Effect of noise on Bloch oscillations and Wannier-Stark localization

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We calculate an exact expression for the probability propagator for a noisy electric field driven tight-binding lattice. The noise considered is a two level jump process or a telegraph process (TP) which jumps randomly between two values $\pm\mu$. In the absence of a static field, and in the limit of zero jump rate of the noisy field, we find that the dynamics yields Bloch oscillations with frequency μ, while with an additional static field ϵ we find oscillatory motion with a superposition of frequencies $(\epsilon \pm \mu)$. On the other hand, when the jump rate is rapid, and in the absence of a static field, the stochastic field averages to zero if the two states of the TP are equally probable *a priori*. In that case we see a delocalization effect. The intimate relationship between the rapid relaxation case and the zero field case seems to be a generic effect found in a wide variety of systems. It is interesting to note that even for zero static field and rapid relaxation, Bloch oscillations ensue if there is a bias δp in the probabilities of the two levels. Remarkably, the Wannier-Stark localization caused by an additional static field is destroyed if the latter is tuned to be exactly equal and opposite to the average stochastic field $\mu\delta p$. This is an example of *incoherent* destruction of Wannier-Stark localization.

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I. INTRODUCTION

In a variety of physical phenomena where a rapidly changing external field is involved, an equivalence with the corresponding zero ac field phenomenon is found [1–6]. A useful anecdotal analogy for this generic effect may be found in an extended version of this paper [7]. In the specific context of an ac electric field applied onto a one-dimensional tight-binding lattice, the large frequency limit can be shown to be mathematically equivalent to simply renormalizing the hopping parameter [6,8], thus corresponding to the zero-field case. However, for certain delicate choices of the ratio of the amplitude and frequency, a *dynamical* localization [8–11] may be engineered via a *band collapse* mechanism. On the other hand, the zero frequency limit when the electric field is time independent is characterized by the familiar Bloch oscillations [12–15]. Other phenomena such as coherent destruction of Wannier-Stark (WS) localization [16,17] and super Bloch oscillations [18–21] arise when an additional static field is added onto an existing sinusoidal field. The former occurs when the static field is resonantly tuned with the frequency of the sinusoidal field while the latter for a slight detuning from the resonance condition.

Random disorder, in the zero electric field case, is known to localize the particle via the famous phenomenon of Anderson localization [22]. Since the work of Anderson, transport in the presence of a fluctuating environment has also been studied [23–27] both analytically and numerically. The aim here has been to understand the diffusion of a quantum particle in the presence of dynamic disorder. This dynamic disorder originates from the lattice vibrations where the modes of phonons are randomly excited and the process is modeled by a stochastic process [28]. In the presence of an electric field, disorder dephases the Bloch oscillations depending on the strength of disorder [29–31]. However, for a slowly varying disordered potential the Bloch oscillations are known to survive [32,33]. An increased diffusion has been found to be the effect of scattering on the motion of a charged particle with a time-dependent field [34].

There are numerous experimental realizations of Bloch oscillations [35–39]. To realize pure Bloch oscillations, often a lot of effort is expended experimentally to produce clean systems since disorder and noise are inherent in physical systems [17]. Advances in cold-atom technology have now made it possible to in fact control noise [40–42] in order to capture special features. Therefore on the one hand, it is important to understand theoretically the effects of noise so that clever experimental techniques may be devised to get rid of them. On the other hand, it may be useful to understand them better so that they may even be exploited, given the high degree of control that modern technology has brought in [43]. Here we consider the effect of a stochastic noise on top of an electric field on the motion of the particle and focus on how the Bloch oscillations are influenced by this type of dynamic disorder. The particular form of the stochastic noise is the *telegraphic noise* [28,44–46], where the noise consists of jumps randomly between two levels $\pm\mu$. Telegraph noise is one of the simplest realizations of fluctuations in the battery. When such a telegraphic noise term appears as fluctuations in the site energies without any linear variation (the limit when the electric field is zero), exact analytical results for the diffusion coefficient have been obtained [27]. Also the effect of noise on dynamical localization has been studied [47]. Here we consider the case where the noise term acts as fluctuations to an electric field. This noise can also be thought of as an aperiodic form of

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TABLE I. The table contrasts the various phenomena that arise due to ac drive and telegraph noise.

	Deterministic $F = c_0 F_{\rm dc} + c_1 F_{\rm ac}$ (high frequency regime)	Stochastic $F = c_0 F_{\rm dc} + c_1 F_{\rm TP}$ (rapid relaxation regime)
dc ($c_1 = 0$)	Bloch oscillations (WS localization).	Bloch oscillations (WS localization).
ac/TP ($c_0 = 0$)	Equivalent to no-field case. Dynamical localization with proper tuning.	Equivalent to no-field case.
dc + ac/TP ($c_0, c_1 \neq 0$)	Coherent destruction of WS localization at resonance.	Incoherent destruction of WS localization with proper tuning of bias.
	Super Bloch oscillations at off-resonance.	Bloch oscillations with bias-dependent renormalized frequency (another case of equivalence to no-field case).

a square wave driving (periodic square wave driving with proper tuning can yield dynamic localization [48,49]). Perfect periodic drive is impossible to achieve [50,51] in realistic experimental situations and therefore it is important to study the effects of noisy drive [5,52–54].

