Tailoring one-dimensional layered metamaterials to achieve unidirectional transmission and reflection

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We investigate elastic-wave propagation in a spatially dispersive multilayered, totally passive metamaterial system. At oblique incidence a longitudinal (acoustic) wave can convert to transverse in the solid material comprising the layers, but when the incident wave enters the multilayer from a solid as opposed to a liquid medium, the incident transverse component supported by the solid medium indirectly causes the longitudinal transmission response to be greatly modified, with a similar result for the transverse wave exiting the multilayer into a solid medium in response to an incident longitudinal wave. The conversion between longitudinal and transverse waves is found to lead to the emulation of a characteristic nonreciprocal phenomenon at some frequencies: a directionality in the transmission response, sometimes simultaneously with the reflection response. The directionality can be exploited, for example, in the construction of antiseismic structures or breakwater structures. The inclusion of gain and loss elements can strongly enhance the directionality. Periodicity-breaking defects can cause a great variability in the response, enabling the use of devices based on this phenomenon as sensors.

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I. INTRODUCTION

Controlling elastic wave propagation in layered media has been the scope of pioneering theoretical and, recently, experimental studies. While the propagation of acoustic waves in phononic lattices has many parallels with the propagation of electromagnetic waves in photonic lattices, showing similar phenomena and applications, such as acoustic cloaking [1] and illusions [2] and, more generally, transformation acoustics [3,4], the more general case of wave propagation in elastodynamic media has, as yet, not been widely studied.

The interconversion between longitudinal and transverse modes has been exploited in the design of metasurfaces in order to emulate nonreciprocal phenomena in electromagnetic systems [5,6].

Nonreciprocal effects such as asymmetric transmission may be emulated by spatially dispersive [7] metasurfaces. In Ref. [5] metasurfaces comprised of asymmetrically aligned electric dipoles were studied analytically and numerically with respect to their effect on the polarization of the incident electromagnetic radiation. It was found that for some surface topologies at oblique incidence the metasurface exhibits spatial dispersion, that is, the two-dimensional surface parameters (in this case electric and magnetic susceptibilities and magnetoelectric parameters linking the in-plane electric and magnetic fields to the respective currents), depend on the transverse momentum of the incident wave. The transverse momentum thus acts as a self-biasing mechanism, and this dependence of the surface parameters on this quantity is necessary for the presence of tangential (in-plane) polarization. Along with oblique incidence, tangential polarization was found to be necessary for the transmission to be asymmetric (dependent on the direction at which the metasurface is approached). As noted in Ref. [6], such devices do not break time-reversal symmetry. Generally, the directionality of the transmission occurs whenever there is a transverse component to an incident wave and the metasurface is spatially dispersive. The transverse momentum of the wave incident on a spatially dispersive metasurface functions as the magnetic bias in a nonreciprocal magneto-optic material. Devices may be designed for achieving asymmetric transmission by solving the inverse problem, that is, designing the surface parameters to fit the desired scattering parameters [8]. Examples of nonreciprocal phenomena which can be emulated using the transverse momentum of plane waves obliquely incident on a spatially dispersive metasurface include Faraday rotators and isolators [6]. In addition, vortex beams carrying orbital angular momentum carry transverse momentum even if they are normally incident on a metasurface, and thus, devices which emulate nonreciprocal phenomena such as unidirectional transmission can operate even at normal incidence [6].

Most of the studies on emulation of nonreciprocal behavior have been conducted on electromagnetic systems such as those described above. The few studies thus far on acoustic systems have considered surface waves (Lamb waves) in superlattices [9,10]. Reference [9] found that in some superlattice designs there can be a conversion between the two modes, while the design of the superlattice can be adjusted in order to transmit one mode of Lamb waves but not the other in a given frequency range. These two behaviors may then be combined in order to achieve unidirectional transmission. In Ref. [10] the design's purpose is more for manipulating Lamb waves. Mode conversion is avoided by diffracting the two modes into different propagation directions.

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The name given to systems which yield a result in one mode in response to excitation from two types of modes is bianisotropic metasurfaces [6]. For example, a magnetic response from electric and magnetic excitation in electromagnetic systems or a transverse response from transverse and longitudinal excitation in the case of elastic systems ("acoustic" activity [11]) may be implemented by asymmetric unit cells [12,13] or by using $\mathcal{PT}$-symmetric systems [14]. In the latter studies, all based on reciprocal acoustic systems, bianisotropic metasurfaces generate different phases in the reflection coefficient in different directions [12]. With the addition of loss (or gain) the magnitude of the reflection coefficient also becomes different. In all cases dealing with reciprocal materials, the transmission coefficients in both directions are the same. Recently, elastic-wave propagation under oblique incidence was studied for the transverse polarization which is independent of the other two modes [15], which are coupled under oblique incidence and are the object of the present study. Up until now, studies of propagating longitudinal and transverse modes in *elastic* systems have not been undertaken to achieve asymmetry in the transmission coefficients.

