

Spin polarization and orbital effects in superconductor-ferromagnet structures

A. F. Volkov*

Theoretische Physik III, Ruhr-Universität Bochum, D-44780 Bochum, Germany

F. S. Bergeret†

*Centro de Física de Materiales (CFM-MPC), Centro Mixto CSIC-UPV/EHU, Manuel de Lardizabal 5, E-20018 San Sebastian, Spain
and Donostia International Physics Center, Manuel de Lardizabal 4, E-20018 San Sebastian, Spain*

K. B. Efetov

Theoretische Physik III and Ruhr-Universität Bochum, D-44780 Bochum, Germany



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We study theoretically spontaneous currents and magnetic field induced in a superconductor-ferromagnet (S-F) bilayer due to direct and inverse proximity effects. There are two types of contributions to the induced currents. One is the Meissner current, which appears even in the absence of an external magnetic field due to the magnetic moment in the ferromagnet and to the magnetization in the superconductor. The second contribution is due to a space-dependent magnetization in the superconductor which is induced by the inverse proximity effect over a distance of the order of the superconducting correlation length ξ_S . In contrast, the magnetic induction B , caused by the Meissner currents, penetrates the S film over the London length λ_S . Even though λ_S usually considerably exceeds the correlation length, the amplitude and sign of B at distances much larger than ξ_S depend crucially on the strength of the exchange energy in the ferromagnet and on the magnetic moment induced in the S layer.

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I. INTRODUCTION

Besides the orbital effects, it is well known that conventional superconducting pairing is also suppressed by a magnetic field when acting on the spins of electrons via the Zeeman interaction. Whereas superconducting correlations couple pairs of electrons in a singlet state (Cooper pairs), the Zeeman interaction tends to align both spins in the direction parallel to the magnetic field. These two antagonistic tendencies can, nevertheless, coexist when the Zeeman energy is small enough in comparison to the superconducting gap. This coexistence implies the appearance of Cooper pairs in a triplet state. This situation occurs, for example, in thin superconducting films (S films) in the presence of an in-plane magnetic field and also in the superconductor-ferromagnet (S-F) heterostructure in which Cooper pairs from the S layer can penetrate into the ferromagnet, where the intrinsic exchange field J acts on the spins of electrons (see review articles [1–5]).

Leakage of Cooper pairs from S to F is the so-called proximity effect. The wave function of the Cooper pairs penetrating into the F region with a uniform magnetic moment \mathbf{M} contains not only the singlet but also the triplet component with zero spin projection on the vector \mathbf{M} . In this case, Cooper pairs penetrate into a diffusive F over a short length $\xi_{F,s-r} \approx \sqrt{D_F/J}$ and experience oscillations in space. This leads to

the appearance of the so-called π state in Josephson SFS junctions [6–12]. If the magnetic moment \mathbf{M} is nonuniform, the triplet Cooper pairs with a nonzero projection penetrate into F over a much longer distance $\xi_{F,l-r} \approx \sqrt{D_F/2\pi T}$ [13–24]. Interestingly, if the S-F interface is transparent enough, the triplet pairs can leak into the superconductor. This is the inverse proximity effect that leads to a finite magnetic moment in S [25–29].

The size of this spin-polarized region within the superconductor is of the order of the superconducting correlation length, which in the diffusive limit is given by $\xi_S \approx \sqrt{D_S/2\pi T_c}$. The magnetic moment \mathbf{M}_S induced in S has a direction opposite to the magnetization vector \mathbf{M}_F in the F layer. Under certain conditions the total magnetic moment in the S region compensates the total magnetic moment of the F film, resulting in a full spin screening [25,26,30]. In the ballistic case, the induced magnetization $\mathbf{M}_S(x)$ may spatially change sign [27,28].

These predictions for the inverse proximity effect were eventually confirmed experimentally [31,32]. However, quantitative interpretation of the experimental results is quite subtle [33,34] since the magnetic field arising in S is caused not only by the induced magnetization \mathbf{M}_S but also by Meissner currents that arise in the S/F structure [26,35,36]. For this reason a detailed understanding of the inverse proximity effect is a key issue for interpretation of experimental data on S/F structures.