The central findings (Table I) of our article are as follows. For a stochastic electric field characterized by telegraph noise, we find the exact expression for the probability propagator $\mathcal{P}_m(t)$, defined as the probability of a particle to remain at site m at any time t given that it was at the origin at $t = 0$. The limit of the rapidly changing stochastic field is given particular emphasis. Denoting the bias in the probabilities of the two levels of the field to be δp, we show that this is equivalent to an effective dc field of $\mu\delta p$, yielding Bloch oscillations with frequency $\mu\delta p$ (although these oscillations are exponentially damped in the infinite time limit). If an additional static field is present, we recover Bloch oscillations with a renormalized frequency in the rapid relaxation limit—this is another instance of equivalence to zero field phenomenon. Remarkably, by choosing the additional static field to have a precise magnitude, a destruction of WS localization [16,17] may be engineered. Since no frequency is involved in the present context, and rather the noise may be a result of connection to a bath, this may be termed an *incoherent* destruction of WS localization. When the two levels of the stochastic field are equiprobable ($\delta p = 0$), we recover the well-known scenario that the rapid relaxation limit is equivalent to the zero-field limit. A complementary numerical approach is used to independently verify our findings.

II. MODEL HAMILTONIAN AND PROBABILITY PROPAGATOR

The Hamiltonian for a 1D tight-binding model with a time-dependent electric field is

$$H = -\frac{\Delta}{4}\sum_{n=-\infty}^{\infty} c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n + \mathcal{F}(t)\sum_{n=-\infty}^{\infty} n c_n^\dagger c_n, \quad (1)$$

where $\mathcal{F}(t)$ is the electric field. The lattice constant is kept at unity and natural units ($\hbar = e = 1$) are adopted for all the calculations. For a constant electric field, the dynamics gives the well-known Bloch oscillations, while a periodic driving can give rise to dynamical localization when the amplitude and frequency are tuned appropriately. Here we consider the case where the time-dependent electric field is described by a two state jump process or a *telegraph process*.

It is useful to define the unitary operators $\hat{K}$, $\hat{K}^\dagger$, and $\hat{N}$ [15], and their operations on the state $|n\rangle$ as

$$\hat{K} = \exp(-i\kappa) = \sum_{n=-\infty}^{\infty} |n\rangle\langle n+1|, \quad \hat{K}|n\rangle = |n-1\rangle,$$
$$\hat{K}^\dagger = \exp(i\kappa) = \sum_{n=-\infty}^{\infty} |n+1\rangle\langle n|, \quad \hat{K}^\dagger|n\rangle = |n+1\rangle,$$
$$\hat{N} = \sum_{-\infty}^{\infty} n|n\rangle\langle n|. \quad (2)$$

These operators follow the commutation rules

$$[\hat{K}, \hat{N}] = \hat{K}, \quad [\hat{K}^\dagger, \hat{N}] = -\hat{K}^\dagger, \quad [\hat{K}, \hat{K}^\dagger] = 0. \quad (3)$$

The eigenvectors of $\hat{K}$ are the Bloch states $|\kappa\rangle$ with eigenvalues $e^{i\kappa}$. The connection between the Wannier basis and the Bloch basis is given by $|k\rangle = \sqrt{\frac{1}{2\pi}}\sum_n e^{-ink}\ |n\rangle$ and $|n\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} dk\ e^{ink}\ |k\rangle$.

In terms of these new operators, the tight-binding Hamiltonian can be written as

$$\hat{H}(t) = V^+ + H_0(t), \quad (4)$$

where $V^\pm = -\frac{\Delta}{4}(\hat{K} \pm \hat{K}^\dagger)$ and $H_0(t) = \mathcal{F}(t)\hat{N}$.

The time evolution of the density matrix ρ in Heisenberg picture is given by

$$\frac{\partial\rho}{\partial t} = -i[H(t), \rho(t)]. \quad (5)$$

By considering the transformation $\tilde{\rho}(t) = e^{i\int_0^t H_0(t')dt'}\rho(t)\, e^{-i\int_0^t H_0(t')dt'}$, the equation of motion for $\tilde{\rho}(t)$ can be written as

$$\frac{\partial\tilde{\rho}}{\partial t} = -i[\tilde{V}^+(t), \tilde{\rho}(t)], \quad (6)$$

where $\tilde{V}^+(t) = e^{i\int_0^t H_0(t')dt'}V^+ e^{-i\int_0^t H_0(t')dt'}$. The time evolution of $\tilde{\rho}$ can now be solved to

$$\tilde{\rho}(t) = e^{-i\int_0^t \tilde{V}^+(t')dt'}\rho(0)e^{i\int_0^t \tilde{V}^+(t')dt'}, \quad (7)$$

where $\tilde{\rho}(0) = \rho(0) = |0\rangle\langle 0|$. It turns out that $[\tilde{V}^+(t), \tilde{V}^+(t')]$ even for $t \neq t'$, and therefore no complicated time ordering is essential.

