}\beta_\lambda^{(i)2}+V_c(r,\beta_\lambda^{(i)}), \quad (5)$$

where μ is the reduced mass of the colliding nuclei, r is the magnitude of the position vector $\mathbf{r}$ that originates from the center of nucleus 2 to the center of nucleus 1, $D_\lambda^{(i)}$ and $C_\lambda^{(i)}$ are the effective-mass and stiffness parameters for the λth degree of freedom in the ith nucleus, and $V_c(r,\beta_\lambda^{(i)})$ represents the mutual Coulomb interactions between the two nuclei. The latter can be expanded in terms of the multipole moments $M_1(\lambda_1)$ and $M_2(\lambda_2)$ of the two charge distributions, as in Alder and Winther[4]:

$$V_c=\sum_{\lambda_1\lambda_2\mu_1\mu_2}\left[C(\lambda_1\lambda_2)\begin{pmatrix}\lambda_1 & \lambda_2 & \lambda_1+\lambda_2\\ \mu_1 & \mu_2 & -(\mu_1+\mu_2)\end{pmatrix}\times Y_{\lambda_1+\lambda_2,-(\mu_1+\mu_2)}(\mathbf{r})\right]M_1(\lambda_1\mu_1)M_2(\lambda_2\mu_2)/r^{\lambda_1+\lambda_2-1}, \quad (6)$$

where

$$C(\lambda_1\lambda_2)=(-1)^{\lambda_2}(4\pi)^{3/2}\times[(2\lambda_1+2\lambda_2)!/(2\lambda_1+1)!(2\lambda_2+1)!]^{1/2} \quad (7)$$

and the multipole moments are defined as

$$M(\lambda\mu)=\int r^\lambda Y_{\lambda\mu}(\theta,\phi)\rho(r,\theta)\,d^3r.$$

To the second order in the deformation parameters β_λ and a/R_0, we have for our charge distribution

$$M(\lambda\mu)=\delta_{\mu0}(3ZeR_0^\lambda/4\pi)\{(4\pi)^{1/2}\delta_{\lambda0}/(\lambda+3)+\beta_\lambda[1+(\lambda+4)(\lambda-1)\pi^2a^2/6R_0^2]+\tfrac{1}{2}(\lambda+2)\sum_{\lambda',\lambda''=0}^{N}\beta_{\lambda'}\beta_{\lambda''}t(\lambda',\lambda'',\lambda)\times[1+(\lambda+3)(\lambda-2)\pi^2a^2/6R_0^2]\}, \quad (8)$$

where

$$t(\lambda',\lambda'',\lambda)=\int Y_{\lambda'0}Y_{\lambda''0}Y_{\lambda0}\,d\Omega=\begin{pmatrix}\lambda' & \lambda'' & \lambda\\ 0 & 0 & 0\end{pmatrix}\times[(2\lambda+1)(2\lambda'+1)(2\lambda''+1)/4\pi]^{1/2}. \quad (9)$$

The first factor in Eq. (6) can also be written explicitly for head-on collisions. We have

$$C(\lambda_1\lambda_2)\begin{pmatrix}\lambda_1 & \lambda_2 & \lambda_1+\lambda_2\\ 0 & 0 & 0\end{pmatrix}Y_{\lambda_1+\lambda_2,0}(\theta=0)=(-1)^{\lambda_1}4\pi\times[(\lambda_1+\lambda_2)!/\lambda_1!\lambda_2!]/[(2\lambda_1+1)(2\lambda_2+1)]^{1/2}. \quad (10)$$

The Hamiltonian [Eq. (5)] leads to the following equations of motion:

$$D_\lambda^{(i)}\ddot{\beta}_\lambda^{(i)}+C_\lambda^{(i)}\beta_\lambda^{(i)}+\partial V_c/\partial\beta_\lambda^{(i)}=0, \qquad \lambda=2,3;\ i=1,2 \quad (11)$$

and

$$\mu\ddot{r}+\partial V_c/\partial r=0.$$

This set of coupled equations can be integrated to give the deformation $\beta_\lambda^{(i)}$ as a function of time. For this integration, we start at a distance $r\gg R$, so that the influence of higher multiple moments on the initial orbital is negligible. The initial values of r and dr/dt at a time t are then given as in Alder *et al.*[5]

III. DETAILS OF CALCULATION

Many quantities must be specified in the equations presented above before one can carry out the program of tracing the history of deformations of two colliding nuclei.

