Beta Decay of Na^{25} and Al^{29}†

A. D. W. JONES, J. A. BECKER, R. E. MCDONALD, AND A. R. POLETTI*
Lockheed Palo Alto Research Laboratory, Palo Alto, California 94304
(Received 30 October 1969)

The β-decay modes of Na^{25} and Al^{29} have been studied by observation of delayed γ rays with a Ge(Li) γ-ray spectrometer system. The nuclei were produced via the $Na^{23}(t, p)Na^{25}$ and $Al^{27}(t, p)Al^{29}$ reactions, respectively, at the incident triton energy of 2.7 MeV. For Na^{25}, the previously reported β branches to the 0.975- and 1.612-MeV states of Mg^{25} are confirmed, while new branches were found to the states at 1.965 and 2.801 MeV. Decay modes (labeled by the final-state energy in Mg^{25}) and their relative intensities, assuming a ground-state branch of 65%, are 0.975 MeV, 25.5%; 1.612 MeV, 8.8%; 1.965 MeV, 0.4%; and 2.801 MeV, 0.3%. Accurate excitation energies in Mg^{25} deduced from the γ-ray pulse-height distribution are (in keV) 585.9±0.4, 975.3±0.3, 1612.0±0.4, 1965.2±0.9, and 2801.0±0.7. In the decay of Al^{29}, previously reported branches to the 1.273- and 2.427-MeV states of Si^{29} were confirmed, while a new branch is reported to the state at 2.028 MeV. Assuming zero ground-state branching, the decay modes with their relative intensities are 1.273 MeV, 89.1%; 2.028 MeV, 4.1%; and 2.426 MeV, 6.8%. The state at 3.069 MeV is populated with an intensity ≤0.1%. Excitation energies in keV of 1273.2±0.8, 2028.2±0.8, and 2426.3±0.8 have been obtained for these states in Si^{29}. For both nuclei, experimentally observed logft values have been compared with nuclear-model predictions. An adequate description of the Na^{25} β decay in terms of the Nilsson model is obtained by introducing mixing of the two $K=\frac{1}{2}$ bands based on neutrons in orbits 9 and 11 in Mg^{25}. The $J^{\pi}=\frac{1}{2}^{+}$ states of these bands are at 0.586 and 2.562 MeV, respectively. The collective-model interpretation of Si^{29} is strengthened by the observation of a β branch to the 2.028 MeV level in Si^{29}; quantitative agreement, however, with the Al^{29} β-decay modes could not readily be obtained using the Nilsson-model wave functions. Possible spherical-shell-model configurations to explain the decay modes are suggested. The β-decay strengths deduced from these observations for Na^{25} and Al^{29} are compared with analogous γ-ray strengths observed in Al^{25} and P^{29}, respectively.

I. INTRODUCTION

THE availability of a triton beam from the Lockheed Research Laboratory Van de Graaff accelerator[1] affords an opportunity to produce some β unstable nuclei which otherwise could only be formed by methods that would also produce a great deal of other unwanted activity. Examination of the radiation with a Ge(Li) γ-ray spectrometer of high intrinsic resolution is a powerful technique for studying the properties of neutron-rich nuclei that lie off the stability line. In particular, through observation of delayed γ radiation, we can study the β decay of these nuclei; in the present report we describe such an investigation of the decay properties[2] of Na^{25} and Al^{29}.

Na^{25} decays by β^- emission with a half-life of 60 sec to Mg^{25}, a nucleus whose properties have been successfully described[3] by the strong coupling unified model of Nilsson.[4] We attempt to interpret our results for Na^{25} in the light of this model. The nuclide Al^{29} is also a β^- emitter decaying with a half-life of 6.5 min. In this case the daughter nucleus Si^{29} is in the middle of the s-d shell which is a region where the unified model has not been too successful in interpreting the nuclear properties. The simplest shell-model configurations for Si^{29} consist of a closed $1d_{5/2}$ shell plus one valence neutron, whereas the parent nucleus Al^{29} can be considered to have a ground-state structure of two particles and one hole with respect to the Si^{28} closed shell. The β decay of Al^{29} is discussed in terms of this model and also the unified model.

The experimental basis of our work is the investigation of the delayed γ rays observed after activation. No attempt is made to measure or detect β branches to the ground states of the daughter nuclei. The strength of such ground-state transitions is taken from the work of other investigators.[5,6] In Sec. II we describe the experimental procedure and results, while in Sec. III we discuss these results in terms of both the Nilsson and spherical shell models. In Sec. IV the β-decay strengths are compared with strengths of the corresponding $\Delta T=1$, $M1$ γ-ray transitions.

II. EXPERIMENT

A. Decay of Al^{29}

The Lockheed Van de Graaff accelerator was used to produce the Al^{29} activity by the $Al^{27}(t, p)Al^{29}$ reaction ($Q=8.68$ MeV).[2] A 0.001-in. aluminum foil mounted

† Research supported in part by the Lockheed Independent Research program and in part by the U.S. Office of Naval Research.

* Now at Department of Physics, University of Auckland, Auckland, New Zealand.

[1] L. F. Chase, Jr., in *Nuclear Research With Low Energy Accelerators*, edited by J. B. Marion and D. M. Van Patter (Academic Press Inc., New York, 1967).

[2] A compilation of properties of these nuclei is contained in the summary of Endt and van der Leun [P. M. Endt and C. van der Leun, Nucl. Phys. **A105**, 1 (1967)].

[3] A. E. Litherland, H. McManus, E. B. Paul, D. A. Bromley, and H. E. Gove, Can. J. Phys. **36**, 378 (1958); A. E. Litherland, in *The Structure of Low-Medium Mass Nuclei*, edited by J. P. Davidson (University of Kansas Press, Lawrence, Kansas, 1968).

[4] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, No. 16 (1955).

[5] D. Maeder and P. Stähelin, Helv. Phys. Acta **28**, 193 (1955).

[6] L. Seidlitz, E. Bleuler, and D. J. Tendam, Phys. Rev. **76**, 861(L) (1949).

