Comments and Addenda

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$^{16}O(p, d)^{15}O$ Form Factors*

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(Received 18 July 1969)

Two methods of obtaining distorted-wave form factors which provide fits to the $^{16}O(p, d)$ reaction are investigated. Both apply a radial damping term to a Woods-Saxon eigenfunction. One method employs a correction for the density dependence of the interaction along with the usual local-energy-approximation correction terms for nonlocality and finite range. The other method applies a simple empirical two-parameter damping term. The calculations are compared with data for $1p_{1/2}$ and $1p_{3/2}$ neutron transfers at energies from 25 to 45 MeV.

In order to obtain theoretical fits to angular distributions from (p, d) reactions on light nuclei, the standard approach using the distorted-wave (DW) theory has often been modified. These modifications have consisted of lower cutoffs in the radial integration,[1,2] adjustments of the parameters in the optical potentials,[3] and local-energy-approximation finite-range and nonlocal damping terms employing parameters which are factors of 2 or more larger than the usual ranges and nonlocality strengths.[1,4] This paper presents the results of some calculations which do not use the above modifications but which do achieve acceptable fits to angular distributions for the $^{16}O(p, d)^{15}O$ reaction to the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states in ^{15}O measured at four incident energies[1] (25.52, 31.82, 38.63, and 45.34 MeV).

Guided by the fact that the above *ad hoc* corrections to the DW calculations obtain results by either neglecting or greatly diminishing the contribution from the nuclear interior, a correction was sought whose effect would yield a damping of the form factor entering the expression for the DW transition amplitude. This form factor is usually taken as a Woods-Saxon eigenfunction whose eigenenergy is the separation energy of the transferred neutron.[5]

First, the form factor was damped with the usual nonlocality[6,7] and finite-range[7,8] correction terms. Using values of 1.5 F for the range of the interaction, nonlocality ranges of 0.54 F for the deuteron, and 0.85 F for the proton and the transferred neutron, the DW calculations are improved over the local, zero-range calculations, but they still do not reproduce the slope of the experimental angular distributions.[9] However, one can obtain more damping if the interaction V_{pn} is taken to be dependent on the nuclear matter density. Using the density-dependent term $(\rho^{2/3})$ found by Green[10] for the triplet-even interaction (without the normalization of 1.623), and including the factors for nonlocality and finite range with the optical potentials for 45.45 MeV, one obtains the form factor labeled DFRNL in Fig. 1. The effect is to push the peak to larger radii and to make it more diffuse.

* Work supported in part by the National Science Foundation.

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TABLE I. Optical-model parameters describing proton elastic scattering from ^{16}O ($r_{s0}=r_R=1.12$ F, $a_{s0}=a_R=0.69$ F, $r_{0c}=1.15$ F, and $V_{s0}=7.0$ MeV).

E_p (lab) (MeV)	V (MeV)	W_S (MeV)	W_D (MeV)	r_I (F)	a_I (F)	σ_R (expt) (mb)	σ_R (theor) (mb)	χ^2/N
25.46	48.4	0.0	6.80	1.19	0.550	507	535	36.7
32.07	45.5	0.0	5.31	1.44	0.490	473	487	32.3
35.20	45.0	0.91	5.70	1.45	0.450	485	498	45.3
38.43	44.4	2.00	4.89	1.40	0.430	441	446	16.5
45.3	42.7	3.11	5.65	1.28	0.415	407	407	4.9

Two different density distributions for ^{16}O were investigated. They were a Gaussian distribution and one determined from the shell model. Their forms and the constants for ^{16}O are given by Green.[10] It was found that the resulting form factors were different only for radii less than 2 F and that they predicted essentially the same angular distributions both in shape and magnitude. The curve shown in Fig. 1 uses the Gaussian density distribution.