For the half-density radius of a nucleus, we have the

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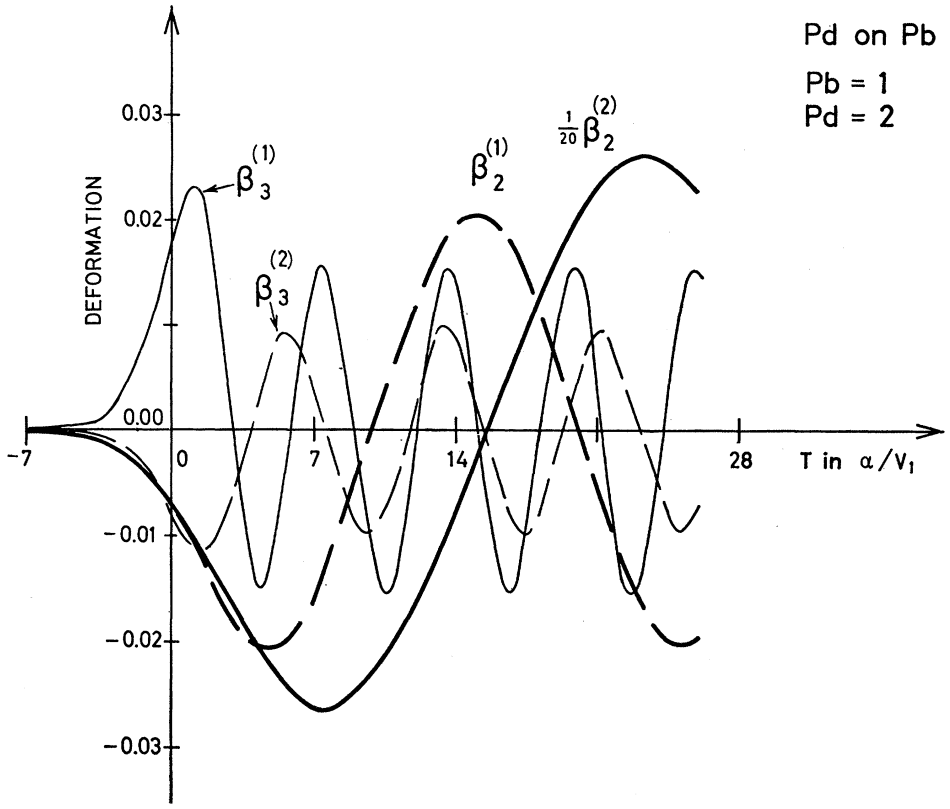

FIG. 1. Time dependence of the deformations for the collision of ^{106}Pd and ^{206}Pb. The superscript 1 is for the nucleus ^{206}Pb and 2 is for the nucleus ^{106}Pd. The time is measured in units of the collision time α/v_1 where α is half the distance of closest approach and v_1 is the relative velocity at infinite separation.

following expression from Myers[6,7]:

$$R_0=\Re[1-1/\Re^2+\cdots], \tag{12}$$

where

$$\Re=1.16A^{1/3}(1+\bar{\epsilon})\text{ fm}$$

and $\bar{\epsilon}$ is the local-density deviation correction as determined from the droplet model.[6] The diffuseness is taken to be

$$a=0.55\text{ fm}.$$

The skin thickness τ, as defined in Elton,[8] is therefore 2.4 fm.