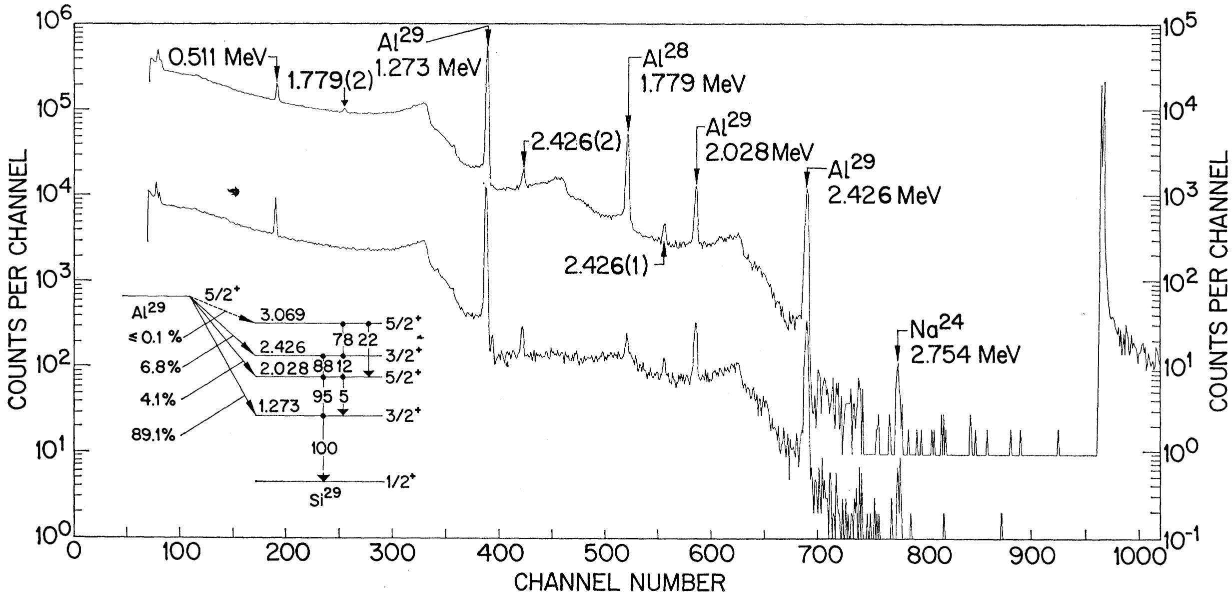

FIG. 1. Delayed γ-ray spectra obtained after six 13.3-min bombardments of a thick Al^{27} target with 2.7-MeV tritons as described in the text. The top spectrum, for which the right-hand scale applies, was accumulated during a 6.6-min time period, initiated 6.6 min after each bombardment was completed. All the prominent γ rays are assigned to the parent nuclei as shown. The weak line at 2.754 MeV arises from the long-lived Na^{24} isotope formed by the $Na^{23}(t, d)Na^{24}$ reaction arising from a slight Na^{23} contamination. The peak in channel 965 is a pulser peak used to stabilize the electronic gain. The bottom spectrum, for which the left-hand scale applies, was accumulated during a 6.6-min time period initiated 19.8 min after each bombardment. The different decay rate of the γ rays associated with the parent nucleus Al^{29} ($t_{1/2}=6.52$ min) and the 1.779-MeV γ ray associated with Al^{28} ($t_{1/2}=2.3$ min) can be seen. The Al^{28} is mostly produced via the $Al^{27}(t, d)Al^{28}$ reaction, although it is also produced via the $Al^{27}(n, \gamma)Al^{28}$ reaction occurring in Al material near the target. The deduced β-ray branches are summarized, as shown, together with the γ-ray branching ratios used to calculate the results. The level scheme for Mg^{25} shown in the insert is taken from Ref. 2 and Ref. 15.

on a thick tantalum backing was bombarded with a beam (~500 nA) of 2.7-MeV tritons for 13.3 min. After a delay of 6.6 min, four γ-ray spectra were accumulated sequentially, each for 6.6 min. After a further delay of 10 sec, this sequence of operations was repeated. γ-ray spectra were accumulated by the use of conventional electronics and a 4096-channel analog-to-digital converter interfaced to a Systems Engineering Laboratories 810A computer. The detector was a 22.5-cm^3 Ge(Li) diode, located with its front face 10 cm from the source. The gain of the γ-ray spectrometer was stabilized by a digital stabilizer and pulser combination. Radioactive sources of Eu^{152}, RaTh, Co^{56}, and Co^{60} were used for efficiency determinations and for energy calibration of the spectrometer.[7]

The γ-ray spectra obtained in the first and third counting intervals are shown in Fig. 1. This data was obtained during six complete bombardment cycles. The various transitions were identified from the calculated energies of the peaks, the relative efficiency of the γ-ray spectrometer, and also from the half-life of the γ rays obtained from the decay curves. The half-lives of all lines assigned to the decay of Al^{29} were consistent with the accepted half-life for this nucleus of 6.52 ± 0.05 min.[6,8] We have summarized our measurements of the energies of the observed γ rays and listed these results in Table I. In Table I we have also identified the levels involved in these transitions in Si^{29}, and compared our measurements with the results of other investigators. The transition energies are all seen to be in excellent agreement. Finally, we have deduced weighted averages for excitation energies in Si^{29} from the measured γ-ray energies. The level scheme of Si^{29} included in Fig. 1 was taken from Ref. 2. Other γ rays whose presence in Fig. 1 may be inferred from this decay scheme are not prominent in these pulse-height distributions.

Results relevant to the β deday are summarized and presented in Table II, where we have used the previously reported result[6] that there is no allowed β branch to the ground state of Si^{29} ($J^\pi=\frac{1}{2}^+$) from the Al^{29} ground state ($J^\pi=\frac{5}{2}^+$). These calculations were carried out using the branching ratios of Ref. 2 for Si^{29} and the observed relative intensities of those γ rays listed in Table I. The log*ft* values for these transitions were calculated after Moszkowski.[9] Our results are in

[7] J. B. Marion, Nucl. Data **A4**, 301 (1968).

[8] W. J. Henderson and R. L. Doran, Phys. Rev. **56**, 123 (L) (1939); H. A. Bethe and W. J. Henderson, *ibid.* **56**, 1060 (L) (1939).

[9] S. A. Moszkowski, Phys. Rev. **82**, 35 (1951).

TABLE I. Delayed γ rays from the β decay of Al^{29}.

E_γ[a] (keV)	Excitation energy in Si^{29} (keV) a	b	c	d
2426.2±0.8	2426.3±0.8	2426.9±1.0	2425.7±0.4	2425.8±0.4
2028.2±0.8	2028.2±0.8	2031.5±1.0	2027.8±0.4	2027.9±0.4
1273.2±0.8	1273.2±0.8	1273.0±0.5	1273.3±0.2	1273.2±0.2

[a] This experiment.
[b] H. Lycklama, L. B. Hughes, and T. J. Kennett, Can. J. Phys. **45**, 1871 (1967).
[c] Reference 13.
[d] Adopted value.

TABLE II. The β decay of Al^{29}.

$E_x(Si^{29})$ (MeV)	J^π	β branching ratios[a] (%) b	c	d	e	Adopted $\log ft$[f]
1.273	$\frac{3}{2}^+$	85±9	93.8	90.1±1.0	89.1±0.7	5.1
2.028	$\frac{5}{2}^+$	≤3.8	<2	3.6±0.4	4.1±0.3	5.7
2.426	$\frac{3}{2}^+$	15±9	6.2±0.6	6.3±0.5	6.8±0.5	5.0
3.069	$\frac{5}{2}^+$			0.027±0.011	≤0.1	6.16

[a] Ground-state transition assumed to be zero. See Ref. 6.
[b] H. Roderick, O. Lönsjö, and W. E. Meyerhof, Phys. Rev. **97**, 97 (1955).
[c] Reference 10.
[d] Reference 13.
[e] Present experiment.
[f] After S. A. Moszkowski, Ref. 9.

