Perturbation theory for two-dimensional hydrodynamic plasmons

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Perturbation theory is an indispensable tool in quantum mechanics and electrodynamics that handles weak effects on particle motion or fields. However, its extension to plasmons involving complex motion of *both* particles and fields remained challenging. We show that this challenge can be mastered if electron motion obeys the laws of hydrodynamics, as recently confirmed in experiments with ultraclean heterostructures. We present a unified approach to evaluate corrections to plasmon spectra induced by carrier drift, magnetic field, scattering, viscosity, and Berry curvature. The developed theory enables us to resolve the long-standing stability problem for direct current in confined two-dimensional electron systems against self-excitation of plasmons. We show that arbitrarily weak current in the absence of dissipation is unstable provided the structure lacks mirror symmetry. On the contrary, we find that in extended periodic systems—plasmonic crystals—weak carrier drift is always stable. Instead, this drift induces anomalous Doppler shift, which can be both below and higher than its value in uniform systems.

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I. INTRODUCTION AND OUTLINE

In quantum mechanics, there are a limited number of potential landscapes that allow exact solutions for energy spectra and wave functions. Fortunately, weak potentials of arbitrary form can be handled with perturbation theory (PT) [1]. Several decades after the success in quantum mechanics, PT was formulated for classical electrodynamics [2,3]. Currently, electrodynamic PT represents an indispensable tool for analysis of nonuniform laser cavities and inhomogeneous waveguides [4].

The simplicity of perturbation theory in electrodynamics stems from the fact that the state of the field is characterized by two vectors, **E** and **H**. Waves propagating near conductive surfaces—plasmons—involve not only field oscillations but charge carrier oscillations as well. The state of carriers is characterized by the distribution function generally having an infinite number of harmonics in momentum space. For this reason, the state of plasmon is more complex, and the formulation of plasmonic PT represents a challenging task. Its solution promises a unified approach for the treatment of various perturbations on plasma resonances in metal and semiconductor nanostructures, including magnetic fields, electric currents, electron scattering, and others. Previous attempts to construct PT for plasmonic structures required the synthesis of auxiliary equations of motion for polarization and velocity fields in materials that provide a necessary form of dielectric function [5].

In this paper, we show that formulation of a simple PT for plasmon eigenfrequencies and field distributions is possible when charge carriers in conductors obey the laws of hydrodynamics. While being a common approximation for analysis of carrier motion for nearly a century [6], the true hydrodynamic phenomena in solids were demonstrated only recently with the advent of high-mobility two-dimensional (2D) heterostructures [7–10]. The reason is that the prerequisite of hydrodynamics is the dominance of carrier-carrier momentum-conserving collisions over all other collisions (electron-impurity and electron-phonon) [11]. Under these conditions, only three harmonics of the distribution function survive, and the state of charge carriers is characterized only by three variables: density, velocity, and temperature.

Having constructed the PT, we apply it to resolve the stability problem for confined hydrodynamic electron flows against excitation of plasma waves. From a fundamental viewpoint, the emergence of unstable modes under direct currents signifies a transition to a turbulent flow [12], an intriguing and weakly explored transport regime in solids [13,14]. From an applied point of view, excitation of plasmons by direct current with their subsequent radiative decay can form the basis for new voltage-tunable terahertz sources [15]. Previously, the problem of flow stability was attacked with numerical simulators [16,17], mainly focusing on incompressible flows [18], where the propagation of plasma waves is impossible. An analytical approach to the problem dates back to Dyakonov and Shur [19], where electron flow in 2DES with a grounded source and open-circuited drain was shown to be unstable with respect to plasmon excitation. Later, several other 2DES-based confined structures were shown to support plasma instability [20–23], but a general criterion of instability remained unknown.

The developed perturbation theory enables a transparent formulation of bulk plasmon (in)stability conditions in arbitrary confined 2DES. The key to the problem lies in treating convective, scattering, and viscous terms in hydrodynamic equations as small perturbations, and finding the corrections to plasmon eigenfrequencies (though the developed theory can handle other perturbations, as we show in Appendix A). We indicate that structural asymmetry of confined 2DES is a necessary and (in the absence of dissipation) sufficient condition for self-excitation of plasmons in a one-dimensional flow

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(Sec. III). This finding resolves the long-standing experimental puzzle about the relation between structural asymmetry and the strength of plasmon-assisted terahertz emission from FETs [24–26].

In Sec. IV, we elucidate the effect of carrier drift on plasmon modes in periodic two-dimensional electron systems, i.e., plasmonic crystals. Electron drift in this case does not lead to instabilities, which follows from the Hermiticity of a "drift operator." However, the wave eigenfrequencies are shifted by direct current, and the value of the shift can be both above and below that given by Doppler relations.