The addition of balanced gain (G) and loss (L) to metamaterials constitutes an approach to achieving reduced losses [16]. Parity-time- ($\mathcal{PT}$) or the more general category of pseudo-Hermitian- (PH) [17] symmetric systems, despite being non-Hermitian, can have real eigenvalues, which is necessary for propagation, as well as bound states. By modifying the system parameters, the system may migrate from the $\mathcal{PT}$- or PH-symmetric phase into the broken phase and vice versa.

Defects, in the form of interfacial wrinkling, have been found to be able to control the wave propagation in layered materials by introducing complete band gaps in one-dimensional phononic materials [18]. The transfer-matrix technique has long been applied to the study of elastic-wave propagation in multilayer systems in order to study the effect of defects or absorption [19]. In recent work [20] we used transfer-matrix methods to investigate the response of a multilayered metamaterial system containing periodicity-breaking defects to an incident acoustic plane wave at normal or oblique incidence. The transmission response was found to be composed of pass bands with oscillatory behavior, separated by band gaps, and covers a wide frequency range. The presence of gain and loss in the layers was found to lead to the emergence of symmetry-breaking and re-entrant phases. While defects in general were found to lead to a near or complete loss of PH symmetry at all frequencies, it was shown that they can be exploited to produce highly sensitive responses, making such systems good candidates for sensor applications. The presence of defects as well as their location within the system was found to have a profound effect on the transmission response: changing the thickness of one passive layer shifts transmission resonances to different frequencies, while even very small changes in thickness were found to produce great sensitivity in the responses.

In this work, we extend the above study to the more general investigation of propagation of elastic waves, whereby we exploit interconversions between longitudinal and transverse modes to optimize the directionality in not just the reflection but also the transmission response and also the sensitivity to periodicity-breaking defects.

II. METHODS

We summarize our formalism from our previous work [20], noting where it becomes generalized owing to the consideration of both longitudinal and transverse incident modes.

We consider a system of $n-1$ layers, extending along the negative $\hat{z}$ direction, as in Fig. 1. Each layer is described by homogeneous mass density ρ and elastic properties: the shear modulus μ and Lamé constant λ. The longitudinal and transverse wave speeds are thus $c=\sqrt{(\lambda+2\mu)/\rho}$ and $b=\sqrt{\mu/\rho}$, respectively. We further assume isotropic elasticity where the Lamé constant $\lambda=\frac{2\mu\nu}{1-2\nu}$ and the shear modulus $\mu=\frac{E}{2(1+\nu)}$ are expressed in terms of Young's modulus E and the Poisson ratio ν. A plane wave of frequency ω is incident on the multilayer system from a general (solid or liquid) ambient medium either normally or at an angle θ in the xz plane and exits again in the same or a different medium.

The particle displacement is in the xz plane, and there may be both shear and longitudinal modes present. The displacement field may be split into longitudinal ϕ and transverse $\vec{\psi}$ potentials,

$$\vec{u}=\vec{\nabla}\phi+\vec{\nabla}\times\vec{\psi}, \tag{1}$$

and we set $\vec{\psi}=\psi\hat{y}$. The wave equations for the potentials, assuming time-harmonic plane waves, are

$$\begin{aligned}\nabla^2\phi+k^2\phi=0,\quad k\equiv\omega/c,\\ \nabla^2\psi+\kappa^2\psi=0,\quad \kappa\equiv\omega/b\end{aligned} \tag{2}$$

and have the solutions

$$\begin{aligned}\phi_j=\phi_j'e^{i\alpha_jz}+\phi_j''e^{-i\alpha_jz},\ \alpha_j=\left(k_j^2-\xi^2\right)^{1/2},\ \xi=k_j\sin\theta,\\ \psi_j=\psi_j'e^{i\beta_jz}+\psi_j''e^{-i\beta_jz},\ \beta_j=\left(\kappa_j^2-\chi^2\right)^{1/2},\ \chi=\kappa_j\sin\theta\end{aligned} \tag{3}$$

for each layer j, including the terminating ambient media $j=1$ and $j=n+1$, where the quantities with a prime are amplitudes. For example, for a transverse wave incident at the half-space $n+1$, $r_l=\phi_{n+1}'$ and $t_l=\phi_1''$ are the expressions for the longitudinal-mode reflection and transmission coefficients.

