Glass-induced enhancement of superconducting T_c: Pairing via dissipative mediators

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With substantial evidence of glassy behavior in the phase diagram of high-T_c superconductors and its coexistence with superconductivity, we attempt to answer the following question: what are the properties of a superconducting state where the force driving Cooper pairing becomes dissipative? We find that when the bosonic mediator is local, dissipation acts to reduce the superconducting critical temperature (T_c). On the other hand, contrary to naïve expectations, T_c behaves nonmonotonically with dissipation for a nonlocal mediator—weakly dissipative bosons at different energy scales act coherently to give rise to an increase in T_c and eventually destroy superconductivity when the dissipation exceeds a critical value. The critical value occurs when dissipative effects become comparable to the energy scale associated with the spatial stiffness of the mediator, at which point T_c acquires a maximum. We outline consequences of our results to recent proton-irradiation experiments (M. Leroux *et al.*, arXiv:1808.05984) on the cuprate superconductor $La_{2-x}Ba_xCuO_4$ (LBCO), which observe a disorder-induced increase in T_c even when the transition temperature of the proximate charge density wave (CDW) seems to be unaffected by irradiation. Our mechanism is a way to raise T_c that does not require a "tug-of-war" type scenario between two competing phases.

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I. INTRODUCTION

In s-wave superconductors (SCs) where the quasiparticle excitation spectrum is fully gapped and has a constant sign of the pairing form factor across the Fermi surface, Anderson's magic theorem keeps the critical temperature (T_c) robust to weak, nonmagnetic impurities. In higher angular momentum SCs (p, d wave, etc.) or SCs where the sign of the gap changes across parts of the Fermi surface (such as $s_\pm$ pnictide SCs), T_c is drastically suppressed with the addition of impurities—magnetic or otherwise [1]. These effects hold in the independent disorder limit and in the absence of electron correlations.

At a collective level when electron correlations are taken into account, randomness can yield several interesting phases of matter [2]. Among these is the spin-glass (SG) phase widely observed in the phase diagram of many strongly correlated systems such as high-T_c SCs [3–19]. The SG phase exhibits a remarkable phenomenology [20]—a transition into the SG defined by a broad cusp in the specific heat, a split in the dc magnetization at the SG transition depending on whether the SG phase is field cooled (FC) or zero field cooled (ZFC), linear temperature dependence of the ac susceptibility peak, and aging. Theoretically, SGs are described by an order parameter where the spin average on each site is nonvanishing but goes to zero when averaged over the lattice [21]. Important to the discussions that follow, spin correlators at the SG critical point follow a power law of the form [21–24]

$$D(\tau) \equiv \left[\langle S_{i\mu}(\tau) S_{i\mu}(0)\rangle\right] \sim \frac{1}{\tau^2}, \tag{1}$$

which in frequency space reads $D(\omega) \sim |\omega|$. Here, $S_{i\mu}$ is the μth component of the spin at site i, and the angular and square brackets denote thermal and site averages, respectively. The linear frequency dependence of the spin correlators indicates that dissipative dynamics is a necessary—albeit not sufficient—ingredient of SGs.

In this work, we explore the robustness of T_c and properties of a superconducting state where the dynamics of the pairing mediator is rendered dissipative due to *collective* disorder (in the aforementioned sense). To this end, we add to the Lagrangian describing the mediator a dissipative term [25–28], $\Delta L = \sum_{\mathbf{k},\omega_n} \eta|\omega_n||\Psi(\mathbf{k},\omega_n)|^2$. Here, $\mathbf{k}$ and ω_n are the momenta and Matsubara frequencies, η is a measure of dissipation, and $\Psi(\mathbf{k},\omega_n)$ is the bosonic field. We find that dissipative effects generally act to suppress T_c when the mediator is local. This occurs because dissipation has the effect of reducing the attractive interaction mediating Cooper pairs. However, contrary to naïve expectations, T_c behaves nonmonotonically with dissipation for a nonlocal mediator. In this scenario, weakly dissipative bosons at different energy scales act coherently to give rise to an increase in T_c and eventually destroy superconductivity when the dissipation exceeds a critical value. The critical value occurs when the dissipation parameter η becomes comparable to the energy scale associated with the velocity of the mediating bosons (or the spatial stiffness); at this crossover, T_c acquires a maximum value. We also study the effects of the dissipative mediator on the ratio $\frac{2\Delta(0)}{T_c}$ and the heat-capacity jump at the superconducting transition and find departures from values predicted by BCS theory.