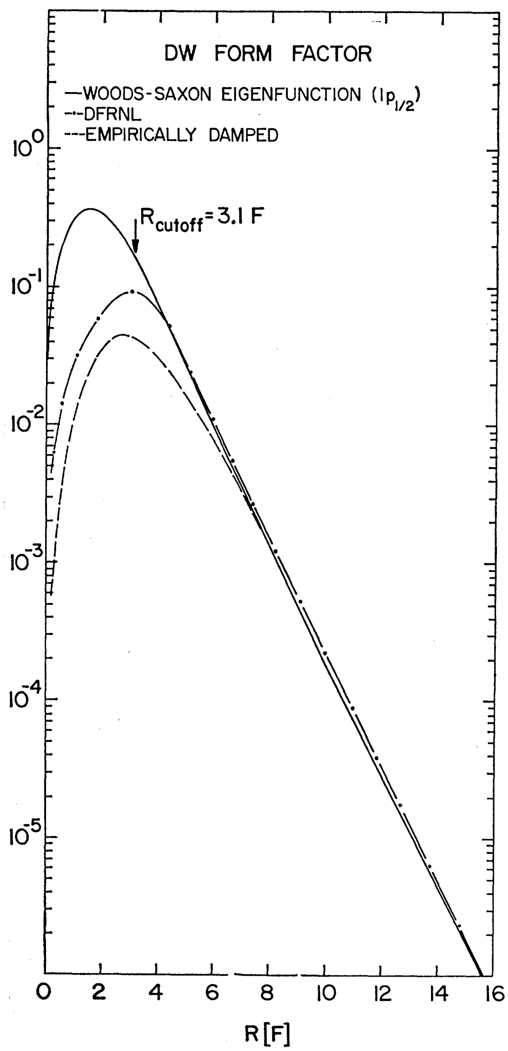

FIG. 1. Form factors used in DW calculations for the transfer of a $1p_{1/2}$ neutron.

Also shown in Fig. 1 is a form factor which results from applying a simple empirical two-parameter damping factor to the Woods-Saxon wave function. The damping factor is both simple and smoothly varying, and yet it allows enough freedom to cover a wide range of forms. This factor, employing ρ_0 as a range and α as a diffuseness, can be written as

$$F(r)=1-C(1+e^x)^{-1},$$

where $x=(r-\rho_0A^{1/3})/\alpha$ and $C=1+e^{-\rho_0A^{1/3}/\alpha}$. This factor can produce the effects of a very slow damping (α large) or a sharp cutoff (α small) at any radius. The curve shown in Fig. 1 uses values of $\rho_0=1.7$ F and $\alpha=1.2$ F. These values were determined to give the best fit to the shape of the experimental angular distributions. They were selected from the results of a search in which ρ_0 varied from 0.5 to 2.0 F and α from 0.1 to 1.5 F. This same damping factor gave a good fit for both a $1p_{1/2}$ and a $1p_{3/2}$ transfer at each of the four incident energies. Although this factor is probably not unique nor is it *a priori* justified, it indicates the general features that a successful form factor should have.

In Fig. 2, four calculations are compared with the data at the energies 31.82 and 45.34 MeV. The cases for both a $1p_{1/2}$ and a $1p_{3/2}$ neutron transfer are shown. The curves are normalized to the data at the first

TABLE II. Deuteron optical parameters used in the DWBA analysis of the $^{16}O(p,d)^{15}O$ reaction ($r_{s0}=r_R$, $a_{s0}=a_R$, $V_{s0}=7.57$ MeV, and $r_{0c}=1.30$ F).

E_d (MeV)	V (MeV)	r_R (F)	a_R (F)	W_D (MeV)	r_I (F)	a_I (F)
33.2	92.8	1.03	0.80	8.84	1.41	0.70
26.2	98.0	1.00	0.80	7.95	1.45	0.70
19.0	104.0	0.98	0.80	7.05	1.50	0.70
10.4	114.0	0.95	0.80	6.00	1.57	0.70
4.5	130.0	0.90	0.80	5.00	1.66	0.70

TABLE III. Spectroscopic strengths for $^{16}O(p, d)$.[a]