Through transformations from the $\{\phi',\phi'',\psi',\psi''\}$ basis, a transfer matrix for the passage of a wave through one layer and then, by repeated application, through the whole system of $n-1$ layers may be constructed in terms of the displacement equation (1) and the stresses

$$\begin{aligned}Z_x=\mu(\partial u_x/\partial z+\partial u_z/\partial x),\\ Z_z=\lambda(\partial u_x/\partial x+\partial u_z/\partial z)+2\mu\partial u_z/\partial z\end{aligned} \tag{4}$$

as

$$\begin{pmatrix}u_x^{(n)}\\u_z^{(n)}\\Z_z^{(n)}\\Z_x^{(n)}\end{pmatrix}=\underline{\underline{A}}\begin{pmatrix}u_x^{(1)}\\u_z^{(1)}\\Z_z^{(1)}\\Z_x^{(1)}\end{pmatrix}, \tag{5}$$

where $\underline{\underline{A}}$ is the transfer matrix through the entire multilayer (see Ref. [21]).

The boundary conditions applied are the continuity of the displacements u_x and u_z and that of the stresses Z_x, Z_z across

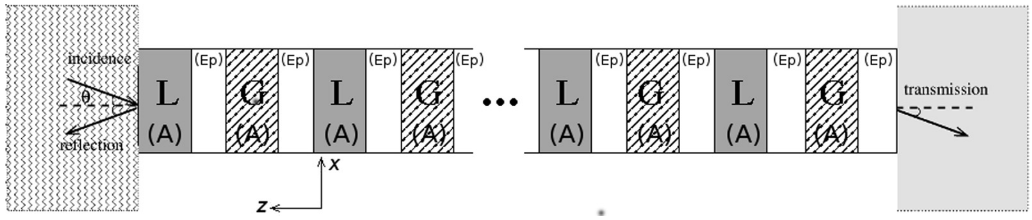

FIG. 1. Metamaterial system under study: oblique wave impinging onto a multilayer stack composed of alternating gain (G) and loss (L) layers, separated by passive material. The materials comprising the layers, A (alumina) and Ep (epoxy thermoset), are discussed in the text. The angle of incidence θ shown corresponds to an incident longitudinal or transverse wave. The orientation shown corresponds to the forward (F) direction.

the $(n, n+1)$ boundary. When this is done and after inverting the system of equations for the stress and displacement to obtain the amplitudes in Eq. (3), we arrive at expressions for the longitudinal and transverse-mode reflections and transmissions from the incident and outgoing sides, respectively, of the multilayer system.

A. System

In the system depicted in Fig. 1 we show a system of alumina (A)/epoxy thermoset (Ep) layers from which an elastic plane wave is incident and exits from arbitrary ambient media (half-spaces at the ends). The propagation direction shown is the "forward" (F) direction. When the ambient media at both ends are different or the same but not comprised of one of the materials of the multilayer, then the total system is spatially asymmetric, even without the inclusion of gain and loss (G/L) parameters.

Here, the response of a multilayer system composed of alternating A and Ep layers to an obliquely incident longitudinal or transverse wave is examined. In our previous study [20], we examined the effect of the incidence angle on the response. In this study we focus almost exclusively on an incident angle of $\theta = \pi/16$ as the results are generalizable to other angles. The parameters of the materials used are given in Table I. When including G/L, through the addition of an imaginary component with a positive (G) or negative (L) sign to Young's modulus, we work in the region of balanced G/L by imposing alternating G/L on the A layers. Loss is easily achieved by adding dissipation, but adding gain represents a major challenge for acoustic/elastic materials. However, piezoelectric materials have been demonstrated to be able to tune the G/L character [22] of these materials. The amount of G/L imparted in our system is in accordance with what has been demonstrated to be achievable for the tunable effective bulk modulus in a recent implementation of an acoustic metamaterial [23].

The units of the frequency ω in all the figures are m kHz/l, where l is the layer thickness (see Table I), which in this study is taken to be the same for all the layers.

B. Unidirectional transmission

The transmission at some frequencies can be unidirectional. When the solid half-spaces at the ends are identical, the forward/backward transmissions under oblique incidence are different only (a) when G/L is present or (b) in the presence of spatial asymmetry such as (i) a periodicity-breaking defect or (ii) when the identical half-spaces are composed of a material different from those present in the multilayer (in the prototype of Fig. 1 studied here). However, the transmission will be asymmetric in the two directions only for the mode which is different from the induced mode (i.e., not that which impinges on the system). The asymmetric or even unidirectional transmission of the induced mode is a very interesting result which seems counterintuitive given that the angle of transmission is technically the same as the angle of incidence. But due to the fact that for the induced mode there is no incident wave, the outgoing angle for the induced mode is, in general, different than that of the incident mode. In this study in addition to asymmetry, we also examine the effect of periodicity-breaking defects. The defect need not be liquid for the asymmetry in the transmission response to occur, although in this study we consider only liquid periodicity-breaking defects.

TABLE I. Parameters of the A/Ep multilayer system. E (ImE) is the real (imaginary, when activated) part of Young's modulus, ν is the Poisson ratio, ρ is the mass density, and l is the layer thickness (uniform in this study). The composition of the half-spaces at the ends was varied (see text) among Ep and one of the other materials which follow Ep in the list.

