Bell-state correlations of quasiparticle pairs in the nonlinear current of a local Fermi liquid

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We study Bell-state correlations for quasiparticle pairs excited in nonlinear current through a double quantum dot in the Kondo regime. Exploiting the renormalized perturbation expansion in the residual interactions of the local Fermi liquid and Bell's inequality for cross correlation of spin currents through distinct conduction channels, we derive an asymptotically exact form of Bell's correlation for the double dot at low bias voltages. We find that pairs of quasiparticles and holes excited by the residual exchange interaction violate Bell's inequality for the spin currents.

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I. INTRODUCTION

Recent advancement in current observation has realized ultrasensitive current noise measurements on current through a Kondo dot, spin current, and current cross correlation [1–5]. Low-energy properties of quantum dots with magnetic moments that strongly interact with conduction electrons in connected lead electrodes exhibit the Kondo effect, which has been a central issue of condensed-matter physics over the 50 years [6]. The low-energy properties of the Kondo effect are described well by the local Fermi-liquid theory. The local Fermi liquid is an extension of Landau's Fermi liquid to cover quantum impurities, in which free quasiparticles and residual interactions account for the underlying physics [7–13]. In electric current through the Kondo dot at low applied bias voltages, residual interactions excite quasiparticle pairs that have an effective charge of $2e$ [10,14–20]. This doubly charged state has been observed as enhancements of the shot noise [1,2,21–25].

This paper will explore the nature of the correlation between the quasiparticles that are excited by the residual interactions within the current. In a previous work of ours [26], we found that the residual exchange interaction of a quantum dot excites spin-entangled quasiparticles and holes. However, it remains a question how the entanglement can be observed. We exploit Bell's inequality with current correlations to investigate the quasiparticle's entanglement. Bell's theorem draws an essential distinction between the correlations found in quantum mechanics and those found in classical mechanics. As a *no-go theorem*, Bell's theorem places limits on physical possibility [27–33]. Bell-state correlation of electrons involved in tunneling currents through mesoscopic devices has been studied for the past 20 years [34–37]. Several studies have focused on Bell-state correlations caused by many-body effects. For example, Bell-state correlations of superconducting electron pairs have been studied with the Cooper pair splitter both theoretically and experimentally [4,38,39]. Bell-state correlations have also been predicted for electrons scattered by the Kondo exchange interaction at temperatures near the Kondo temperature [40]. Our work paves a way to investigate quantum entanglement in a variety of correlated materials, and will bring deeper understanding and different applications of local Fermi liquid.

This paper is organized as follows. First, we introduce a double quantum dot to generate quasiparticles' entanglements between the channels in Sec. II. Then, we briefly describe Bell's inequality with current correlations in Sec. III, and introduce the source term to systematically calculate the current correlations in Sec. IV. We also describe the renormalized perturbation theory to correctly treat the low-energy excited states of the local Fermi liquid in the nonlinear current in Sec. V. We discuss the restriction on the measurement time interval, and the Bell's inequality for an effective current that carries spin entanglements in Sec. VI. A measurable form of the Bell's inequality for the full current and the interaction dependence are investigated. A brief summary is given in Sec. VII.

II. MODEL

Consider the double dot illustrated in Fig. 1. The system is described by the action of the Anderson impurity model given as $\mathcal{S} = \sum_\mu \int_{-\mathcal{T}/2}^{\mathcal{T}/2} dt\,(\sigma_3)^{\mu\mu}\mathcal{L}_{\mathrm{A}}^{\mu}$, where the Lagrangian is given as $\mathcal{L}_{\mathrm{A}}^{\mu} = \mathcal{L}_0^{\mu} + \mathcal{L}_{\mathrm{T}}^{\mu} + \mathcal{L}_{\mathrm{I}}^{\mu}$ with

$$\mathcal{L}_0^{\mu} = \sum_{\alpha m\sigma} \int_{-D}^{D} d\varepsilon\, \bar{c}_{\varepsilon\alpha m\sigma}^{\mu}\left(i\frac{\partial}{\partial t} - \varepsilon\right)c_{\varepsilon\alpha m\sigma}^{\mu} + \sum_{m\sigma} \bar{d}_{m\sigma}^{\mu}\left(i\frac{\partial}{\partial t} - \epsilon_{\mathrm{d}}\right)d_{m\sigma}^{\mu}, \tag{1}$$