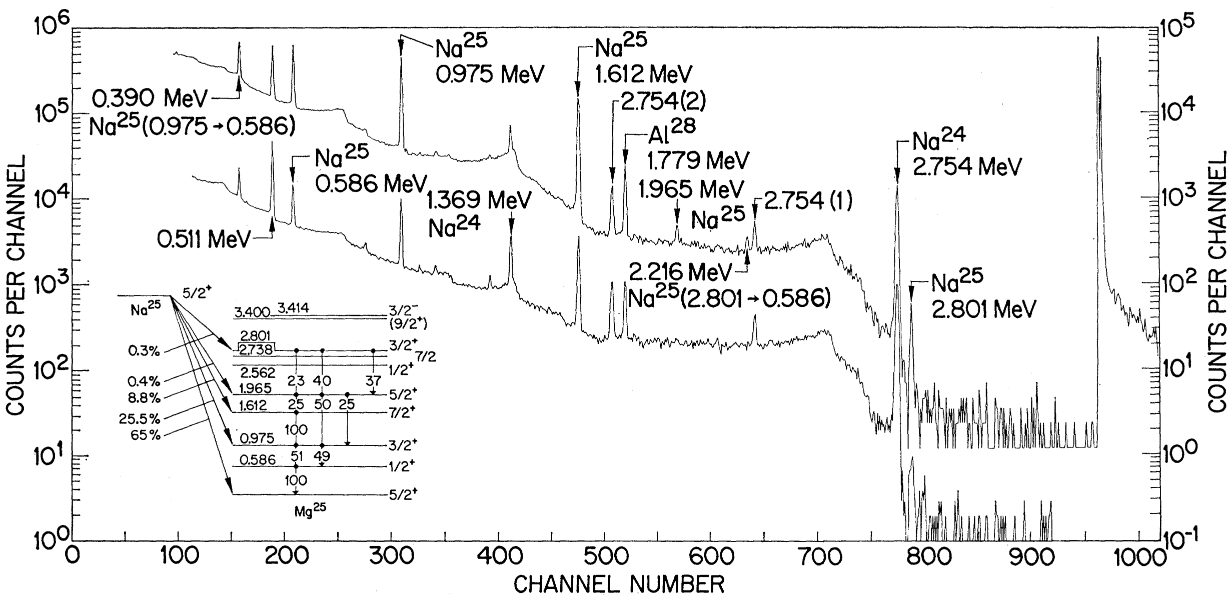

FIG. 2. Delayed γ-ray spectra obtained after 80 bombardments, each of 2-min duration, of a thick Na^{23} target with 2.7-MeV tritons as described in the text. The top spectrum for which the right-hand scale applies was accumulated during a 60-sec time period beginning 20 sec after each bombardment was completed. All the prominent γ rays are assigned to the parent nuclei as shown. The peak in channel 965 is a pulser peak which was used to stabilize the electronic gain. The bottom spectrum, for which the left-hand scale applies, was accumulated during a 60-sec time period initiated 140 secs after each bombardment. The long-lived Na^{24} decay ($t_{1/2}=15$ h) dominates this second spectrum, and to minimize its effect on the spectrum the Na target was replaced after every 10 bombardment cycles. The deduced β-decay branchings together with the γ-ray branching ratios used to calculate these values are summarized in the insert. The line labeled $E_\gamma=1.779$ MeV arises from the decay of Al^{28}, produced via the n, γ reaction in the Al material located near the target. The level scheme for Si^{29} shown in the insert is taken from Ref. 2.

good agreement with the early work of Bromley *et al.*[10] In addition, we report a branch of 4.1% to the 2.028-MeV, $J^{\pi}=\frac{5}{2}^{+}$ state in Si^{29} and an upper limit of 0.1% for a possible branch to the state at 3.069 MeV, reported[11,12] to have $J^{\pi}=\frac{5}{2}^{+}$.

The branch to the 2.028-MeV level has recently been confirmed by the results of Harris *et al.*[13] and of Hirko *et al.*,[14] while our limit of 0.1% for a branch to the 3.069-MeV level, although better than the 0.5% limit set in a recent experiment,[14] is a factor of 3 larger than the positive branch 0.03% quoted by Harris *et al.*[13]

B. Decay of Na^{25}

The experimental procedure was similar to that described in Sec. II A. The nucleus Na^{25} was produced by the $Na^{23}(t,p)Na^{25}$ reaction ($Q=7.49$ MeV) [2] at an incident triton energy of 2.7 MeV. The target was a thick piece of metallic sodium, which was freshly sliced in a helium atmosphere in the target chamber. This technique prevented excessive oxidation of the target. The time intervals for irradiation, delay, counting, and delay were 120, 20, 60, and 0.3 sec, respectively. During the experiment the Na target was replaced every hour ($\sim$10 cycles) to minimize the background radiation from the long-lived Na^{24} isotope ($t_{1/2}\approx 15$ h) produced by the $Na^{23}(t,d)Na^{24}$ reaction. In Fig. 2 we show the spectra obtained in the first and third counting intervals after 80 complete bombardment cycles. Prominent γ-ray lines observed in these spectra are identified and labeled in Fig. 2, according to isotope and energy. The γ rays associated with the Na^{25} decay are identified from their computed energies and characteristic half-lives, as was described previously. These energy determinations are summarized in Table III; also shown in this table are the γ rays assigned to transitions in Mg^{25}. A comparison of γ-ray energy measurements with other results is made here also and the agreement in general is good. The level scheme of Na^{25} included in Fig. 2 was taken from Ref. 2, except for the branching of the 2.801-MeV level where we quote the results of Ref. 15. Other γ rays whose presence may be inferred from this level scheme are either not prominent or are obscured in the pulse-height distributions, e.g., the 1.380 (1.965→0.586) transition is not resolved from the 1.369-MeV γ ray arising from the Na^{24} decay.

TABLE III. Observed γ rays in the $Na^{25}(\beta^-)Mg^{25}$ decay.

E_γ [a] (keV)	Excitation energy in Mg^{25} (in keV) a	b	c	d
2800.6±1.1	2800.8±1.1			2801.0±0.7
2215.6±0.7	2801.1±0.8			
1965.1±0.9	1965.2±0.9	1962		1965.2±0.9
1611.9±0.4	1612.0±0.4	1611	1613.7±1.5	1612.1±0.4
975.2±0.4	975.2±0.4		974.7±0.3	975.0±0.2
585.9±0.4	585.9±0.4		585.2±0.3	585.5±0.3
390.7±0.5	975.5±0.4		389.7±0.3	

[a] This experiment.
[b] R. W. Ollerhead, J. F. Sharpey-Schafer, A. J. Ferguson, and A. E. Litherland, Bull. Am. Phys. Soc. **12**, 554 (1967).
[c] P. Spilling, H. Gruppelaar, and A. M. F. Op den Kamp, Nucl. Phys. **A102**, 209 (1967).
[d] Adopted value.

TABLE IV. The β decay of Na^{25}.

E_x (MeV)	J^π	β branching ratio (%) b	c	d	logft [a] This work
0	$\frac{5}{2}^+$	65	65	65	5.3
0.586	$\frac{1}{2}^+$	3.5	<1	<1	>6.8
0.975	$\frac{3}{2}^+$	25	30	25.5±0.5	5.1
1.612	$\frac{7}{2}^+$	6.5	5	8.8±0.5	5.1
1.965	$\frac{5}{2}^+$		<1	0.4±0.1	6.2
2.801	$\frac{3}{2}^+$			0.3±0.1	5.2

[a] After S. A. Moszkowski, Ref. 9.
[b] Reference 5. The ground-state branch of 65% reported here is used to calculate branching ratios.
[c] Reference 15.
[d] This work.

The β-branching modes of Na^{25} calculated using the γ-ray branches shown in Fig. 2 and 65% for the intensity of the ground state branch[5] are summarized in Table IV, where we compare our results with those reported earlier.[5,16] We report two new branches, a 0.4% branch to the $J^{\pi}=\frac{5}{2}^{+}$ state at 1.965 MeV and a 0.3% branch to the $J^{\pi}=\frac{3}{2}^{+}$ state at 2.801 MeV. The remaining strength is distributed as shown in Table IV.