Material	E (GPa)	Im(E) (GPa)	ν	ρ (Mg/m^3)	Layer thickness
A (alumina)	390	±20	0.26	3.9	l
Ep (epoxy thermoset)	3.5	0	0.25	1.2	l
HDPE (polyethylene)	0.7	0	0.42	0.95	
Glass	65	0	0.23	2.5	
Polycarbonate	2.7	0	0.42	1.2	
SiC	450	0	0.15	2.8	
Cermet	470	0	0.30	11.5	
Cork	0.032	0	0.25	0.18	

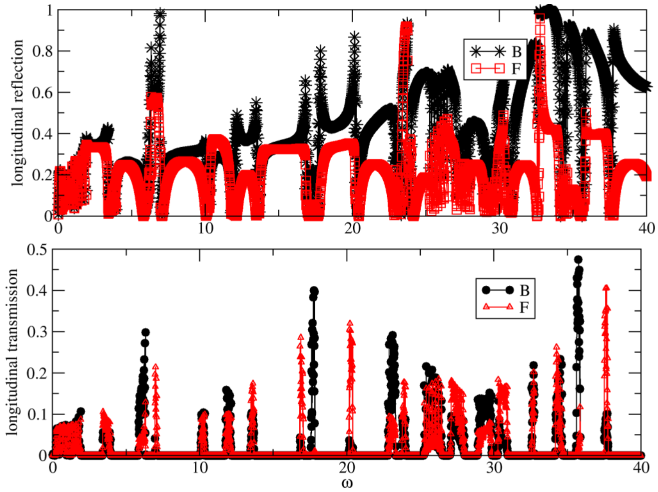

FIG. 2. Longitudinal-mode response for an A/Ep multilayer system (16 total layers) with no G/L present between two glass half-spaces on which a transverse mode is incident at an angle of $\pi/16$ in either the forward (F) or backward (B) direction.

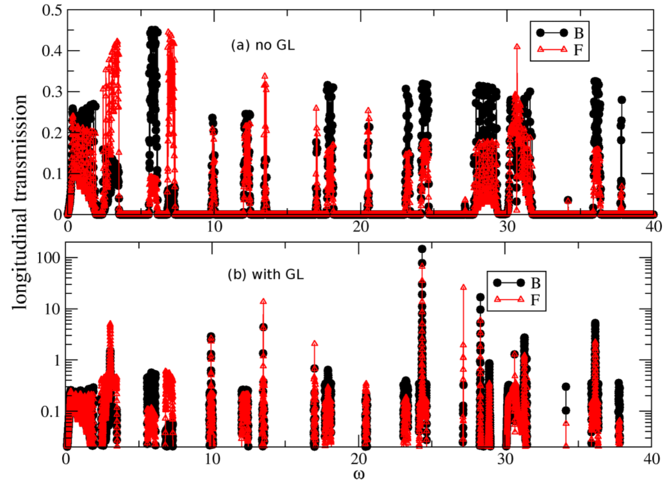

FIG. 3. Longitudinal-mode response for an A/Ep multilayer system (16 total layers) between two glass half-spaces on which a transverse mode is incident at an angle of $\pi/8$ in either the forward (F) or backward (B) direction, (a) with no G/L present and (b) with GL.

The surface band structure of the system depicted in Fig. 1 is of the same type as those depicted in Fig. 2 of Ref. [20] but with the horizontal axis transformed to show k_x, the wave vector in the plane of the layers, equal to $k \sin\theta$, where the wave vector k is for propagation in both the x and z directions and corresponds to solutions of the dispersion relation analogous to those shown in Fig. 1 of Ref. [20]. It is clear that for the cases of oblique incidence, $\theta \neq 0$, such a transformation results in spatial dispersion, a precondition for unidirectional transmission [5].

III. RESULTS

In this section we present our main results upon either changing the composition of the half-spaces or adding defects. The purpose is to highlight the directionality in the transmission and reflection responses. When the half-spaces admit only longitudinal waves, i.e., for liquid ambient media (see, for example, the systems investigated in Ref. [20]), no asymmetry in the transmission is found, while the magnitude of the reflections is different only upon the inclusion of G/L elements (PH symmetric system) [20]. Both longitudinal and transverse modes need to be transmitted for there to be an asymmetry in the transmission responses. When the half-spaces admit both longitudinal and transverse modes, i.e., are solid, then directionality in the reflection is seen when the total system is asymmetric in space (Fig. 1 when the ambient media are different or not the same as either of the materials in the multilayer). However, the response to the same mode type as the incident wave showed no directionality when the half-spaces were identical. When the half-spaces were different, the response to either the incident or the induced mode was found to display a directionality as the incident and outgoing angles for this type of mode were no longer identical.