II. PERTURBATION THEORY FOR ELECTRON HYDRODYNAMICS

The confinement of a 2DES to a characteristic length L leads to the emergence of collective modes (plasma modes) with frequencies $\omega_p \sim [n_0 e^2/m\varepsilon L]^{1/2}$, where $e > 0$ is the elementary charge, n_0 is the electron density, m is the effective mass, and ε is the background dielectric constant. These modes have been extensively studied since the pioneering works of Stern [27] and Chaplik [28] and host a variety of phenomena when exposed to external influence, e.g., magnetic field [29] or carrier drift [30]. Interestingly, many of these effects can be incorporated inside a single theoretical shell of plasmonic perturbation theory (PT), which we develop in this section.

The formulation of plasmonic PT is possible if oscillating charge carriers obey the laws of hydrodynamics. The hydrodynamic approach grants a straightforward formulation of operator eigenvalue problem on 2D plasmon eigenfrequencies, allowing the isolation of drift, magnetic field, viscosity, and pressure operators—which is unachievable in other transport regimes. This property enables us to construct an analogy of quantum-mechanical perturbation theory with respect to these operators; such treatment is possible when the corrections to the eigenfrequency are small as compared to the frequency itself. In this regard, electric charge of electron fluid plays the central role in our theory. It leads to the existence of plasma modes setting the largest frequency scale in the problem (compared, e.g., to the plasmon decay time). As a result, the unperturbed dynamic matrix is Hermitian, which simplifies the formulation of PT. Such simplicity is lacking in PT for neutral incompressible fluids, the motion of which is strongly affected by viscous dissipation [31,32].

Formally, the inequalities for the perturbative treatment to be possible are $\{u_0/L, \omega_c, \tau_p^{-1}, \nu/L^2\} \ll \omega_p$, where ν is the kinematic viscosity and $\omega_c = eB/m$ is the cyclotron frequency (B denotes the magnetic inductance). Taking the realistic parameters $u_0 \simeq 10^5$ m/s, $L \simeq 1$ μm, $\tau_p \simeq 10^{-11}$ s^{-1}, $B = 0.1$ T, and estimating the viscosity as $\nu \simeq v_0^2 \tau_{ee}/4 \simeq 250$ cm^2/s [10], we see that the inequalities are fulfilled for $\omega_p/2\pi \simeq 1$ THz.

After these preliminary remarks, we are ready to construct the theory. The set of HD equations for a two-dimensional motion of a charged fluid has the form

$$\partial_t \mathcal{N} + \partial_i J_i = 0, \tag{1}$$

$$\partial_t J_i + \partial_j \mathcal{P}_{ij} = F_i/m - J_i/\tau_p, \tag{2}$$

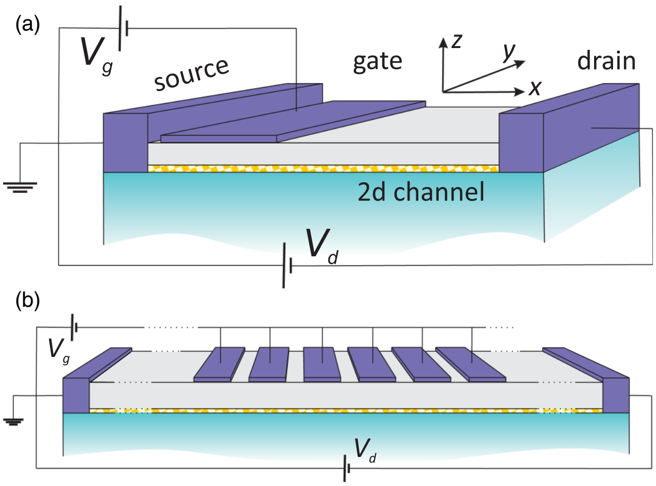

FIG. 1. Schematics of 2DES realizations. (a) Bounded 2DES, namely partly gated field-effect transistor [22]. (b) Plasmonic crystal.

where t denotes time, $\mathcal{N}$ is the electron density, $J_i = \mathcal{N}\mathcal{U}_i$ is the current, $\mathcal{U}_i$ is the drift velocity, F_i is the Lorentz force, and $\mathcal{P}_{ij}$ is the stress tensor of the electron fluid (we neglect the bulk viscosity and pressure terms, which can be easily restored if needed):

$$\mathbf{F} = e\mathcal{N}\nabla\varphi + \omega_c[\hat{\mathbf{z}}, \mathcal{U}], \tag{3}$$

$$\mathcal{P}_{ij} = \mathcal{N}\mathcal{U}_i\mathcal{U}_j - \frac{\eta}{m}(\partial_j \mathcal{U}_i + \partial_i \mathcal{U}_j - \delta_{ij}\partial_k \mathcal{U}_k), \tag{4}$$

where $\hat{\mathbf{z}}$ is the unitary vector in the direction perpendicular to 2DEG (see Fig. 1), η denotes dynamic viscosity, and δ_{ij} is the Kronecker delta, $\{i, j, k\} = 1, 2$. The set (1)–(4) is completed by the expression for the electric potential φ determined by the 2DEG surroundings (consider Fig. 1) through the electrostatic Green function $G(\mathbf{r}, \mathbf{r}')$:

$$\varphi(\mathbf{r}) = \varphi_{\rm ext}(\mathbf{r}) - e\,\mathcal{G}[\mathcal{N}], \tag{5}$$

where $\mathcal{G}[f] = \int d^2\mathbf{r}' G(\mathbf{r}, \mathbf{r}') f(\mathbf{r}')$ is the self-consistent field, the $\mathbf{r}$-vector lies in the 2DEG plane, the contribution $\varphi_{\rm ext}(\mathbf{r})$ is fixed at the contacts by the voltage source, and the integration is performed over the whole 2DEG.