Using the Baker -Campbell-Hausdorff (BCH) formula $e^X Y e^{-X} = Y + [X, Y] + \frac{1}{2!}[X, [X, Y]] + \cdots$, and the commutation relations [Eq. (3)], we can simplify the effective

Hamiltonian which governs the dynamics of the density matrix $\tilde{\rho}(t)$ as

$$\tilde{V}^+(t) = V^+ \cos[\eta(t)] + iV^- \sin[\eta(t)], \quad (8)$$

where $\eta(t) = \int_0^t \mathcal{F}(t')\,dt'$. Substituting the expressions of V^+ and V^-, we get

$$\tilde{V}^+(t) = -\frac{\Delta}{4}(\hat{K}^\dagger e^{i\eta(t)} + \hat{K}e^{-i\eta(t)}). \quad (9)$$

It can be seen that the effective Hamiltonian has the time dependence appearing as a phase term and it can be easily diagonalized in the momentum basis. In the k representation, $\tilde{V}^+(t)$ can be expressed as

$$\langle k|\tilde{V}^+(t)|k'\rangle = -\frac{\Delta}{4}\delta(k-k')[e^{ik+i\eta(t)} + e^{-ik-i\eta(t)}]. \quad (10)$$

Furthermore, the transformed density matrix $\tilde{\rho}(t)$ can also be written in k basis as

$$\langle k|\tilde{\rho}(t)|k'\rangle = e^{-i\int_0^t dt' V_k^+(t')}\langle k|0\rangle\langle 0|k'\rangle e^{i\int_0^t dt' V_{k'}^+(t')}, \quad (11)$$

where $V_k^+(t) = -\frac{\Delta}{4}[e^{i[k+\eta(t)]} + e^{-i[k+\eta(t)]}]$.

In Wannier space the probability propagator is given by

$$\mathcal{P}_m(t) = \langle m|\rho(t)|m\rangle = \langle m|\tilde{\rho}(t)|m\rangle, \quad (12)$$

where we have used the fact that H_0 is diagonal in the Wannier basis. Going into the momentum basis the expression for the probability can be simplified to

$$\mathcal{P}_m(t) = \iint dk\,dk'\,\langle m|k\rangle\langle k|\tilde{\rho}(t)|k'\rangle\langle k'|m\rangle, \quad (13)$$

which using Eq. (11) takes the simplified form as

$$\mathcal{P}_m(t) = \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi} dk \int_{-\pi}^{\pi} dk' e^{-i(k-k')m} \times e^{-i\int_0^t dt'[V_k^+(t') - V_{k'}^+(t')]}. \quad (14)$$

The mean squared width of the wave packet is then expressed in terms of the probability propagator as $\sigma^2(t) = \langle m^2\rangle = \sum_m m^2\mathcal{P}_m(t)$.

III. EFFECT OF RANDOM TELEGRAPH NOISE

The particular form of the field is taken as a telegraph noise where electric field is time dependent and randomly fluctuates between two levels $\pm\mu$. Let σ and τ be the rate of switching from level $+\mu$ to $-\mu$ and $-\mu$ to $+\mu$, respectively. The probability of being at any time in state $+\mu$ is given by $p_+ = \tau/(\tau+\sigma)$, whereas the probability of being in state $-\mu$ is $p_- = \sigma/(\tau+\sigma)$. It is useful to define $\lambda = \sigma + \tau$.

A clever way to make progress is to elevate $i\eta(t) = i\int_0^t \mathcal{F}(t')\,dt'$ to a 2×2 matrix [55]

$$i\eta(t) = it\epsilon\mathcal{I} + it\mu\sigma_z + \lambda t W, \quad (15)$$

in which $\mathcal{I}$ is the identity matrix, σ_z is the Pauli z matrix, and the relaxation matrix [44,45] is defined as

$$W = \begin{bmatrix} -p_- & p_+ \\ p_- & -p_+ \end{bmatrix} = \lambda\begin{bmatrix} -\frac{\sigma}{\tau+\sigma} & \frac{\tau}{\tau+\sigma} \\ \frac{\sigma}{\tau+\sigma} & -\frac{\tau}{\tau+\sigma} \end{bmatrix}. \quad (16)$$

As a consequence of this operation, the probability propagator [Eq. (12)] is also now a 2×2 matrix. The first term added in Eq. (15) is to account for the static electric field ϵ and the two stochastic states are $|+\rangle = \binom{1}{0}$, $|-\rangle = \binom{0}{1}$ corresponding to the fields $+\mu$ and $-\mu$, respectively. Equation (15) can be decomposed in terms of Pauli matrices as

$$i\eta(t) = -t(\gamma - i\epsilon)\mathcal{I} + t\sigma_z(\gamma\delta p + i\mu) + \gamma t(\sigma_x + i\delta p\sigma_y), \quad (17)$$

where $\gamma = \frac{\lambda}{2}$ and $\delta p = (p_+ - p_-)$. The exponential of Eq. (17) can be written in a compact form: $e^{i\eta(t)} = e^{-t(\gamma-i\epsilon)}\,e^{t(\mathbf{h}\cdot\boldsymbol{\sigma})}$, where $h_x = \gamma$, $h_y = i\gamma\delta p$, and $h_z = (\gamma\delta p + i\mu)$ and $|\mathbf{h}| = \sqrt{\gamma^2 - \mu^2 + 2i\gamma\mu\delta p} = \nu$. Using the Pauli spin identity: $e^{i(\mathbf{a}\cdot\boldsymbol{\sigma})} = \mathcal{I}\cos|\mathbf{a}| + i(\hat{n}\cdot\vec{\sigma})\sin|\mathbf{a}|$, the above expression can be written as

$$e^{i\eta(t)} = \tfrac{1}{2}e^{-t(\gamma-i\epsilon)}[e^{\nu t}(1+\hat{h}\cdot\vec{\sigma}) + e^{-\nu t}(1+\hat{h}\cdot\vec{\sigma})]. \quad (18)$$

Similarly, an equation for the complex conjugation can be written with $h'_x = \gamma$, $h'_y = -i\gamma\delta p$, $h'_z = \gamma\delta p - i\mu$.