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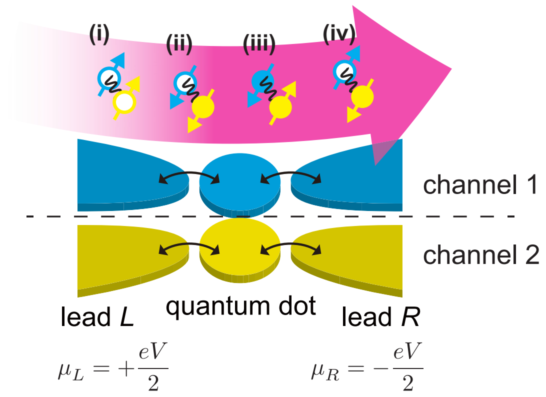

FIG. 1. Schematic of the double dot and quasiparticle pairs excited within the two channels of the current. The bias voltage eV is applied between the left and right leads. The filled and unfilled circles represent quasiparticles and holes, respectively, and the arrows attached to them indicate their spin degrees of freedom. Cyan and yellow indicate channels 1 and 2, respectively.

$$\mathcal{L}_{\mathrm{T}}^{\mu} = \sum_{\alpha m\sigma} \left[v \bar{d}_{m\sigma}^{\mu}\, \psi_{\alpha m\sigma}^{\mu} + v^{*} \bar{\psi}_{\alpha m\sigma}^{\mu}\, d_{m\sigma}^{\mu} \right], \tag{2}$$

$$\mathcal{L}_{\mathrm{I}}^{\mu} = U \sum_{m} n_{\mathrm{d}m\uparrow}^{\mu}\, n_{\mathrm{d}m\downarrow}^{\mu} + W n_{\mathrm{d}1}^{\mu}\, n_{\mathrm{d}2}^{\mu} + 2J \boldsymbol{S}_{\mathrm{d}1}^{\mu} \cdot \boldsymbol{S}_{\mathrm{d}2}^{\mu}. \tag{3}$$

Here, $\mathcal{T}$ is a measurement time, $\sigma_3 = ((1,0)^t, (0,-1)^t)$ is the third element of the Pauli matrix $\boldsymbol{\sigma}$, and the superscripts $\mu = -$ and $+$ represent the forward and backward paths of the Keldysh contour, respectively. Throughout this paper, the time argument t in the Lagrangian and the Grassmann numbers are suppressed. $\mathcal{L}_0^{\mu}$ represents electrons in the lead electrodes and the double dot. $c_{\alpha\varepsilon m\sigma}^{\mu}$ and $\bar{c}_{\alpha\varepsilon m\sigma}^{\mu}$ are the Grassmann numbers for electrons with spin $\sigma = \uparrow, \downarrow$ and energy ε in the conduction band of the left and right leads $\alpha = L, R$ of channel $m = 1, 2$. $d_{m\sigma}^{\mu}$ and $\bar{d}_{m\sigma}^{\mu}$ are the Grassmann numbers for electrons with spin σ in level ϵ_{d} of dot m. $\mathcal{L}_{\mathrm{T}}^{\mu}$ represents electron tunneling between the leads and the dots. They are connected by tunneling matrix element v through $\psi_{\alpha m\sigma}^{\mu} := \int_{-D}^{D} d\varepsilon \sqrt{\rho_{\mathrm{c}}} c_{\varepsilon\alpha m\sigma}$ and $\bar{\psi}_{\alpha m\sigma}^{\mu} := \int_{-D}^{D} d\varepsilon \sqrt{\rho_{\mathrm{c}}} \bar{c}_{\varepsilon\alpha m\sigma}^{\mu}$ with D the half width of the conduction band and $\rho_{\mathrm{c}} = \frac{1}{2D}$ the density of state for the conduction electrons. Electron tunneling causes an intrinsic linewidth of the dot levels to be $\Gamma = 2\pi \rho_{\mathrm{c}} |v|^2$.

$\mathcal{L}_{\mathrm{I}}^{\mu}$ represents the electron interactions in the double dot. U and W are the intra- and interdot Coulomb interactions, respectively, and J is the exchange interaction. The Grassmann number corresponding to the electron occupations and the total spin in the dot m are given by $n_{\mathrm{d}m\sigma} = \bar{d}_{m\sigma} d_{m\sigma}$, $n_{\mathrm{d}m} = \sum_{\sigma} n_{\mathrm{d}m\sigma}$, and $\boldsymbol{S}_{\mathrm{d}m} = \frac{1}{2} \sum_{\sigma\sigma'} \bar{d}_{m\sigma} \boldsymbol{\sigma}_{\sigma\sigma'} d_{m\sigma'}$. We impose the particle-hole symmetry $\epsilon_{\mathrm{d}} = -\frac{U}{2} - W$ and the absolute zero temperature $T = 0$ to eliminate the thermal and partition noises and maximize the effect of J. The bias voltage eV is applied symmetrically: the chemical potentials of the left and right leads are $\mu_L = +\frac{1}{2}eV$ and $\mu_R = -\frac{1}{2}eV$, respectively. With no loss of generality, a positive bias voltage $eV > 0$ can be assumed. We also use the natural units $\hbar = k_{\mathrm{B}} = 1$.