The following analysis is based on linearization $\mathcal{N}(\mathbf{r}, t) = n_0(\mathbf{r}) + n(\mathbf{r})e^{-i\Omega t}$, $\mathcal{U}(\mathbf{r}, t) = \mathbf{u}_0(\mathbf{r}) + \mathbf{u}(\mathbf{r})e^{-i\Omega t}$, and reformulation of (1)–(5) as an operator eigenvalue problem:

$$(\hat{\Omega} + \hat{V}_{\rm drift} + \hat{V}_{\rm sc} + \hat{V}_{\rm visc} + \hat{V}_{\rm mag})\mathbf{\Phi} = \Omega\mathbf{\Phi}. \tag{6}$$

Above, we have introduced the "three-component wave function" $\mathbf{\Phi} = \{n, \mathbf{u}\}^{\rm T}$ describing density and velocity variations in plasma modes. The unperturbed motion is described by the "hydrodynamic" operator:

$$\hat{\Omega} = -i\begin{pmatrix} 0 & \nabla[n_0(\mathbf{r})\cdot] \\ \frac{e^2}{m}\nabla\mathcal{G}[\cdot] & 0 \end{pmatrix}, \tag{7}$$

and we consider the steady carrier drift, magnetic field, scattering, and viscosity as small perturbations given by the operators

$$\hat{V}_{\rm drift} = -i\begin{pmatrix} (\nabla, \mathbf{u}_0\cdot) & 0 \\ 0 & (\cdot, \nabla)\mathbf{u}_0 + (\mathbf{u}_0, \nabla)\cdot \end{pmatrix}, \quad (8)$$

$$\hat{V}_{\rm visc} = \frac{i}{mn_0}\begin{pmatrix} 0 & 0 \\ 0 & (\nabla, \eta(\nabla\cdot + (\nabla\cdot)^T - (\nabla, \cdot)\delta_{ij})) \end{pmatrix}, \quad (9)$$

$$\hat{V}_{\rm sc} = -i\tau_p^{-1}\begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} \end{pmatrix}, \quad \hat{V}_{\rm mag} = -i\omega_c\begin{pmatrix} 0 & 0 \\ 0 & e_{ij} \end{pmatrix}, \quad (10)$$

where δ_{ij} and e_{ij} are the Kronecker delta and the two-dimensional absolutely antisymmetric tensor, respectively; $i, j = 1, 2$.

At this stage, one might be willing to apply a standard Schrödinger perturbation theory for correction to the eigenfrequency $\delta\Omega_\lambda = \langle\Phi_\lambda|\hat{V}|\Phi_\lambda\rangle$, where λ enumerates the plasmon modes. However, this step is premature until the inner product is specified. Apparently, a standard definition $\langle\Phi_\lambda|\Phi_{\lambda'}\rangle = \int dx[n_\lambda^* n_{\lambda'} + u_\lambda^* u_{\lambda'}]$ fails: it does not ensure that matrix $\hat{\Omega}$ is Hermitian. We resolve this issue by reformulating the initial problem: we apply the Hamilton operator $\hat{H}$ of a charged fluid to Eq. (6) and obtain a generalized eigenvalue problem:

$$\hat{H}(\hat{\Omega} + \hat{V}_{\rm drift} + \hat{V}_{\rm sc} + \hat{V}_{\rm visc} + \hat{V}_{\rm mag})\mathbf{\Phi} = \Omega\hat{H}\mathbf{\Phi}, \quad (11)$$

$$\hat{H} = \begin{pmatrix} e^2/m\,\mathcal{G}[\cdot] & 0 \\ 0 & \hat{I}_2 n_0(\mathbf{r}) \end{pmatrix}. \quad (12)$$

Here, the dynamic matrix $\hat{H}\hat{\Omega}$ is Hermitian (i.e., $\langle\Phi_1|\hat{H}\hat{\Omega}|\Phi_2\rangle = \langle\Phi_2|\hat{H}\hat{\Omega}|\Phi_1\rangle$) as well as the Hamiltonian $\hat{H}$; this fact can be shown explicitly by evaluating the corresponding matrix elements. Hence, we are now able to apply the standard perturbative expansion that leads to the following expression for the first correction to the eigenfrequency:

$$\delta\Omega_\lambda = -i\frac{\langle\Phi_\lambda|\,\hat{H}(\hat{V}_{\rm drift} + \hat{V}_{\rm sc} + \hat{V}_{\rm visc} + \hat{V}_{\rm mag})\,|\Phi_\lambda\rangle}{\langle\Phi_\lambda|\,\hat{H}\,|\Phi_\lambda\rangle}. \quad (13)$$

We stress that the whole procedure does not require any additional boundary conditions (BCs) apart from the two natural ones: (i) the Green function vanishes at the electrodes and (ii) $J_\perp = 0$ at the 2DES edges (no carrier leakage). In such a way, our analysis holds for 2DES with *arbitrary* BCs, which are hardly known in real experimental setups.