In this work we study the proximity effect in S-F structures, taking into account explicitly the generated spontaneous currents. This topic was first addressed by the authors in

*volkov@tp3.rub.de

†fs.bergeret@csic.es

2004 [26] and more recently in Ref. [36]. These two works predict a magnetic induction $B_S(x)$ induced in the S layer which penetrates over the London penetration depth λ_S . The authors of Ref. [26] focused on the magnetic field caused by $\mathbf{M}_S(x)$ and estimated the orbital effects. They showed that the spin polarization effects are stronger than those related to the Meissner currents screening the magnetic moment \mathbf{M}_F . In Ref. [36] the orbital effects were studied in more detail, but the inverse proximity effect was completely neglected. Since in dirty superconducting films λ_S is usually larger than the coherence length ξ_S characterizing penetration of a magnetic moment M_S into S, it might look at first glance as if the magnetic field measured in a superconducting film with a large thickness d_S ($\xi_S \ll d_S \lesssim \lambda_S$) cannot be affected by the magnetic moment M_S localized close to the S-F interface. In contrast to this scenario, we demonstrate here that for a correct interpretation of the experimental data one does need to take into account the magnetic moment in the S layer induced by the inverse proximity effect.

To be specific, we show that the spatial dependence of the magnetic induction $B_S(x)$ induced in the S region consists of a long-range component $B_{l-r}(x)$ which decreases over the London penetration depth λ_S and of a short-range component B_{s-r} caused by the induced magnetization which decays over the superconducting coherence length $\sim \xi_S$. The magnetic inductance B_S in a thick superconducting film ($\xi_S \ll d_S \lesssim \lambda_S$) thus has the form

$$B_S(x) = B_{l-r}(0) \exp(x/\lambda_S) + B_{s-r}(x). \quad (1)$$

At large distances from the S/F interface, $|x| \gg \xi_S$, $B_S(x)$ is mainly determined by the long-range term $B_{l-r}(x)$. Its amplitude consists of two contributions:

$$\mathbf{B}_{l-r}(\mathbf{0}) = \mathbf{B}_{\text{orb}}(\mathbf{0}) + \mathbf{B}_{\text{sp}}(\mathbf{0}). \quad (2)$$

The first term is the contribution from the spontaneous Meissner currents (orbital effects) and equals

$$B_{\text{orb}}(0) = -4\pi M_0 \theta_F^2 / 2, \quad (3)$$

where $\theta_F = (d_F/\lambda_F)$ and d_F and λ_F are the thickness of the F layer and the London penetration depth in the ferromagnet, respectively. This expression coincides with the result for the magnetic induction obtained in Ref. [36]. One of our main findings is that there is an additional contribution to the magnetic induction, the term B_{sp} in Eq. (2). This contribution is caused by the inverse proximity effect that was neglected in Ref. [36]. For a wide range of parameters this contribution due to spin polarization near the S/F interface is much larger than that due to orbital effects, i.e., $B_{\text{sp}}(0) \gg B_{\text{orb}}(0)$. Moreover, this contribution might be crucial in determining the sign of the magnetic induction in the S layer since, as we show below, $B_{\text{sp}}(0)$ and $B_{\text{orb}}(0)$ have different signs. Moreover, the relative magnitude between these two contributions depends on the exchange field J in the F layer. The contribution $B_{\text{sp}}(0)$ due to spin polarization in S can be neglected only in the case of the F film with sufficiently large exchange energy J .

In the next sections we investigate the spatial distribution of the Meissner currents $j_S(x)$ and the fields $B(x)$, $H(x) = B(x) - 4\pi M(x)$. Our main findings are the following: (i) Eq. (3) describes the orbital effect only in the case of rather large exchange energy J . (ii) In the full-screening case, both

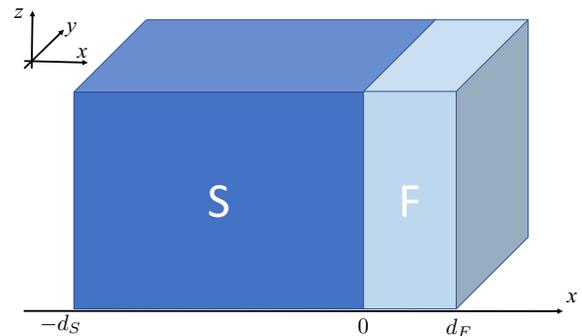


FIG. 1. The S-F structure under consideration. We assume that the ferromagnet has a homogeneous magnetization in the z direction.

short- and long-range components in Eq. (2) are determined by spin polarization effects. (iii) Meissner currents in the S region change sign at some point $x_0 \sim \xi_S$ away from the S/F interface. (iv) The total Meissner currents in the F (or S) film calculated with or without accounting for the spin screening effect may have opposite directions, and (v) in the case of an out-of-plane magnetization of the F layer no spontaneous currents, and hence no magnetic induction, are induced.

This article is organized as follows. In the next section we consider a diffusive S/F bilayer and derive the expressions for the magnetic moment M_S induced by the spin polarization. Although this was presented in our earlier publications [25,26], for completeness and to set the notation, we rederive it here. In Sec. III we solve the magnetostatic equations for the vector potential and find the spatial distribution of the spontaneous supercurrents j in the system which arise in the absence of an external magnetic field. In particular we show that the current density j_S in the S film can change its sign near the S/F interface. In the last section we summarize our results.