III. INTERPRETATION IN TERMS OF NUCLEAR MODELS

The spirit of the following discussion is to compare the experimentally determined β-ray branching modes with the branching modes predicted from model considerations. We consider both the unified model of Nilsson and the spherical shell model for our interpretation of the Al^{29} β decay, but restrict ourselves solely to the Nilsson model for interpreting the Na^{25} β decay.

A. $Al^{29}(\beta^-)Si^{29}$ Decay

1. Unified-Model Interpretation

The ground state of Al^{29} has recently been interpreted[17] as being the head of a $K=\frac{5}{2}^{+}$ band based on

[10] D. A. Bromley, H. E. Gove, E. G. Paul, A. E. Litherland, and E. Almqvist, Can. J. Phys. **35**, 1042 (1957).
[11] J. A. Becker, L. F. Chase, Jr., and R. E. McDonald, Phys. Rev. **157**, 967 (1967).
[12] A. J. Ferguson, P. J. M. Smulders, T. K. Alexander, C. Broude, J. A. Kuehner, A. E. Litherland, and R. W. Ollerhead, J. Phys. Soc. Japan **23**, 1 (S) (1968).
[13] W. R. Harris, K. Nagatani, and D. Alburger, Bull. Am. Phys. Soc. **14**, 18 (1969); Phys. Rev. **187**, 1445 (1969).
[14] R. G. Hirko, R. A. Lindgren, A. J. Howard, J. G. Pronko, M. W. Sachs, C. A. Whitten, Jr., and D. A. Bromley, Bull. Am. Phys. Soc. **13**, 1371 (1968).
[15] B. D. Sowerby, and G. J. McCallum, Nucl. Phys. **A112**, 453 (1968).
[16] H. E. Gove, G. A. Bartholomew, E. B. Paul, and A. E. Litherland, Nucl. Phys. **2**, 132 (1956); **3**, 344 (1957).
[17] A. D. W. Jones, J. A. Becker, and R. E. McDonald (to be published).

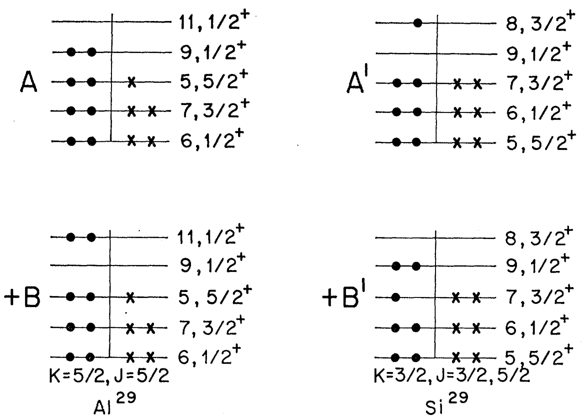

FIG. 3. Schematic representation of the Nilsson wave functions of the ground state of Al^{29} and the $J^\pi=\frac{3}{2}^+$ and $\frac{5}{2}^+$ state of the $K=\frac{3}{2}$ band based on the 1.273-MeV state of Si^{29}. To explain the observed β decay between Al^{29} and the $K=\frac{3}{2}$ band in Si^{29}, a B' component must be mixed with the major $K=\frac{3}{2}$ component shown as A'.

Nilsson orbit number 5. The deformation of the band is prolate and evidence suggests that the deformation parameter[4] η is of the order of $+3$. Within the framework of the same model for Si^{29}, the evidence[18] favors this nucleus having an oblate deformation with $\eta=-3$. In this interpretation, the states at 0-, 2.426-, and 2.028-MeV are described as the $J^\pi=\frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$ members of a $K=\frac{1}{2}$ band, respectively, based on a neutron in Nilsson orbit number 9, while the 1.273- and 3.069-MeV levels are interpreted as the $J^\pi=\frac{3}{2}^+$ and $\frac{5}{2}^+$ members of a $K=\frac{3}{2}$ band, respectively, based on a neutron in Nilsson orbit number 8. As previously noted,[18] it is not possible to properly calculate β-ray transition probabilities between bands of different deformation using the Nilsson wave functions because the core wave functions are different. We will, however, apply the β-decay selection rules[19] applicable to deformed nuclei to learn something about these states. Since β-ray transitions with $\Delta K=2$ are forbidden, transitions between the Al^{29} ground state $(J^\pi, K)=(\frac{5}{2}^+, \frac{5}{2})$ and the $(\frac{3}{2}^+, \frac{1}{2})$ and $(\frac{5}{2}^+, \frac{1}{2})$ states in Si^{29} at 2.426- and 2.028-MeV, respectively, are not allowed. Branches are, however, observed to both these states with log*ft* values of 5.0 and 5.7, respectively. This apparent discrepancy with the strong coupling rotational-model interpretation of the states of Al^{29} and Si^{29} is best removed by including rotational particle coupling in the model description[20]; the states in the $K=\frac{1}{2}$ and $K=\frac{3}{2}$ bands in Si^{29} will then mix and the 2.426- and 2.028-MeV states will have some $K=\frac{3}{2}$ component in their wave functions. Previous to this work, this explanation, proposed earlier by Bromley *et al.*[18] was rather tentative because it implied a β branch to the 2.028-MeV level; no branch had been previously reported, however, and the apparent K purity of this state was of some concern. This 4.1% branch reported here and confirmed elsewhere[13,14] makes a consistent interpretation of the Al^{29} β decay possible. It is noted that the wave function of the Al^{29} ground state is expected to be pure $K=\frac{5}{2}$ since there are no $K=\frac{3}{2}$ orbitals nearby to mix with it.

It is instructive to consider the Nilsson wave functions[4] that would allow β-ray transitions between the $(J^\pi, K)=(\frac{5}{2}^+, \frac{5}{2})Al^{29}$ ground state and the states in Si^{29} based on the $K=\frac{3}{2}$ band head at 1.273 MeV. Illustrated in Fig. 3 is a representation of the Al^{29} ground-state wave function denoted by components A and B and the Si^{29} $K=\frac{3}{2}$ wave function with components A' and B'. Component A' probably contributes mostly to the states of the $K=\frac{3}{2}$ band built on the first excited state of Si^{29}, but because of the single-particle nature of the β-decay operator, no decay is possible from the Al^{29} ground state (A or B components) to the A' component. Component B' represents a wave function with $T=T_z=\frac{1}{2}$, $K=\frac{3}{2}$ to which the decay can proceed from component A of the Al^{29} wave function. It is noted that because of the need to antisymmetrize component B' taking into account isospin, the wave function schematically represented as B' does not completely define the state and, in fact, two separate states exist with $T=T_z=\frac{1}{2}$ which can be viewed as a hole in orbit 7 coupled, respectively, to the $T=0$ and $T=1$ coupling of two particles in orbit 9. Of the components of these two wave functions, however, only the B' component of Fig. 3 is connected to the Al^{29} wave function by the β-decay operator. Thus, a possible way of explaining the decay is to postulate that the Si^{29} wave functions for the $J^\pi=\frac{3}{2}^+$, $\frac{5}{2}^+$ states at 2.426- and 2.028-MeV, respectively, contain a $K=\frac{3}{2}$ component based on a hole in Nilsson orbit number 7.