III. BELL'S INEQUALITY WITH CURRENT CORRELATIONS

We investigate quasiparticles that become correlated across the two channels. In the original argument of Bell's theorem, the spin correlation of two particles was studied [41]. However, one-by-one detection of every spin of the quasiparticles in a quantum-scale current remains is still difficult to be achieved in solid-state devices. Thus, we exploit Bell's inequality for two correlated currents, derived by Chtchelkatchev *et al.* [39]. This approach is outlined below.

The key idea of Bell's theorem is that determinism with a hidden variable is assumed to describe any correlations [41]. The violation of this assumption gives a sufficient condition for the quantum entanglement. For our double dot, the correlation between channel 1 and 2 are assumed to be described by a hidden variable η. Then, the density matrix of the whole system can be written in the form

$$\rho_{\mathrm{HVT}} = \int d\eta\, f(\eta) \rho_1(\eta) \otimes \rho_2(\eta), \tag{4}$$

where the distribution function for the hidden variable is satisfied with $f(\eta) \geqslant 0$ and $\int d\eta\, f(\eta) = 1$, and $\rho_m(\eta)$ is the density matrix for channel m. Integration of the current can give the average of spin angled to θ per a particle in the current of channel m in a measurement time from $t - \frac{\mathcal{T}}{2}$ to $t + \frac{\mathcal{T}}{2}$:

$$\bar{A}_{m\theta}(t,\eta) = \frac{\int_{t-\mathcal{T}/2}^{t+\mathcal{T}/2} dt' \, \mathrm{tr}\left[\rho_m(t',\eta) J_{m\theta}^{\mathrm{s}}(t')\right]}{\int_{t-\mathcal{T}/2}^{t+\mathcal{T}/2} dt' \, \mathrm{tr}\left[\rho_m(t',\eta) J_{m}^{\mathrm{c}}(t')\right]}. \tag{5}$$

$J_{m\theta}^{\mathrm{s}} = J_{m\theta} - J_{m\theta+\pi}$ and $J_m^{\mathrm{c}} = J_{m\theta} + J_{m\theta+\pi}$ are the spin and charge current, respectively, where $J_{m\theta}$ is the current with spin angled to the θ direction in channel m. For a current which effectively carries the spin correlation, the average spin is normalized as $|\bar{A}_{m\theta}(t,\eta)| \leqslant 1$. Then, the conventional derivation of Bell's inequality for two incident entangled particles is applicable to the averaged spin in the currents through the two channels. We obtain the Clauser-Horne-Shimony-Holt Bell inequality for two correlated currents as

$$0 \leqslant \mathcal{C} \leqslant 2, \tag{6}$$

where Bell's correlation is given in the form

$$\mathcal{C} = |F(\theta,\varphi) - F(\theta',\varphi) + F(\theta,\varphi') + F(\theta',\varphi')|. \tag{7}$$

Here, $F(\theta,\varphi) = h^{\mathrm{s}}(\theta,\varphi)/h^{\mathrm{c}}$ is given by a cross correlation of the spin current

$$h^{\mathrm{s}}(\theta,\varphi) = \int_{-\mathcal{T}/2}^{\mathcal{T}/2} dt\, dt' \left\langle J_{1\theta}^{\mathrm{s}}(t) J_{2\varphi}^{\mathrm{s}}(t') \right\rangle_{\mathrm{HVT}}, \tag{8}$$

and that of the charge current,

$$h^{\mathrm{c}} = \int_{-\mathcal{T}/2}^{\mathcal{T}/2} dt\, dt' \left\langle J_{1}^{\mathrm{c}}(t) J_{2}^{\mathrm{c}}(t') \right\rangle_{\mathrm{HVT}}, \tag{9}$$

with the average by the density matrix of the hidden variable theory, $\langle \ldots \rangle_{\mathrm{HVT}} := \mathrm{tr}[\rho_{\mathrm{HVT}} \ldots]$. Therefore, violation of Eq. (6) for Bell's correlation $\mathcal{C}_{\mathrm{QM}}$ calculated with the fully quantum-mechanical density matrix ρ_{QM} gives a sufficient condition for quantum correlation.