II. EXPERIMENTAL BASIS

We now make our case for a dissipative or "glassy" mediator from experiments on a variety of high-T_c SCs. The SG phase has been observed extensively in the underdoped

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and other regions proximate to superconductivity in the phase diagrams of both the cuprate [3–13] and iron-based superconductors [14–19]. Existing evidence is also spread over several techniques such as dc magnetization [3,4,16–19], nuclear magnetic resonance (NMR)/nuclear quadrupole resonance (NQR) [6,10–14], μSR [8,9], and neutron scattering [8,15]. Given the strong evidence of a SG phase and its proximity to the superconducting dome in high-T_c SCs, it is already reasonable to consider its effect on the pairing problem. Additionally, there is ample experimental evidence lending credence to a dissipative character of fluctuations that mediate Cooper pairing. First, disorder causes the d-electron spins (Cu spins in the cuprates and Fe spins in the iron superconductors) to exhibit glassy behavior and not the dopant spins [3,7,14,18] (although in certain iron-based systems, it is the dopant spins that become glassy [29]). Second, SG and SC phases actually coexist in a variety of high-T_c SCs [11,14,18]. This indicates a strong intermixing of properties of the two phases, similar to what is expected in the context of other mean-field orders (such as density waves) acquiring a glassy behavior [30]. Third, neutron-scattering and NMR/NQR measurements in the cuprate SCs $La_{2-x}Sr_xCuO_2$ (LSCO) and $La_{2-x}Ba_xCuO_4$ (LBCO) have found a direct "slowing" of spin fluctuations in the vicinity of glassy orders [6,8,13,15]. Finally, early theoretical predictions on doping La_2CuO_4 clearly point to a frustration-induced glassy behavior of the Cu d-orbital spins in the phase diagram [31].

Hence, the notion of a dissipative pairing mediator in high-temperature superconductors has firm foundations in both experiment and theory. As will be argued later in this paper, nonlocal dissipative mediators can help throw light on recent proton irradiation experiments [57] on LBCO, which observe a disorder-induced increase in T_c *even when the transition temperature of the proximate charge density wave (CDW) is unaffected by the presence of radiation disorder*. The mechanism we propose in this paper forms an alternative way to raise T_c of a superconductor that does not require a "tug-of-war" type scenario between two competing phases.

III. MODEL AND GAP EQUATION

We begin by writing the conjectured model for the bosonic propagator. The total action consists of a free part $S_0[\Psi, \Psi^*]$ and a dissipative part $S_{\rm dis}[\Psi, \Psi^*]$ defined by

$$S[\Psi,\Psi^*] = S_0[\Psi,\Psi^*] + S_{\rm dis}[\Psi,\Psi^*],$$
$$S_0[\Psi,\Psi^*] = \int d^d\mathbf{r}\, d\tau[\kappa|\nabla\Psi(\mathbf{r},\tau)|^2 + |\partial_\tau\Psi(\mathbf{r},\tau)|^2 + M^2|\Psi(\mathbf{r},\tau)|^2],$$

where κ is the spatial stiffness or energy scale associated with the boson velocity, and the squared mass M^2 is proportional to the inverse correlation length. As outlined in Sec. I, we take the dissipative term to be the form $S_{\rm dis}[\Psi,\Psi^*] = \sum_{\mathbf{k},\omega_n}(2\eta\,|\omega_n|)|\Psi(\mathbf{k},\omega_n)|^2$ in Fourier space with the various quantities defined previously. With this total action, the bosonic propagator $D(\mathbf{q}, i\omega_n - i\omega_m)$ takes the form

$$D(\mathbf{q}, i\omega_n) = \frac{\alpha}{\kappa q^2 + \omega_n^2 + 2\eta|\omega_n| + M^2}.$$

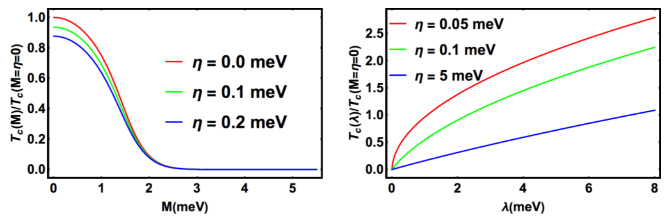

FIG. 1. Effect of a local dissipative mediator. Left: Superconducting critical temperature T_c (normalized to its value at $M = \eta = 0$) as a function of the mass parameter M. The coupling constant λ is chosen to be equal to 1 meV. Right: Same quantity now plotted as a function of the coupling constant λ for $M = 0.1$ meV. A crossover from $T_c \sim \sqrt{\lambda}$ to $T_c \sim \lambda$ occurs as a function of η.

