possibly as much as 15% might be the result of a source impurity; higher order accidentals cannot account for more than ∼1%; and double-photon decay (see Ref. 1) contributes a maximum of ∼15%. Furthermore, the coincident incident photon spectrum must be fairly continuous.

APPENDIX B

A particularly favorable case for the detection of DIB would seem to be the $P^{32} \rightarrow S^{32}$ β decay, which is an allowed transition with an endpoint energy of 1.71 MeV. We have taken data with a P^{32} source that at the start had an activity of about 2 mC. Coincidences were recorded at 180 and 90°, for a total of 100 h each, and with the lower discriminators moved down to 545 keV. The results, however, were inconclusive. The net diagonal-sum coincidence spectra do not approach an endpoint at 1.71 MeV, but extend beyond it, and the calculated values of $W_{\text{DIB}}(E_\alpha)/W_{\text{IB}}(E_\alpha)$ are roughly three to five times larger than the theoretical values. Also, theoretically we should find for the anisotropy, averaged over energy from 1160 to 1720 keV, $\eta_{\text{DIB}}=0.573$. This compares with an experimental value of 0.83 ± 0.37. These discrepancies were thought to be due to a radioactive contaminant in the P^{32} source. Later tests verified this, showing the presence of Ag^{110}.

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Beta-Gamma-Gamma Direction-Correlation Function for Two-Photon Radiation Emitted During Allowed Beta Decay

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Two Feynman diagrams for double internal bremsstrahlung are evaluated for an allowed β decay. The nucleons are treated nonrelativistically, and Coulomb effects are not considered.

1. INTRODUCTION

THE process of internal bremsstrahlung (IB), where a photon is emitted during the β decay of a nucleus, was first observed by Aston[1] in 1927. The classic theoretical studies of IB were published by Knipp and Uhlenbeck[2] and by Block[3] in 1936.

Occasionally, two photons should be emitted during β decay. This double internal bremsstrahlung (DIB) process should occur with a probability per β decay of order α^2, where $1/\alpha \approx 137$.

Recently, Jastram and Vanderleeden[4] observed a continuous two-photon coincidence spectrum in the β decay of Y^{90}. The question of whether they were observing DIB provided the impetus for the present calculation of the general DIB β-γ-γ direction-correlation function.

In the only previously published DIB calculation, that of Thun *et al.*,[5] the authors restrict themselves to that special case in which the electron is emitted in a direction perpendicular to two oppositely directed photons. We have verified that when applied to this special case our result agrees with theirs.

2. DERIVATION OF THE CORRELATION FUNCTION

In Fig. 1, we present the DIB Feynman diagrams to be considered. Assuming an *allowed* β decay, treating the nucleons nonrelativistically, and adopting the set of units in which $c=\hbar=m_e=1$, we evaluate the diagrams of Fig. 1. Employing standard techniques,[6] we quickly deduce the following expression for the differential probability $dP_{\beta\gamma\gamma}$ of the emission of an antineutrino of energy q whose direction and polarization are unspecified, and an electron and two photons, of energy E, k_1, and k_2, respectively, of direction specified by the solid angles $d\Omega_e$, $d\Omega_1$, and $d\Omega_2$, respectively, but of unspecified polarization:

$$\begin{aligned} dP_{\beta\gamma\gamma} &= dk_1 dk_2 dE d\Omega_1 d\Omega_2 d\Omega_e \alpha^2 P_\beta(E+k_1+k_2) F \\ &\quad \times \sum_{\text{pol}} \tfrac{1}{4}\,\text{Tr}[T\gamma_4 T'(\mathfrak{p}+1)] \\ &= dP_{\beta\gamma\gamma}(q;\, p,\, \Omega_e;\, k_1,\, \Omega_1;\, k_2,\, \Omega_2), \end{aligned} \tag{1}$$