IV. CURRENT CORRELATIONS

In our quantum dot, the current of the electrons with spin angled to the θ direction in channel m is given as

$$I_{m\theta} = -ie(v\bar{d}_{m\theta}\psi_{Rm\theta} - v^*\bar{\psi}_{Rm\theta}d_{m\theta}). \tag{10}$$

To calculate current correlation, we introduce the source term

$$\mathcal{L}^{\mu}_{\rm sou}(\boldsymbol{\lambda}) = -i\sum_{m\gamma}[(e^{i\lambda^{\mu}_{m\gamma}}-1)v\bar{d}_{m\gamma}\psi_{Rm\gamma} + (e^{-i\lambda^{\mu}_{m\gamma}}-1)v^*\bar{\psi}_{Rm\gamma}d_{m\gamma}] \tag{11}$$

in $\mathcal{L}^{\mu}_{\rm A}$. Here, $\lambda^{\mu}_{m\gamma} = (\sigma_3)^{\mu\mu}\lambda_{m\gamma}$ is a contour-dependent source field, and $\gamma(=\theta, \theta+\pi)$ is the spin index defined with respect to the θ direction. The Grassmann number for an electron in the dot with spin γ can be given by a rotational transformation as $d^{\mu}_{m\theta} = \cos\frac{\theta}{2}d^{\mu}_{m\uparrow} + \sin\frac{\theta}{2}d^{\mu}_{m\downarrow}$. $\bar{d}^{\mu}_{m\theta}$, $\psi^{\mu}_{\alpha m\theta}$, and $\bar{\psi}^{\mu}_{\alpha m\theta}$ are also defined in the same manner. Current correlations can be calculated by differentiating the generating function $\ln\mathcal{Z}(\boldsymbol{\lambda})$ with the corresponding source fields. The partition function is given in the form

$$\mathcal{Z}(\boldsymbol{\lambda}) = \int\mathcal{D}(\bar{c}_{\varepsilon\alpha m\sigma})\mathcal{D}(c_{\varepsilon\alpha m\sigma})\mathcal{D}(\bar{d}_{m\sigma})\mathcal{D}(d_{m\sigma})e^{i\mathcal{S}(\boldsymbol{\lambda})} \tag{12}$$

with

$$\mathcal{S}(\boldsymbol{\lambda}) = \sum_{\mu}\int_{-\mathcal{T}/2}^{\mathcal{T}/2}dt\,(\sigma_3)^{\mu\mu}\big[\mathcal{L}^{\mu}_{\rm A} + \mathcal{L}^{\mu}_{\rm sou}(\boldsymbol{\lambda})\big]. \tag{13}$$

The specific form of $\ln\mathcal{Z}(\boldsymbol{\lambda})$ up to order V^3 is given in Ref. [26].

V. RENORMALIZED PERTURBATION THEORY

To take electron correlations into account, we use the renormalized perturbation theory [42–44]. At low energies, perturbation expansion in $\mathcal{L}^{\mu}_{\rm I}$ provides an exact result if all the terms in the series are accounted for. However, this expansion is difficult, except for some special cases. Below, employing the idea of the renormalized perturbation theory, we reorganize the perturbation expansion and effectively carry out all-order calculations at low energies.

First, we formulate the quasiparticle's Lagrangian $\widetilde{\mathcal{L}}^{\mu}_{\rm qp}$ by replacing $\epsilon_{\rm d}$, v, U, W, J, $d^{\mu}_{m\sigma}$, and $\bar{d}^{\mu}_{m\sigma}$ of $\mathcal{L}^{\mu}_{\rm A}$ with the renormalized parameters and the Grassmann numbers of the quasiparticle given by $\tilde{\epsilon}_{\rm d}$, $\tilde{v}$, $\widetilde{U}$, $\widetilde{W}$, $\widetilde{J}$, $\tilde{d}^{\mu}_{m\sigma}$, and $\bar{\tilde{d}}^{\mu}_{m\sigma}$. These renormalized parameters and Grassmann numbers are defined by sets of perturbation series given by the self-energy and the four vertex at $T = eV = 0$ [26]. Note that the renormalized linewidth given by $\widetilde{\Gamma} := 2\pi\rho_{\rm c}|\tilde{v}|^2$ corresponds to the characteristic energy scale, namely, the Kondo temperature: $T_{\rm K} = \pi\widetilde{\Gamma}/4$. We can evaluate $\tilde{\epsilon}_{\rm d}$, $\widetilde{\Gamma}$, $\widetilde{U}$, $\widetilde{W}$, and $\widetilde{J}$ by using the numerical renormalization group (NRG) approach [44–46]. The nonequilibrium effects at low bias voltages $eV \ll T_{\rm K}$ arise through perturbation expansions in the renormalized interactions.