Here, $q = |\mathbf{q}|$ and α is a constant with dimensions of energy that can be absorbed into an effective coupling constant (similar to spin fluctuations; see, for example, [32]).

T_c, *local case* ($\kappa = 0$). We choose a quadratic electron dispersion $\xi_\mathbf{k}$ and gap function denoted by $\Delta(i\omega_n, \mathbf{k})$. Substituting $D(\mathbf{q}, i\omega_n - i\omega_m)$ into the gap equation,

$$\Delta(i\omega_n,\mathbf{k}) = \frac{|g|^2}{\beta V}\sum_{\mathbf{q},\omega_m}\frac{D(\mathbf{q}, i\omega_n - i\omega_m)\Delta(i\omega_m,\mathbf{k}+\mathbf{q})}{\omega_m^2 + \xi_{\mathbf{k}+\mathbf{q}}^2 + \Delta(i\omega_m,\mathbf{k}+\mathbf{q})^2}, \quad (2)$$

and assuming an isotropic, frequency-independent s-wave gap (defined by the $\mathbf{k} = 0$ value and denoted by Δ henceforth), the equation determining T_c (setting $\Delta = 0$) reduces to $1 = \pi\lambda T\sum_{\omega_m<\Lambda}\frac{1}{|\omega_m|(\omega_m^2+2\eta|\omega_m|+M^2)}$, where T is set to T_c. Here, β is the inverse temperature, g is the interaction strength, $\lambda \equiv N(0)|g|^2\alpha$ is the coupling constant, Λ is the high-energy cutoff, and $N(0)$ is the density of states at the Fermi energy. The sum over ω_m can be performed exactly to yield the equation for T_c as (c.c. is complex conjugate)

$$1 = \frac{\lambda(\eta - i\bar{M})^{-1}}{2i\bar{M}}\left[\psi\left(\frac{1}{2} + \frac{\eta}{2\pi T_c} - i\frac{\bar{M}}{2\pi T_c}\right) - \psi\left(\frac{1}{2}\right)\right] + \text{c.c.}, \quad (3)$$

where $\bar{M} \equiv \sqrt{M^2 - \eta^2}$ and $\psi(x)$ denotes the digamma function. The solutions for T_c as a function of the parameters M and η are shown in the left panels of Figs. 1 and 2. The reduction in T_c as a function of the mass and dissipation can be intuitively understood by taking the limit of $M \gg T, \eta$

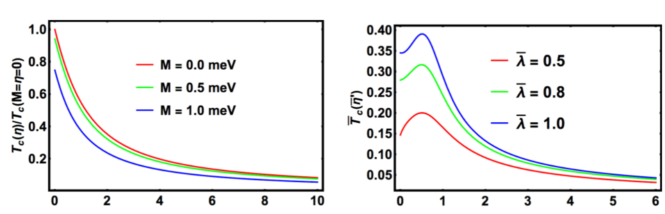

FIG. 2. Left: Superconducting critical temperature T_c (normalized to its value at $M = \eta = 0$) as a function of the dissipation parameter η for different masses M when the mediator is local. The coupling constant λ is chosen to be equal to 1 meV. Right: The case when the mediator is nonlocal for $M = 0$: plot of the dimensionless $\bar{T}_c = T_c/\kappa$ as a function of $\bar{\eta}' = \eta'/\kappa$ for different dimensionless coupling strengths $\bar{\lambda} = \lambda/\kappa^2$. The peak in T_c is set by κ, the energy scale associated with bosonic velocity.

and $\eta \gg T, M$, respectively. In these limits, η and M^2 can be factored out of the Matsubara sum which, in effect, reduces the coupling constant λ and hence suppresses T_c.