The constructed perturbation theory is a powerful tool that can be used to uncover the underlying principles of many plasmonic phenomena in 2DES. We apply its formalism to examine the drift-originated plasmonic effects in bounded systems (Sec. III) and plasmonic crystals (Sec. IV). An illustrative example of handling boundary condition perturbations is presented in Appendix A.

III. CURRENT-DRIVEN INSTABILITIES IN BOUNDED SYSTEMS

Direct current passing in confined 2DES can supply energy to plasmon modes and lead to their self-excitation (plasma instability). One of the first examples of such instabilities in confined 2DES was demonstrated by Dyakonov and Shur; such instability occurred in 2DES with the grounded source, and drain held at a fixed current [19]. Later, other geometries and boundary conditions were analyzed, including loaded drain [23], partly gated FETs [21,22], and Corbino disks [20]. This search for instabilities had been excursive and there was no general understanding of whether a given FET structure supports an instability or not. At the same time, both existing models [20,21] and experimental data [24–26] hinted that structural asymmetry somehow promotes an instability.

The developed perturbation theory enables us to formulate the instability criteria in a very general form. From the prospective of wave mechanics, the operator of electron drift $\hat{V}_{\rm dr}$ in bounded 2DES is generally non-Hermitian. Therefore, the eigenfrequencies of drifting plasmons are complex, which implies plasmon amplitude growth/decay with time. The eigenfrequencies may remain real only under specific symmetry constraints, which we will obtain below.

We restrict our consideration to one-dimensional oscillations of 2D electrons assuming the transverse modes to be inactive. This assumption is justified at least when the channel width is below the plasmon wavelength. Evaluating the matrix elements in Eq. (13), we find the correction to the plasmon frequency in confined 2DES induced by a combined action of direct current, scattering, and viscosity:

$$\delta\Omega_\lambda = i\frac{j_0[K_\lambda(0) - K_\lambda(L)] - Q_{\rm loss}}{|\Pi_\lambda|}, \quad (14)$$

where $K(x) = m|u_\lambda(x)|^2/2$ is the local kinetic energy in a plasmon mode,

$$\Pi = e^2\int_0^L dx\,dx' n_\lambda^*(x)G(x,x')n_\lambda(x') \quad (15)$$

is the potential energy of interacting charge density fluctuations in a 2DES of length L, and

$$Q_{\rm loss} = \frac{1}{2}\int_0^L dx\{{\rm Re}\,\sigma|E_\lambda|^2 + \eta|\partial_x u_\lambda|^2\} + \left.\frac{\eta u_\lambda^*\partial_x u_\lambda}{4}\right|_0^L \quad (16)$$

is the energy loss due to viscous friction and scattering-induced dissipation, where $\sigma = ie^2 n_0/m(\Omega + i/\tau_{\rm p})$ is the Drude conductivity. The last "viscous-boundary" term is also dissipative in systems with time-reversal symmetry; if the latter is broken, shear viscosity will contribute to frequency shift as well.

From Eq. (14), we readily observe that the plasmon growth rate ${\rm Im}\,\delta\Omega_\lambda$ originates from the excess of kinetic energy entering the mode at the source over the energy drawn out at the drain. The Dyakonov-Shur instability growth rate appears as a particular case of Eq. (14), where $K(0) - K(L)$ is nonzero due to inequivalent source and drain contacts.

Apart from the energy interpretation, Eq. (14) immediately indicates that only zero-order plasma modes without definite parity can be excited by a weak external drift. Indeed, the even/odd mode profiles satisfy $u^2(L) = u^2(0)$, which forces the gain term in Eq. (14) to vanish. Therefore, FETs with mirror symmetry do not support unstable modes, but asymmetric structures generally do (in the absence of dissipation). The origin of asymmetry can be arbitrary; this can be either asymmetric placement of gates, asymmetric loading of source and drain, nonuniform carrier density in the channel, or all of them.

