Coupling-assisted Landau-Majorana-Stückelberg-Zener transition in a system of two interacting spin qubits

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We analyze a system of two interacting spin-qubits subjected to a Landau-Majorana-Stückelberg-Zener (LMSZ) ramp. We prove that LMSZ transitions of the two spin qubits are possible without an external transverse static field since its role is played by the coupling between the spin qubits. We show how such a physical effect could be exploited to estimate the strength of the interaction between the two spin qubits and to generate entangled states of the system by appropriately setting the slope of the ramp. Moreover, the study of effects of the coupling parameters on the time behavior of the entanglement is reported. Finally, our symmetry-based approach allows us to discuss also effects stemming from the presence of a classical noise or non-Hermitian dephasing terms.

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I. INTRODUCTION

The Landau-Majorana-Stückelberg-Zener (LMSZ) scenario [1] and the Rabi one [2] represent two milestones among exactly solvable time-dependent semiclassical models for two-level systems. A common fundamental property of these two models is the possibility of realizing a full population inversion in a two-state quantum system: in the former case through an adiabatic passage via a level crossing and in the second case thanks to the application of a resonant π pulse.

It is important to underline that the LMSZ scenario, different from the Rabi case, is an ideal model. The word "ideal" refers to the fact that it consists of a process characterized by an infinite time duration, resulting, then, in being practically unrealizable. This fact leads, indeed, to not physical properties such as the fact that the energies of the adiabatic states diverge at initial ($-\infty$) and final ($+\infty$) instants. As a consequence, both mathematical and physical problems arise when amplitudes and not only probabilities are necessary, e.g., when initial states present coherences [3,4]. In such cases one can alternatively use either the exact solutions of the finite LMSZ scenario [5] or the Allen-Eberly-Hioe model [6], the Demkov-Kunike model [7], or other models [8,9] where no divergency problems arise and the transition probability is rather simple.

However, despite this circumstance, it is a matter of fact that the LMSZ handles peculiar dynamical aspects of a lot of physical systems [10]. This relevant aspect has increased the popularity of the LMSZ model, and several efforts have been made towards its generalization to the case of N-level quantum systems [3,11,12] and total crossing of bare energies [13]. Moreover, its experimental feasibility gave it a basic role in the area of quantum technology thanks also to several sophisticated techniques developed for precise local manipulation of the state and the dynamics of a single qubit in a chain [14–19].

In such an applicative scenario, as we know, several sources of incoherence can be present [20–23]: incoherent (mixed) states, relaxation processes (e.g., spontaneous emission), and interaction with a surrounding environment (e.g., nuclear spin bath). They generate incoherent excitation, leading to a departure from a perfect (ideal) population transfer. Therefore, more realistic descriptions of quantum systems subjected to the LMSZ scenario comprising such effects have been proposed [24–29].

In this respect, the most relevant influence on the dynamics of a spin-qubit primarily stems from the coupling with its nearest neighbors. Recently, attention has been focused on double interacting spin-qubit systems subjected to the LMSZ scenario [30–35]. These papers investigated the coupling effects in the two-spin system dynamics in view of possible experimental techniques and protocols. Moreover, such systems, under specific conditions, behave effectively as a two-level system with relevant applicability in quantum information and computation sciences [36]. In the references cited above, indeed, generation of entangled states [30] or the singlet-triplet transition [15,31,32] in the two-qubit system under the LMSZ scenario was studied.

With the same objective in mind, that is, to characterize physical effects stemming from the coupling between two spin qubits subjected to a LMSZ scenario, in this paper we study a two-spin-1/2 system described by a C_2-symmetry Hamiltonian model. We consider coupling terms compatible with the symmetry of the Hamiltonian, namely, isotropic and anisotropic exchange interaction. The two spins 1/2 are,

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moreover, subjected only to a LMSZ ramp with no transverse static field. We show that LMSZ transitions for the two spin-qubits are still possible thanks to the presence of the coupling playing the role of an effective transverse field. Such an effect, which we call *coupling-assisted LMSZ transition*, deserves particular attention for two reasons. First, it can be exploited to estimate the presence and the relative weight of different coupling parameters determining the symmetry of the Hamiltonian and then the dynamics of the two spins. Second, through such an estimation, it is possible to set the slope of the field ramp in such a way to generate asymptotic entangled states of the two qubits.

This paper is organized as follows. In Sec. II we introduce the model and its symmetry properties on which the dynamical reduction is based. In Sec. III the application of the LMSZ scenario to both the subdynamics (that is, the two-qubit dynamics restricted to the invariant subspaces) is performed. Moreover, physical effects stemming from the (an)isotropy of the exchange interaction are brought to light. In Sec. IV, we emphasize the possibility of estimating the values of the coupling parameters. The generation of asymptotic entangled states of the two spins through coupling-based LMSZ transitions is reported in Sec. V. Some effects of a possible interaction with a surrounding environment, providing for either a classical noisy field component or non-Hermitian terms in the Hamiltonian model, are taken into account in Sec. VI. Finally, some conclusive comments and further remarks can be found in Sec. VII.

II. THE MODEL

Let us consider the following model describing two interacting spin qubits:

$$H = \hbar\omega_1(t)\hat{\sigma}_1^z + \hbar\omega_2(t)\hat{\sigma}_2^z + \gamma_x\hat{\sigma}_1^x\hat{\sigma}_2^x + \gamma_y\hat{\sigma}_1^y\hat{\sigma}_2^y + \gamma_z\hat{\sigma}_1^z\hat{\sigma}_2^z, \tag{1}$$

where $\hat{\sigma}_i^x$, $\hat{\sigma}_i^y$, and $\hat{\sigma}_i^z$ ($i = 1, 2$) are the Pauli matrices and all the parameters may be thought of as time dependent. The matrices are represented in the following ordered two-spin basis: $\{|{+}{+}\rangle, |{+}{-}\rangle, |{-}{+}\rangle, |{-}{-}\rangle\}$ ($\hat{\sigma}^z|\pm\rangle = \pm|\pm\rangle$).