For pure Gamow-Teller decay which is applicable in the present case because the β decay of Al^{29} to Si^{29} involves a change of isotopic spin, the relative *ft* values to members of the same rotational band in the final nucleus is given by the ratio of vector coupling coefficients. For a transition from a state J_1K_1 to states J_2 and J_3 in band K_2, this ratio is

$$\frac{ft(J_1K_1\rightarrow J_2K_2)}{ft(J_1K_1\rightarrow J_3K_2)}=\frac{(J_1\,K_1\,1\,K_2-K_1\,|\,J_3K_2)^2}{(J_1\,K_1\,1\,K_2-K_1\,|\,J_2K_2)^2}. \quad (1)$$

This implies that transitions from Al^{29} to the $J^\pi=\frac{5}{2}^+$ and $J^\pi=\frac{3}{2}^+$ members of the $K=\frac{3}{2}$ band in Si^{29} should be in the ratio of 7/3, the $J^\pi=\frac{5}{2}^+$ transition having the larger *ft* value. The ratio observed is 5.0, which is more than twice the predicted value. For the transitions to the $J^\pi=\frac{3}{2}^+$ and $J^\pi=\frac{5}{2}^+$ states of the $K=\frac{1}{2}$ band, the experimentally observed ratio is 10. Transitions to this band, of course, occur only through the $K=\frac{3}{2}$ impurity in these states. The similar *ft* values for transitions to the two $J^\pi=\frac{3}{2}^+$ states suggests that the $K=\frac{1}{2}$ and $K=\frac{3}{2}$ bands are strongly mixed, with the $K=\frac{3}{2}$ strength being

[18] D. A. Bromley, H. E. Gove, and A. E. Litherland, Can. J. Phys. **35**, 1057 (1957).
[19] G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, No. 9 (1955).
[20] A. K. Kerman, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **30**, No. 15 (1956).

distributed almost equally among the two bands. However, even by considering mixed wave functions as described above for the states in Si^{29}, it is not possible to reproduce the observed relative strengths of the ft values to the $\frac{5}{2}^+$ and $\frac{3}{2}^+$ members of the same band if the wave functions are restricted to those shown in Fig. 3.

2. *Shell-Model Interpretation*

The ground state of Al^{29} ($J^\pi=\frac{5}{2}^+$) can be interpreted as being a two-particle one-hole state with respect to the Si^{28} core. We write the ground-state wave function of Al^{29} as

$$\Psi_{T_z=3/2}{}^{T=3/2}(J=\tfrac{5}{2}) = [A\Phi(s_{1/2})_0{}^2{}_{T_z=1}{}^{T=1}+B\Phi(d_{3/2})_0{}^2{}_{T_z=1}{}^{T=1}] \times\Phi(d_{5/2})^{-1}{}_{T_z=1/2}{}^{T=1/2}. \quad (2)$$

In this model, the ground and 1.273-MeV states in Si^{29} would be interpreted as single-particle $2s_{1/2}$ and $1d_{3/2}$ states. The β decay between the Al^{29} ground state and the Si^{29} 1.273-MeV state is then allowed through the B component of Eq. (2); this decay is observed with a logft value of 5.1.

To explain a $J^\pi=\frac{5}{2}^+$ state in Si^{29} we need a wave function with a hole in the closed $d_{5/2}$ subshell. We can then write two wave functions for these states if we restrict the particles in the $s_{1/2}$ and $d_{3/2}$ shells to be zero coupled pairs:

$$\Psi_{T_z=1/2}{}^{T=1/2}(J=\tfrac{5}{2}) = (\sqrt{\tfrac{1}{3}})\Phi(d_{5/2})^{-1}{}_{T_z=1/2}{}^{T=1/2} \times\Phi(s_{1/2})_0{}^2{}_{T_z=0}{}^{T=1}-(\sqrt{\tfrac{2}{3}})\Phi(d_{5/2})^{-1}{}_{T_z=-1/2}{}^{T=1/2} \times\Phi(s_{1/2})_0{}^2{}_{T_z=1}{}^{T=1}, \quad (3)$$

$$\psi_{T_z=1/2}{}^{T=1/2}(J=\tfrac{5}{2}) = (\sqrt{\tfrac{1}{3}})\Phi(d_{5/2})^{-1}{}_{T_z=1/2}{}^{T=1/2} \times\Phi(d_{3/2})_0{}^2{}_{T_z=0}{}^{T=1}-(\sqrt{\tfrac{2}{3}})\Phi(d_{5/2})^{-1}{}_{T_z=-1/2}{}^{T=1/2} \times\Phi(d_{3/2})_0{}^2{}_{T_z=1}{}^{T=1}. \quad (4)$$

We assign the wave functions of Eqs. (3) and (4) to the states at 2.028- and 3.069-MeV, respectively. The β decay from Al^{29} can proceed to both wave functions.

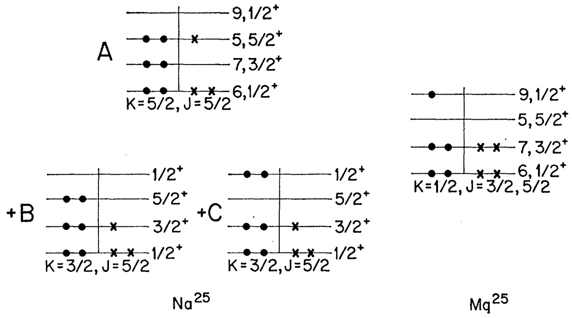

FIG. 4. Schematic representation of the Nilsson wave functions of the ground state of Na^{25} and the $J^\pi=\frac{3}{2}^+$ and $\frac{5}{2}^+$ state of the $K=\frac{1}{2}$ band based on the 0.586-MeV state of Mg^{25}. The β decay can only proceed from the C component of the Na^{25} wave function to the wave function shown for Mg^{25}.

Population of these states by a direct reaction mechanism in the $Si^{28}(d,p)Si^{29}$ reaction can only proceed through a two-particle–two-hole component in the Si^{28} ground state, and this explains the weak strength of the observed direct-reaction cross sections.[2] The major features of the γ decay are similarly explained—the 2.028-MeV state is expected to decay strongly to the ground state; experimentally it is found to decay 95% this way. The 3.069-MeV state is expected to decay to the 1.273-MeV level. Experimentally a 78% branch is observed to this level.[11]

For the Si^{29} $J=\frac{3}{2}$ states, we can write the following wave functions with $T=T_z=\frac{1}{2}$:

$$\Psi_{T_z=1/2}{}^{T=1/2}(J=\tfrac{3}{2}) = \Phi[(2s_{1/2})_0{}^2(d_{5/2})_0{}^{-2}]_{T_z=0}{}^{T=0} \times\Phi[(d_{3/2})]_{T_z=1/2}{}^{T=1/2}, \quad (5a)$$

$$\Psi_{T_z=1/2}{}^{T=1/2}(J=\tfrac{3}{2}) = (\sqrt{\tfrac{1}{3}})\Phi[(2s_{1/2})_0{}^2(d_{5/2})_0{}^{-2}]_{T_z=0}{}^{T=1} \times\Phi[(d_{3/2})]_{T_z=1/2}{}^{T=1/2}-(\sqrt{\tfrac{2}{3}})\Phi[(2s_{1/2})_0{}^2 \times(d_{5/2})_0{}^{-2}]_{T_z=1}{}^{T=1}\Phi[(d_{3/2})]_{T_z=-1/2}{}^{T=1/2}, \quad (5b)$$

$$\Psi_{T_z=1/2}{}^{T=1/2}(J=\tfrac{3}{2}) = \Phi(d_{5/2})^{-1}{}_{T_z=1/2}{}^{T=1/2}\Phi[(s_{1/2})_1{}^2]_{T_z=0}{}^{T=0}. \quad (5c)$$

The 2.43-MeV level is populated in an allowed β decay (logft=5.0), but the direct-reaction cross section observed in the $Si^{28}(d,p)Si^{29}$ reaction is extremely weak. Of these wave functions the one given in Eq. (5c) satisfies both requirements and we suggest it for the dominant configuration of this state. This configuration also is compatible with the γ-ray decay of this state, 88% to the ground state.