After some lengthy calculations (detailed in the Appendix), the exponential part of Eq. (14) can be written as

$$e^{-i\int_0^t dt'[V_k^+(t') - V_{k'}^+(t')]} = i\{g_0(t)\mathcal{I} + \alpha(t)\sigma_x + i\delta p\alpha(t)\sigma_y + [\delta p\alpha(t) + \beta(t)]\sigma_z\}, \quad (19)$$

where the complicated expressions for $g_0(t)$, $\alpha(t)$ and $\beta(t)$ are relegated to the Appendix. Finally, we have the compact form

$$e^{-i\int_0^t dt'[V_k^+(t') - V_{k'}^+(t')]} = e^{ig_0(t)}\,e^{i(\mathbf{H}\cdot\boldsymbol{\sigma})}, \quad (20)$$

where $H_x = \alpha(t)$, $H_y = i\delta p\alpha(t)$, and $H_z = \delta p\alpha(t) + \beta(t)$ and $|\mathbf{H}| = \sqrt{\alpha^2(t) + \beta^2(t) + 2\delta p\alpha(t)\beta(t)}$. Again using the Pauli spin identity, we get

$$e^{i(\mathbf{H}\cdot\boldsymbol{\sigma})} = [\mathcal{I}\cos|\mathbf{H}| + i(\hat{H}\cdot\vec{\sigma})\sin|\mathbf{H}|]. \quad (21)$$

For the final expression of probability, we need to calculate the restricted average $\bar{\mathcal{P}}_m(t) = \sum_{ab} p_a(b|\mathcal{P}_m(t)|a)$ [55]. The averages of σ_y and σ_z give $i\delta p$ and δp, respectively, whereas σ_x averages to unity. While $\mathcal{P}_m(t)$ is a matrix, the average $\bar{\mathcal{P}}_m(t)$ is just a number. The final expression for the average probability can be written as

$$\bar{\mathcal{P}}_m(t) = \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi} dk \int_{-\pi}^{\pi} dk'\, e^{-i(k-k')m}\, e^{ig_0(t)} \times \left[\cos|\mathbf{H}| + i\sin|\mathbf{H}|\frac{\alpha(t)}{|\mathbf{H}|} + i\delta p\sin|\mathbf{H}|\frac{\beta(t)}{|\mathbf{H}|}\right], \quad (22)$$

which is one of our key results. It is straightforward to verify the well-known results for the zero field ($\epsilon = 0$) and static field ($\epsilon \neq 0$) case where μ vanishes, hence $\beta(t) = 0$ and $\nu \to \gamma$. In the former case the probability propagator decays in time and the mean squared width becomes unbounded in time. Hence, an initially localized particle will delocalize. In the latter case of static field, both the probability and the mean squared width are bounded and exhibit the familiar Bloch oscillations with frequency $\omega_B = \epsilon$. Furthermore, considering the effect of telegraph noise the zero relaxation limit where $\gamma = 0$, $\nu = i\mu$ [and hence $\alpha(t) = 0$] is straightforward. A simplification of the probability propagator in this limit yields a superposition of probabilities for the two "static fields" $(\epsilon \pm \mu)$.

The rapid relaxation condition $\gamma \gg \mu, \epsilon$ is the core emphasis of our article, and will be imposed in the rest of the discussion ahead. In this limit, $\alpha^2(t) \gg \beta^2(t)$ and an expansion of $|\mathbf{H}|$, $\frac{\alpha(t)}{|\mathbf{H}|}$, and $\frac{\beta(t)}{|\mathbf{H}|}$ simplifies the integrand of the probability propagator [Eq. 22)] as

$$e^{ig_0(t)}\left[\cos|\mathbf{H}| + i\sin|\mathbf{H}|\frac{\alpha(t)}{|\mathbf{H}|} + i\delta p\sin|\mathbf{H}|\frac{\beta(t)}{|\mathbf{H}|}\right] \approx e^{ig_0(t)+i\alpha(t)+i\delta p\beta(t)}e^{i\left(\frac{\beta^2(t)}{2\alpha(t)}\right)}. \quad (23)$$

We consider separately the cases where both the levels of the stochastic field are equally probable $(\delta p = 0)$ and where one level is more probable than the other $(\delta p \neq 0)$.

With $\delta p = 0$, and $\gamma \gg \mu$, the expression for ν can be expanded up to $O(\frac{\mu^2}{\gamma})$ as $\nu = \sqrt{\gamma^2 - \mu^2} \approx \gamma - \frac{\mu^2}{2\gamma}$. For the zero static field case $(\epsilon = 0)$, the expressions for $g_0(t)$, $\alpha(t)$, and $\beta(t)$ can be written as [up to $O(\mu/\gamma)$]

$$g_0(t) \approx \eta_+(\cos k - \cos k'), \quad \alpha(t) \approx \eta_-(\cos k - \cos k'),$$
$$\beta(t) \approx -\frac{\mu}{\gamma}\eta_-(\sin k - \sin k'), \quad (24)$$

where $\eta_\pm(t) = \frac{\Delta}{4}\frac{1}{2\gamma}[2\gamma t \pm (1 - e^{-2\gamma t})]$. Substituting the values of $g_0(t)$, $\alpha(t)$, and $\beta(t)$ and taking the long time limit, we get