II. PROXIMITY EFFECT IN THE S-F HETEROSTRUCTURE

In this section we study the proximity effect in an S-F structure. We assume the diffusive limit, such that the conditions $\Delta\tau \ll 1$ and $J\tau \ll 1$ are satisfied, where τ is the momentum relaxation time. The presence of a spin-dependent term in the F region means that the condensate induced in this layer consists of singlet and triplet components. In turn, triplet Cooper pairs may penetrate into the S region and induce a finite spin polarization. To describe this processes in a diffusive system, we use the quasiclassical Green's functions (GF) $\hat{g}_S(\omega)$ [37,38] and the generalized Usadel equation [3,39].

Specifically, we consider the structure shown in Fig. 1. It consists of a ferromagnetic layer of thickness d_F and a superconducting layer of thickness d_S . In the absence of the proximity effect the Green's function in S corresponds to the bulk BCS matrix Green's function $\hat{g}_S(\omega)$, which has the form

$$\hat{g}_S(\omega) = G_S \hat{\tau}_3 + F_S \hat{\tau}_1, \quad (4)$$

where $\hat{\tau}_{1,3}$ are the Pauli matrices operating in the particle-hole space and $G_S = \omega/\zeta_\omega = (\omega/\Delta)F_S$, $\zeta_\omega = \sqrt{\omega^2 + \Delta^2}$. Here ω is the fermionic Matsubara frequency.

In the presence of an exchange field J , the quasiclassical Green's function \hat{g} maintains its structure in the particle-hole space, but its components are matrices in the spin space. We consider here only a monodomain ferromagnet with a homogeneous J , and therefore the general form of \hat{g} in S and F is

$$\hat{g}_a(\omega) = (g_{a0}\hat{1} + g_{a3}\hat{\sigma}_3)\hat{\tau}_3 + (f_{a0}\hat{1} + f_{a3}\hat{\sigma}_3)\hat{\tau}_1, \quad (5)$$

where $\hat{\sigma}_3$ is the third Pauli matrix in the spin space and the index a means $a = S, F$.

In Eq. (5) the terms proportional to τ_3 are the normal GFs which determine the electronic charge and spin densities. The terms proportional to τ_1 are the anomalous GF describing the singlet and zero-spin-projection triplet components of the condensate. Without losing generality we assume that the exchange field J points in the z direction, $J = J\hat{z}$.

The GFs can be calculated by solving the Usadel equation complemented with proper boundary conditions (see Appendix A for details). The GFs calculated in this way determine the current and electron magnetization density $M = M\hat{z}$ as follows:

$$\mathbf{j}_a = \frac{1}{4e}\sigma(2\pi i)T\text{Tr}\sum_{\omega}(\tau_3\hat{g}_a\nabla\hat{g}_a), \quad (6)$$

$$M_a(x) = M_0(x) + \frac{1}{4}(2\pi i)T\mu_B\nu\text{Tr}\sum_{\omega}(\tau_3\sigma_3\hat{g}_a). \quad (7)$$

Here σ_a is the conductivity of the S and F layers ($a = S, F$), μ_B is the effective Bohr magneton, and ν is the normal density of states at the Fermi level. M_0 is the magnetization in the normal state, which is finite and spatially homogeneous only in the F layer.

Clearly, the matrix \hat{g} defined in Eq. (5) is diagonal in the spin space. This simplifies the calculation of \hat{g} since the equations for up and down spins decouple from each other. In other words, one can write the GF as $\hat{g}_{a\pm} = g_{a\pm}\tau_3 + f_{a\pm}\tau_1$ and solve the problem independently for \pm spins. Qualitatively, due to the conventional proximity effect, a spin-dependent condensate function $\hat{f}_{F\pm}$ is induced in the F layer. Such a spin-polarized condensate can penetrate back into the S region, inducing a local magnetic moment described by the corrections to the GF $\delta\hat{g}_{S\pm}$ defined as $\delta\hat{g}_{S\pm} = \hat{g}_{S\pm} - \hat{g}_S$. All these functions can be obtained from the Usadel equation, as explained in Appendix A.