	Incident energy (MeV)	Standard DW calculation	Radial cutoff[b]	Empirically damped	Empirically damped with unit renorm	Finite range and nonlocality	Density-dependent effective interaction and finite range and nonlocality[c]
$S(1p_{1/2})$	25.52	2.7	3.2	6.2	0.29	3.2	3.1
	31.82	2.5	2.3	5.5	0.26	2.7	2.6
	38.63	2.8	1.8	5.0	0.24	2.4	2.3
	45.34	3.5	1.8	5.1	0.24	2.5	2.3
$S(1p_{3/2})$	25.52	4.0	8.1	44	1.7	16	18
	31.82	2.0	3.5	16	0.61	5.3	5.7
	38.63	2.2	3.0	11	0.42	3.7	3.9
	45.34	2.2	2.6	12	0.46	3.7	3.8
$S(1p_{3/2})/S(1p_{1/2})$	25.52	1.5	2.5	7.1	5.9	5.0	5.8
	31.82	0.88	1.5	2.9	2.3	2.0	2.2
	38.63	0.79	1.7	2.2	1.8	1.5	1.7
	45.34	0.63	1.4	2.4	1.9	1.5	1.7

[a] Spectroscopic strengths are extracted from the data according to the expression $\sigma_{\text{expt}} = 2.25\ S(nlj)\sigma_{\text{DW}}$.
[b] Radial cutoff = 3.1 F.
[c] The normalization of 1.623 for the density-dependent interaction (Ref. 10) has not been included.

maximum. The DW calculation[11] uses optical parameters that describe the elastic scattering in both channels at their respective energies,[1,9] and there are no radial cutoffs. It has been assumed here that the DW method is indeed applicable to the problem and that "good" energy-dependent proton and deuteron optical potentials have been used. The parameters for the optical potentials for protons are listed in Table I and for the deuterons in Table II.

The dot-dash curve, labeled standard DW calculation in Fig. 2 is a local zero-range calculation using a neutron wave function bound in the Woods-Saxon well previously described.[5] It is quite apparent that this calculation is unsatisfactory.

The dashed curve results from introducing a lower cutoff at 3.1 F ($\sim 1.2A^{1/3}$) in the integration of the above case. This calculation falls off like the data out to the region between 90° and 120°. However, the oscillatory nature of this curve does not match that of the data. The different locations of the peaks and valleys, i.e., the spacing of the oscillations, shows that the contributions to the momentum transfer from the different partial waves are treated incorrectly.

The solid curve is obtained from calculations using the single-particle wave functions of the above calculations modified by the empirical damping factor. The integration can now include the interior region, and the correct momentum transfer is predicted. The slope of the data is also well reproduced.

The cross-hatched curve results from calculations with corrections for finite-range, nonlocality, and the density-dependent interaction discussed above. Although indicating that more damping of the form factor is necessary, it is seen that this fit is relatively good.

The spectroscopic strengths obtained for the four form factors at each of the incident energies are presented in Table III. Also presented are the values

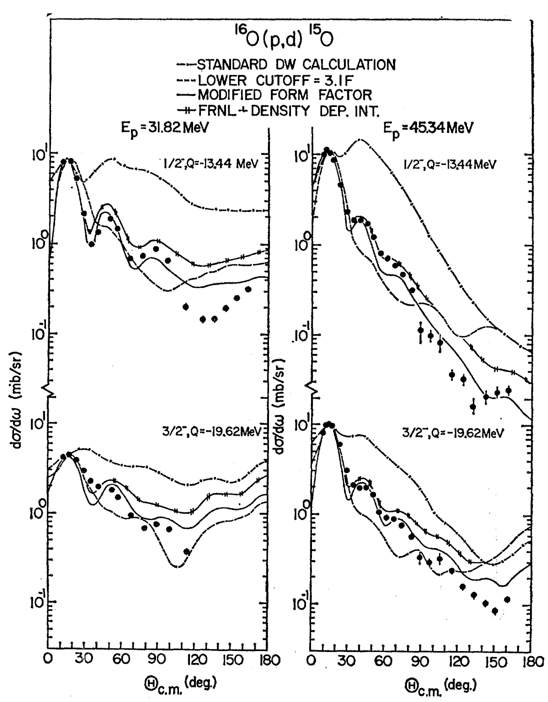

FIG. 2. Comparison of DW calculations with data for the $^{16}O(p, d)^{15}O$ reaction.