$$e^{ig_0(t)+i\alpha(t)}e^{i\frac{\beta^2(t)}{2\alpha(t)}} \approx e^{i\frac{\Delta_{\rm eff}t}{2}(\cos k - \cos k')}, \quad (25)$$

where $\Delta_{\rm eff} = \Delta[1 + \frac{1}{8}(\frac{\mu}{\gamma})^2(\frac{\sin k - \sin k'}{\cos k - \cos k'})^2]$. Hence in this limit, the effect is identical to the case of no field. This is the case where the electric field is so rapidly fluctuating between $\pm\mu$, that for all practical purposes the system feels no effect at all. This effect is shown in Fig. 1, where the probability propagator and the mean squared width of the wave packet are plotted with time. The return probability decays in time and the wave-packet width becomes unbounded signifying the delocalization of an initially localized wave packet. In the presence of the static field $(\epsilon \neq 0)$, we have the approximation

$$g_0(t) + \alpha(t) \approx \frac{\Delta}{2\epsilon}\left[e^{-\frac{\mu^2}{2\gamma}t}\sin(k + \epsilon t) - \sin k\right] - \frac{\Delta}{2\epsilon}\left[e^{-\frac{\mu^2}{2\gamma}t}\sin(k' + \epsilon t) - \sin k'\right]. \quad (26)$$

In the limit $\gamma \gg \mu$, the ratio $\frac{g_3^2(t)}{2g_2(t)}$ becomes very small and can be neglected. Also the term $e^{-\frac{\mu^2}{2\gamma}t}$ becomes unity, unless t is very large. So in this limit one obtains Bloch oscillations with frequency ϵ for small times (Fig. 1); however the rapidly fluctuating noise causes in the long time limit for these oscillations to damp out exponentially.

Another interesting case of rapid relaxation arises when the two levels are not equiprobable $(\delta p \neq 0)$. Here $\nu = \sqrt{\gamma^2 - \mu^2 + 2i\gamma\mu\delta p}$. We can expand ν up to $O(\frac{\mu^2}{\gamma^2})$ as $\nu = \gamma(1 + i(\frac{\mu}{\gamma})\delta p - \frac{\mu^2}{2\gamma^2})$ and $\gamma - \nu = \frac{\mu^2}{2\gamma^2} - i\mu\delta p$. With these approximations and defining $\xi = \epsilon + \mu\delta p$, the exponent of the first part of Eq. (23) can be simplified to

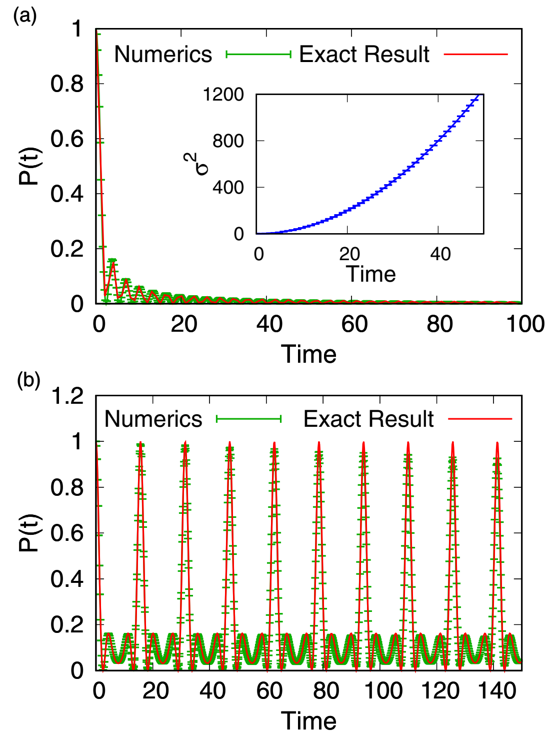

FIG. 1. The return probability of an initially localized wave packet $(\delta_{m,0})$ from the exact calculation and exact numerics. Here we present data for the case of zero bias $(\delta p = 0)$ in the rapid relaxation regime $(\sigma = \tau = 100)$ with $\Delta = 2.0$. (a) Equivalence to zero field case in the zero static field limit $\epsilon = 0.0$. The inset shows the unbounded growth of mean squared width, analogous to the zero-field scenario. (b) Bloch oscillations in the finite static field limit $\epsilon = 0.4$. In both the figures the numerics are performed for a system of size $L = 400$ with averaging carried out over 100 realizations of the disorder.

$$g_0(t) + \alpha(t) + \delta p\beta(t) \approx \frac{\Delta}{2\xi}\left[e^{-\frac{\mu^2}{2\gamma}t}\sin(k + \xi t) - \sin k\right] - \frac{\Delta}{2\xi}\left[e^{-\frac{\mu^2}{2\gamma}t}\sin(k' + \xi t) - \sin k'\right]. \quad (27)$$

The above expression is similar to Eq. (26) with ϵ replaced by ξ. Hence, Bloch oscillations with the average field and frequency ξ appear, which in the long time limit damp out exponentially. Also, unlike the case of $\delta p = 0$, Bloch oscillations with frequency $\mu\delta p$ arise even in the zero static field case. Tuning the bias $\delta p = -\frac{\epsilon}{\mu}$ in order to precisely cancel the effect of the static field, causes the average electric field to become zero, as a consequence of which Bloch oscillations are destroyed. This can be termed as *incoherent* destruction of WS localization as no frequency is involved in this scenario. This is to be contrasted with *coherent* destruction of WS localization [16,17], where a resonant tuning of the drive provides the mechanism in a system that is subjected to a combined dc and time periodic ac field. The incoherent destruction of localization here is to be seen as a contrast with the equivalence to the zero ac field case seen elsewhere. All