In Fig. 2 we show the reflection and transmission when both half-spaces are glass and without any G/L present for a transverse mode incident at an angle of $\pi/16$. The reflections are different because the system is asymmetric and the ends are solid, enabling the interconversion between modes for the incoming and outgoing waves, viz., the asymmetric system but liquid ambient media in Ref. [20], where the reflections showed directionality only when G/L was present. There are several narrow frequency ranges in which the transmission in particular is virtually unidirectional, namely, around $\omega = 6.1, 7.0, 16.9, 17.7, 20.3, 35.7, 37.6$. Similarly for Fig. 3(a) where the angle is $\pi/8$, the transmission here is also unidirectional at several frequencies. When G/L is added, however, as in Fig. 3(b), many of these resonances are not responsive and remain at the same low level. In particular those narrow bands centered around $\omega = 5.8$ and 7.1, displaying strong unidirectional transmission, did not respond to G/L. The responses at other frequencies are amplified in both directions rather than being attenuated in one direction (e.g., at $\omega = 3.0$, 36.2), so these are obviously not useful for operation as a unidirectional device. In the case of Fig. 3(b) the system, extrapolated to an infinite multilayer, is in the broken PH regime for all frequencies as the angle is relatively large.

In Fig. 4 we show the transverse-mode transmission response of the same A/Ep system located between Ep half spaces at both ends of an incident longitudinal wave. Since the total system without the inclusion of G/L [the case in Fig. 4(a)] here is symmetric, the transmission is independent of direction. Once G/L is added, however [Fig. 4(b)], notable asymmetry in the transmission at some frequencies occurs. In particular, at $\omega = 1.76$, the F direction gives a transmission of 0.18, while the backward (B) direction gives 0.82, but this behavior is not seen over a wide range, as it is caused by a shift in overlap in the transmission spectra of the two directions. Other cases of strongly asymmetric transmission, such as the band around $\omega = 12$, show low absolute values overall, in this case a value of 0.2 for the F direction and 0.01 for the B direction, making them not very exploitable. In the band around $\omega = 24.2$, the transmission of the F direction reaches 5.3, and that of the B direction reaches 0.36 in this example, making the transmission not quite unidirectional but highly

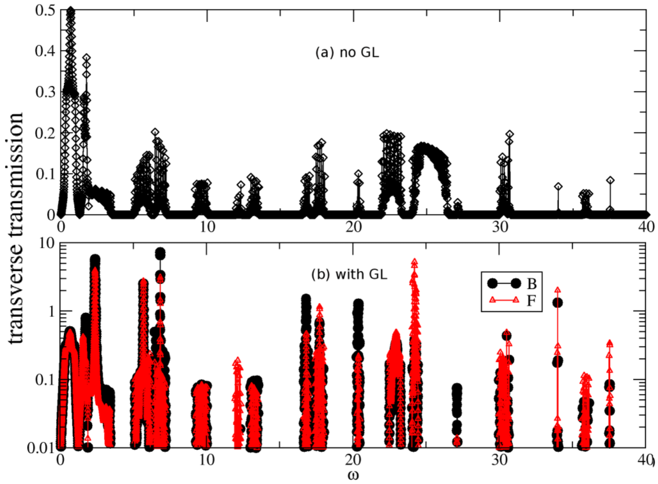

FIG. 4. Transverse-mode transmission response for an A/Ep multilayer system (16 total layers): (a) without G/L and (b) with G/L between two Ep half-spaces on which a longitudinal mode is incident at an angle of $\pi/16$ in either the forward (F) or backward (B) direction. In (a) both directions give the same result.

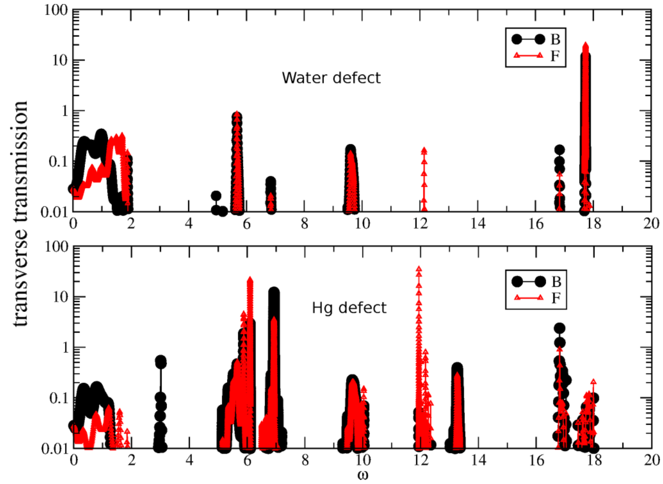

FIG. 6. Transverse-mode transmission response for the same system and conditions as in Fig. 4(b), but with water or Hg replacing the second Ep, as taken from the F direction.

asymmetric. In the case in Fig. 4(b) there are some regions of ω where an analysis of the transfer matrix shows that the system is in the PH-symmetric regime.