In order to solve the problem analytically we assume that the F film is thin enough, $d_F \ll \sqrt{D_F/J}$, and therefore the matrix \hat{g}_F can be considered almost constant in space. We also assume that the S-F interface has a finite interface resistance per unit area R_b . This allows us to use the Kupriyanov-Lukichev boundary condition (A4). Then we can integrate spatially the Usadel equation, Eq. (A2), in the F region to obtain the following algebraic equation for $\hat{g}_{F\pm}$:

$$[\tilde{\omega}_{\pm}\hat{\tau}_3 + \tilde{\Delta}\hat{\tau}_1, \hat{g}_{F\pm}] = 0, \quad (8)$$

where $\tilde{\omega}_{\pm} = \omega + \epsilon_{bF}G_S \pm iJ$, $\tilde{\Delta} = \epsilon_{bF}F_S$, and σ_F is the conductivity of the F layer. The characteristic energy ϵ_{bF} is defined as

$$\epsilon_{bF} = \frac{D_F}{R_b\sigma_F d_F}, \quad (9)$$

with R_b being the S/F interface resistance per unit area. Equation (8) has to be solved together with the normalization condition $\hat{g}_{\pm}^2 = 1$. The solution has the same structure as the bulk BCS solution with the renormalized ω and Δ [see Eq. (4)],

$$\hat{g}_{F\pm} = (\tilde{\omega}_{\pm}\hat{\tau}_3 + \tilde{\Delta}\hat{\tau}_1)/\tilde{\zeta}_{\omega\pm}, \quad (10)$$

where $\tilde{\zeta}_{\omega\pm} = \sqrt{\tilde{\omega}_{\pm}^2 + \tilde{\Delta}^2}$.

On the superconducting side of the interface, the GFs are modified due to the inverse proximity effect. Provided the transmission of the S/F interface is finite, a correction $\delta\hat{g}_S$ to the BCS Green's functions arises in the S film. We assume that the elements of the matrix $\delta\hat{g}_S$ are small: $|\delta\hat{g}_S| \ll 1$. Then, in the leading-order approximation we obtain $g_{S\pm} \approx G_S + \delta g_{S\pm}$ and $f_{S\pm} \approx F_S + \delta f_{S\pm}$. So the magnetization density induced in the S film is given by

$$\begin{aligned} M_S(x) &= 2i\pi T\mu_B\nu_S \sum_{\omega \geq 0} g_{S3}(\omega, 0) \exp(x\kappa_{\omega}) \\ &\equiv - \sum_{\omega \geq 0} m_S(\omega) \exp(x\kappa_{\omega}), \end{aligned} \quad (11)$$

where $m_S(\omega) \equiv -2\pi iT\mu_B\nu_S g_{S3}^{(S)}(\omega, 0)$ and $\kappa_{\omega}^2 = 2\sqrt{\omega^2 + \Delta^2}/D_S$. The function $g_{S3}(\omega, 0)$ is defined in Eq. (5) and explicitly given in the Eq. (A9). The total magnetic moment induced in the superconductor \mathcal{M}_S is obtained by integrating the previous expression in the interval $-\infty < x < 0$,

$$\mathcal{M}_S = \int_{-\infty}^0 dx M_z(x) = - \sum_{\omega \geq 0} \frac{m_S(\omega)}{\kappa_{\omega}}. \quad (12)$$

Using Eqs. (5) and (A9), we reduce Eq. (12) to the form

$$\mathcal{M}_S = -M_0 d_F (\epsilon_{bF}/J) (2\pi T) \text{Im} \sum_{\omega \geq 0} \frac{\Delta^2}{\zeta_{\omega}^3} \frac{\omega + iJ}{\tilde{\zeta}_{\omega}}, \quad (13)$$

where the functions ζ_{ω} and $\tilde{\zeta}_{\omega}$ are defined in Eqs. (4) and (10). In Fig. 2 we represent the dependence of the normalized total magnetic moment in the S region M_S on the exchange energy J for two values of ϵ_{bF} : $\epsilon_{bF} > \Delta$ and $\epsilon_{bF} < \Delta$. One can clearly see a kink at $J \cong \epsilon_{bF}$. When $\epsilon_{bF} < \Delta$, the characteristic energy ϵ_{bF} describes the subgap induced in the F film due to the proximity effect. In this case the induced magnetization is small, and there is no full screening. In contrast, in the limit $\epsilon_{bF} > \Delta$ an almost full screening takes place provided $J < \epsilon_{bF}$.