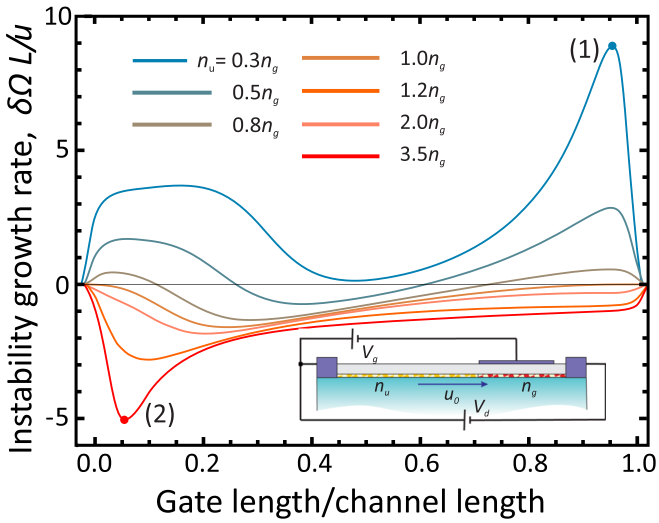

FIG. 2. Calculated instability growth rates for the first plasmon mode in a partly gated FET (shown in the inset) with different gate lengths and carrier densities. The carrier density distribution is taken to be $n(x) = n_1 + (n_2 - n_1)[1 + e^{(x-x_0)/l_j}]^{-1}$. The growth rates are normalized by u_0/L, where u_0 is the drift velocity at the drain (same for all curves), and the density at the drain n_0 is also fixed. The instability benefits if the drift is directed from the low- into the high-density region and is especially pronounced if the low-density region is short and ungated.

We can go beyond the symmetry constraints and specify the requirements on 2DES aiming to maximize the plasmon instability growth rate. This is equivalent to maximization of $u^2(0) - u^2(L)$. The velocity is proportional to the electric field and inverse carrier density. Thus, a 2DES with a high field and small carrier density at the source and a low field and high density at the drain would be most suitable for "plasmonic turbulence." The simplest realization of this scheme is a partly gated field-effect transistor (FET) with a short ungated depleted region at the source, and a long gated and enriched region at the drain (see the inset in Fig. 2).

To prove this hypothesis, we develop a model for plasmon modes in a partly gated FET. The contacts of such a structure are connected to voltage sources, which is a typical experimental situation, and variation of the gate-source bias allows us to form a carrier density step in the channel ($n^+ - n$ junction). We numerically solve the governing equations in the absence of scattering (Appendix B), and we plot in Fig. 2 the drift-induced corrections to plasmon eigenfrequencies for a set of structures with varying junction location x_0 and density modulation factor. In accordance with our expectations, the highest growth rate (point 1) is achieved for a structure with a short, depleted, ungated source region. If we swap the contacts and thus change the sign of the drift velocity, the magnitude of a new maximum (point 2) will be predictably lower due to the gate screening. In addition, Fig. 2 shows that the source regions should not be too short, or else the structure would approach the symmetric limits of open ($x_0 < 0$) or fully gated ($x_0 > 1$) FETs with uniform density that are not subject to instabilities.

Experimental data support our findings. Thus, the first terahertz sources exploiting the excitation of plasmons by direct current [24] were symmetric and provided broadband radiation only at 4 K. Further implication of asymmetric partly gated FETs [26] enabled the observation of resonant room-temperature emission due to efficient plasmon-to-drift coupling. Moreover, it was demonstrated in [26] that the threshold current for THz emission is significantly reduced with depletion of the near-source region, which is in agreement with our analysis.

However, this depletion should be kept in proper bounds. Indeed, a sharp density step not only increases the source field, but also causes highly nonuniform distribution of plasma wave velocity in the channel. This nonuniformity promotes viscous losses, which are proportional to the velocity gradient, and at some degree of asymmetry the viscosity takes over gain.

IV. DRIFT-INDUCED PHENOMENA IN PLASMONIC CRYSTALS

It turns out that drift-induced phenomena in periodic systems—plasmonic crystals (PCs)—are completely different from those in bounded 2DES. The reason is that in periodic systems the drift operator is Hermitian due to translational invariance and thus conserves energy. Still, plasmon instabilities in PCs can emerge—but only at high drift velocities, at least in the depleted regions [33–35]. In that way, moderate carrier drift in PCs does not affect the stability of plasma modes; instead, it changes their spectrum.

To determine these spectrum modifications, we apply the constructed perturbation theory accounting for the periodicity of the structure. Namely, we exploit the Bloch decomposition for the drift velocity carrier density variations and arrive at the expression for modified frequency (13) with a generalized derivative operator $\partial_x \to \partial_x + iq_B$, where q_B is the Bloch vector. The Green's function in the periodic system is defined according to

$$G(x, x') = \sum_{n=-\infty}^{n=+\infty} G_0(x, x' + nL) e^{iq_B(x'-x)} e^{iq_B nL}, \tag{17}$$

where x and x' lie within a unit cell of length L, and G_0 is the true Green's function of the periodic system. The unknowns n, u, φ should now be understood as periodic parts of the corresponding Bloch functions.

These remarks allow us to apply Eq. (13) to drift-induced effects in PCs. We arrive at

$$\frac{\delta\Omega_\lambda}{\Omega_\lambda} = j_0 \frac{\int_0^L dx\, \mathrm{Im}(m u_\lambda D_x u_\lambda^*)}{\int_0^L dx\, \mathrm{Re}(-e n_\lambda \varphi_\lambda^*)}. \tag{18}$$

As expected, the correction is real and corresponds to the zero-mode Doppler shift.