The C_2 symmetry with respect to the z direction, possessed by the Hamiltonian, causes the existence of two dynamically invariant Hilbert subspaces related to the two eigenvalues of the constant of motion $\hat{\sigma}_1^z\hat{\sigma}_2^z$ [37]. Based on such a symmetry, the time evolution operator, the solution of the Schrödinger equation $i\hbar\dot{U} = HU$, may be formally put in the following form [37]:

$$U = \begin{pmatrix} a_+(t) & 0 & 0 & b_+(t) \\ 0 & a_-(t) & b_-(t) & 0 \\ 0 & -b_-^*(t) & a_-^*(t) & 0 \\ -b_+^*(t) & 0 & 0 & a_+^*(t) \end{pmatrix}. \tag{2}$$

The condition $U(0) = \mathbb{1}$ is satisfied by setting $a_\pm(0) = 1$ and $b_\pm(0) = 0$. It is worth noticing that $a_\pm(t)$ and $b_\pm(t)$ are the time-dependent parameters of the two evolution operators,

$$U_\pm = e^{\mp i\gamma_z t/\hbar}\begin{pmatrix} a_\pm(t) & b_\pm(t) \\ -b_\pm^*(t) & a_\pm^*(t) \end{pmatrix}, \tag{3}$$

the solutions of two independent dynamical problems of a fictitious single spin 1/2, namely, $i\hbar\dot{U}_\pm = H_\pm U_\pm$, $U_\pm(0) = \mathbb{1}_\pm$, with

$$\begin{aligned} H_\pm &= \begin{pmatrix} \hbar\Omega_\pm(t) & \gamma_\pm \\ \gamma_\pm & -\hbar\Omega_\pm(t) \end{pmatrix} \pm \gamma_z\mathbb{1}_\pm \\ &= \hbar\Omega_\pm(t)\hat{\sigma}^z + \gamma_\pm\hat{\sigma}^x \pm \gamma_z\mathbb{1}_\pm, \end{aligned} \tag{4}$$

where

$$\Omega_\pm(t) = [\omega_1(t) \pm \omega_2(t)], \quad \gamma_\pm = (\gamma_x \mp \gamma_y), \tag{5}$$

and $\mathbb{1}_\pm$ represent the identity operators within the two-dimensional subspaces. Thus, the solution of the dynamical problem of the two interacting spins 1/2 is traced back to the solution of two independent problems, each one of a single (fictitious) spin 1/2 [37].

The explicit expressions of $a_\pm(t)$ and $b_\pm(t)$ depend on the specific time dependences of $\omega_1(t)$ and $\omega_2(t)$. It is well known that it is not possible to find the analytical solution of the Schrödinger equation for a spin 1/2 subjected to a generic time-dependent field. Therefore, specific exactly solvable time-dependent scenarios for a single spin 1/2 might be of great help to investigate the dynamics of the two interacting spin systems under scrutiny [37].

III. COUPLING-BASED LMSZ TRANSITION

In this section we investigate the case in which a LMSZ ramp is applied on either just one or both the spins. Our following theoretical analysis is based on the possibility of experimentally addressing at will the spin systems exploiting, for example, scanning tunneling microscopy (STM). It appears hence appropriate to furnish a sketch of such a technique.

STM proved to be an excellent experimental technique for controlling the dynamics of spin-qudit systems for two main reasons: (1) the possibility of building, atom by atom, atomic-scale structures [38], such as spin chains and nanomagnets [39], and (2) the possibility of controlling the whole system by addressing a single element (qudit) while it interacts with the others [39–41], succeeding in realizing, for example, logic operations [38]. The manipulation of a single-qudit dynamics is performed through the exchange interaction between the atom on the tip of the scanning tunneling microscope and the target atom in the chain. It is possible to show that such an interaction is equivalent to a magnetic field applied to the atom we want to manipulate [18,39]. In this way, it is easy to guess that a time-dependent distance between the tip and the target atom generates a time-dependent exchange coupling, giving rise, in turn, to a time-dependent effective magnetic field on the atom of the chain, as analyzed in Ref. [18]. Based on such an observation, in Ref. [19] the authors studied the spin dynamics and entanglement generation in a spin chain of Co atoms on a surface of $Cu_3N/Cu(110)$. Precisely, they considered a LMSZ ramp along the z direction produced in a time window of 20 ps and a short Gaussian pulse in the x direction (half width of 10 ps).

A. Collective LMSZ dynamics

In light of the STM experimental scenario, we take into account first the case of a LMSZ ramp applied to the first spin such that

$$\hbar\omega_1(t) = \alpha t/2, \quad \hbar\omega_2(t) = 0, \quad t \in (-\infty, \infty), \tag{6}$$

where α is related to the velocity of variation of the field, $\dot{B}_z \propto \alpha$, and it is considered a positive real number without loss of generality. Let us consider, moreover, the two spins initialized in the state $|--\rangle$. In this instance, the subdynamics governed by H_+ is characterized by a LMSZ scenario where the longitudinal (z) magnetic field produces the standard LMSZ ramp $\hbar\Omega_+(t) = \hbar\omega_1(t) = \alpha t/2$ and the transverse effective magnetic field along the x direction is given by γ_-. It is well known that the dynamical problem for such a time-dependent scenario can be analytically solved. The transition probability of finding the two-spin system in the state $|++\rangle$ coincides with the probability of finding the fictitious spin 1/2 subjected to H_+ in its state $|+\rangle$ starting from $|-\rangle$ and reads [1]

$$P_+ = |\langle ++|U_+(\infty)|--\rangle|^2 = 1 - \exp\{-2\pi\gamma_+^2/\hbar\alpha\}. \tag{7}$$

If we now, instead, consider the two spins initially prepared in $|-+\rangle$, the probability for each spin 1/2 of undergoing a LMSZ transition, that is, the probability of finding the two-spin system in the state $|+-\rangle$, results:

$$P_- = |\langle +-|U_-(\infty)|-+\rangle|^2 = 1 - \exp\{-2\pi\gamma_-^2/\hbar\alpha\}. \tag{8}$$

This time the transition probability is governed by the fictitious magnetic field given by γ_-. The effective longitudinal magnetic field, instead, is the same, namely, $\hbar\Omega_-(t) = \hbar\omega_1(t) = \alpha t/2$. We see that in both cases, although a constant transverse magnetic field is absent, the LMSZ transition of both spins is possible thanks to the presence of the coupling between them. It is important to stress that, for the cases considered before, if $\gamma_x = \gamma_y$ (as often happens experimentally), we cannot have a transition in the first case, that is, in the subdynamics involving $|++\rangle$ and $|--\rangle$. In this instance, indeed, P_+ happens to be zero at any time.