As a part of the interaction effects are taken into account *ab initio* in the quasiparticle's Lagrangian during renormalized perturbation expansion, a counter term has to be introduced to avoid overcounting in the perturbation expansion. In the other words, the total Lagrangian has to be satisfied with $\mathcal{L}^{\mu}_{\rm A} = \widetilde{\mathcal{L}}^{\mu}_{\rm qp} + \mathcal{L}^{\mu}_{\rm CT}$. The counter term $\mathcal{L}^{\mu}_{\rm CT}$, can be expressed in terms of the renormalized parameters and the renormalized Grassmann numbers, which are determined by the renormalized condition for the renormalized self-energy and the renormalized four-vertex. In the particle-hole symmetric case, the perturbation expansion up to only the second order in the renormalized interactions provides an asymptotically exact form of the self-energy at $T = 0$ up to the second order in ω and eV because of the counter term. As a result, asymptotically exact forms of currents and current correlations up to order $(eV)^3$ are obtained. We shall calculate the current correlations using perturbation expansion in the residual interactions.

TABLE I. Signs of spin/charge current correlations of particle-particle (*p*-*p*), hole-hole (*h*-*h*), and particle-hole (*p*-*h*) pairs with parallel and antiparallel spins. The pairs excited in the current are shown in Fig. 1.

	p-*p* or *h*-*h* pairs	*p*-*h* pair
Parallel spin	(i) +/+	(ii) −/−
Antiparallel spin	(iii) −/+	(iv) +/−

VI. RESULTS AND DISCUSSION

Let us calculate $\mathcal{C}_{\rm QM}$ in terms of the quasiparticle parameters. Since $\langle I^{\rm s}_{m\theta}\rangle = 0$ in our model, the correlation of the spin currents can be rewritten into the correlation of spin current fluctuations $\delta I^{\rm s}_{m\theta} = I^{\rm s}_{m\theta} - \langle I^{\rm s}_{m\theta}\rangle$ as

$$h^{\rm s}_{\rm QM}(\theta,\varphi) = \int_{-\mathcal{T}/2}^{\mathcal{T}/2}dt\,dt'\big\langle\delta I^{\rm s}_{1\theta}(t)\delta I^{\rm s}_{2\varphi}(t')\big\rangle. \tag{14}$$

At low energies, namely, $\mathcal{T} \gg (eV)^{-1} \gg t_{\rm K}$, differentiation of $\ln\mathcal{Z}(\boldsymbol{\lambda})$ with the source fields yields

$$h^{\rm s}_{\rm QM}(\theta,\varphi) = -\mathcal{T}\frac{e^3V}{2\pi}\left(\frac{eV}{\widetilde{\Gamma}}\right)^2\left(\frac{1}{4}\tilde{j}^2 - \frac{1}{3}\tilde{w}\tilde{j}\right)\cos(\theta-\varphi) + \mathcal{O}\big(V^5\big), \tag{15}$$

where $t_{\rm K} \propto \widetilde{\Gamma}^{-1}$ is the Kondo time scale, and $\tilde{w} = \frac{\widetilde{W}}{\pi\widetilde{\Gamma}}$ and $\tilde{j} = \frac{\widetilde{J}}{\pi\widetilde{\Gamma}}$. Note that the spin correlation measured by $h^{\rm s}_{\rm QM}(\theta,\varphi)$ comes from only a portion of the entangled quasiparticle pairs within the current. As seen in the specific form of $\ln\mathcal{Z}(\boldsymbol{\lambda})$ [26], the residual interactions can excite four types of the quasiparticle pairs in the current (see Fig. 1). As Table I shows, the spin and charge current correlations of these pairs have different signs from each other. Consequently, some of the spin and charge correlations due to these pairs are independently canceled in the full current. Therefore, the correlation of the charge current $I'^{\rm c}_m$ that effectively carries the spin current correlation must be calculated, rather than that of the full current given by $h^{\rm fcc} = \int_{-\mathcal{T}/2}^{\mathcal{T}/2}dt\,dt'\langle I^{\rm c}_1(t)I^{\rm c}_2(t')\rangle$ with $I^{\rm c}_m = \sum_{\gamma}I_{m\gamma}$. The current correlation can be written in terms of current fluctuation of $I'^{\rm c}_m$ as