Significant shifts can be useful in resonant photodetection exploiting the plasmonic drag effect [36]. Indeed, in a typical PC a normally incident light excites plasma wave packets around $q_B = 0$, where the group velocity distribution is symmetrical in the absence of drift. For a sufficient detection, however, one needs a substantial asymmetry in this distribution, which can be introduced via carrier drift. The latter breaks the dispersion curve symmetry, and the greater the Doppler shift is, the greater is the group velocity difference.

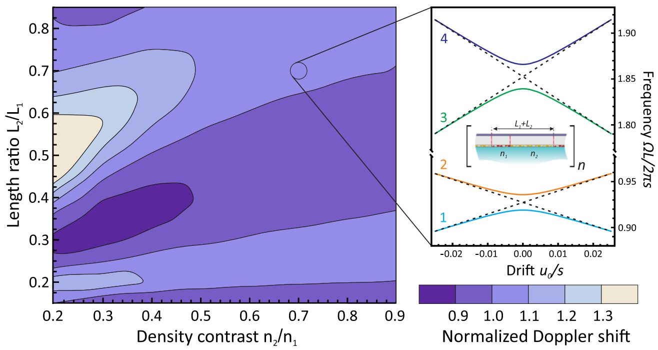

FIG. 3. Left and lower right panels: Normalized Doppler shift for the third and fourth modes in fully gated PCs with a density step inside a unit cell. Upper right panel: $\Omega(u_0)$ dependencies for a PC with $L_2 = 0.7L_1$ and $n_2 = 0.7n_1$. Their character coincides with the predictions of degenerate perturbation theory, Eq. (19). Inset: scheme of a fully gated PC.

A detailed inspection of Eq. (18) shows, however, that the linear Doppler shift turns to zero in the standing-wave limit ($q_B = 0$). Indeed, the velocity eigenfunctions $u_\lambda(x)$ can be chosen to be real in this case, which turns the numerator to zero. At the same time, the value of the Doppler shift can be readily predicted using the degenerate perturbation theory that is applicable when two interacting modes (labeled by 1 and 2) are well separated in frequency from the others. In this case, the drift-modified plasmon frequencies are given by

$$\Omega_\pm = \tfrac{1}{2}\{\Omega_1 + \Omega_2 \pm \sqrt{(\Omega_1 - \Omega_2)^2 + 4|V_{12}|^2}\}, \qquad (19)$$

where V_{12} is the normalized matrix element of the drift operator,

$$V_{12} = \frac{\langle \Phi_1 | \hat{H}\hat{V}_{\rm dr} | \Phi_2 \rangle}{\sqrt{\langle \Phi_1 | \hat{H} | \Phi_1 \rangle \langle \Phi_2 | \hat{H} | \Phi_2 \rangle}}. \qquad (20)$$

Figure 3 illustrates our findings. In the left panel we plot the calculated Doppler shifts for third and fourth modes that exist in a range of fully gated PCs (see the inset in the upper right panel) with a density step inside the unit cell. The shifts are normalized by the expected shift value

$$\frac{\Omega}{2\pi sN/L} = \bar{u}_0/N = j_0\frac{L_1/n_1 + L_2/n_2}{NL}, \qquad (21)$$

where s is the plasma wave velocity under the first gate, L_1 and L_2 are the gate lengths, $L_1 + L_2 = L$, n_2 and n_1 denote carrier densities, and N enumerates the pairs of modes; in our case, $N = 2$. The upper right panel shows $\Omega(u_0)$ dependencies for the first four modes in a PC with $L_2 = 0.7L_1$ and $n_2 = 0.7n_1$. In full accordance with Eq. (19), we observe the parabolic spectrum at very low velocities that transforms into the linear spectrum when the perturbation magnitude exceeds the degeneracy contribution. Hence, the slope of the linear part is determined by the nondiagonal matrix element, and in a certain parameter range it leads to higher than the Doppler shift.

The linear Doppler shift would reappear in plasmonic structures where the velocity and density profiles cannot be chosen real so that $\mathrm{Im}(mu_\lambda D_x u_\lambda^*) \neq 0$. As an example, the magnetic field, being included in the $\hat{\Omega}$ operator, efficiently entangles the real and imaginary parts of the zero-order wave functions. Thus, the plasma modes become coupled to drift in the first order (their stability is not affected as $\hat{V}_{\rm mag}$ is Hermitian). A more detailed discussion of this influence will be given elsewhere.

V. DISCUSSION AND CONCLUSION

The developed plasmonic perturbation theory has the same functionality as its elder brother in quantum mechanics: given the exact solution of an unperturbed problem, we can accurately find corrections to the eigenfrequency under small perturbations. Unfortunately, exact solutions in 2D plasmonics are unique among nontrivial cases such as the edge modes [37] and plasma oscillations in gated 2DES with infinite conductive walls [38]. Fortunately, however, the unperturbed problem of plasma oscillations due to a self-consistent electric field is readily solved with commercial electromagnetic simulators, and the resulting field profiles can be supplied to the developed perturbation theory.