[1] G. H. Aston, Proc. Cambridge Phil. Soc. **22**, 935 (1927).
[2] J. K. Knipp and G. E. Uhlenbeck, Physics **3**, 425 (1936).
[3] F. Block, Phys. Rev. **50**, 272 (1936).
[4] P. S. Jastram and J. C. Vanderleeden, Phys. Letters **19**, 29 (1965).
[5] J. E. Thun, W. D. Hamilton, K. Siegbahn, and K. E. Eriksson, Arkiv Fysik **22**, 55 (1962).
[6] S. DeBenedetti, in *Nuclear Interactions* (John Wiley & Sons, Inc., New York, 1964), Chap. 6.

where $\sum_{\rm pol}$ indicates a summation over photon (transverse) polarizations,

$$P_\beta(E)=\text{const}(E_0-E)^2E(E^2-1)^{1/2}, \tag{2}$$

which is the probability of an allowed β decay of end-point energy E_0 and electron energy E,

$$\mathfrak{p}=p_\mu\gamma_\mu. \tag{3}$$

(This use of German type to abbreviate such terms is continued in the next equation.)

$$T=[\tfrac{1}{2}(p\cdot k_2)^{-1}\,\mathfrak{e}_2(\mathfrak{p}+\mathfrak{k}_2+1)\,\mathfrak{e}_1(\mathfrak{p}+\mathfrak{k}_1+\mathfrak{k}_2+1)]+[1\leftrightarrow 2], \tag{4}$$

where a four-vector dot product is to be understood as

$$p\cdot k=Ek-\mathbf{p}\cdot\mathbf{k}. \tag{5}$$

T' is the same as T except that the order of the γ matrices is reversed, and we have introduced the abbreviation

$$F=\tfrac{1}{8}k_1k_2(2\pi)^{-5}(E^2-1)^{1/2}(E+k_1+k_2)^{-1}\times[(E+k_1+k_2)^2-1]^{-1/2}[p\cdot k_1+p\cdot k_2+k_1\cdot k_2]^{-2}. \tag{6}$$

The lengthy trace evaluation and polarization summation of Eq. (1) yields the result

$$\sum_{\rm pol}\tfrac{1}{4}\,\mathrm{Tr}[T\gamma_4T'(\mathfrak{p}+1)]=A+A^{\sim}, \tag{7}$$

where we have introduced the abbreviation

$$\begin{aligned}A={}&2(k_1\cdot k_2)(p\cdot k_2)^{-1}[2(E+k_1+k_2)-k_2(1+(p\cdot k_1)^{-1})(1-\lambda_{12}^2)]+(p\cdot k_1+p\cdot k_2+k_1\cdot k_2)[(p\cdot k_1)(p\cdot k_2)]^{-1}\\&\times\{Ek_1k_2(\lambda_{12}^2-1)+4k_1[p^2(1-\lambda_2^2)(p\cdot k_1)(p\cdot k_2)^{-1}+pk_2(\lambda_2-\lambda_1\lambda_{12})]\\&+2k_2[pk_1(\lambda_1+\lambda_2\lambda_{12})-Ek_1(1+\lambda_{12}^2)+2p^2(1-\lambda_1^2)+2pk_2(\lambda_2-\lambda_1\lambda_{12})]]\}\\&+2(E+k_1+k_2)(p\cdot k_2)^{-2}\{(p\cdot k_2)^2(1+\lambda_{12}^2)+2p^2(1-\lambda_2^2)[p^2(1-\lambda_1^2)+k_2^2(1-\lambda_{12}^2)+2pk_2(\lambda_2-\lambda_1\lambda_{12})]\\&+2p(p\cdot k_2)[2p(1-\lambda_1^2-\lambda_2^2+\lambda_1\lambda_2\lambda_{12})+2k_2\lambda_{12}(\lambda_2\lambda_{12}-\lambda_1)\\&+p(k_1(\lambda_1-\lambda_2\lambda_{12})+p(1-\lambda_2^2))(k_2(\lambda_2-\lambda_1\lambda_{12})+p(1-\lambda_1^2)(p\cdot k_1)^{-1})]\}\end{aligned} \tag{8}$$

with $A^{\sim}$ given by interchanging 1 and 2 in A, and where we have abbreviated the following cosines:

$$\lambda_1=\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}_1,\quad \lambda_2=\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}_2,\quad \lambda_{12}=\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2. \tag{9}$$