[11] All DW calculations use the FORTRAN-IV version of the DW code JULIE. R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240, 1962 (unpublished); Oak Ridge National Laboratory Memorandum to the Users of the Code JULIE, 1966 (unpublished).

which result from renormalizing the empirically damped form factor to unity and those which are obtained when only the finite range and nonlocality corrections are made.

It is seen that there is a large discrepancy between the $p_{3/2}$ strengths obtained for 25.52 MeV and those at the higher energies. This is attributed to the fact that the Q value to the $\frac{3}{2}^-$ state is -19.62 MeV, and therefore the exit-channel energy is quite low. Possibly this effect exists in the $p_{3/2}$ strengths at 31.82 MeV also. These effects are mirrored in the relative $p_{3/2}$ to $p_{1/2}$ ratios. It is seen, however, that the $p_{1/2}$ strengths remain relatively constant for each energy. The strengths obtained for the modified form factor are quite high because of the large damping necessary to obtain the fits to the angular distribution. Renormalizing the modified form factor to unity leads to spectroscopic strengths much too small. The values obtained from the cutoff method as well as those from the calculations employing finite range and nonlocality with and without the density dependence are in the neighborhood of probable values for the occupation of the shells in ^{16}O. It should be noted, however, that the normalization strength of 1.623 used by Green[10] for the density-dependent effective interaction has not been included here. If it were, the $p_{1/2}$ and $p_{3/2}$ strengths would both be reduced by 2.634, but the ratios would be left unchanged.

Comparing the values obtained for the relative $p_{3/2}$ to $p_{1/2}$ strengths, it is seen that the standard DW calculation fails to give even a plausible ratio. The relative strengths for all but one of the other calculations, ignoring the 25.52-MeV values, are within $\pm 30\%$ of the value 2. An indication of what values of this ratio of neutron strengths might be reasonable can be obtained from the measured values for the proton configuration of the ^{16}O ground state. In the $^{16}O(d, {}^3He){}^{15}N$ reaction,[12] the relative $p_{3/2}$-$p_{1/2}$ strength was determined to be 1.58 when a local zero-range DW calculation was used, and 1.74 when finite-range effects were included.

We would like to acknowledge many helpful discussions with our colleagues at the Cyclotron Laboratory. We also wish to thank Professor R. M. Drisko for his very enlightening comments. We are also grateful to Professor H. McManus for many stimulating conversations and especially for his suggestion to investigate the possible density dependence of the interaction.

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Addendum to Delayed Neutron Emission in the Decays of Short-Lived Separated Isotopes of Gaseous Fission Products*

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A correction is reported to the mass-formula predictions for the onset of delayed neutron emission using the formula of Garvey and Kelson. The predictions of the Garvey-Kelson and Seeger mass formulas are given for delayed-neutron-emission thresholds which have been recently reported in elements not included in the original report.

SINCE the publication of a previous paper on delayed neutron emission in the decays of short-lived separated isotopes of gaseous fission products,[1] it has been pointed out that the β-decay energies were incorrectly inferred from the Garvey-Kelson mass formula.[2] The reported values for the β-decay energy of the precursor nuclei in Table VI of Ref. 1 are too small by the neutron-proton mass difference of 0.78 MeV (under the column for formula h). Adjustment of these values leads to agreement with the experimentally reported thresholds for delayed neutron emission as predicted with this formula,[3] with the exception of the precursor nucleus As^{84}, changing the predictions for the precursor nuclei Kr^{92}, Rb^{92}, Xe^{141}, Xe^{142}, and Cs^{141}.

Delayed neutron emission has been reported recently for isotopes of Se, Y, and Te.[4,5] The predictions of the

* Work performed in the Ames Laboratory of the U.S. Atomic Energy Commission, Contribution No. 2644.

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