Previous tabulation of the effective-mass and stiffness parameters for quadrupole vibration[9] indicates that they are very sensitive functions of the intrinsic structure of the nuclei. This may also be expected for octupole vibrations. The interaction of one nucleus with another during collision must be treated individually. However, it is out of place to consider the collision of all the known spherical nuclei, not only because it will present nothing essential, but also because experimental information on the octupole vibration is scanty. As heavy vibrational nuclei cluster around mass number $A\sim60$, 110, and 210, we shall consider collisions of ^{60}Ni, ^{106}Pd, and ^{206}Pb with themselves and with each other.

The vibrational parameters for quadrupole and octupole vibrations of ^{60}Ni, ^{106}Pd, and ^{206}Pb are listed in Table I in units of the liquid-drop stiffness parameter C_λ(LD) and the mass parameter D_λ(LD) for irrotational flow. The parameters for octupole vibrations are obtained from the experimental data of Fricke *et al.*,[10] Robinson *et al.*,[11] and Ziegler *et al.*[12] It should be kept in mind that while, on the average, the quadrupole stiffness of nuclei is close to that of the liquid drop, the quadrupole mass parameters are normally an order of magnitude larger than that of a liquid drop.[9] Not much can be said about the octupole vibration because comprehensive data are lacking.

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[7] Strictly speaking, one should use the half-density radius of the protons in the present calculation. However, the difference between the radii is small.

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[11] R. L. Robinson, J. L. C. Ford, Jr., P. H. Stelson, and G. R. Satchler, Phys. Rev. **146**, 816 (1966).

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TABLE I. Effective-mass and stiffness parameters for quadrupole and octupole oscillations in units of their respective values for the classical liquid-drop with an irrotational flow.

	C_2/C_2(LD)	D_2/D_2(LD)	C_3/C_3(LD)	D_3/D_3(LD)
^{60}Ni	1.12	11.1	5.10	23.6
^{106}Pd	0.331	9.90	2.60	22.9
^{206}Pb	23.9	74.9	2.13	7.46

In Table I, it can be observed that ^{106}Pd is very soft against quadrupole deformation and has a relatively small effective mass, while ^{206}Pb is very stiff and has a very large effective mass. The ^{60}Ni nucleus is intermediate but resembles ^{106}Pd more than ^{206}Pb. For octupole vibrations, ^{206}Pb is relatively soft and has a small relative mass parameter among the three nuclei, while ^{60}Ni has large relative stiffness and mass parameters. The relative octupole vibrational parameters for ^{106}Pd are between these two extremes.

Now, with the knowledge of all the parameters and the initial conditions, we can proceed with the integration of the equations of motion [Eq. (11)]. Figure 1 shows the results of calculations of the collision of ^{106}Pd with ^{206}Pb at an energy of $E=344$ MeV (which is about 10 MeV below the Coulomb-barrier height defined later). The time is expressed in units of the collision time, the time origin being approximately the moment of closest approach. It can be seen that the quadrupole deformations of the two nuclei are small at the moment of closest approach. They become much larger after the two nuclei are appreciably separated from each other. This is because the collision time for the process is only about 1/25 of the natural quadrupole-oscillation period of ^{106}Pd. For the octupole vibrations, the deformation at the moment of closest approach is about the same as that of the oscillation amplitudes after the two nuclei are parted. This is expected, as the natural octupole-oscillation period is now only a few units of the collision time.