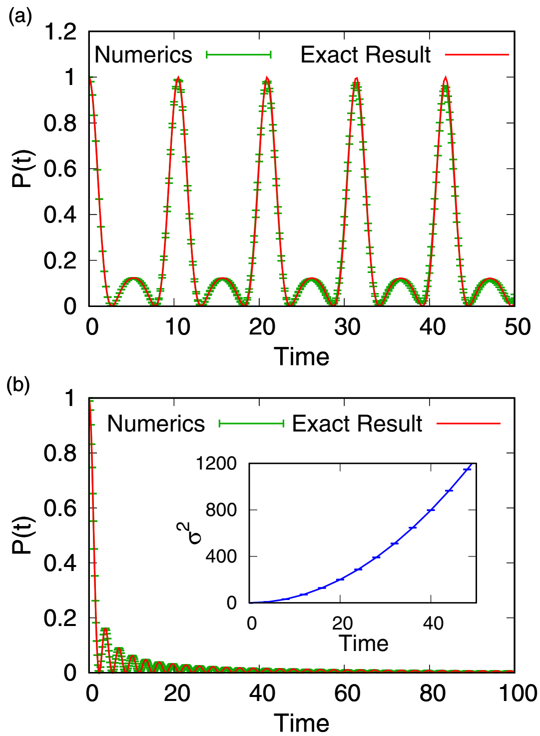

FIG. 2. The return probability of a wave packet initially localized state at the center of the chain ($m = 0$), from exact calculation and exact numerics. (a) Bloch oscillations with renormalized frequency ($\epsilon + \mu\delta p$) are seen in the rapid relaxation regime ($\sigma = 50,\ \tau = 150$) and with bias $\delta p = 0.5$ and static field $\epsilon = 0.5$. (b) Incoherent destruction of WS localization by the stochastic field for $\epsilon = 0.1,\ \sigma = 150,\ \tau = 50,\ \delta p = -0.5$. The inset shows the unbounded growth of mean squared width. In both the figures, the other parameters are $\mu = 0.2,\ L = 400,\ \Delta = 2.0,\ dt = 0.01$, and the exact numerics average over 100 realizations of disorder.

these effects are plotted in Fig. 2, where the return probability and the mean squared width of the wave packet are given as a function of time. The details of the numerical generation of the telegraph noise are given in the Appendix.

IV. NUMERICAL IMPLEMENTATION OF TELEGRAPH NOISE

The different cases considered above for the telegraphic noise can be verified independently from an exact numerical approach. The numerical approach involves the implementation of telegraph noise followed by the diagonalization of the Hamiltonian at each instant of time. The probability propagator can then be calculated by looking at the dynamics of an initial state.

For the numerical generation of the telegraph noise we follow Refs. [28,44,56–58]. The method works as follows: Let σ and τ be the rate of switching from level a to b and b to a, respectively. The probability of being at any time in state a is given by $\tau/(\tau+\sigma)$, whereas the probability of being in state b is $\sigma/(\tau+\sigma)$. Furthermore, let $w_{ij} = (i|W|j)$ with $i, j = \{a, b\}$ be the matrix elements of the relaxation matrix which gives the transition rate to jump from a state j to i. The condition of detailed balance implies

$$p_b(a|W|b) = p_a(b|W|a), \tag{28}$$

where p_a and p_b are the probability to remain in state a and b, respectively. Invoking conservation of probability along with Eq. (28), the matrix element of the relaxation matrix can be expressed as

$$w_{ab} = \lambda p_a, \quad w_{ba} = \lambda p_b, \tag{29}$$

where $\lambda = w_{ab} + w_{ba}$.

The relaxation matrix can thus be written as

$$W = \lambda \begin{bmatrix} -p_b & p_a \\ -p_b & -p_a \end{bmatrix}. \tag{30}$$

By substituting the values of p_a and p_b, the relaxation matrix W can be expressed as

$$W = \lambda \begin{bmatrix} -\frac{\sigma}{\tau+\sigma} & \frac{\tau}{\tau+\sigma} \\ \frac{\sigma}{\tau+\sigma} & -\frac{\tau}{\tau+\sigma} \end{bmatrix}, \tag{31}$$

where $\lambda = \tau + \sigma$. The difference of the probabilities between the two levels can be extracted as $\delta p = \frac{\tau-\sigma}{\tau+\sigma}$.