One can analytically calculate the total magnetization M_S in the superconducting region in the limit $\Delta(T), T \ll J \ll D_F/(R_b\sigma_F d_F) \equiv \epsilon_{bF}$. This condition combined with Eq. (A10) can be written as

$$\rho_S \xi_S \ll R_b \ll \rho_F d_F \frac{D_F}{d_F^2 \Delta}, \quad (14)$$

$$J \ll E_{\text{Th}} \equiv D_F/d_F^2, \quad (15)$$

where $\rho_a = 1/\sigma_a$ is the resistivity of the $a = S$ or F film. In this limit one obtains $\tilde{\omega}_{\pm} \approx \epsilon_{bF}(G_S + i\tilde{J})$ and $\tilde{\zeta}_{\omega\pm} \approx \epsilon_{bF}(1 + i\tilde{J}G_S)$, where $\tilde{J} = J/\epsilon_{bF}$. The term $m_S(\omega)/\kappa_{\omega}$ in Eq. (12) is

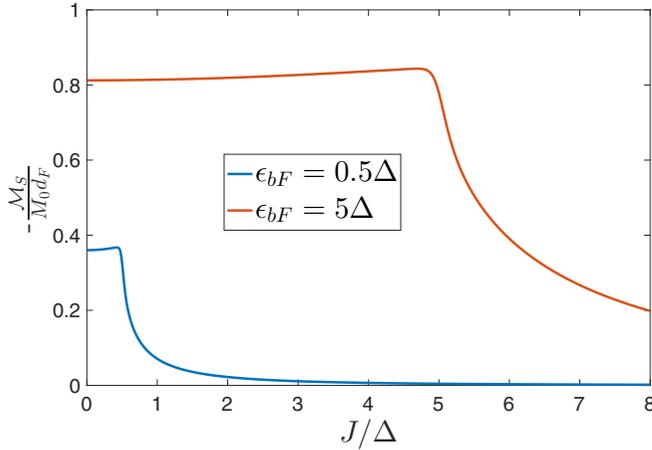


FIG. 2. The total magnetization induced in the S layer as a function of J for two different values of ϵ_{bF} , defined in Eq. (9), and $d_F = \xi_S$, $d_S = 2\xi_S$, and $\kappa_{Fb} = 1/(R_b\sigma_F) = 5\kappa_{Sb} = 5/(R_b\sigma_S) = 0.5\xi_S^{-1}$.

approximately equal to

$$\frac{m_S(\omega)}{\kappa_\omega} \cong (2\pi T \mu_B v_S) \frac{2\kappa_{Sb}}{\kappa_\omega^2} F_S^2 \tilde{J} = M_0 d_F \frac{(2\pi T)\Delta^2}{(\Delta^2 + \omega^2)^{3/2}}, \quad (16)$$

where $M_0 = v_F \mu_B J$ is the magnetic moment in F in the absence of the proximity effect and we used the relation $\sigma_F/\sigma_S = v_F D_F/v_S D_S$. At low temperatures $T \ll \Delta$, the summation in Eq. (12) is transformed into integration over ω , which gives [26]

$$\mathcal{M}_S = -M_0 d_F = -\mathcal{M}_F. \quad (17)$$

This means that in the limiting case considered here the total magnetic moment of Cooper pairs induced in the S region compensates the magnetic moment of the F film [29]. If the condition (14) is not fulfilled or the temperature is not low enough, the screening is not complete, and $|\mathcal{M}_S| < |\mathcal{M}_F|$. In the other limiting case of a large exchange field $J \gg \epsilon_{bF}$ the induced magnetic moment in S is given by

$$\mathcal{M}_S = -M_0 d_F \epsilon_{bF}^2 / 2J^2 \ll \mathcal{M}_F, \quad (18)$$

and therefore the screening is very weak.

The magnetic moment $M_S(0)$ induced to the right of the S-F interface can be calculated from Eq. (16) in the limit of low temperatures when the function $m_S(\omega)$ is approximated,

$$m_S(\omega) \cong M_0 \frac{d_F}{\xi_S} \frac{2\pi T}{\Delta [1 + (\omega/\Delta)^2]^{5/4}}, \quad (19)$$

with $\xi_S^2 = D_S/\sqrt{2}\Delta$. We then obtain

$$M_S(0) = -M_0 \frac{d_F}{\xi_S} c_0, \quad (20)$$

with $c_0 = \int_0^\infty dt (1+t^2)^{-5/4} \cong 1.18$.

The magnetization in the F film can be written in the form $M_F = M_0 + \delta M_F$, where M_0 is the uniform magnetization of the ferromagnet in the absence of the proximity effect and δM_F is a correction due to the proximity effect. As we consider a thin F layer with $d_F \ll \xi_F \approx \sqrt{D_F/J}$, the correction δM_F can be assumed to be constant in space. It can be

shown that δM_F is negative, which leads to a decrease in the magnetization of the F film [25]. However, in what follows we neglect δM_F since it does not affect qualitatively the main results.