B. $Na^{25}(\beta^-)Mg^{25}$ Decay

1. *Unified-Model Interpretation*

We turn now to the question of how well the Na^{25} β branches may be described in terms of the unified-model[4] interpretation of the $A=25$ system, first given by Litherland *et al.*[3] If we assume that Na^{25} has a prolate deformation similar to that observed for Mg^{25} then the ground state would be expected to have spin and parity $J^\pi=\frac{3}{2}^+$, corresponding to the head of a $K=\frac{3}{2}$ band based on a proton in Nilsson orbit number 7 (see Fig. 4). The Na^{25} ground-state spin and parity is, however, $\frac{5}{2}^+$.[5] An explanation for this apparent anomaly was given by Litherland *et al.*[3] and later discussed by Morrison *et al.*[21] They assume that the $J^\pi=\frac{5}{2}^+$ state of the $K=\frac{3}{2}$ band discussed above, in the unperturbed position, is close in energy to the $J^\pi=\frac{5}{2}^+$ band head based on Nilsson orbit 5. The interaction of these states forces the $J^\pi=\frac{5}{2}^+$ state of the $K=\frac{3}{2}$ band below the $J=\frac{3}{2}$ band head, and so the ground state is predicted to be primarily $K=\frac{3}{2}$ (orbit 7) but with an admixture of $K=\frac{5}{2}$ (orbit 5).

[21] G. C. Morrison, D. H. Youngblood, R. C. Bearse, and R. E. Segel, Phys. Rev. **174**, 1366 (1968).

In the β decay of Na^{25}, transitions are observed to states identified as members of $K=\frac{5}{2}$ and $K=\frac{1}{2}$ bands in Mg^{25}. Since the $K=\frac{1}{2}$ band in Mg^{25} is expected to be unmixed with $K=\frac{5}{2}$ levels,[3] the observation of allowed β transitions to states with both $K=\frac{5}{2}$ and $K=\frac{1}{2}$ requires the presence of a $K=\frac{3}{2}$ component in the ground-state wave function of Na^{25}. Possible wave functions for the Na^{25} ground state are shown in Fig. 4. No β transition is possible from component A of the Na^{25} wave function to states based on the $(J^\pi, K)=(\frac{1}{2}+, \frac{1}{2})$ 0.585-MeV state in Mg^{25} because selection rules[19] forbid $\Delta K=2$ β-decay transitions. Component B represents the $K=\frac{3}{2}$ wave function that would be the ground state if the Nilsson orbitals were filled as expected, and one can immediately see that here also no transition is possible to states in the Mg^{25} $K=\frac{1}{2}$ band (orbit 9), because this involves changing the orbits of two nucleons. Das Gupta[22] has recently reviewed the situation presented by this case and has concluded that the decay of Na^{25} requires one to modify the Nilsson approach of rotational symmetry and introduce some measure of nuclear asymmetry; K is not a good quantum number in this model and states with $\Delta K=2$ can mix. The β decay proceeds through the mixed components; qualitative features of the β-decay data are explained. Quantitatively the agreement obtained is not very good. An alternative explanation of the decay within the framework of a symmetric nuclear model is still possible if we assume that there is a contribution to the Na^{25} ground-state wave function such as denoted by C in Fig. 4. This component has $K=\frac{3}{2}$ and a β transition is now possible to the $K=\frac{1}{2}$ band in Mg^{25} within the β-decay selection rules for deformed nuclei as formulated by Alaga *et al.*[19] The relative strengths of the B and C components may be estimated from direct-reaction studies with the neighboring even-even nucleus, Mg^{26} as target. If we take the ground-state wave function of this nucleus to be given by similar B and C components as shown for Na^{25} in Fig. 4, with the provision that an extra proton fills orbit 7 $(K=\frac{3}{2})$, the ratio of C to B may be deduced from analysis of the cross sections measured in neutron pickup direct-reaction investigations. Such information has been summarized recently by Dehnhard and Yntema.[23] They collate information on the p, d, d, t, and He^3, α reactions on Mg^{26}; summing the spectroscopic strengths to members of the $K=\frac{5}{2}$ ground-state band and the $K=\frac{1}{2}$ band based on the 0.585-MeV Mg^{25} state results in a ratio of $C^2/B^2 \approx 0.20$. If we take this value as a guideline for similar components in the Na^{25} wave function, then a $K=\frac{3}{2}$ component of the form given by C in Fig. 4 contributes significantly to the ground-state wave function.

Quantitative calculations of the ft values under discussion according to the Nilsson model are summarized in Table V. The calculations were done with the various components of the Na^{25} wave functions A, B, and C as in Fig. 4. The major β-decay branch occurs between component B of the Na^{25} wave function and the $J^\pi=\frac{5}{2}+$ ground state of Mg^{25}. As can be seen from Table V, agreement with experiment is obtained with a large value of B^2, confirming that this is the major term in the Na^{25} ground-state wave function. The ratio of the ft values to the ground and 1.61-MeV states should be given by vector coupling coefficients as mentioned previously in the discussion of the $Al^{29}(\beta^-)Si^{29}$ decay. In this case the calculated ratio is 0.4, whereas the experimental value is $\sim$0.6. Previous to the present work the experimental ratio was quoted as $\sim$1.3.[15] The agreement between experiment and theory is now considerably better, but it still depends on the β-decay branch to the ground state of Mg^{25} reported by Maeder and Stähelin[5] which has not been remeasured.

We consider next the β decays to the levels at 0.98- and 1.96-MeV, both members of the $K=\frac{1}{2}$ band with $J^\pi=\frac{3}{2}+$ and $J^\pi=\frac{5}{2}+$, respectively. These decays take place via the C component of the Na^{25} wave function. The measured logft to the 0.98-MeV level is 5.1, while the logft to the 1.96-MeV level is 6.2, an order of magnitude weaker. According to the Nilsson model, this ratio of decay strengths should be $\frac{7}{8}$ [see Eq. (1)] and is independent of deformation. One reasonable way to explain this failure is to postulate mixing of the two $K=\frac{1}{2}$ bands in Mg^{25} based on neutrons in Nilsson orbitals 9 and 11 identified with the $J^\pi=\frac{1}{2}+$ states at 0.586- and 2.562-MeV, respectively. For purposes of further discussion we note that our calculations require an amplitude $C^2=0.06$ to reproduce the β-decay strength to the 0.98-MeV level if it is assumed to be a pure $J^\pi=\frac{3}{2}+$, $K=\frac{1}{2}$ orbit-9 level.