Also the various conditional probabilities can be expressed in terms of the elements of the relaxation matrix as follows [57,58]:

$$\begin{aligned} P_{aa} &= P(a, t_{n+1}|a, t_n) = \frac{\sigma}{\tau+\sigma} + \frac{\tau}{\tau+\sigma}\exp[-(\tau+\sigma)dt], \\ P_{ba} &= P(a, t_{n+1}|b, t_n) = \frac{\sigma}{\tau+\sigma} - \frac{\sigma}{\tau+\sigma}\exp[-(\tau+\sigma)dt], \\ P_{bb} &= P(b, t_{n+1}|b, t_n) = \frac{\tau}{\tau+\sigma} + \frac{\sigma}{\tau+\sigma}\exp[-(\tau+\sigma)dt], \\ P_{ab} &= P(b, t_{n+1}|a, t_n) = \frac{\tau}{\tau+\sigma} - \frac{\tau}{\tau+\sigma}\exp[-(\tau+\sigma)dt]. \end{aligned} \tag{32}$$

Finally, the numerical simulation is done as follows. Let the starting state be a. A random number between 0 and 1 is generated from the computer, and is compared against the conditional probability P_{aa}. If the conditional probability is greater than the random number, the next state will remain a, otherwise the next state will be changed to b. If the state changes to b, then for the next time, a random number is again generated and contrasted against the conditional probability P_{ba}. If this conditional probability is greater than the random number, the next state is taken as a else it will remain b. If the starting state is b, the random number is compared against the conditional probability P_{bb}. Again if this conditional probability is greater than the random number, the next state will remain b, otherwise it will be changed to a. If a flip happens to a, then a random number is generated and compared against the conditional probability P_{ab}. If this conditional probability is greater than the random number, the next state will flip to b, else it will remain a. This process is repeated in time units of length dt until the final time is reached. The different cases of the telegraphic noise can then be generated by setting the values σ and τ.

V. SUMMARY AND CONCLUSIONS

To summarize, we studied the effect of an electric field subjected to random telegraphic noise on a nearest-neighbor tight-binding chain. Our first result is the derivation of an exact general expression for the probability propagator, which is then employed to illuminate several special cases. As expected, in the zero relaxation case, the probability shows oscillatory behavior, with a superposition of the frequencies $\epsilon \pm \mu$. The rapid relaxation scenario forms the core emphasis of our work, and may be subdivided into two cases: one where the rates for the two levels are the same and the other where one level has greater lifetime than the other. In the former case, a delocalization effect is obtained in zero static field and Bloch oscillations in the presence of a static field. We identify this limit as a manifestation of equivalence of a rapidly changing field phenomenon with the corresponding zero ac field limit. In the latter case, a finite difference in the probabilities of the two levels renormalizes the Bloch frequency to $\omega_B = \epsilon + \mu\delta p$. A precise tuning of the bias δp leads to *incoherent* destruction of WS localization. The exact results are also verified by an independent numerical approach as well.

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APPENDIX: PROBABILITY CALCULATION

For a telegraph noise we have [with $\eta(t) = \int_0^t \mathcal{F}(t')dt'$]

$$i\eta(t) = -t(\gamma - i\epsilon).\mathcal{I} + t\sigma_z(\gamma\delta p + i\mu) + \gamma t(\sigma_x + i\delta p\sigma_y), \tag{A1}$$

where $\gamma = \frac{\lambda}{2}$ and $\delta p = (p_+ - p_-)$. Using a Pauli spin identity: $e^{i(\mathbf{a}\cdot\sigma)} = \mathcal{I}\cos|\mathbf{a}| + i(\hat{n}\cdot\vec{\sigma})\sin|\mathbf{a}|$, the exponential of Eq. (A1) can be written as

$$e^{i\eta(t)} = \tfrac{1}{2}e^{-t(\gamma-i\epsilon)}[e^{\nu t}(1+\hat{h}\cdot\vec{\sigma} + e^{-\nu t}(1+\hat{h}\cdot\vec{\sigma})]. \tag{A2}$$

Also, the conjugate equation is ($h'_x = \gamma$, $h'_y = -i\gamma\delta p$, $h'_z = \gamma\delta p - i\mu$)

$$e^{-i\eta(t)} = \tfrac{1}{2}e^{-t(\gamma+i\epsilon)}[e^{\nu t}(1+\hat{h}'\cdot\vec{\sigma} + e^{-\nu t}(1+\hat{h}'\cdot\vec{\sigma})]. \tag{A3}$$

Introducing $z = e^{ik}$ and $z' = e^{ik'}$, the expression for $V_k^+(t) - V_{k'}^+(t)$ can be solved to

$$\begin{aligned} V_k^+(t) - V_{k'}^+(t) = -\frac{\Delta}{8}e^{-\gamma t}\{&(z-z')\,e^{i\epsilon t}[e^{\nu t}(1+\hat{h}\cdot\vec{\sigma}) + e^{-\nu t}(1-\hat{h}\cdot\vec{\sigma})] \\ &+ (z^* - z'^*)\,e^{-i\epsilon t}[e^{\nu t}(1+\hat{h}'\cdot\vec{\sigma}) + e^{-\nu t}(1-\hat{h}'\cdot\vec{\sigma})]\}. \end{aligned} \tag{A4}$$

Finally, we need to solve the integration

$$\begin{aligned} -i\int_0^t dt'[V_k^+(t') - V_{k'}^+(t')] = \frac{i\Delta}{8}\bigg\{&(z-z')\left[\frac{1-e^{-(\gamma-\nu)t+i\epsilon t}}{(\gamma-\nu)-i\epsilon} + \frac{1-e^{-(\gamma+\nu)t+i\epsilon t}}{(\gamma+\nu)-i\epsilon}\right] \\ &+ (z-z')(\hat{h}\cdot\vec{\sigma})\left[\frac{1-e^{-(\gamma-\nu)t+i\epsilon t}}{(\gamma-\nu)-i\epsilon} - \frac{1-e^{-(\gamma+\nu)t+i\epsilon t}}{(\gamma+\nu)-i\epsilon}\right] + c.c\bigg\}. \end{aligned} \tag{A5}$$