We note that the condition, Eq. (14), can be fulfilled in experiments with weak ferromagnets, for example, in Nb/CuNi structures such as those used in Ref. [9]. By taking $R_b = RL_y L_z \cong 30 \mu\Omega \times 10 \times 10 \mu\text{m}^2$, $d_F \cong 20 \text{ \AA}$, $D_F \cong 5 \text{ cm}^2/\text{s}$, and $\rho_F \cong 60 \mu\Omega \text{ cm}$, we obtain $d_F \rho_F / R_b \cong 0.4$ and $\epsilon_{bF} = D_F \rho_F / R_b d_F = 0.4 D_F / d_F^2 \cong 1200 \text{ K}$. For these parameters the condition in Eq. (14) is satisfied, provided the energy J is not too large.

In this section we analyzed the proximity effect on the magnetic moment induced in S. In the next section we find the spatial distribution of the Meissner currents and magnetic fields in the whole S-F bilayer.

III. MAGNETOSTATICS OF A S-F BILAYER

In this section we determine the currents and fields induced in the S/F structure shown in Fig. 1. The total current consists of two contributions: the Meissner contribution and the current \mathbf{j}_m stemming from the finite magnetization in the system, $\mathbf{j}_m = c \nabla \times \mathbf{M}$ [40]. The magnetic induction \mathbf{B} and the magnetic field \mathbf{H} obey the Maxwell equation in both the F and S films,

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + 4\pi \nabla \times \mathbf{M}, \quad (21)$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j}, \quad (22)$$

where \mathbf{j} is the Meissner current denoted as $\mathbf{j}_{S,F}$ in S and F films, respectively. The second term on the right-hand side of Eq. (21) is the magnetization current. In the F film, the current \mathbf{j}_F is carried by Cooper pairs induced due to the proximity effect. Both currents $\mathbf{j}_{S,F}$ are related to the vector potential $\mathbf{A}_{S,F}$ via the London equation:

$$\mathbf{j}_{S,F} = -\frac{c}{4\pi} \frac{1}{\lambda_{S,F}^2} \mathbf{A}_{S,F}, \quad (23)$$

where $\lambda_{S,F}$ is the London penetration length. We neglect variation of $\lambda_{S,F}$ due to the proximity effect and assume that it is constant. Moreover, in the case of superconducting films with a short mean free path l the coherence length ξ_S is usually much smaller than λ_S , which is equivalent to the limit of a large Ginzburg-Landau parameter κ_{G-L} ,

$$\kappa_{G-L}^{-2} = (\xi_S/\lambda_S)^2 = e^2 n l^2 / m c^2 \ll 1, \quad (24)$$

where n , m , and $l = v\tau$ are the concentration of free carriers, effective mass, and mean free path, respectively. For typical values of the electron density the above approximation is valid for values of the mean free path up to $l \lesssim 1000 \text{ \AA}$.

As follows from Eqs. (21)–(23) the vector potential $\mathbf{A}_{S,F}$ satisfies the equation

$$\nabla^2 \mathbf{A}_{S,F} - \frac{1}{\lambda_{S,F}^2} \mathbf{A}_{S,F} = -4\pi \nabla \times \mathbf{M}. \quad (25)$$

In what follows we solve Eq. (25) for two different cases: in-plane and out-of-plane orientations of \mathbf{M}_0 .

A. In-plane magnetization

In this case $\mathbf{M}_0 \parallel \mathbf{e}_z$, $\mathbf{B} \parallel \mathbf{M} \parallel \mathbf{e}_z$, and $\mathbf{A} \parallel \mathbf{e}_y$. This means that $\mathbf{B} = (0, 0, B)$ and $\mathbf{A} = (0, A, 0)$. Then, Eq. (25) reduces to

$$\partial_{xx}^2 A_{S,F} - \frac{1}{\lambda_{S,F}^2} A_{S,F} = 4\pi \partial_x M_{S,F}. \quad (26)$$

As mentioned above, in the thin F film the magnetization is assumed to be almost constant, and therefore one can neglect the right-hand side of the previous equation in the F region. The solution of Eq. (26) in the F layer within the limit $d_F \ll \lambda_F$ can be written as

$$A_F(x) = a_0 \left(1 + \frac{1}{2} \frac{x^2}{\lambda_F^2} \right) + (h_0 + 4\pi M_0) x \left(1 + \frac{1}{6} \frac{x^2}{\lambda_F^2} \right), \quad (27)$$

where the coefficients a_0 and h_0 are integration constants.