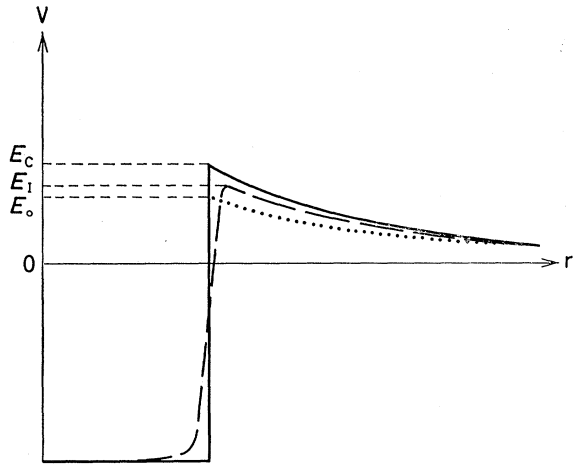

FIG. 2. The position of the Coulomb-barrier height E_c as compared with the interaction-barrier height E_I. The quantity E_c is defined with respect to a square-well nuclear potential, whereas E_I is defined with respect to a diffused potential. The total nucleus-nucleus interaction potentials in these cases are shown as a solid and a dashed curve, respectively. For the case when there is no distortion and a square-well potential is assumed, the total potential is shown as a dotted curve. The Coulomb-barrier height is E_0 in that case.

TABLE II. Percentage increase in the Coulomb-barrier height for various combinations of targets and projectiles.

$\Delta E/E_c$	^{60}Ni	^{106}Pd	^{206}Pb
^{60}Ni	0.4	1.8	0.8
^{106}Pd		4.1	3.2
^{206}Pb			1.6

IV. COULOMB BARRIER AND INTERACTION BARRIER

The nucleus-nucleus interaction consists of both the Coulomb interaction and the nuclear interaction. If the nuclear interaction is of the form of a square well with a sharp edge at a definite separation, the Coulomb-barrier height is well defined and is equal to the energy needed to bring the two nuclei to that separation. We know that nuclear interaction, although being short range, still can extend quite far as the surface of a nucleus is diffused. The total potential has a peak whose position and height depend not only on the Coulomb energy but also on the nuclear potential. The situation is illustrated in Fig. 2. The height of this peak can be conveniently called the interaction-barrier height E_I. This is also the energy that may be measured by experiment.[13] When the imaginary potential is relatively

TABLE III. Deformations for various combinations of colliding nuclei at their Coulomb-barrier height E_c. For each combination of nuclei, the deformations are such that the nucleus on the left side has a negative value of β_3, while the nucleus on the right side has a positive value of β_3.

	E_c (MeV)		$+10^2\beta_2$	$10^2\beta_3$
^{60}Ni+^{60}Ni	92.0	Ni	−0.80	−0.23
		Ni	−0.80	0.23
^{60}Ni+^{106}Pd	141.6	Ni	−1.1	−0.28
		Pd	−6.5	0.52
^{60}Ni+^{206}Pb	225.1	Ni	−1.4	−0.32
		Pb	−0.47	1.4
^{106}Pd+^{106}Pd	221.2	Pd	−8.5	−0.63
		Pd	−8.5	0.63
^{106}Pd+^{206}Pb	354.2	Pd	−12	−0.76
		Pb	−0.65	1.8
^{206}Pb+^{206}Pb	569.1	Pb	−0.96	−2.3
		Pb	−0.96	2.3

[13] U. Smilánsky (private communication).

unimportant, this is the energy at which the probability of absorbing the projectile reaches a value of one-half, as given by the Hill-Wheeler penetration formula.[13,14] Lacking any knowledge on the nuclear part of the nucleus-nucleus interaction, we shall consider only the Coulomb barrier in our calculations, keeping in mind its distinction from the interaction barrier.

Accordingly, we shall define the Coulomb-barrier height as the energy needed to bring the two nuclei into a "touching" position. Two nuclei are said to be touching each other if the separation of centers is

$$s(\beta)=R^{(1)}(\theta=180°,\beta_\lambda^{(1)})+R^{(2)}(\theta=0°,\beta_\lambda^{(2)}) +\tfrac{1}{2}\sum_i \tau^{(i)}+\lambda\!\!\!^{-}_\pi, \quad (13)$$

where $\lambda\!\!\!^{-}_\pi$ is the Compton wavelength of the pion, included here to take into account approximately the range of nuclear forces. This condition also coincides approximately with that used in the traditional estimate given by $s=1.5\text{ fm}\times(A_1^{1/2}+A_2^{1/3})$.