T_c, nonlocal case ($\kappa \neq 0$). We can make similar assumptions on the superconducting gap for the $\kappa \neq 0$ case. To maintain analytical tractability and focus on the effect of dissipation parameter η, we will later set the mass (now renormalized by the chemical potential; we use the same symbol for ease of notation) to zero. We can now substitute the bosonic propagator with $\kappa \neq 0$ back into the gap equation (2). The resulting energy integral can be solved exactly by the method of residues and takes the form $\int_{-\infty}^{\infty} \frac{d\xi}{(\xi^2+r^2)(\kappa\xi+s)} = \frac{\pi s}{(\kappa^2 r^2+s^2)r}$, where $r^2 = \omega_m^2 + \Delta^2$ and $s = \omega_m^2 + \eta'|\omega_m| + M^2$. Performing the remaining Matsubara sum, we obtain the equation for T_c as

$$1 = -\lambda\left[\frac{\psi\left(\frac{1}{2} + \frac{\eta' - i\kappa}{2\pi T_c}\right)}{2(\eta' - i\kappa)^2} + \frac{\psi\left(\frac{1}{2} + \frac{\eta' + i\kappa}{2\pi T_c}\right)}{2(\eta' + i\kappa)^2} + \frac{\kappa^2 - \eta'^2}{(\kappa^2 + \eta'^2)^2}\psi\left(\frac{1}{2}\right) - \frac{\pi^2 \eta'}{4\pi T_c(\eta'^2 + \kappa^2)}\right], \quad (4)$$

where $\eta' \equiv 2\eta$. The solution for $\bar{T}_c = T_c/\kappa$ is plotted in the right panel of Fig. 2 as a function of $\bar{\eta}' = \eta'/\kappa$. As is evident, for the case of a nonlocal mediator, T_c behaves nonmonotonically with dissipation and rises up to 40% of the initial $\eta = 0$ value. This happens because weakly dissipative bosons at different energy scales act coherently to give rise to an increase in T_c, but eventually destroy superconductivity for large dissipation. The critical value occurs when the dissipation parameter is of the order of the stiffness constant ($2\eta \sim \kappa$); at this point, T_c acquires a maximum with respect to η. This physics follows from the energy integral leading to Eq. (4) above. To see this, notice that the role of the stiffness parameter κ is to induce nonmonotonicity in an "effective" coupling constant as a function of η; while η acts only to reduce the effective coupling constant for the local case, the energy integral [leading to Eq. (4)] for the nonlocal mediator forces the gap equation to acquire dissipative contributions that both increase and decrease the effective coupling constant. Consequently, this translates into a nonmonotonic behavior in T_c.

IV. GAP AND SPECIFIC-HEAT JUMP

We now study the variation of the gap with temperature and the specific-heat jump at T_c. In Fig. 3, we plot the temperature dependence of the superconducting gap as a function of the dissipation and mass parameters for $\kappa = 0$. Both η and M reduce the zero-temperature gap $\Delta(0)$ and T_c; however, dissipation (mass) has a greater (smaller) effect on T_c compared to $\Delta(0)$. Hence, the BCS ratio $\frac{\Delta(0)}{T_c}$ increases (decreases) with the dissipation (mass) parameter. To get an analytical handle for the gap near T_c, we begin with the case of $\eta = M = 0$, where the gap equation becomes

$$1 = \lambda \int_{-\infty}^{\infty} d\xi \left[\frac{1}{4T(\xi^2 + \Delta^2)} - \frac{\tanh\frac{\sqrt{\xi^2+\Delta^2}}{2T}}{2(\xi^2 + \Delta^2)^{3/2}}\right]. \quad (5)$$

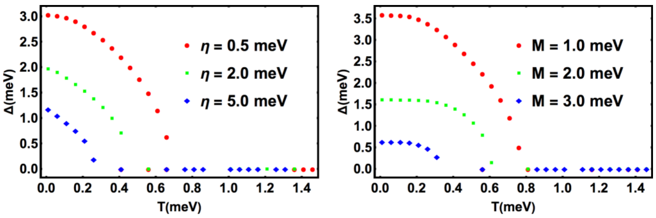

FIG. 3. Temperature dependence of the superconducting gap for $\lambda = 5$ meV, as a function of the dissipation parameter η and $M = 1$ meV (left) and as a function of the mass parameter M and $\eta = 0.1$ meV (right). The BCS ratio $\frac{\Delta(0)}{T_c}$ increases (decreases) with the dissipation (mass) parameter.