B. Isotropy effects: Local LMSZ transition by nonlocal control and state transfer

The symmetry-based dynamical decomposition and the application of the STM LMSZ scenario in each subdynamics allow us to bring to light peculiar evolutions of physical interest. For example, if we consider $\gamma_x \neq \gamma_y$ and the initial condition

$$|+\rangle \otimes \frac{|+\rangle + |-\rangle}{\sqrt{2}}, \tag{9}$$

the two states $|++\rangle$ and $|--\rangle$ evolve independently, and applying the LMSZ ramp, we have the probability $P = P_+P_-$ to find asymptotically the two-spin system in the state

$$|-\rangle \otimes \frac{|+\rangle + |-\rangle}{\sqrt{2}}. \tag{10}$$

We see that such a dynamics leaves unaffected the second spin, while it produces a LMSZ transition only in the first spin. The dynamical evolution of the symmetric initial condition

$$\frac{|+\rangle + |-\rangle}{\sqrt{2}} \otimes |+\rangle \tag{11}$$

is also relevant. This time, we get the same probability, $P = P_+P_-$, of finding asymptotically the two-spin system in

$$\frac{|+\rangle + |-\rangle}{\sqrt{2}} \otimes |-\rangle. \tag{12}$$

This case results less intuitive, even though we are reproducing the same dynamics but with interchanged roles of the two spins. In this instance, in fact, we generate a LMSZ transition only in the second spin by locally applying the field to the first one. This shows that the coupling between the two spins plays a key role in achieving nonlocal control of the second spin by locally manipulating the first ancilla qubit.

If we consider, instead, $\gamma_x = \gamma_y = \gamma/2$, we know that the transition $|--\rangle \leftrightarrow |++\rangle$ is suppressed. This means that if we consider as initial conditions the states in Eqs. (9) and (11), we get asymptotically, this time, the states

$$\frac{|+\rangle + |-\rangle}{\sqrt{2}} \otimes |+\rangle, \tag{13a}$$

$$|+\rangle \otimes \frac{|+\rangle + |-\rangle}{\sqrt{2}}, \tag{13b}$$

respectively, with probability $P = 1 - \exp\{-2\pi\gamma^2/\hbar\alpha\}$. We see that the isotropic properties of the exchange interaction consistently change the dynamics of the system. When the exchange interaction is isotropic, indeed, the asymptotic states reached by the initial conditions (9) and (11) radically change. In these cases, the resulting physical effect is a state transfer or a state exchange between the two spin qubits. Therefore, the different state transitions from state (9) [(11)] to state (10) or (13a) [(12) or (13b)] can reveal the level of isotropy of the exchange interaction.

IV. COUPLING PARAMETER ESTIMATION

It is interesting to notice that the coupling-based LMSZ transition could be used to estimate the coupling parameters. By measuring P_+ and P_- [Eqs. (7) and (8), respectively] in a physical scenario describable by the Hamiltonian model (1), we get an estimation of γ_+ and γ_- and then of the two coupling parameters γ_x and γ_y. Supposing we know P_+ and P_-, we have indeed

$$\gamma_x = \frac{1}{2}\sqrt{\frac{\hbar\alpha}{2\pi}}\left[\sqrt{\ln\left(\frac{1}{1-P_-}\right)} + \sqrt{\ln\left(\frac{1}{1-P_+}\right)}\right],$$
$$\gamma_y = \frac{1}{2}\sqrt{\frac{\hbar\alpha}{2\pi}}\left[\sqrt{\ln\left(\frac{1}{1-P_-}\right)} - \sqrt{\ln\left(\frac{1}{1-P_+}\right)}\right]. \tag{14}$$

We wish to emphasize that we may estimate the coupling parameters also through the Rabi oscillations occurring in the two subspaces. Applying, indeed, a constant field ω_1 on the first spin, the two probabilities P_+ and P_- become

$$P_+ = \frac{\gamma_+^2}{\hbar^2\omega_1^2 + \gamma_+^2}\sin^2\left(\sqrt{\omega_1^2 + \gamma_+^2/\hbar^2}\, t\right),$$
$$P_- = \frac{\gamma_-^2}{\hbar^2\omega_1^2 + \gamma_-^2}\sin^2\left(\sqrt{\omega_1^2 + \gamma_-^2/\hbar^2}\, t\right). \tag{15}$$

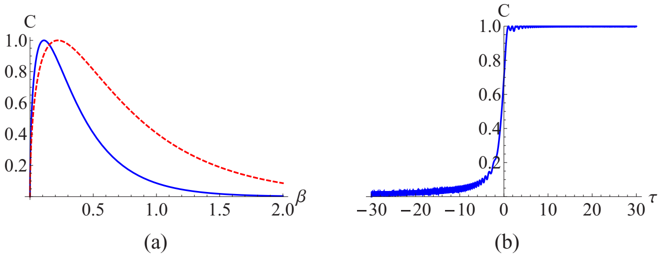

FIG. 1. (a) The two curves of the concurrence in Eq. (16a) (solid blue line) and Eq. (16b) (red dashed line) for $\beta_-/2 = \beta_+ = \beta$. (b) Time behavior of concurrence for the initial condition $|--\rangle$ and $\beta_+ = 0.1$ plotted against the dimensionless time $\tau = \sqrt{\alpha/\hbar}\, t$.