The Mg^{25} 2.801-MeV level has a β branch to it, with logft=5.2. Since this level is interpreted as the $J^\pi=\frac{3}{2}+$ member of the $K=\frac{1}{2}$ band based on a neutron in orbit 11, the observed β decay requires another component, D, in the Na^{25} ground-state wave function. This component is similar to C except that two neutrons are in orbit 11 rather than orbit 9, orbit 11 being next highest in energy

TABLE V. The logft values for $Na^{25}(\beta^-)Mg^{25}$ calculated with no band mixing.

Transition[a]	Final state K(orbit)J^π	Calc logft with $\eta=4$		Expt logft
$B \rightarrow 0$	$\frac{5}{2}(5)\frac{5}{2}+$	5.0[b]	5.3[c]	5.3
$B \rightarrow 1.61$ MeV	$\frac{5}{2}(5)\frac{7}{2}+$	4.6[b]	4.9[c]	5.1
$C \rightarrow 0.98$ MeV	$\frac{1}{2}(9)\frac{3}{2}+$	3.9[d]	5.1[e]	5.1
$C \rightarrow 1.96$ MeV	$\frac{1}{2}(9)\frac{5}{2}+$	3.8[d]	5.0[e]	6.2
$D \rightarrow 2.80$ MeV	$\frac{1}{2}(11)\frac{3}{2}+$	4.5[f]	5.2[g]	5.2

[a] Refer to Fig. 4 for identification of B and C. D is defined in text.
[b] Calculated with $B^2=1$.
[c] Calculated with $B^2=0.50$.
[d] Calculated with $C^2=1$.
[e] Calculated with $C^2=0.06$.
[f] Calculated with $D^2=1$.
[g] Calculated with $D^2=0.21$.

[22] S. Das Gupta, Nucl. Phys. **A97**, 481 (1967).
[23] D. Dehnhard and J. L. Yntema, Phys. Rev. **160**, 964 (1967).

after orbit 9. The observed β-decay strength is reproduced with an amplitude $D^2=0.21$ (see Table V).

The analysis of the β-decay branches therefore yields a ratio $D^2/C^2=3.5$. We can also obtain an estimate from direct-reaction studies[23] of this ratio for the same components in the Mg^{26} ground-state wave function, as previously done for our estimate of C^2/B^2, with the result $D^2/C^2=0.5$. There is therefore an inconsistency, but we note that the latter ratio of 0.5 is more in accord with what one would expect from the relative positions of the energy levels.

We can attempt to remove this discrepancy by considering mixing of the two $K=\frac{1}{2}$ bands based on orbits 9 and 11 in Mg^{25}. We write wave functions as follows:

$$\Psi(0.98\text{ MeV}, J^\pi=\tfrac{3}{2}^+)=\alpha(K=\tfrac{1}{2}, 9)-\beta(K=\tfrac{1}{2}, 11), \tag{6a}$$

$$\Psi(2.80\text{ MeV}, J^\pi=\tfrac{3}{2}^+)=\beta(K=\tfrac{1}{2}, 9)+\alpha(K=\tfrac{1}{2}, 11), \tag{6b}$$

$$\Psi(1.96\text{ MeV}, J^\pi=\tfrac{5}{2}^+)=\alpha_1(K=\tfrac{1}{2}, 9)-\beta_1(K=\tfrac{1}{2}, 11). \tag{6c}$$

Taking the observed ratio of the β decay to the $J^\pi=\frac{3}{2}^+$ states and assuming $2D^2=C^2$ as suggested by the direct-reaction evidence, we extract values of $\alpha^2=0.84$ and $\beta^2=0.16$: the observed logft values require that $C^2=0.10$ and $D^2=0.05$. With these values of C^2 and D^2, we can attempt to fit the ft value to the 1.96-MeV level by letting the values α_1 and β_1 given by Eq. (6c) vary. Such a calculation results in agreement with experiment, with $\alpha_1^2=0.24$ and $\beta_1^2=0.76$.

Thus band-mixing calculations can make the experimental and theoretical ft values agree. Acceptable mixing is predicted for the $J^\pi=\frac{3}{2}^+$ states in Mg^{25}, such that each state retains the major component of its unmixed wave function. The $J^\pi=\frac{5}{2}^+$ state at 1.96 MeV, however, is calculated to have only 24% of the $K=\frac{1}{2}$ band strength based on a neutron in orbit 9. It is difficult to understand the apparent stronger mixing of the $J^\pi=\frac{5}{2}^+$ states in comparison with the $J^\pi=\frac{3}{2}^+$ states, especially since the properties[3] of this state at 1.96 MeV are consistent with it, being mostly the $J^\pi=\frac{5}{2}^+$ state of the rotational band built on Nilsson orbit number 9.

In summary, the transitions observed in the $Na^{25}(\beta^-)Mg^{25}$ decay suggest a minimum of four components in the Na^{25} ground-state wave function as discussed above. A simple application of the Nilsson model fails to describe the relative transition strengths to the $K=\frac{1}{2}$ levels at 0.98- and 1.96-MeV ($J^\pi=\frac{3}{2}^+$ and $\frac{5}{2}^+$, respectively), suggesting that the $K=\frac{1}{2}$ bands based on orbits 9 and 11 in Mg^{25} are mixed. The β-decay strength to the 1.96-MeV level is very weak, log$ft=6.2$, and quantitatively difficult to account for even when $K=\frac{1}{2}$ band mixing is included in the calculation.

TABLE VI. Comparison of measured strengths in the $Al^{29}(\beta^-)Si^{29}$ decays and the $M1$ γ-ray decays of the P^{29} 8.374-MeV level.

$E_x(Si^{29})$ (MeV)	J^π	logft	$\Lambda(GT)$ Expt	$\Lambda(M1)$ Expt	$\Lambda(M1)/\Lambda(GT)$
1.273	$\frac{3}{2}^+$	5.1	0.035	0.16	4.6
2.028	$\frac{5}{2}^+$	5.7	0.009	0.62	71.0
2.426	$\frac{3}{2}^+$	5.0	0.044	0.38	8.6
3.069	$\frac{5}{2}^+$	6.2	0.003	···	···

IV. COMPARISON OF ANALOG $T=\frac{3}{2}\rightarrow T=\frac{1}{2}$ β^- AND γ-RAY TRANSITION STRENGTHS

It is instructive to compare strengths of the measured Gamow-Teller $T=\frac{3}{2}\rightarrow T=\frac{1}{2}$ β decays reported here with strengths of the analog $T=\frac{3}{2}\rightarrow T=\frac{1}{2}$ $M1$ γ-ray decays.[24,25] Because of the similarity of the β and $M1$ operators, expressions that are model-dependent may be deduced relating the strengths of the β-decay matrix element $\Lambda(GT)$ [defined as $\Lambda(GT)=4390/ft$] and the γ-ray strength $\Lambda(M1)$ [defined as $\Lambda(M1)=3.62\times 10^2\times\Gamma_\gamma(eV)/E_\gamma^3(\text{MeV})$], and so one can obtain further insight into the structure of the nuclei being considered.