$$h^{\rm c}_{\rm QM} = H^{\rm c}_{\rm QM} + \mathcal{T}^2\big\langle I'^{\rm c}_1\big\rangle\big\langle I'^{\rm c}_2\big\rangle, \tag{16}$$

where $H^{\rm c}_{\rm QM} = \int_{-\mathcal{T}/2}^{\mathcal{T}/2} dt dt' \, \langle \delta I'^{\rm c}_1(t) \delta I'^{\rm c}_2(t') \rangle$ with $\delta I'^{\rm c}_m(t) = I'^{\rm c}_m(t) - \langle I'^{\rm c}_m \rangle$. Although an explicit expression of $I'^{\rm c}_m$ is not easy to derive, the correlation can be evaluated readily using the terms of spin-correlated carriers in $\ln \mathcal{Z}(\boldsymbol{\lambda})$:

$$H^{\rm c}_{\rm QM} = -\mathcal{T}\frac{e^3V}{2\pi}\left(\frac{eV}{\widetilde{\Gamma}}\right)^2\left(\frac{1}{4}\tilde{j}^2 - \frac{1}{3}\tilde{w}\tilde{j}\right) + \mathcal{O}(V^5). \quad (17)$$

The leading term of the charge current is of the third order in the applied bias voltage, $\langle I^{\rm c}_m \rangle \propto eV(\frac{eV}{\widetilde{\Gamma}})^2$. Thus,

$$t_{\rm b} \propto \left[eV\left(\frac{eV}{\widetilde{\Gamma}}\right)^2\right]^{-1},$$

a boundary value of the measurement time, divides the behavior of $\mathcal{C}_{\rm QM}$ into two regions. One is $\mathcal{T} \gg t_{\rm b}$, where $H^{\rm c}_{\rm QM} \ll \mathcal{T}^2 \langle I^{\rm c}_1 \rangle \langle I^{\rm c}_2 \rangle$. Then, the correlation function can be given simply as $h^{\rm c}_{\rm QM} \sim \mathcal{T}^2 \langle I^{\rm c}_1 \rangle \langle I^{\rm c}_2 \rangle$. This results in $\mathcal{C}_{\rm QM} \sim 0$, and $\mathcal{C}_{\rm QM}$ never violates Bell's inequality in this region. In the opposite region $\mathcal{T} \ll t_{\rm b}$, the correlation of the fluctuations is dominant, namely, $H^{\rm c}_{\rm QM} \gg \mathcal{T}^2 \langle I^{\rm c}_1 \rangle \langle I^{\rm c}_2 \rangle$, which leads to $h^{\rm c}_{\rm QM} \sim H^{\rm c}_{\rm QM}$. Then, Bell's correlation is given in the form

$$\mathcal{C}_{\rm QM} \sim K(\theta, \theta'; \varphi, \varphi') \quad (18)$$

with

$$K(\theta, \theta'; \varphi, \varphi') = |\cos(\theta - \varphi) - \cos(\theta' - \varphi) + \cos(\theta - \varphi') + \cos(\theta' - \varphi')|. \quad (19)$$

Since $K(\theta, \theta'; \varphi, \varphi')$ is bounded within $[0, 2\sqrt{2}]$, it is concluded that the exchange interaction of the Fermi liquid can violate Bell's inequality. We note that the limit $\mathcal{T} \to \infty$ can be taken to evaluate $F_{\rm QM}(\theta, \varphi) = h^{\rm s}_{\rm QM}(\theta, \varphi)/H^{\rm c}_{\rm QM}$ although the measurement time is bounded within $t_{\rm K} \ll \mathcal{T} \ll t_{\rm b}$, because the $\mathcal{T}$ dependencies of $h^{\rm s}_{\rm QM}(\theta, \varphi)$ and $H^{\rm c}_{\rm QM}$ cancel out each other for $\mathcal{T} \gg t_{\rm K}$.

However, $\mathcal{C}_{\rm QM}$ may be difficult to measure experimentally, because $h^{\rm c}_{\rm QM}$ is the current correlation of the carriers that effectively carry the correlated spins. Next we suggest a measurable form of Bell's correlation. Multiplying each side of Eq. (6) for our double dot by $r = |h^{\rm c}/h^{\rm fcc}|$, we derive a measurable form of Bell's inequality and correlation that are composed of the cross correlations of the full current as

$$\mathcal{C}^* = |F^*(\theta, \varphi) - F^*(\theta', \varphi) + F^*(\theta, \varphi') + F^*(\theta', \varphi')| \quad (20)$$

with $F^*(\theta, \varphi) = h^{\rm s}(\theta, \varphi)/h^{\rm fcc}$. Then, Bell's inequality for $\mathcal{C}^*$ is given by a deformed boundary:

$$0 \leqslant \mathcal{C}^* \leqslant 2r. \quad (21)$$

For the quantum-mechanical density of states and $t_{\rm K} \ll \mathcal{T} \ll t_{\rm b}$, $\mathcal{C}^*$ and r take the forms

$$\mathcal{C}^*_{\rm QM} = r_{\rm QM} K(\theta, \theta'; \varphi, \varphi'), \;\; r_{\rm QM} = \left|\frac{1 - \frac{4}{3}\left(\frac{\tilde{w}}{\tilde{j}}\right)}{1 + \frac{4}{3}\left(\frac{\tilde{w}}{\tilde{j}}\right)^2}\right|, \quad (22)$$

respectively. Since $\mathcal{C}^*$ is simply given by a product of $\mathcal{C}$ and r, $\mathcal{C}^*_{\rm QM}$ can also violate Bell's inequality given by Eq. (21). The maximum value of $\mathcal{C}^*_{\rm QM}$ is given by $\mathcal{C}^*_{\rm QM,max} = 2\sqrt{2} r_{\rm QM}$,

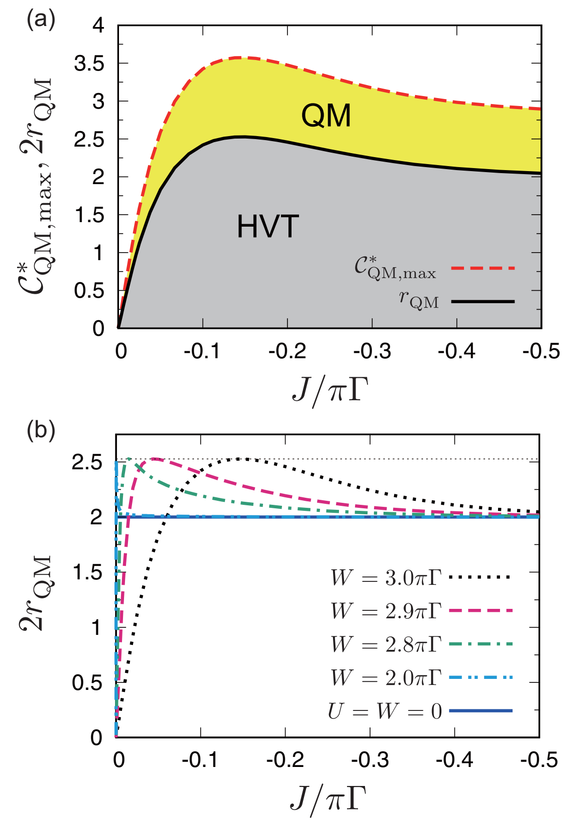

FIG. 2. (a) $\mathcal{C}^*_{\rm QM,max}$ and $2r_{\rm QM}$ as a function of ferromagnetic $J(<0)$ for $U = W = 3.0\pi\Gamma$. The gray area is covered by the hidden variable theory, and the yellow area represents the sufficient condition for the quantum correlation. (b) $2r_{\rm QM}$ as a function of ferromagnetic J for $U = 3.0\pi\Gamma$ and several choices of $W = 3.0\pi\Gamma$, $2.9\pi\Gamma$, $2.8\pi\Gamma$, and $2.0\pi\Gamma$, and $U = W = 0$. The thin dotted line indicates the maximum value $2r_{\rm QM} = 1 + \frac{\sqrt{21}}{3} \approx 2.528$.

which corresponds to Tselson's bound [47] in our model. This bound gives the upper limit for the correlation in the quantum regime. $\mathcal{C}^*_{\rm QM,max}$ and $2r_{\rm QM}$ are plotted as a function of J; $J \leqslant 0$ and $J \geqslant 0$ for $U = W = 3.0\pi\Gamma$ in Figs. 2(a) and 3(a), respectively. A critical point appears at $J = J_{\rm c} > 0$ [46,48]. For $J > J_{\rm c}$, two electrons occupying in the double dot form an isolated singlet state and decouple from the conduction electrons, and then no charge currents can flow through the double dot. Thus, we focus on the region $J < J_{\rm c}$, in which the low-energy state is accounted for by the local Fermi liquid, and electric current flows through the dot. The region between $\mathcal{C}^*_{\rm QM,max}$ and $2r_{\rm QM}$ represents a sufficient condition that the correlation of spin currents across the two channels is quantum mechanical in nature. For $J > 0$, the value $\mathcal{C}^*_{\rm QM,max}$ takes a local minimum to zero, where the excited quasiparticle pairs with parallel and antiparallel contributions to the spin correlation cancel each other out. Thus, Bell's test is not applicable with this value of J.