Substituting Eq. (7) into Eq. (1) we obtain the final form of the DIB β-γ-γ direction-correlation function:

$$dP_{\beta\gamma\gamma}=\alpha^2P_\beta(E+k_1+k_2)\times[A+A^{\sim}]F\,dk_1dk_2dEd\Omega_1d\Omega_2d\Omega_e. \tag{10}$$

It may be verified that Eq. (10) reduces to the result of Thun *et al.* [Ref. 5, Eq. (14)], when applied to their special case, i.e., $\lambda_1=\lambda_2=0$ and $\lambda_{12}=-1$.

3. COMPARISON WITH EXPERIMENT

In the preceding paper,[7] the experimental observations of Jastram and Vanderleeden are presented and compared with theoretical predictions based on Eq. (10). For a comparison with experiment, the necessity for an accurate knowledge of the β-decay matrix element [the constant of Eq. (2)] can be eliminated by considering the ratio of the probability of DIB to that of IB or to that of β decay not accompanied by internal bremsstrahlung. The β-decay matrix element will cancel in such ratios. Also, while the probability of IB is relatively sensitive to Coulomb effects, the ratio of IB to β decay without internal bremsstrahlung is not.[8] This insensitivity to Coulomb effects can be expected to apply also to the ratio of DIB to IB and to the ratio of DIB to β decay without internal bremsstrahlung. Thus the neglect of Coulomb effects in the present calculation should have little effect on either of these ratios.

For the reasons discussed in Ref. 7, the ratio of the probability of DIB to that of IB was selected as the quantity for comparison between experiment and theory. The expression for the IB differential probability is calculated in Refs. 2 and 3, and more recently (Ref 5) using modern techniques. It is

$$dP_{\beta\gamma}=\alpha P_\beta(E+k)B\,dk\,dE\,d\Omega\,d\Omega_e, \tag{11}$$

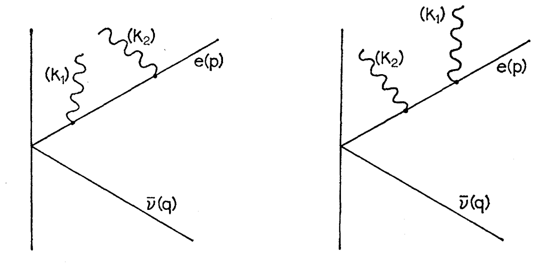

FIG. 1. Feynman diagrams for β decay with double internal bremsstrahlung. Four-momenta are indicated in parentheses.

[7] P. S. Jastram and J. C. Vanderleeden, preceding paper, Phys. Rev. C **1**, 1036 (1970).

[8] L. Spruch and W. Gold, Phys. Rev. **113**, 1060 (1959); R. R. Lewis and G. W. Ford, *ibid.* **107**, 756 (1957); A. Pytte, *ibid.* **107**, 1681 (1957).

where P_β is the β spectrum defined in Eq. (2), k and E are the respective energies of the photon and electron, $d\Omega$ and $d\Omega_e$ their respective solid angles, and B is given by

$$B=p[(E+k)p^2(1-\lambda^2)+k^2(E-p\lambda)]/[16\pi^3k(E+k)(E-p\lambda)^2((E+k)^2-1)^{1/2}], \quad (12)$$

where

$$p^2=E^2-1$$

and

$$\lambda=\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}.$$

For comparison with the experimental findings of Jastram and Vanderleeden,[7] the following integrals of $dP_{\beta\gamma\gamma}$ and $dP_{\beta\gamma}$ are needed:

$$W_{\mathrm{DIB}}=\alpha^2 d\Omega_1 d\Omega_2 dk\int_{k_{\min}}^{k-k_{\min}}dk_1\int_1^{E_0-k}dE\int d\Omega_e P_\beta(E+k)[A+A^{\sim}]F, \quad (13)$$

where

$$k=k_1+k_2,$$

and

$$W_{\mathrm{IB}}=\alpha d\Omega dk\int_1^{E_0-k}dE\int d\Omega_e P_\beta(E+k)B. \quad (14)$$