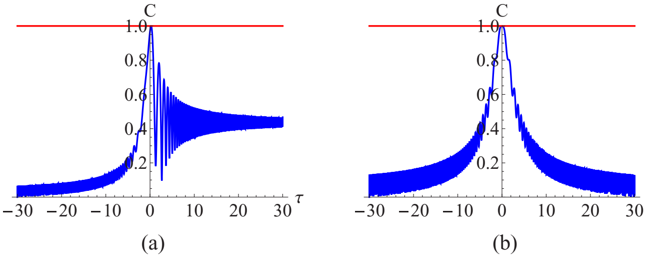

FIG. 2. Time behavior of the concurrence against the dimensionless parameter $\tau = \sqrt{\alpha/\hbar}\, t$ during a LMSZ process when the system starts from the state $|--\rangle$ for (a) $\beta_+ = 1/2$ and (b) $\beta_+ = 2$. The upper straight curve corresponds to $C(\tau) = 1$.

So, by measuring the frequency and the amplitude of the oscillations in the two cases, we may get information about the relative weights of the coupling parameters.

V. ENTANGLEMENT

A precise estimation of the coupling parameters is useful also to generate entangled states of the two spins. Through knowledge of them, indeed, we may set the parameter α in order to get asymptotically $P_\pm = 1/2$, generating an entangled state. Indeed, if the two spins start from state $|--\rangle$ or $|-+\rangle$, they reach asymptotically the pure state $(|++\rangle + e^{i\phi}|--\rangle)/\sqrt{2}$ in the first case and $(|+-\rangle + e^{i\phi}|-+\rangle)/\sqrt{2}$ in the second case, which are maximally entangled states. The asymptotic curves of the concurrence (the entanglement measure for two spins 1/2 introduced in Ref. [42]), when the two-spin system is initialized in $|--\rangle$ or $|-+\rangle$, read, respectively,

$$\begin{aligned} C &= 2|c_{++}c_{--}| = 2\sqrt{P_+(1-P_+)} \\ &= 2\sqrt{(1-e^{-2\pi\beta_+})e^{-2\pi\beta_+}}, \end{aligned} \tag{16a}$$

$$\begin{aligned} C &= 2|c_{+-}c_{-+}| = 2\sqrt{P_-(1-P_-)} \\ &= 2\sqrt{(1-e^{-2\pi\beta_-})e^{-2\pi\beta_-}}, \end{aligned} \tag{16b}$$

and they exhibit a maximum for $\beta_+ = \beta_- = \ln(2)/2\pi \approx 0.11$. In the above expressions we set $\beta_+ = \gamma_+^2/\hbar\alpha$ and $\beta_- = \gamma_-^2/\hbar\alpha$, while c_{++} and c_{--} (c_{+-} and c_{-+}) are the asymptotic amplitudes of the states $|++\rangle$ and $|--\rangle$ ($|+-\rangle$ and $|-+\rangle$), respectively. Therefore, $\ln(2)/2\pi$ is exactly the value the LMSZ parameters β_+ and β_- must have to realize the generation of the entangle states $(|++\rangle + e^{i\phi}|--\rangle)/\sqrt{2}$ and $(|+-\rangle + e^{i\phi}|-+\rangle)/\sqrt{2}$ when the two spins start from $|--\rangle$ and $|-+\rangle$, respectively. Figure 1(a) reports the two curves for $\beta_-/2 = \beta_+ = \beta$.

We may verify this fact by investigating the behavior of the concurrence in time. To this end, the exact solutions of the two time-dependent parameters determining the two time evolution operators U_+ and U_- in Eq. (3), related to each subdynamics, are necessary, and they read [5]

$$\begin{aligned} a_\pm &= \frac{\Gamma_f(1-i\beta_\pm)}{\sqrt{2\pi}} \\ &\quad\times [D_{i\beta_\pm}(\sqrt{2}e^{-i\pi/4}\tau)\, D_{-1+i\beta_\pm}(\sqrt{2}e^{i3\pi/4}\tau_i) \\ &\quad + D_{i\beta_\pm}(\sqrt{2}e^{i3\pi/4}\tau)\, D_{-1+i\beta_\pm}(\sqrt{2}e^{-i\pi/4}\tau_i)], \\ b_\pm &= \frac{\Gamma_f(1-i\beta_\pm)}{\sqrt{2\pi\beta_\pm}} e^{i\pi/4} \\ &\quad\times [-D_{i\beta_\pm}(\sqrt{2}e^{-i\pi/4}\tau)\, D_{-1+i\beta_\pm}(\sqrt{2}e^{i3\pi/4}\tau_i) \\ &\quad + D_{i\beta_\pm}(\sqrt{2}e^{i3\pi/4}\tau)\, D_{-1+i\beta_\pm}(\sqrt{2}e^{-i\pi/4}\tau_i)]. \end{aligned} \tag{17}$$

Γ_f is the gamma function, while $D_\nu(z)$ are the parabolic cylinder functions [43] and $\tau = \sqrt{\alpha/\hbar}\, t$ is a time dimensionless parameter; τ_i identify the initial time instant. If the system starts, e.g., from the state $|--\rangle$, the amplitudes are

$$c_{++} = b_+, \quad c_{--} = a_+^*, \quad c_{+-} = c_{-+} = 0, \tag{18}$$

and the related time behavior of the concurrence $C = 2|b_+||a_+|$ for $\beta_+ = 0.1$ is reported in Fig. 1(b). We see, as expected, that such a choice of the LMSZ parameter generates a maximally entangled state of the two spin qubits. It is important to point out that, on the basis of Eqs. (17), the parameter β_+ determines not only the asymptotic value of the concurrence but also its time behavior. This fact is confirmed and can be seen in Figs. 2(a) and 2(b), which report the concurrence against the dimensionless parameter τ for $\beta_+ = 1/2$ and $\beta_+ = 2$, respectively. The physical meaning of the asymptotic vanishing of C in Fig. 2(b) is that for the specific value of β_+ the system evolves quite adiabatically towards the factorized states $|--\rangle$. On the contrary, in Fig. 2(a) the slope of the ramp induces a nonadiabatic evolution towards a coherent, not factorizable superposition of $|++\rangle$ and $|--\rangle$.

We would get analogous results by studying the LMSZ process when the two spin qubits start from the state $|-+\rangle$. In this case, only states $|-+\rangle$ and $|+-\rangle$ would be involved, and the LMSZ parameter determining the different concurrence regimes would be β_-. For such initial conditions, then, the ratio β_+/β_-, which imposes precise relationships between the coupling parameters γ_x and γ_y, does not matter.