The assumption of hydrodynamic transport used in the derivation limits the frequencies ω below the inverse electron-electron scattering time $\tau_{\rm ee}^{-1}$. This may seem restrictive as $\tau_{\rm ee}^{-1}$ is an order of 1 THz at room temperature [39] and scales as T^2. Most experimental observations of plasmons correspond to the opposite ballistic limit $\omega\tau_{\rm ee} \gg 1$ [29,40]. However, the difference between predictions of hydrodynamic and ballistic approaches is important only when treating thermal corrections to plasmon velocity and Landau damping [41]. Therefore, we can speculate that the developed PT would be applicable in the ballistic limit as well.

Another assumption of the developed PT is the neglect of retardation effects or, formally, setting the velocity of light c to infinity. This assumption is justified for typical 2D plasmons once their frequency ω_0 lies below the light cone $\omega = cq \sim c/L$ [42]. Renouncing this assumption immediately leads to radiative plasmon damping and non-Hermiticity of the unperturbed problem. Formulation of PT in this case is also possible [43] but requires dealing with diverging fields far away from 2DES.

The presented examples were related to first-order or degenerate perturbation theory. Higher-order corrections can also be derived and are important when first-order effects are absent due to symmetry (such as the Doppler shift in the center of the plasmon Brillouin zone). Another nontrivial application of higher-order corrections is the analysis of weak steady-state plasma turbulence for direct current slightly exceeding the threshold [32,44]. So far, the solution of such problems in 2DES was achieved with numerical simulations [18] or it was limited to model systems [45]. The general analysis is possible with the developed PT, and it will be reported elsewhere.

In conclusion, we have developed the perturbation theory for two-dimensional hydrodynamic plasmons and demonstrated its utility by revealing the instability conditions of direct current against excitation of plasmons. We have derived a constitutive relation between the current-induced growth rate of plasmon and its steady-state field distribution. This relation clarifies the crucial role of structural asymmetry for efficient excitation of plasmons by direct current. In periodic systems—plasmonic crystals—the current does not lead to instabilities in the first order, but it does induce a Doppler shift that can be both above and below the conventional value.

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APPENDIX A: CHIRAL BERRY PLASMONS

Apart from treating the perturbations that directly enter the equations of motion (1)–(5), our theory also accounts for the boundary condition perturbations. In particular, this property can be useful for the description of chiral Berry plasmons (CBPs)—a new type of edge plasmonic states in 2D materials with nonzero Berry flux [46]. CBPs arise as follows. Nonzero Berry flux provides an "anomalous" contribution J_a to the total particle current $J_{\rm tot} = J + J_a$, where $J = \mathcal{N}\mathcal{U}$ is the usual current density. The anomalous current does not affect the governing HD equations (due to $\nabla J_a = 0$), but it changes the boundary condition on the 2DEG edge from the usual $J_\perp = 0$ to $J_{{\rm tot},\perp} = 0$. Song and Rudner [47], and later Zhang and Vignale [48], found this BC change to split the frequencies of right- and left-propagating plasmons, which were therefore named chiral Berry plasmons. However, they did that in a restrictive quasilocal electrostatic approximation [49] that does not, in particular, reproduce the logarithmic divergence of edge magnetoplasmon group velocity at short wave vectors in strong magnetic fields [37]. This divergence stems from the 2D Coulomb potential divergence, which is violated in the quasilocal approximation. We demonstrate below how the perturbation theory, given precise electrostatics, reproduces the CBP frequency gap for small Berry flux and corresponding anomalous current.

We consider a semi-infinite 2DEG with nonzero Berry flux $\mathcal{F}$ confined to an $\{x > 0, y\}$ half-plane and require the potential and electric field continuity at the $x = 0$ boundary along with $J_{{\rm tot},x}|_{x=+0} = 0$. The latter BC is the only difference that distinguishes the CBP problem from the edge plasmon problem solved by Volkov and Mikhailov [37], who obtained the edge plasmon frequency $\Omega_0 = \omega_{2D}/\sqrt{1.217}$, $\omega_{2D} = \sqrt{2\pi e^2 n_0|q|/m}$, where q is the wave vector in the y direction. Replacing J in Eqs. (1) and (2) with $J_{\rm tot}$ and using $\nabla J_a = 0$, we arrive at the following problem:

$$(\hat{\Omega} + \hat{V}_a)\Phi = \Omega\Phi, \tag{A1}$$

where

$$\hat{V}_a = \Omega_0 \frac{e\mathcal{F}}{\hbar n_0}\begin{pmatrix} 0 & 0 \\ -e\nabla_a \mathcal{G}_q[\cdot] & 0 \end{pmatrix} \tag{A2}$$

is the "anomalous" perturbation operator induced by the (dimensionless) Berry flux $\mathcal{F}$, $\nabla_a = \hat{\mathbf{x}} iq - \hat{\mathbf{y}}\partial_x$, $\mathcal{G}_q[f] = 2\int_0^\infty dx' K_0(q|x - x'|) f(x')$, $K_0(x)$ is the modified Bessel function of the second kind.