We have made use of the summation identity,

$$\sum_m \frac{1}{(|\omega_m|^2 + x^2)|\omega_m|^2} = \frac{x - 2T\tanh\frac{x}{2T}}{4x^3T^2}, \quad (6)$$

above. We next expand for small gaps near T_c to obtain

$$\frac{1}{\lambda} = \int_{-\infty}^{\infty} d\xi \left[\frac{\beta\xi - 2\tanh\left(\frac{\beta\xi}{2}\right)}{4\xi^3} \times \frac{-3\beta\xi + 6\tanh\left(\frac{\beta\xi}{2}\right) + \beta\xi\tanh^2\left(\frac{\beta\xi}{2}\right)}{8\xi^5}\Delta^2 + \cdots\right] \simeq \frac{\beta^2 a(T)}{4\pi^2} - b\Delta^2, \quad (7)$$

where

$$a(T) = \frac{1}{2}\left[\psi\left(2, \frac{3}{2} + \frac{\beta\Lambda}{2\pi}\right) - \frac{1}{2}\psi\left(2, \frac{1}{2}\right)\right] \quad (8)$$

is weakly temperature dependent in the limit of $\beta\Lambda \to \infty$, $b \simeq \frac{31}{32}\frac{\beta_c^4 \xi(5)}{\pi^4}$, and $\psi(n, x)$ is the nth-order digamma function. Setting the gap to zero in Eq. (7), we can read off the dependence of T_c on the coupling as $T_c \sim \sqrt{\lambda}$, which grows faster than the conventional BCS relation. The temperature dependence of the gap can be derived as $\Delta^2(T) = \frac{2a(0)T_c\pi^4}{4\pi^2(31/32)\xi(5)}(T_c - T)$, and, therefore, the normalized specific-heat jump at T_c is ($\gamma = 2\pi^2 N(0)/3$ is the normal-state specific heat) $\frac{\Delta C}{\gamma T_c} = \frac{3a(0)}{4\xi(5)}\left(\frac{32}{31}\right) \simeq 6$, which is greater than the BCS value. Similarly, in the limit where the dissipation is much larger than the temperature and mass ($\eta|\omega_m| \gg |\omega_m|^2, M^2$), we have $\frac{1}{\lambda} \simeq \frac{u(T)}{4\pi T\eta} - \frac{v}{8\eta\pi^4 T_c^3}\Delta^2$. Here,

$$u(T) = \pi^2 - 2\psi\left(1, \frac{3}{2} + \frac{\beta\Lambda}{2\pi}\right), \quad (9)$$

$$v = \int_0^{\infty} dx \left\{\frac{H\left(-\frac{1}{2} - ix\right) + \text{c.c.} + \ln 16}{x^4} + \frac{\frac{ix}{2}\left[\psi\left(1, \frac{1}{2} - ix\right) - \text{c.c.}\right]}{x^4}\right\}, \quad (10)$$

and $H(z)$ is the harmonic number. T_c can be evaluated again by setting $\Delta = 0$ and we see that in this limit, $T_c \sim \frac{\lambda}{\eta}$. The crossover from $T_c \sim \sqrt{\lambda}$ to $T_c \sim \lambda$ as a function of η is shown in Fig. 1 (right). The temperature dependence of the gap can be evaluated from above as $\Delta(T)^2 = \frac{2\pi^5 T_c}{v}(T_c - T)$; hence, the specific-heat jump at T_c takes the value $\frac{\Delta C}{\gamma T_c} = \frac{3\pi^3}{v} \sim 3.64$, which is again greater than the BCS value.

On the other hand, expanding the gap equation for a nonlocal mediator ($\kappa \neq 0$) in the limit of $\eta = M \to 0$, we obtain $\frac{\kappa^2}{\lambda} \simeq F(\frac{\kappa}{2\pi T}) - G(\frac{\kappa}{2\pi T})\tilde{\Delta}^2$. The dimensionless functions $F(x)$, $G(x)$, and $\tilde{\Delta}$ are defined as

$$F(x) = \tfrac{1}{2}\big[H\big(-\tfrac{1}{2} - ix\big) + \text{c.c.} + \ln 16\big], \tag{11}$$

$$G(x) = \tfrac{-1}{x^2}\big[10(\gamma_E + \ln 4) + 5\psi\big(0, \tfrac{1}{2} - ix\big) + \text{c.c.} + ix\psi\big(1, \tfrac{1}{2} - ix\big) + \text{c.c.} - 42x^2\xi(3)\big], \tag{12}$$