However, in the S region the equation for A is obtained by using Eq. (11) for the $M_S(x)$ induced in S:

$$\partial_{xx}^2 A_S - \frac{A_S}{\lambda_S^2} = -4\pi \sum_{\omega \geq 0} m_S(\omega) \kappa_\omega \exp(x\kappa_\omega). \quad (28)$$

As demonstrated in the previous section, the induced magnetization [right-hand side of Eq. (28)] decays on a length of the order ξ_S . Since we consider the case $\xi_S \ll \lambda_S$, the solution in the superconductor can be written as

$$A_S(x) = a_S \frac{\cosh[(x+d_S)/\lambda_S]}{\cosh \theta_S} - 4\pi \sum_{\omega \geq 0} \frac{m_S(\omega)}{\kappa_\omega} [1 + \delta_S^2] \exp(x\kappa_\omega), \quad (29)$$

where a_S is a third integration constant, $\delta_S = 1/\lambda_S \kappa_\omega$ is a small parameter [see Eq. (24)], and $\theta_S = d_S/\lambda_S$. From Eq. (29), one can obtain the expressions for $B_S(x) = \partial_x A_S(x)$ and $H_S(x) = B_S(x) - 4\pi M_S(x)$, as shown in Appendix B.

The integration constants a_0 , b_0 , and a_S in Eqs. (27) and (29) are determined by the following boundary conditions:

$$\begin{aligned} [A]|_{x=0} &= 0, & [H]|_{x=0} &= 0, \\ H_S(-d_S) &= 0, & H_F(d_F) &= H_{\text{ext}}, \end{aligned} \quad (30)$$

where $[A]|_{x=0} \equiv A_F(0+) - A_S(0-)$. The first and second equations provide the continuity of the vector potential A and the field H at the interface. The condition assumes the presence of an external magnetic field, but in what follows we assume that $H_{\text{ext}} = 0$.

In the main approximation we find three coupled equations determining a_S , a_0 , and h_0 . Their solution is given by

$$a_S = -4\pi \frac{M_0 \lambda_S \theta_F^2 / 2 + \mathcal{M}_S \theta_F \lambda_S / \lambda_F - \Lambda}{D_S}, \quad (31)$$

$$a_0 = a_S + 4\pi \mathcal{M}_S, \quad (32)$$

$$h_0 = \frac{a_S}{\lambda_S} \tanh \theta_S - 4\pi \frac{\Lambda}{\lambda_S}, \quad (33)$$

where $\theta_{F,S} = (d/\lambda)_{F,S} \ll 1$, $D_S = \tanh \theta_S + \theta_F \lambda_S / \lambda_F$, and $\Lambda = \sum m_S \delta_S^2 \lambda_S$. In deriving these equations we have used Eq. (12) for the total magnetization \mathcal{M}_S induced in the S region.

Before analyzing the full spatial solution of the boundary problem let us focus on the value of the vector potential at the outer interface, $x = -d_S$. The expression can be straightforwardly obtained from the above equation and reads

$$A_S(-d_S) = -4\pi \frac{\mathcal{M}_S \theta_F \lambda_S / \lambda_F + (\theta_F^2 / 2) M_0 \lambda_S - \Lambda}{\cosh \theta_S D_S}. \quad (34)$$

From this expression one can already draw important conclusions regarding the vector potential and supercurrents at large distances from the boundary. Let us consider two cases.

(a) If one neglects the inverse proximity effect as done in Ref. [36], the first and third terms in the numerator of Eq. (34) are zero, and one obtains ($\mathcal{M}_S = 0$)

$$A_{S,a}(-d_S) \approx -4\pi M_0 \lambda_S \frac{\theta_F^2}{2 \sinh \theta_S}. \quad (35)$$

The corresponding magnetic induction at large distances from the S/F interface coincides with Eq. (3)

(b) If one takes into account the inverse proximity effect, then the second contribution anticipated in Eq. (2) appears. Specifically, in the full-screening situation ($\mathcal{M}_S = -M_0 d_F$), one obtains

$$A_{S,b}(-d_S) = 4\pi M_0 d_F \frac{\gamma_S + \theta_F \lambda_S / \lambda_F}{\sinh \theta_S}, \quad (36)$$

where $\gamma_S = (c_2 \kappa_S \xi_S) = 0.85(\xi_S / \lambda_S) \ll 1$ and $c_2 = \int_0^\infty dt (1+t^2)^{-7/4} \approx 0.85$. Clearly, these two limiting cases describe very different situations, in which the spontaneous currents have even different signs. It is important to emphasize that even though the magnetization induced in the S layer occurs over the coherence length $\xi_S \ll \lambda_S$, it changes drastically the vector potential at distances of the order of λ_S .