In Fig. 5 we show the transverse transmission and reflection responses of the otherwise passive system, where we have also added a water periodicity-breaking defect, replacing the second Ep in the F direction. The incidence is again of a longitudinal mode at an angle of $\pi/16$. At low ω, clear directionality is evident, especially in the transmission, demonstrating that even if the total system including the ambient media is symmetric, the addition of a periodicity-breaking defect can generate asymmetry in the transmission and the reflection responses. However, the direction with the high transmission has low reflection and vice versa, rather than the system being unidirectional in both transmission and reflection responses. When a defect of water or Hg is added to the system in Fig. 4(b), including G/L, we can achieve large responses as in Fig. 6. However, the responses here, particularly for the case of water, were found to be highly variable with respect to defect location, successively alternating between large responses and sinking below 1 depending on which Ep was replaced. What is most notable is that the response with the Hg defect is not only greatly magnified but highly unidirectional at $\omega = 12$, with one direction having a transmission of 0.05 and the other having a transmission of 33. This same peak is also present in the undefected system [Fig. 4(b)], where it is asymmetric but nevertheless has a weak response. With the water defect this peak maintains a similarly low response, while in one direction it is nonexistent on the scale used. The cases with the defects studied (Fig. 6) were in the broken PH phase at all frequencies.

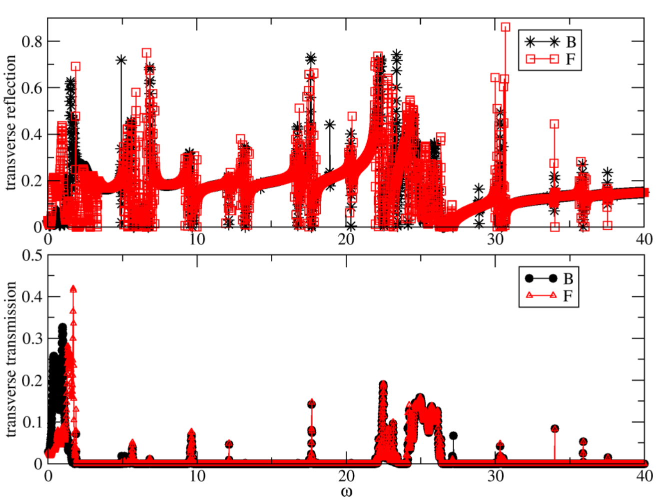

FIG. 5. Same system and incident conditions as in Fig. 4(a), but with water replacing the second Ep, as taken from the F direction. Shown are both the transverse-mode reflection and transmission responses.

In Fig. 7 we depict the extent of the directional response of the induced-mode response to an incident longitudinal or transverse wave at an angle of $\pi/16$ for the periodic multilayer system without any defects when the ambient media, the same at both ends, are varied. More specifically, in Fig. 7 we show the difference in the transmission response of the induced mode between the forward and backward directions. We see that for longitudinal incidence, cork ends give the largest asymmetry in the response, and SiC gives the smallest. For transverse incidence, cork gives almost no difference between the two directions, while high-density polyethylene (HDPE) gives the largest difference. At $\omega = 6$ and 20 in Fig. 7(b) the peak for glass is barely visible, behind that of cermet but at the same depth. Similar plots but for relative directionality, i.e., the difference in the transmissions with respect to the largest of the two transmissions, may lead to misleading conclusions wherein, for frequencies for which both transmissions are low but one is essentially zero, the result yields perfect directionality, while in situations with a higher absolute difference, in which one transmission is high and the other is merely low, a lower value of the directionality results. We plot the relative directionality in Fig. 8 for the same systems as in Fig. 7. We have included only data for

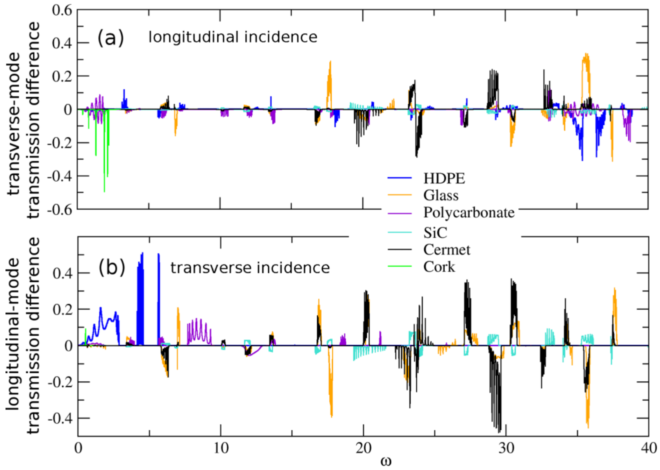

FIG. 7. (a) Transverse-mode and (b) longitudinal-mode transmission response difference between the F and B directions for an A/Ep multilayer system (16 total layers) without G/L between two half-spaces of varying composition. In (a) a longitudinal mode is incident at an angle of $\pi/16$, while in (b) a transverse mode is incident at $\pi/16$.

which one of the two transmission values exceeds a "cutoff," set here to 0.1, in order to avoid fictitiously high values of directionality for the case mentioned above, where both transmissions are very low. In the plots in Fig. 8, glass in particular yields several perfect directionalities at the same frequencies where some small differences between the two directions are seen, but this is due to the fact that one of the directions had a near-zero response, while the other had a value which was low but above the cutoff used.