Using the relations

$$\hat{h}\cdot\vec{\sigma} = \frac{\gamma}{\nu}\sigma_x + \frac{i\gamma\delta p}{\nu}\sigma_y + \frac{(\gamma\delta p + i\mu)}{\nu}\sigma_z, \quad \hat{h}'\cdot\vec{\sigma} = \frac{\gamma}{\nu^*}\sigma_x + \frac{i\gamma\delta p}{\nu^*}\sigma_y + \frac{(\gamma\delta p - i\mu)}{\nu^*}\sigma_z, \tag{A6}$$

the exponential of the above equation can be written as

$$e^{-i\int_0^t dt'[V_k^+(t') - V_{k'}^+(t')]} = i[g_0(t).\mathcal{I} + g_1(t)\sigma_x + g_2(t)\sigma_y + g_3(t)\sigma_z], \tag{A7}$$

where

$$\begin{aligned} g_0(t) &= \frac{\Delta}{8}\left\{(z-z')\left[\frac{1-e^{-(\gamma-i\epsilon)t+\nu t}}{(\gamma-i\epsilon)-\nu} + \frac{1-e^{-(\gamma-i\epsilon)t-\nu t}}{(\gamma-i\epsilon)+\nu}\right] + (z^*-z'^*)\left[\frac{1-e^{-(\gamma+i\epsilon)t+\nu^* t}}{(\gamma+i\epsilon)-\nu^*} + \frac{1-e^{-(\gamma+i\epsilon)t-\nu^* t}}{(\gamma+i\epsilon)+\nu^*}\right]\right\}, \\ g_1(t) &= \frac{\Delta\gamma}{8}\left\{\frac{(z-z')}{\nu}\left[\frac{1-e^{-(\gamma-i\epsilon)t+\nu t}}{(\gamma-i\epsilon)-\nu} - \frac{1-e^{-(\gamma-i\epsilon)t-\nu t}}{(\gamma-i\epsilon)+\nu}\right] + \frac{(z^*-z'^*)}{\nu^*}\left[\frac{1-e^{-(\gamma+i\epsilon)t+\nu^* t}}{(\gamma+i\epsilon)-\nu^*} - \frac{1-e^{-(\gamma+i\epsilon)t-\nu^* t}}{(\gamma+i\epsilon)+\nu^*}\right]\right\}, \\ g_2(t) &= \frac{i\Delta\gamma\delta p}{8}\left\{\frac{(z-z')}{\nu}\left[\frac{1-e^{-(\gamma-i\epsilon)t+\nu t}}{(\gamma-i\epsilon)-\nu} - \frac{1-e^{-(\gamma-i\epsilon)t-\nu t}}{(\gamma-i\epsilon)+\nu}\right] + \frac{(z^*-z'^*)}{\nu^*}\left[\frac{1-e^{-(\gamma+i\epsilon)t+\nu^* t}}{(\gamma+i\epsilon)-\nu^*} - \frac{1-e^{-(\gamma+i\epsilon)t-\nu^* t}}{(\gamma+i\epsilon)+\nu^*}\right]\right\}, \\ g_3(t) &= \frac{\Delta\gamma\delta p}{8}\left\{\frac{(z-z')}{\nu}\left[\frac{1-e^{-(\gamma-i\epsilon)t+\nu t}}{(\gamma-i\epsilon)-\nu} - \frac{1-e^{-(\gamma-i\epsilon)t-\nu t}}{(\gamma-i\epsilon)+\nu}\right] + \frac{(z^*-z'^*)}{\nu^*}\left[\frac{1-e^{-(\gamma+i\epsilon)t+\nu^* t}}{(\gamma+i\epsilon)-\nu^*} - \frac{1-e^{-(\gamma+i\epsilon)t-\nu^* t}}{(\gamma+i\epsilon)+\nu^*}\right]\right\} \\ &\quad + \frac{i\Delta\mu}{8}\left\{\frac{(z-z')}{\nu}\left[\frac{1-e^{-(\gamma-i\epsilon)t+\nu t}}{(\gamma-i\epsilon)-\nu} - \frac{1-e^{-(\gamma-i\epsilon)t-\nu t}}{(\gamma-i\epsilon)+\nu}\right] - \frac{(z^*-z'^*)}{\nu^*}\left[\frac{1-e^{-(\gamma+i\epsilon)t+\nu^* t}}{(\gamma+i\epsilon)-\nu^*} - \frac{1-e^{-(\gamma+i\epsilon)t-\nu^* t}}{(\gamma+i\epsilon)+\nu^*}\right]\right\}. \end{aligned} \tag{A8}$$

Also the expressions for $g_2(t)$ and $g_3(t)$ can be related to $g_1(t) = \alpha(t)$ as

$$g_2(t) = i\delta p\alpha(t), \quad g_3(t) = \delta p\alpha(t) + \beta(t), \tag{A9}$$

where $\beta(t)$ is the second part of $g_3(t)$. The expression for $|\mathbf{H}|$ can be solved to

$$|\mathbf{H}| = \sqrt{\alpha^2(t) + \beta^2(t) + 2\delta p\alpha(t)\beta(t)}. \tag{A10}$$

Finally, substituting these into the expression for the probability propagator and taking the restricted averages [55], a simplified expression for the probability propagator for the case of telegraph noise can be obtained.

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