$\tilde{\Delta} = \frac{\Delta}{2\pi T}$, and γ_E is the Euler gamma constant. In the limit $x \ll 1$, the functions $F(x)$ and $G(x)$ satisfy the property $F(x) = C_1 x^2$ and $G(x) = C_2 x^2$, where C_1 and C_2 are numerical constants. The dependence of T_c on λ goes as $T_c \sim \sqrt{\lambda}$ and the temperature dependence of the gap takes the form $\Delta(T)^2 = \frac{8\pi^2 C_1 T_c}{C_2}(T_c - T)$. This implies that the specific-heat jump is $\frac{\Delta C}{\gamma T_c} \sim 5.6$, again larger than the BCS value. However, in the limit $\eta|\omega_m| \gg |\omega_m|^2, M^2$, the expansion of the gap equation gives

$$\frac{1}{\bar{\lambda}} = \frac{1}{2\pi T}\left[\frac{\bar{\eta}'\pi^2}{1+\bar{\eta}'^2} - \frac{\bar{\eta}'\pi^4(3+\bar{\eta}'^2)}{12(1+\bar{\eta}'^2)}\left(\frac{\Delta}{2\pi T_c}\right)^2 + \cdots\right], \tag{13}$$

where $\bar{\lambda} = \lambda/\kappa$. Setting $\Delta = 0$, we see that $T_c \sim \bar{\lambda}$ and the temperature dependence of the gap is given by $\Delta(T)^2 = \frac{24T_c(T_c-T)(1+\bar{\eta}'^2)}{(3+\bar{\eta}'^2)}$. Hence, the specific-heat jump is (weakly) dependent on the dissipation parameter and is given by $\frac{\Delta C}{\gamma T_c} = \frac{36(1+\bar{\eta}'^2)}{\pi^2(3+\bar{\eta}'^2)}$. For small $\bar{\eta}'$, the normalized specific-heat jump is $\sim$1.2 and is *smaller* than the BCS value, consistent with specific-heat experiments in underdoped cuprates [33] and the pnictides [34].

V. DISCUSSIONS AND EXPERIMENTS

Several theoretical works have explored mechanisms that yield an enhancement of T_c with disorder strength. These phenomena range from competition of superconductivity with a proximate density wave phase [35–38], multiorbital effects [39], local inhomogeneities in the pairing interactions and mediators [40–45], to localization [46–49]. A few works have also explored the interplay between glassy phases and superconducting T_c [50,51]. In [51], the authors study a spin glass formed by Ruderman-Kittel-Kasuya-Yosida (RKKY) interactions between paramagnetic spins in a superconductor, and find an interaction-driven enhancement of T_c for a fixed impurity density. As a function of impurity concentration, however, the authors find that the T_c decreases monotonically. Reference [50] also finds a similar decrease in T_c due to a reduction of the effective interaction induced by a SG phase that does not take into account the role of dissipation explicitly. Our results can alternatively be viewed from the perspective of the well-studied spin-fermion model [52–56], where the dissipation parameter is proportional to the inverse spin-fluctuation frequency ω_{SF}. In all of these works, ω_{SF} and the correlation length ξ parameters are held fixed for different materials (see Table I of Ref. [52]) at $T = T_c$. Although these works do not study the effect of ω_{SF} and ξ on T_c, legitimate questions can be raised with regards to whether these quantities can be varied by an experimentally controlled tuning knob. In the case relevant to the present context, the effect of disorder on ω_{SF} and ξ needs further examination, and perhaps the current work brings forth the need for a microscopic understanding of the relationship between disorder strength and the dissipation parameters η and ω_{SF}. In the following paragraphs, we argue for the applicability of the mechanism presented in this paper to the cuprates.