Such a ratio, conversely, plays a decisive role for the time evolution from different initial conditions, e.g., the one considered in Eq. (10). In this case the amplitudes read

$$c_{++} = a_+, \quad c_{--} = -b_+^*, \quad c_{+-} = a_-, \quad c_{-+} = -b_-^*. \tag{19}$$

In Figs. 3(a)–3(f) we may see the influence of both the ratio β_-/β_+ and the free parameter β_+; the former influences only qualitatively the behavior of the concurrence, while the latter influences it both qualitatively and quantitatively. This time,

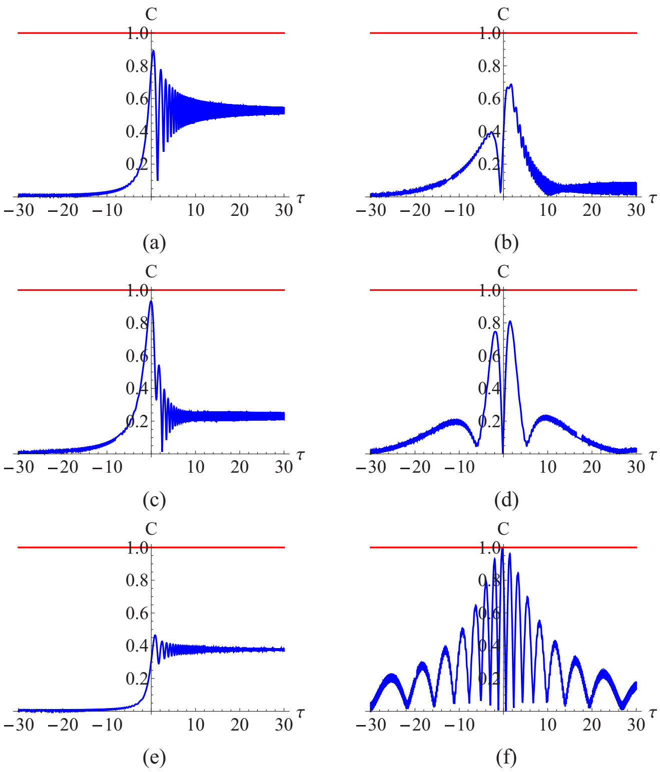

FIG. 3. Time behavior of the concurrence against the dimensionless parameter $\tau = \sqrt{\alpha/\hbar}\,t$ during a LMSZ process when the system starts from the state $(|{++}\rangle + |{+-}\rangle)/\sqrt{2}$ for $\beta_-/\beta_+ = 1/2$ and (a) $\beta_+ = 1/2$ and (b) $\beta_+ = 2$, for $\beta_-/\beta_+ = 2$ and (c) $\beta_+ = 0.5$ and (d) $\beta_+ = 2$, and for $\beta_-/\beta_+ = 2$ and (e) $\beta_+ = 0.1$ and (f) $\beta_+ = 10$. The upper straight curve corresponds to $C(\tau) = 1$.

too, the concurrence vanishes for high values of β_+, showing an asymptotic factorized state. For small values of β_+, instead, positive values of entanglement even for large times indicate a superposition of the four standard basis states.

To conclude this section, we underline that in Ref. [30] the authors considered a system of two spins 1/2 interacting only through the term $\hat{\sigma}_1^z\hat{\sigma}_2^z$ and subjected to the same magnetic field consisting of a Gaussian pulse uniformly rotating in the x-y plane and a LMSZ ramp in the z direction. They showed that the coupling between the two spins enhances significantly the probability to drive adiabatically the two-spin system from the separate state $|{--}\rangle$ to the entangled state $(|{+-}\rangle + |{-+}\rangle)/\sqrt{2}$. In this case the procedure to generate an entangled state is different from the scenario considered here because of the different symmetries of the Hamiltonians ruling the two-spin dynamics. Indeed, in Ref. [30] the Hamiltonian commutes with $\hat{S}^2$, and consequently, two dynamically invariant Hilbert subspaces exist: one of three dimensions and the other of one dimension. The three-dimensional subspace is spanned by the states $|{++}\rangle$, $(|{+-}\rangle + |{-+}\rangle)\sqrt{2}$, and $|{--}\rangle$, making possible the preparation of the entangled state of the two spins 1/2 by an adiabatic passage when they start from the separate state $|{--}\rangle$. In our case, instead, $\hat{S}^2$ is not constant, while the integral of motion is $\hat{\sigma}_1^z\hat{\sigma}_2^z$. The symmetries of the Hamiltonian thus generate two two-dimensional dynamically invariant Hilbert subspaces: one spanned by $|{++}\rangle$ and $|{--}\rangle$ and the other spanned by $|{+-}\rangle$ and $|{-+}\rangle$. Then, in our case, the transition between the states considered in the other work is impossible since such states belong to different invariant subspaces.

VI. EFFECTS OF CLASSICAL NOISE

In experimental physical contexts involving atoms, ions, and molecules investigated and manipulated by the application of lasers and fields, the presence of noise in the system stemming from the coupling with a surrounding environment is unavoidable. Although a lot of technological progress has been made and experimental expedients have been developed, it is necessary to introduce such decoherence effects in theoretical models for a better understanding and closer description of the experimental scenarios. Different approaches exist to treat the influence of a thermal bath; one is to consider the presence of classical noisy fields [26,44] stemming, e.g., from the presence and the influence of a surrounding nuclear spin bath [26].

In Ref. [26] the authors studied a noisy LMSZ scenario for an N-level system. They took into account a time-dependent magnetic field $\eta(t)$ only in the z direction characterized by the time correlation function $\langle \eta(t)\eta(t')\rangle = 2G\delta(t - t')$. The authors showed that the LMSZ transition probability P_-^+ for a spin 1/2 to be found in the state $|+\rangle$ starting from $|-\rangle$, in the case of large values of G, changes as

$$P_-^+ = \frac{1 - \exp\{-2\pi g^2/\hbar\alpha\}}{2}, \tag{20}$$

where g is the energy contribution due to the coupling of the spin 1/2 with the constant transverse magnetic field and α is the ramp of the longitudinal magnetic field. We see that the value of G, provided that it is large, does not influence the transition probability. The unique effect of the noisy component is the loss of coherence. The field, indeed, being in the same direction of the quantization axis, cannot generate transitions between the two diabatic states. In this way the transition probability is reasonably hindered by the presence of the noise since, for $g^2/\alpha \gg 1$, the system reaches at most the maximally mixed state.