In Fig. 9 we show the difference in the defect response between the two directions for three materials, glass, HDPE, and polycarbonate, to an incident transverse wave at $\pi/16$, that is, how the presence of a defect, in this case water replacing Ep in different locations, affects the asymmetry in the response in the two directions. Having an Hg defect rather than a water defect did not significantly alter the results, as Hg shows an improvement in defect response compared with water but not in the directional asymmetry of the response, something which is more affected by the material at the ends. These plots were produced by taking the difference between the defect responses (defined as the difference between the response of the system containing a defect and that without) of the F and B directions. HDPE showed the highest response, and polycarbonate showed almost none, meaning that the difference in the responses between the two directions due to the presence of a defect is mainly determined by the composition of the ends, as in the case of no defects [see Fig. 7(b) for transverse incidence]. It is interesting,

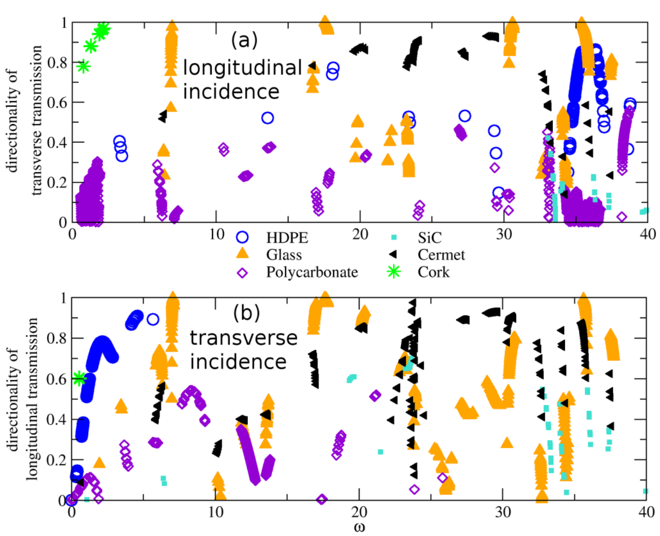

FIG. 8. (a) Transverse-mode and (b) longitudinal-mode transmission response relative directionality between the F and B directions for the same system as depicted in Fig. 7.

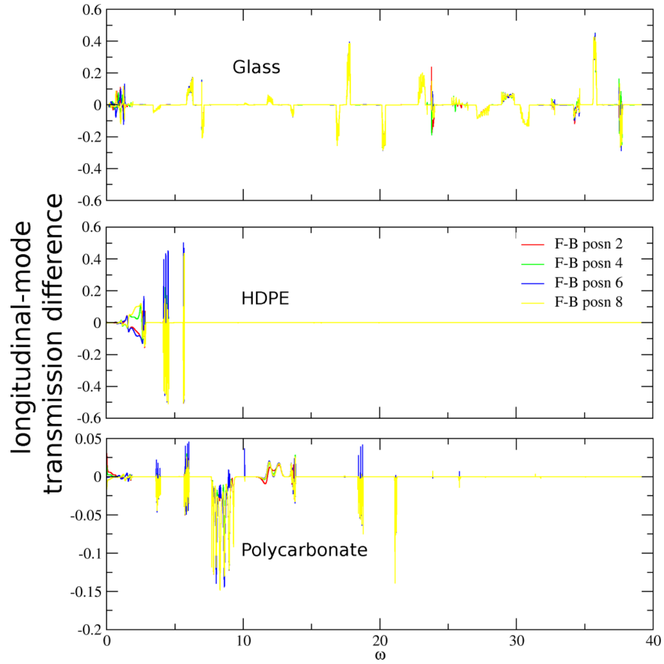

FIG. 9. Longitudinal-mode transmission response difference between the F and B directions compared to the respective undefected system for a transverse mode incident at an angle of $\pi/16$ on an A/Ep multilayer system (16 total layers) without G/L between two half spaces of glass, HDPE, or polycarbonate.