The quantity $k_{\min}$ appearing in Eq. (13) denotes an experimental cutoff energy.

Jastram and Vanderleeden[7] investigate the cases $\lambda_{12}=0$ and -1, and compare the ratio $W_{\mathrm{DIB}}/W_{\mathrm{IB}}$ for various values of k [where k has the same meaning in Eqs. (13) and (14)] with their experimental findings. They also compare the relative probability of DIB with the photons at 180° (to one another) and DIB with the photons at 90° (i.e., the integral of W_{DIB} over k for $\lambda_{12}=-1$ divided by the integral of W_{DIB} over k for $\lambda_{12}=0$) with their experimental values.

One final point should be included in the discussion of the comparison of theory and experiment. Of the two β decays investigated by Jastram and Vanderleeden[7] one (P^{32}) is an allowed transition, while the other (Y^{90}) is first forbidden of unique shape. Since the DIB and IB correlation function Eqs. (10) and (11), respectively, were derived under the assumption of an allowed β decay, we need to consider whether the general form of Eqs. (10) and (11) also applies to forbidden β decays. This question has been discussed by Chang and Falkoff[9] for the case of single internal bremsstrahlung [Eq. (11)]. Using second-order perturbation theory they derive the IB correlation functions for the case of first and second-forbidden β decays. They find that, for k small compared with E, their forbidden β decay correlation functions reduce to Eq. (11) with the allowed β spectrum [Eq. (2)] replaced by the β spectrum appropriate to the degree of forbiddenness. Since the intensity is greatest at the low-energy end of the radiation spectrum and goes rapidly to zero as k approaches E_0, the condition that k be small compared with E is *a posteriori* satisfied for most of the photons.

This fact, that the single internal-bremsstrahlung transition probability is equal (except for large k in forbidden transitions) to the product of the probability of β decay and the probability of accompanying radiation, demonstrates that the β decay and radiative processes are (except for large k in forbidden transitions) independent.[10] By a separate second-order perturbation-theory calculation we have verified that the terms multiplying $P_\beta(E+k_1+k_2)$ in Eq. (10) do indeed represent the conditional probability that a β decay electron of energy $E+k_1+k_2$ will simultaneously radiate two photons of energies k_1 and k_2. Thus for DIB with allowed β decay, the β decay and radiative processes are independent, just as they were for IB with allowed β decay [Eq. (11)]. It therefore seems reasonable to assume that for DIB with forbidden β decay, this independence of the β decay and radiative processes will continue to hold (except possibly for large k).

The theoretical predictions for the Y^{90} transition presented in Ref. 7 are calculated on the basis of this assumption. That is, Eq. (10) and Eq. (11) are used, but with the allowed form of $P_\beta(E+k)$ as defined in Eq. (2) replaced by the first-forbidden unique-shape form of $P_\beta(E+k)$ given by

$$P_\beta(E)=\mathrm{const}(E_0-E)^2E(E^2-1)^{1/2}[(E_0-E)^2+E^2-1]. \quad (15)$$

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[9] C. S. W. Chang and D. L. Falkoff, Phys. Rev. **76**, 365 (1949).

[10] It is important to note that this "independence" applies only when the direction of the antineutrino is not observed. That is, the more general three-particle correlation function $dP_{\nu\beta\gamma}$ can *not* be obtained from Eq. (11) by replacing P_β by the electron-antineutrino angular-correlation function $P_{\beta\nu}d\Omega_\nu$.