This result is of particular interest in our case since the addition of the noisy component $\eta(t)$ leaves completely unaffected the symmetry-based Hamiltonian transformation and the validity of the dynamics-decoupling procedure. Thus, also in this case, the dynamical problem of the two-qubit system may be converted into two independent spin-1/2 problems affected by a random fluctuating z field. Thus, we may easily write the transition probabilities when the two spins are subjected to a unique homogeneous field influenced by the noisy component considered before. We have precisely

$$P_+ = \frac{1 - \exp\{-2\pi\gamma_+^2/\hbar\alpha\}}{2}, \qquad \omega_1(t) = \omega_2(t) = [\alpha t + \eta(t)]/4. \tag{21}$$

We underline that the transition probability P_- vanishes in the case of a unique homogeneous magnetic field. In the related subdynamics, indeed, the effective field ruling the two-spin dynamics is zero, namely, $\Omega_-(t) = 0$. Moreover, for $\gamma_x = \gamma_y$

we would have no physical effects since, in such a case, P_+ would also be zero.

Another way to face the problem of open quantum systems is to use non-Hermitian Hamiltonians, effectively incorporating the fact that the system they describe is interacting with a surrounding environment [45–50]. We may suppose, for example, that the spontaneous emission from the up state to the down one is negligible and that some mechanism makes the up state $|+\rangle$ irreversibly decay out of the system with rates ξ and ξ' for the first and second spins 1/2, respectively. It is well known that we can phenomenologically describe such a scenario by introducing the non-Hermitian terms $i\xi\hat{\sigma}_1^z/2$ and $i\xi'\hat{\sigma}_2^z/2$ in our Hamiltonian model. Analogous to the case of a noisy field component, the introduction of these terms does not alter the symmetry of the Hamiltonian model. The symmetry-based transformation leads us to two independent non-Hermitian two-level models. In the same way we may exploit the results received for a single qubit with a decaying state subjected to the LMSZ scenario [24,25,27] and reread them in terms of the two-spin-1/2 language. We know that the decay rate affects only the time history of the transition probability but not, surprisingly, its asymptotic value [24]. However, this result is valid for the ideal LMSZ scenario; considering the more realistic case with a limited time window, it has been demonstrated, indeed, that a decay rate dependence for the population of the up state arises [25].

VII. CONCLUDING REMARKS

In this work we considered a physical system of two interacting spins 1/2 whose coupling comprises the terms stemming from the anisotropic exchange interaction. Moreover, each of them was subjected to a local field linearly varying over time. The C_2 symmetry (with respect to the quantization axis $\hat{z}$) possessed by the Hamiltonian allowed us to identify two independent single spin-1/2 subproblems nested in the quantum dynamics of the two spin qubits. This fact gave us the possibility of decomposing the dynamical problem of the two spins 1/2 into two independent problems of a single spin 1/2. In this way, our two-spin-qubit system may be regarded as a four-level system presenting an avoided crossing for each pair of instantaneous eigenenergies related to the two dynamically invariant subspaces. This aspect turned out to be the key to solve easily and exactly the dynamical problem, bringing to light several physically relevant aspects.

In the case of time-dependent Hamiltonian models, such a symmetry-based approach and the reduction to independent problems of a single spin 1/2 were also used in other cases [37,51–53]. This fact permits a deep understanding of the quantum dynamics of the spin systems with consequent potential applications in quantum information and computation. We underline, in addition, that the dynamical reduction exposed in Sec. II is independent of the time dependence of the fields. Thus, we may consider different exactly solvable time-dependent scenarios [54–60] for the two subdynamics, resulting, of course, in different two-spin dynamics and physical effects.

In this paper, we showed that, despite the absence of a transverse chirp [30] or constant field, LMSZ transitions are still possible, precisely from $|--\rangle$ to $|++\rangle$ and from $|-+\rangle$ to $|+-\rangle$ (the two couples of states spanning the two dynamically invariant Hilbert spaces related to the symmetry Hamiltonian). Such transitions occur thanks to the presence of the coupling between the spins, which acts as an effective static transverse field in each subdynamics.

It is worth noticing that, in our model, the two LMSZ subdynamics are ruled either by different combinations of the externally applied fields (when the local fields are different) or by the same field (under the STM scenario, that is, when one local field is applied on just one spin). In the latter case we showed the possibility of (1) a nonlocal control, that is, manipulating the dynamics of one spin by applying the field on the other one, and (2) a state exchange/transfer between the two spins. We brought to light how such effects are two different responses of the system depending on the isotropic properties of the exchange interaction.

Concerning the interaction terms, each subdynamics is characterized by different combinations of the coupling parameters. This aspect has relevant physical consequences since, as shown, by studying the LMSZ transition probability in the two subspaces, it is possible both to evaluate the presence of different interaction terms and to estimate their weights in ruling the dynamics of the two-spin system. We brought to light how the estimation of the coupling parameters could be of relevant interest since, through this knowledge, we may set the slope of variation of the LMSZ ramp to generate asymptotically entangled states of the two spins 1/2. Moreover, we reported the exact time behavior of the entanglement for different initial conditions, and we analyzed how the coupling parameters can determine different entanglement regimes and asymptotic values.

Finally, we emphasized how our symmetry-based analysis has proved to be useful to get exact results when a classical random field component or non-Hermitian terms are considered to take into account the presence of a surrounding environment interacting with the system. In this case, the dynamics decomposition is unaffected by the presence of the noise or the dephasing terms, and then we may apply the results previously reported for a two-level system [24–26] and reread them in terms of the two spins 1/2.