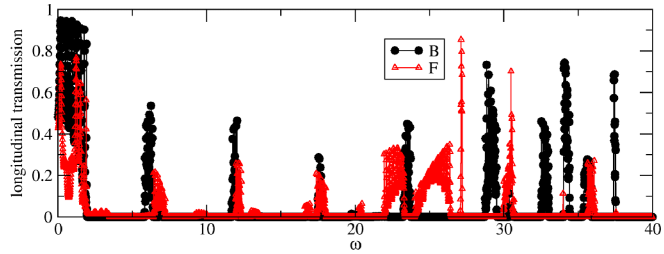

FIG. 10. Longitudinal-mode transmission response for an A/Ep multilayer system (16 total layers) without G/L between Ep/glass half-spaces (referring to the F direction) on which a longitudinal mode is incident at an angle of $\pi/16$ in either the forward (F) or backward (B) direction.

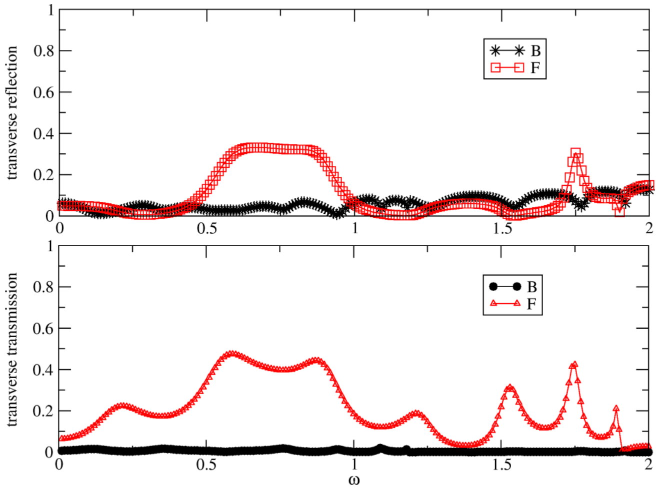

FIG. 11. Transverse-mode reflection and transmission responses for an A/Ep multilayer system (16 total layers) without G/L between Ep/glass half-spaces (referring to the F direction) on which a longitudinal mode is incident at an angle of $\pi/16$ in either the forward (F) or backward (B) direction.

however, that sometimes there is a great variation even in the sign of the response depending on the location of the defect.

We also examined the effects of having different half-spaces at the ends. In all of the previous cases where the ambient media were the same, the transmission response of the same mode as the incident one was unaffected by direction. However, when the ambient media are different, the transmission response displays a directional asymmetry for this mode. As an example, we show in Fig. 10 the longitudinal-mode transmission response to a longitudinal incident mode at $\theta = \pi/16$ for the same multilayer system as before without any G/L or defects. In general, we find that when the half-spaces are different, the asymmetry in the response between the two directions is much more evident owing to the fact that, for example, in this case the transmission angle is different from the incident angle. In Fig. 11 we show, for the same system, the transverse-mode response to the longitudinally incident wave, and here, we find a region with asymmetry simultaneously in both the transmission and reflection responses.

Last, we briefly mention that if just one of the half-spaces is solid and the other is liquid, we still have asymmetry in the longitudinal-mode reflection and transmission responses, without any G/L or defects present, in the case of an incident longitudinal wave. The transverse-mode response to a longitudinal incident mode is a trivial case, resulting in the total suppression of the transmission response in one direction. This situation is more likely to be encountered in an application, for example, wave breaking, while the case of both ends being solid is more applicable to seismology.

IV. CONCLUSIONS

We have investigated the propagation of obliquely incident longitudinal and transverse elastic waves in a completely passive multilayered metamaterial system having a spatial dispersion, located between solid ambient media. We found that the interconversion between longitudinal and transverse modes within the system leads to large responses in the induced (different from the incident) mode. We further found that there can be directionality not just in the reflection response but also in the transmission for the induced mode. This represents an emulation of a property of nonreciprocal propagation, and it is achievable for a completely passive metamaterial system if both transverse and longitudinal modes are able to propagate, as is the case for when the elastic waves originate from and end up in solid ambient media.

In some cases, especially when the elastic wave enters and exits from different solid materials, we find that the transmission and reflection responses can be simultaneously unidirectional, with both the transmission and reflection responses being strongly suppressed in one direction and greatly enhanced in the other. If spatial asymmetry in the total system including the ambient media is present, then the asymmetry in the transmission occurs even in the absence of periodicity-breaking defects or gain and loss in the layers, although it can be greatly modified by including these parameters. The degree of asymmetry in the transmission response, whether or not a periodicity-breaking defect is present, is determined mainly by the composition of the ambient media from which the elastic wave enters or exits the multilayer system. However, there was great variability in the response depending on the location of the defect within the multilayer system as well as some variation in the absolute sensitivity with the type of defect present. Devices built on this property could, for example, function as sensors. The introduction of gain or loss in the system causes some of the responses to be greatly enhanced and sometimes profoundly magnifies their asymmetry. Some uses for the unidirectional transmission and reflection properties displayed by the system studied here could include breakwater structures and seismic protection.

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