In terms of a jj coupling spherical shell model, the expression relating the β^- and γ transition strengths between states $|i\rangle$ and $|f\rangle$ is[25]

$$\Lambda(M1)=5.90[1+0.12\langle f|\mathbf{j}\tau|i\rangle/\langle f|\mathbf{s}\tau|i\rangle]^2\Lambda(GT) \tag{7}$$

for $T=\frac{3}{2}\rightarrow T=\frac{1}{2}$ transitions. Here, $\langle f|\mathbf{j}\tau|i\rangle$ and $\langle f|\mathbf{s}\tau|i\rangle$ are the matrix elements of the j and s parts of the magnetic transition operators, reduced with respect to isospin. The derivation of this expression is based on the fact that only the isovector part of the generalized nuclear Hamiltonian contributes to $\Delta T=1$ transitions,[24] with the underlying assumption that T is a good quantum number. For a single-particle transition for which $j\rightarrow j\pm 1$, the $\langle f|\mathbf{j}\tau|i\rangle$ matrix element is identically zero, leaving an exact relationship between the γ-ray and β-ray widths.[24]

Expressions analogous to Eq. (7) can also be formulated in terms of the Nilsson model. Such an expression, applicable except where both the initial and final bands have $K=\frac{1}{2}$, is derived from Eqs. (36) and (48) of Nilsson's paper. This is given by

$$\Lambda(M1)=0.240(G_{M1}^2/\gamma_1^2)\Lambda(GT), \tag{8}$$

where G_{M1} and γ_1, respectively, are the $M1$ and β matrix elements obtained from Nilsson wave functions.

The β decays investigated in this paper have involved

[24] E. K. Warburton and J. Weneser, in Isospin in Nuclear Physics, edited by D. H. Wilkinson (to be published).
[25] S. S. Hanna, Ref. 25.

TABLE VII. Comparison of strengths computed for $Na^{25}(\beta^-)Mg^{25}$, $\Lambda(GT)$, and the γ decay of Al^{25*} (7.916 MeV), $\Lambda(M1)$ with experiment.

$E_x(Mg^{25})$ (MeV)	J^π	$\Lambda(GT)$ Expt	$\Lambda(M1)$ Expt	$\Lambda(M1)$ Nilsson model[a] $\eta=+2$	$\eta=+4$	$\eta=+6$	$\Lambda(M1)/\Lambda(GT)$ Expt	Spherical shell model	Nilsson model[b] $\eta=+2$	$\eta=+4$	$\eta=+6$
0	$\frac{5}{2}^+$	0.022	0.60	0.61	0.46	0.37	27.0	$5.9R$[c]	18.4	23.0	28.4
0.585	$\frac{1}{2}^+$	<0.001	...	...	...	...	...	...	...	...	...
0.975	$\frac{3}{2}^+$	0.035	0.17	0.12[d]	0.27[d]	0.32[d]	4.9	5.9	7.9	7.8	7.5
1.612	$\frac{7}{2}^+$	0.035	1.48	1.52	1.14	0.92	42.0	5.9	18.4	23.0	28.4
1.965	$\frac{5}{2}^+$	0.003	...	...	...	...	...	...	...	...	...

[a] Calculated with $B^2=0.50$ and $C^2=0.06$. See Fig. 4.
[b] Ratio is defined in Eq. (8) of the text, and is independent of amplitudes of wave functions shown in Fig. 4.
[c] $R=(1+0.12\ \langle f|\mathbf{j}\tau|i\rangle/\langle f|\mathbf{s}\tau|i\rangle)^2$. See Eq. (7) in text.
[d] Calculated assuming the 0.975-MeV state to be a pure $(J^\pi, K)=(\frac{3}{2}^+, \frac{1}{2})$ state based on a particle in orbit 9; i.e., no band mixing.

transitions to Si^{29} and Mg^{25} from Al^{29} and Na^{25}. The $\Delta T=1$, $M1$ widths have not been reported for these nuclei but have been for their mirror nuclei, P^{29} and Al^{25}.[21] A direct comparison, however, is still possible and follows from the rule that corresponding $M1$, $\Delta T=1$ transitions in conjugate nuclei are identical in all properties.[25,26]

In Table VI we present the results of the comparison of the Al^{29} β-decay strengths with the γ decay of the P^{29} 8.374-MeV level. The experimental ratios of the reduced $M1$ width to the reduced β width for the transition to the two $J^\pi=\frac{3}{2}^+$ states in Si^{29} at 1.273- and 2.426-MeV are 4.6 and 8.6, respectively. Since the $\Lambda(M1)$ values are not determined to a better accuracy than 40% and the errors on the branching ratios are $\approx$20%, both these numbers are consistent with a jj coupling spherical shell model. The transition to the $J^\pi=\frac{5}{2}^+$ state at 2.028 MeV in Si^{29} is a $j\rightarrow j$ transition according to our model of this state (see Sec. III A 2) and we therefore would not expect the ratio to be 5.9. The measured value is 71. This value can therefore be taken to yield an estimate of the $\mathbf{j}\tau$ and $\mathbf{s}\tau$ matrix elements of Eq. (7). No comparisons with transition strengths calculated in the Nilsson-model framework were carried out since, as mentioned previously, the core overlap integral is difficult to evaluate properly.

Turning our attention to the mass-25 case in order to compute $\Lambda(M1)$, we first have to write wave functions of good isospin for the analog $T=\frac{3}{2}$ levels in Mg^{25}, so that the evaluated $T=\frac{3}{2}\rightarrow T=\frac{1}{2}$ $M1$ strengths are equal in the $T_z=\pm\frac{1}{2}$ nuclei, i.e., Mg^{25} and Al^{25}. The results are summarized in Table VII. The Nilsson-model estimates of $\Lambda(M1)/\Lambda(GT)$ for any prolate deformation agree quite well with the experimental ratios for the three states to which γ-ray decays are observed. The spherical shell-model estimates disagree for the decays to the 1.612-MeV level, well beyond the errors of the measurements, while for the 0.975-MeV level the model predictions agree within experimental errors. The similar ratio observed for decay to the $J^\pi=\frac{5}{2}^+$ ground state and the $J^\pi=\frac{7}{2}^+$, 1.612-MeV state is evidence for the applicability of the collective model rather than the shell model. The data on the mass-25 system is entirely consistent with a collective-model interpretation. It is noted also that, taking the properly isospin-antisymmetrized wave function for the 7.916-MeV $T=\frac{3}{2}$ state in Al^{25}, the absolute value of the $M1$ width is correctly predicted with the wave-function amplitudes B^2 and C^2 implied from the direct-reaction data and with an acceptable value of the distortion parameter, $\eta=+2$. The β-decay strength is similarly reproduced as described earlier and summarized in Table V. We wish to emphasize the usefulness of this ratio as a test of the proposed models. In contrast to the evaluated γ- and β-decay widths, the ratio is independent of wave-function amplitudes and also the core overlap integral, mentioned previously. In principle, the deformation of Na^{25} could be determined from these measurements. In particular, the matrix element for the β decay and $\Delta T=1$ $M1$ γ decays depends on the deformation of the $K=\frac{3}{2}$ band and is independent of the deformation of the Mg^{25} ground-state band. In the present case the 50% uncertainty in Γ_γ precludes deducing a definite value for η.

[26] G. Morpurgo, Phys. Rev. **114**, 1075 (1959).

ACKNOWLEDGMENT

We wish to thank Dr. R. D. Lawson of the Argonne National Laboratory for consultation and counsel.