For the set of nuclei under consideration, the Coulomb-barrier height E_c can be obtained and compared with the same quantity E_0 when there is no distortion (i.e., when $\beta_\lambda^{(i)}=0$).

The difference $\Delta E=E_c-E_0$ divided by E_c for various combinations of projectile and target is presented in Table II. The distortions at the touching moment are depicted in Table III.

V. DISCUSSION AND SUMMARY

The most important conclusion one can draw from Table II is that the fractional increase $\Delta E/E$ obtained in a dynamical model is, even for the most favorable case, only a few percent, which is much smaller than previous estimates with the liquid-drop model[1,2] or the adiabatic model.[3] The difference is also partly due to the use of a larger and more realistic contact radius [Eq. (13)], so that the distortion effect terminates at a larger separation.

We come now to investigate the effect of dipole distortion on the Coulomb-barrier height. Since the dipole-oscillation period is comparable to the collision time, an adiabatic model can be used to estimate the dipole distortion. Here, Eq. (4) for β_1 has to be abandoned, as now one has to speak of a neutron medium oscillating collectively with respect to a proton medium. Instead, the displacement parameter $\beta_{p1}^{(i)}$ for the protons can be shown in the adiabatic model to be

$$|\beta_{p1}^{(i)}|=(3/4\pi)^{1/2}(Z_1Z_2e^2R_0/r^2C_1^{(i)}).$$

The stiffness parameter $C_1^{(i)}$ can be estimated from the energy of the giant-dipole state and the effective mass for dipole oscillation. In the Goldhaber-Teller model,[15] the latter is

$$D_1=(3/4\pi)(ZA/N)m_pR_0^2,$$

where m_p is the mass of a nucleon. This gives a stiffness of 3.5×10^4 MeV for ^{106}Pd and 7.5×10^4 MeV for ^{208}Pb. The fractional decrease in the Coulomb-barrier height due to dipole distortion is of the order of 10^{-4}. The effect of the dipole oscillation on the Coulomb barrier is therefore very small.

When we compare the importance of the octupole with the quadrupole degree of freedom, we find that, since the octupole-monopole interaction becomes effective at a separation closer than that for the quadrupole-monopole interaction, the octupole distortion is usually less important. However, there are important exceptions for nuclei such as ^{206}Pb, which is soft against octupole and stiff against quadrupole vibrations. For these nuclei, β_3 may be larger than β_2. (See Table III.)

For permanently deformed nuclei, the stiffness and the effective mass for quadrupole oscillation are rather large. One should expect small distortions during the collision. Thus, as far as the Coulomb-barrier height is concerned, the important effects are the geometrical and rotational effects. The first effects arise because the Coulomb energy and the touching condition depend on the orientation of the symmetry axis. The maximum and minimum energies may differ by as much as 10%. This difference may be reflected in the threshold behavior for some reactions, which for colliding deformed nuclei should be different from that for spherical nuclei. The second effects have to do with the dynamical rotational force when one (or both) of the colliding nuclei is endowed with a permanent intrinsic quadrupole moment. This force tends to rotate the nucleus so that its symmetry axis is perpendicular to the collision axis. The averaged Coulomb-barrier height therefore increases.[13]

When the present work was completed and the results reported at the Conference on the Properties of Nuclear States,[16] a similar calculation on Coulomb distortion was published by Holm *et al.*[17] However, with a simpler method using the Alder-Winther interaction,[4] we have treated the collision of other cases. Although the results obtained by Holm *et al.*[17] are qualitatively similar to ours, the fractional increase in the Coulomb-barrier height is substantially larger than what we obtain here. This may be due to the difference in the choice of the contact radius.

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