We begin by emphasizing that the change in T_c in our work is due to modification of the "effective" coupling by dissipation, and is unrelated to pair-breaking effects originating from lowering translational symmetry (say due to magnetic/nonmagnetic inhomogeneities, such as those summarized in Ref. [1]). Hence, it can be intuited that qualitative aspects of our conclusions must hold for higher angular momentum pairing as well (albeit with tedious calculations). This can be more readily seen by noting that when the summand in the gap equation [in Eq. (2)] is decomposed into its partial fractions, there is always at least one (nonzero) term present where the dissipation parameter contributes to enhance the effective coupling strength [similar to Eq. (4)]. This term(s) generally competes with other terms which suppress T_c, but gives rise to a T_c increase when the dissipation is weak enough. Furthermore, according to our proposal, disorder acts as an external tuning knob of the parameter η; hence, increased irradiation leads to larger dissipation. Recent magnetization and tunnel diode (penetration-depth) experiments [57] on proton-irradiated LBCO at $\frac{1}{8}$ doping found up to a 50% increase in T_c as a function of radiation dosage. An increased dosage above a critical value gradually suppressed T_c until the eventual destruction of superconductivity. LBCO also hosts a rich phase diagram with evidence of density wave orders (CDW, spin density wave (SDW) [58–60]) as well as spin-glass behavior [5] in conjunction with superconductivity in the underdoped regime. Hence, it is natural to anticipate an influence of these phases on superconductivity and examine their implications to T_c variation as a function of disorder. Of the aforementioned existing mechanisms of T_c enhancement proposed in the literature, a competition-based scenario between superconductivity and a density wave order seems the most promising at first sight—especially given the close proximity of the CDW phase to the superconducting dome. Indeed, this was the point of view first suggested by Leroux and co-workers in [57]. However, a closer examination of the data points to details that render this mechanism debatable. First, assuming that x-ray scattering is primarily sensitive to long-range CDW order [61], the CDW transition temperature seems unaffected by irradiation [57]. But a mechanism involving the competition between two mean-field phases necessarily involves a tug-of-war scenario, where one phase gains stability at the expense of its competitor [37]. Second, it is unclear how nonmagnetic disorder affects two different mean-field phases (CDW, SDW, SC, etc.) asymmetrically in a parameter-independent manner, except under very specific circumstances [37,38] which do not necessarily hold in the case of LBCO and cuprates. Third, other nonmagnetic impurities are well known to kill d-wave superconductivity monotonically [1]. Thus a consistent picture which distinguishes proton and electron irradiation with other point impurities such as Zn at a microscopic level is absent. Finally, from Anderson's theorem, one can expect that a T_c enhancement that occurs through a competition-based scenario must be more prevalent in *s*-wave SCs rather than higher angular momentum SCs, which are far less robust to nonmagnetic impurities. Experiments on the *s*-wave superconductor 2H-$NbSe_2$, however, draw conclusions that are mixed at best [62–64]. Hence, a reasonable explanation for nonmonotonic T_c dependence as a function of disorder in LBCO must necessarily involve a mechanism that *does not depend on the competition of two mean-field-like phases*. The proposed mechanism in this paper, along with the experimental evidence provided in Sec. I, form a feasible alternative that fits experiments.

In conclusion, motivated by the close proximity of glassy phases to the superconducting dome in high-T_c SCs, we explored the role of dissipation on superconducting properties such as T_c, the temperature dependence of the gap, BCS ratio, and the specific-heat jump at T_c. We found that when the mediator is local, dissipation acts to reduce the effective coupling constant and T_c monotonically. On the other hand, when the mediator is nonlocal, two competing effects of dissipation determine the T_c variation—first, the dissipative contributions of individual bosons at a given energy that act to suppress T_c, and, second, collective contributions where dissipation acts to connect bosons at different energy scales that combine coherently to increase the effective coupling and T_c. The former (latter) contribution dominates when the dissipation parameter is greater (lesser) than the bosonic spatial stiffness, i.e., $\eta > \kappa (\eta < \kappa)$; T_c peaks when these two scales are comparable to each other. We also studied the effects of a dissipative mediator on the ratio $\frac{2\Delta(0)}{T_c}$ and the heat capacity jump at T_c, and found departures from values predicted by BCS theory. In particular, the specific-heat jump at T_c acquires a value smaller than that predicted by BCS theory when the mediator is both dissipative and nonlocal, consistent with experiment. We pointed out consequences of our results to recent proton-irradiation experiments in LBCO [57], where superconducting T_c is enhanced with increased radiation disorder despite a robust CDW transition temperature, and concluded that one does not require a tug-of-war type scenario between two competing phases to enhance superconductivity.

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