We wish to underline, in addition, that our results are valid not only within the STM scenario; they are also applicable to other physical platforms. Indeed, the local LMSZ model for a spin qubit interacting with another neighboring spin qubit may be reproduced also in laser-driven cold atoms in optical lattices where highly selective individual addressing has been experimentally demonstrated [61]. Another prominent example is laser-driven ions in a Paul trap where spatial individual addressing of single ions in an ion chain has been routinely used for many years [62,63]. Yet another example is microwave-driven trapped ions in a magnetic-field gradient where individual addressing with extremely small cross talk has been achieved in frequency space [64,65].

We point out that the results obtained in this paper are deeply different from the ones reported in Refs. [15,31,32], where systems of two spins 1/2 in a LMSZ framework were investigated on the basis of an approximate treatment. In those papers, indeed, the two spin qubits are not directly coupled, but they interact through a common nuclear spin bath which they are coupled to. Such a composite system behaves as a two-level system under several assumptions, and deriving the effective single spin-1/2 Hamiltonian requires several approximations. In Ref. [32], in particular, the effective Hamiltonian describes the coupling between the two-level system and a longitudinal time-dependent field which is not a pure LMSZ ramp, presenting a complicated functional dependence on the original Hamiltonian parameters. There is, in addition, a time-dependent effective interaction between the two states possessing a complicated functional dependence on the confinement energy as well as the tunneling and Coulomb energies. Although such an effective Hamiltonian goes beyond the standard LMSZ scenario, it may be considered to be similar to the LMSZ one since both Hamiltonians describe an adiabatic passage through an anticrossing.

In our case, instead, the two spins 1/2 are directly coupled in addition to being subjected to a random field stemming from the presence of a spin bath. Furthermore, the effective two-state Hamiltonians governing the two-qubit dynamics in the two invariant subspaces are easily determined without involving any assumption and/or approximation. The two two-level Hamiltonians, indeed, are derived only on the basis of a transparent mathematical mapping between the two-qubit states in each subspace and the states of a fictitious spin 1/2. Moreover, they describe exactly a LMSZ scenario with a standard avoided crossing where the transverse constant field is effectively reproduced by the coupling existing between the two qubits. The treatment at the basis of this work remarkably enables us to explore peculiar dynamical aspects of the system described by Eq. (1), leading, for example, to the exact evolution of the entanglement established between the two spins.

We underline, moreover, that our study is not a special case of the one considered in Ref. [34], where a Lipkin-Meskow-Glick (LMG) interaction model for N spin qubits subjected to a LMSZ ramp was considered. The numerical results reported in Ref. [34] were, indeed, based on the mean-field approximation. In addition, there is no possibility of considering in the LMG model effects stemming from the anisotropy between x and y interaction terms.

Finally, two challenging problems naturally extending the investigation here reported are (1) considering the interaction of two qutrits [66] in place of two qubits and (2) taking into account the coupling of the two spins with a quantum bath [67] in place of the interaction with a classical random field.

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[1] L. D. Landau, Phys. Z. Sowjetunion **2**, 46 (1932); E. Majorana, Nuovo Cimento **9**, 43 (1932); E. C. G. Stückelberg, Helv. Phys. Acta **5**, 369 (1932); C. Zener, Proc. R. Soc. London, Ser. A **137**, 696 (1932).
[2] I. I. Rabi, Phys. Rev. **51**, 652 (1937).
[3] G. S. Vasilev, S. S. Ivanov, and N. V. Vitanov, Phys. Rev. A **75**, 013417 (2007).
[4] B. T. Torosov and N. V. Vitanov, Phys. Rev. A **84**, 063411 (2011).
[5] N. V. Vitanov and B. M. Garraway, Phys. Rev. A **53**, 4288 (1996).
[6] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Dover, New York, 1987); F. T. Hioe, Phys. Rev. A **30**, 2100 (1984).
[7] Y. N. Demkov and M. Kunike, Vestn. Leningr. Univ. Fiz. Khim. **16**, 39 (1969).
[8] G. S. Vasilev and N. V. Vitanov, Phys. Rev. A **73**, 023416 (2006).
[9] J. M. S. Lehto and K.-A. Suominen, Phys. Rev. A **94**, 013404 (2016).
[10] S. N. Shevchenko, S. Ashhab, and F. Nori, Phys. Rep. **492**, 1 (2010).
[11] S. S. Ivanov and N. V. Vitanov, Phys. Rev. A **77**, 023406 (2008).
[12] N. A. Sinitsyn, Phys. Rev. A **87**, 032701 (2013).
[13] B. D. Militello and N. V. Vitanov, Phys. Rev. A **91**, 053402 (2015).
[14] S. Thiele, F. Balestro, R. Ballou, S. Klyatskaya, M. Ruben, and W. Wernsdorfer, Science **344**, 1135 (2014).
[15] D. J. Reilly, J. M. Taylor, J. R. Petta, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Science **321**, 817 (2008).
[16] P. Huang, J. Zhou, F. Fang, X. Kong, X. Xu, C. Ju, and J. Du, Phys. Rev. X **1**, 011003 (2011).
[17] J. Randall, A. M. Lawrence, S. C. Webster, S. Weidt, N. V. Vitanov, and W. K. Hensinger, Phys. Rev. A **98**, 043414 (2018).
[18] R. Wieser, V. Caciuc, C. Lazo, H. Hölscher, E. Y. Vedmedenko, and R. Wiesendanger, New J. Phys. **15**, 013011 (2013).
[19] I. N. Sivkov, D. I. Bazhanov, and V. S. Stepanyuk, Sci. Rep. **7**, 2759 (2017).
[20] J. R. Petta, A. C. Johnson, J. M. Taylor, E. A. Laird, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Science **309**, 2180 (2005).
[21] M. Anderlini, P. J. Lee, B. L. Brown, J. Sebby-Strabley, W. D. Phillips, and J. V. Porto, Nature (London) **448**, 452 (2007).
[22] H. Bluhm, S. Foletti, I. Neder, M. Rudner, D. Mahalu, V. Umansky, and A. Yacoby, Nat. Phys. **7**, 109 (2011).
[23] X. Wang, L. S. Bishop, J. P. Kestner, E. Barnes, K. Sun, and S. Das Sarma, Nat. Commun. **3**, 997 (2012).
[24] V. M. Akulin and W. P. Schleich, Phys. Rev. A **46**, 4110 (1992).
[25] N. V. Vitanov and S. Stenholm, Phys. Rev. A **55**, 2982 (1997).
[26] V. L. Pokrovsky and N. A. Sinitsyn, Phys Rev. B **67**, 144303 (2003).
[27] P. A. Ivanov and N. V. Vitanov, Phys. Rev. A **71**, 063407 (2005).
[28] M. Scala, B. Militello, A. Messina, and N. V. Vitanov, Phys. Rev. A **84**, 023416 (2011).
[29] A. V. Dodonov, B. Militello, A. Napoli, and A. Messina, Phys. Rev. A **93**, 052505 (2016).
[30] R. G. Unanyan, N. V. Vitanov, and K. Bergmann, Phys. Rev. Lett. **87**, 137902 (2001).
[31] H. Ribeiro and G. Burkard, Phys. Rev. Lett. **102**, 216802 (2009).