Thus, we transformed the BC perturbation into a more convenient form. Now the formulation of the zero-order problem (A1) coincides with the formulation of the edge plasmon problem [37]. Taking Volkov-Mikhailov mode profiles, we evaluate the Berry-flux-induced correction $\langle\Phi|\,\hat{H}\hat{V}_a\,|\Phi\rangle \,/\, \langle\Phi|\,\hat{H}\,|\Phi\rangle$ and calculate the gap between two branches of CBP:

$$\hbar\Delta\Omega(q) = 3.65\,\mathcal{F}e^2|q| \tag{A3}$$

versus the Song-Rudner result

$$\hbar\Delta\Omega_{\rm SR}(q) = 8\sqrt{2}\pi/9\,\mathcal{F}e^2|q| \approx 3.95\,\mathcal{F}e^2|q|. \tag{A4}$$

Thus, our calculations qualitatively approve the approximate Song-Rudner solution and downshift the gap width by 10%. This change is a minor one and grants all the nonreciprocal implementations of CBPs discussed in [47].

APPENDIX B: NUMERICAL METHOD

To obtain the results shown in Fig. 2, we apply a standard spectral numerical method to the system of hydrodynamic equations (1)–(5) with Chebyshev polynomials of the first kind T_i taken as the basis functions. To be more concrete, after writing the linearized Eqs. (1) and (2) in dimensionless units [$\xi = 2x/L - 1, \tilde{n} = n/n_0(0), \tilde{u} = u/s(0), \omega = \Omega L/s(0)$, where $s(0)^2 = e^2 n_0(0)L/m$] we substitute the Chebyshev expansions in the form

$$\{\tilde{n}, \tilde{u}\} = \sum_{i=0}^{N} C_i^{\{n,u\}} T_i(\xi) \tag{B1}$$

and project the system on each of the polynomials $T_i(\xi)$, $i = 0, \ldots, N$. After these manipulations, we arrive at the matrix equation:

$$\begin{pmatrix} \hat{M}^{(1)} & \hat{M}^{(2)} \\ \hat{M}^{(3)} & \hat{M}^{(4)} \end{pmatrix}\begin{pmatrix} C_i^n \\ C_i^u \end{pmatrix} = i\omega\begin{pmatrix} C_i^n \\ C_i^v \end{pmatrix}, \tag{B2}$$

where

$$\hat{M}_{ij}^{(1)} = \hat{M}_{ij}^{(4)} = t_{ij}\,\langle T_j|\,w(\xi)\partial_\xi\,|u_0 T_i\rangle\,,$$
$$\hat{M}_{ij}^{(2)} = t_{ij}\,\langle T_j|\,w(\xi)\partial_\xi\,|n_0 T_i\rangle\,,$$
$$\hat{M}_{ij}^{(3)} = t_{ij}\,\langle T_j|\,w(\xi)\partial_\xi\left|\int_{-1}^{1} d\xi' G(\xi,\xi')T_i(\xi')\right\rangle,$$
$$t_{ij} = \begin{cases} 1/\pi, & i = 0, \\ 2/\pi & \text{otherwise}, \end{cases}$$

and $w(\xi) = (1 - \xi^2)^{-1/2}$ is the weight function; $\{i, j\} = 0, \ldots, N$ for all the matrices.

Implying boundary conditions of fixed charge density at the contacts $\tilde{n}(-1) = \tilde{n}(1) = 0$, we obtain

$$C_N^n = -\sum_{i=0}^{N/2-1} C_{2i}^n, \tag{B3}$$

$$C_{N-1}^n = -\sum_{i=0}^{N/2-1} C_{2i+1}^n, \tag{B4}$$

where N is supposed to be even. We use these expressions to eliminate C_N^n and C_{N-1}^n from the system (B2) and, in order to keep the matrix dimensions, truncate the first three matrices.

One may face computational difficulties while evaluating matrix elements $\hat{M}_{ij}^{(3)}$ as the Green function is singular on the diagonal $\xi = \xi'$. To overcome this, we decompose the Green function into singular (G_s) and regular (G_r) parts:

$$G = (G - G_s) + G_s \equiv G_r + G_s, \tag{B5}$$

where

$$G_s = \ln \frac{(\xi - \xi')^2}{(\xi + \xi' - 2)^2(\xi + \xi' + 2)^2} \tag{B6}$$

accounts for the singularity provided by the charge itself as well as by the two nearest mirror images in the electrodes. The regular integral is then calculated numerically while the singular one can be significantly simplified via transition to the complex plane.

The Green function of a partly gated structure is given by

$$G_{\mathrm{PG}}(\xi, \xi') = \ln \left[\frac{(\alpha - \alpha')^2 + (\beta + \beta')^2}{(\alpha - \alpha')^2 + (\beta - \beta')^2} \right], \tag{B7}$$

where $\alpha' + i\beta' = z'$,

$$z' = \cos\psi \sqrt{\tanh^2 \frac{\pi(d + i\xi')}{2L} + \tan^2\psi}, \tag{B8}$$

and $\psi = \pi L_g / 2L$.

The values of the first correction obtained by the described procedure and by the perturbation theory [Eq. (14)] fully coincide at small drift velocities.

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