Experimentally, the violation of Bell's inequality can be confirmed through observation with values of $\mathcal{C}^*_{\rm QM}$ larger than the theoretically calculated value of $2r_{\rm QM}$. This parameter $2r_{\rm QM}$ depends on the strength of U, W, and J, which can be evaluated using NRG calculations. $2r_{\rm QM}$ is plotted as a function of J for several choices of U and W for $J \leqslant 0$ and $J \geqslant 0$

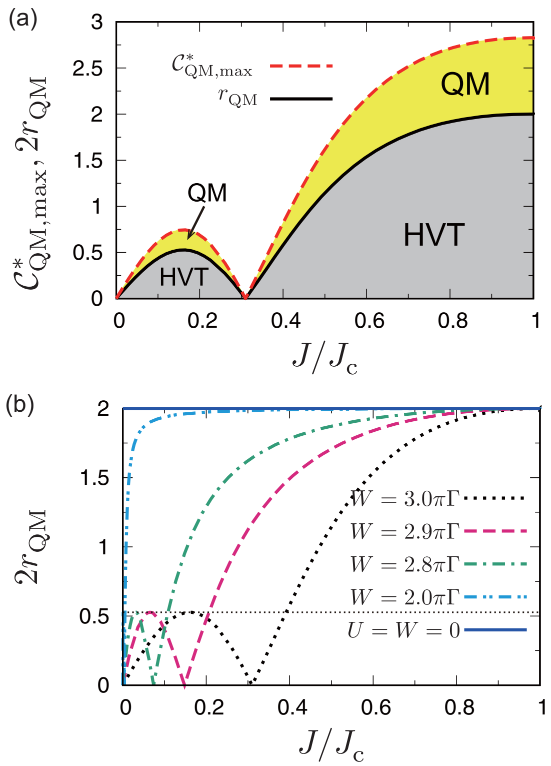

FIG. 3. (a) $\mathcal{C}^*_{\rm QM,max}$ and $2r_{\rm QM}$ as a function of antiferromagnetic $J(>0)$ for $U = W = 3.0\pi\Gamma$. J is normalized by the critical value $J_{\rm c}$. The gray area is covered by the hidden variable theory, and the yellow area represents the sufficient condition for the quantum correlation. (b) $2r_{\rm QM}$ as a function of ferromagnetic J for $U = 3.0\pi\Gamma$ and several choices of $W = 3.0\pi\Gamma,\ 2.9\pi\Gamma,\ 2.8\pi\Gamma$, and $2.0\pi\Gamma$, and $U = W = 0$. The thin dotted line indicates the value of the local maximum $r_{\rm QM} = 1 - \frac{\sqrt{21}}{3} \approx 0.528$.

in Figs. 2(b) and 3(b), respectively. For $|J| \gg T_{\rm K}$, the values of $h^{\rm fcc}_{\rm QM}$ coincide with $h^{\rm c}_{\rm QM}$, which results in $r_{\rm QM} \to 1$ and the $r_{\rm QM}$ independent form of Bell's inequality is recovered. In this region, therefore, Bell's test can be examined without the need for any numerical calculations of $r_{\rm QM}$.

Finally, we discuss the causal locality of Bell's theorem in our model. Bell-state correlations in our model are induced by entangled quasiparticles that are excited by the residual exchange interaction that is scaled by $T_{\rm K}$. Therefore, for the causal locality to hold, the two measurements in channels 1 and 2 must be separated by a distance $d \gg ct_{\rm K}$, where $t_{\rm K} = \frac{\hbar}{k_{\rm B}T_{\rm K}}$ and c is the speed of light. For a typical Kondo temperature of quantum dots $T_{\rm K} \sim 1$ K, d must be much larger than $ct_{\rm K} \sim 4.58 \times 10^{-2}$ m.

VII. SUMMARY

We have found that spin entangled quasiparticles that are excited by the residual exchange interaction of the local Fermi liquid in the double dot leave their trace in the violation of Bell's inequality with correlations of the effective current. By deforming the boundary of the hidden variable theory, we have derived an experimentally measurable form of Bell's inequality that is composed of correlations of the full current. The interaction dependence of the deformed boundary and Bell's correlation has been demonstrated by using the NRG approach. We have also shown that the long measurement-time limit can be taken to both theoretically and experimentally evaluate correlations of current fluctuations, beyond the restriction to extract the meaningful Bell-state correlation.

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