[32] H. Ribeiro, J. R. Petta, and G. Burkard, Phys. Rev. B **87**, 235318 (2013).

[33] M. J. Rancić and D. Stepanenko, Phys. Rev. B **94**, 241301(R) (2016).

[34] J. Larson, Eur. Phys. Lett. **107**, 30007 (2014).

[35] S. N. Shevchenko, A. I. Ryzhov, and F. Nori, Phys. Rev. B **98**, 195434 (2018).

[36] S. Mehl, Phys. Rev. B **91**, 035430 (2015).

[37] R. Grimaudo, A. Messina, and H. Nakazato, Phys. Rev. A **94**, 022108 (2016).

[38] A. A. Khajetoorians, J. Wiebe, B. Chilian, and R. Wiesendanger, Science **332**, 1062 (2011).

[39] S. Yan, D.-J. Choi, J. A. J. Burgess, S. Rolf-Pissarczyk, and S. Loth, Nat. Nanotechnol. **10**, 40 (2015).

[40] B. Bryant, A. Spinelli, J. J. T. Wagenaar, M. Gerrits, and A. F. Otte, Phys. Rev. Lett. **111**, 127203 (2013).

[41] K. Tao, V. S. Stepanyuk, W. Hergert, I. Rungger, S. Sanvito, and P. Bruno, Phys. Rev. Lett. **103**, 057202 (2009).

[42] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).

[43] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1964).

[44] C. Benedetti and M. Paris, Phys. Lett. A **378** 2495 (2014).

[45] H. Feshbach, Ann. Phys. (NY) **5**, 357 (1958); **19**, 287 (1962).

[46] N. Moiseyev, *Non-Hermitian Quantum Mechanics* (Cambridge University Press, Cambridge, 2011).

[47] I. Rotter and J. P. Bird, Rep. Prog. Phys. **78**, 114001 (2015); I. Rotter, J. Phys. A **42**, 153001 (2009).

[48] L. S. Simeonov and N. V. Vitanov, Phys. Rev. A **93**, 012123 (2016).

[49] B. T. Torosov and N. V. Vitanov, Phys Rev. A **96**, 013845 (2017).

[50] R. Grimaudo, A. S. M. de Castro, M. Kuś, and A. Messina, Phys. Rev A **98**, 033835 (2018).

[51] R. Grimaudo, A. Messina, P. A. Ivanov, and N. V. Vitanov, J. Phys. A **50**, 175301 (2017).

[52] R. Grimaudo, Y. Belousov, H. Nakazato, and A. Messina, Ann. Phys. (NY) **392**, 242 (2017).

[53] R. Grimaudo, L. Lamata, E. Solano, and A. Messina, Phys. Rev. A **98**, 042330 (2018).

[54] V. G. Bagrov, D. M. Gitman, M. C. Baldiotti, and A. D. Levin, Ann. Phys. (Berlin, Ger.) **14**, 764 (2005).

[55] M. Kuna and J. Naudts, Rep. Math. Phys. **65**, 77 (2010).

[56] E. Barnes and S. Das Sarma, Phys. Rev. Lett. **109**, 060401 (2012).

[57] A. Messina and H. Nakazato, J. Phys. A **47**, 445302 (2014).

[58] L. A. Markovich, R. Grimaudo, A. Messina, and H. Nakazato, Ann. Phys. (NY) **385**, 522 (2017).

[59] R. Grimaudo, A. S. M. de Castro, H. Nakazato, and A. Messina, Ann. Phys. (Berlin, Ger.) **530**, 1800198 (2018).

[60] T. Suzuki, H. Nakazato, R. Grimaudo, and A. Messina, Sci. Rep. **8**, 17433 (2018).

[61] C. Weitenberg, M. Endres, J. F. Sherson, M. Cheneau, P. Schauss, T. Fukuhara, I. Bloch, and S. Kuhr, Nature (London) **471**, 319 (2011).

[62] T. Monz, D. Nigg, E. A. Martinez, M. F. Brandl, P. Schindler, R. Rines, S. X. Wang, I. L. Chuang, and R. Blatt, Science **351**, 1068 (2016).

[63] A. Bermudez, X. Xu, R. Nigmatullin, J. O'Gorman, V. Negnevitsky, P. Schindler, T. Monz, U. G. Poschinger, C. Hempel, J. Home, F. Schmidt-Kaler, M. Biercuk, R. Blatt, S. Benjamin, and M. Müller, Phys. Rev. X **7**, 041061 (2017).

[64] C. Piltz, T. Sriarunothai, A. F. Varon, and C. Wunderlich, Nat. Commun. **5**, 4679 (2014).

[65] B. Lekitsch, S. Weidt, A. G. Fowler, K. Mølmer, S. J. Devitt, C. Wunderlich, and W. K. Hensinger, Sci. Adv. **3**, e1601540 (2017).

[66] R. Grimaudo, N. V. Vitanov, and A. Messina, arXiv:1901.00322.

[67] A. Sergi, G. Hanna, R. Grimaudo, and A. Messina, Symmetry **10**, 518 (2018).