From the knowledge of the vector potential one can write the current density $j(x)$ using the London equation, Eq. (23), as $j(x) = -(c/4\pi) A(x) / \lambda_{S,F}^2$. The total currents through the F and S layers are then defined as

$$I_{S,F} = \int_{S,F} dx j_{S,F}. \quad (37)$$

From Eqs. (27) and (29) and in the leading order in our approach we find that

$$\frac{4\pi}{c} I_F \approx -4\pi M_0 \frac{\theta_F^2}{2} - 4\pi \mathcal{M}_S \theta_F^2 / d_F \quad (38)$$

and

$$\frac{4\pi}{c} I_S \approx 4\pi M_0 \frac{\theta_F^2}{2} + 4\pi \mathcal{M}_S \theta_F^2 / d_F. \quad (39)$$

As there is no external field the currents $I_{S,F}$ sums to zero, $I_F + I_S = 0$. Remarkably, in the two limiting cases, the total current in the S and F films has a different sign, i.e., $I_{S,a} = -I_{S,b} = 2\pi M_0 \theta_F^2$.

J is large enough, $J \gg \epsilon_{bF}$, that there is a transition from positive to negative $A_S(-d_S)$ determined by a critical J_c . Indeed, we obtain from Eq. (18)

$$A_S(-d_S) = \frac{-2\pi M_0 \theta_F^2 \lambda_S}{D_S \cosh \theta_S} \left[1 - \left(\frac{J_c}{J} \right)^2 \right], \quad (40)$$

with $J_c = \epsilon_{bF} (\lambda_F / \lambda_S) \sqrt{c_2 (\xi_S / d_F)}$.

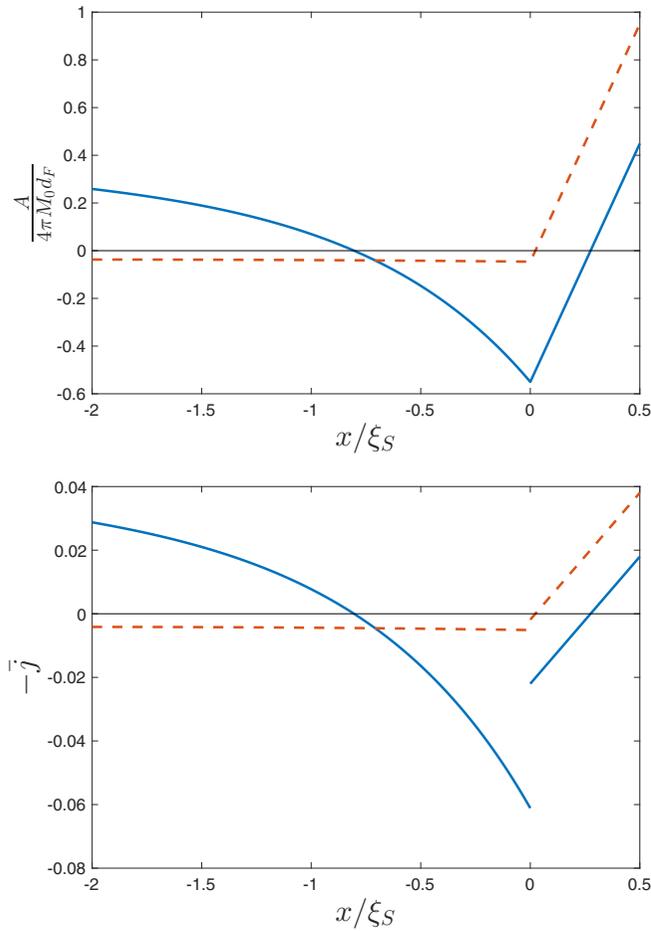


FIG. 3. Spatial dependence of the vector potential A (top panel) and the spontaneous current $\vec{j} \equiv jA\xi_S^2/cM_0d_F$ (bottom panel). The solid line represents the case which accounts for the inverse proximity effect [case (b) in the text], whereas the dashed line shows the behavior when the inverse proximity effect is neglected [case (a)]. Other parameters are chosen to be $\epsilon_{bF} = 5\Delta$, $d_S = 2\xi_S$, $d_F = 0.5\xi_S$, $\kappa_{Sb}\xi_S = 0.1$, $\kappa_{Fb}\xi_S = 1$, and $\lambda_F = (5/3)\lambda_S = 5\xi_S$.

In Fig. 3 we compare the spatial dependence of $A_S(x)$ (top panel) and of the current density $j(x)$ induced in the system (bottom panel) in the two cases, no spin polarization [case (a)], $M_S = 0$, and when the inverse proximity effect is taken into account [case (b)]. One clearly sees qualitative differences between these two cases. In case (b) the spontaneous currents change sign at a certain distance from the interface of the order of ξ_S , whereas in case (a) the sign of the current in S is constant. In addition, the amplitude of the spontaneous currents, and hence of the magnetic inductance [Eq. (2)], generated far from the S/F interface is much larger in case (b). These results demonstrate that for a correct interpretation of experiments the induced magnetization in the superconductor cannot simply be neglected.

B. Out-of-plane magnetization

Finally, in this section we consider the case of a F layer with out-of-plane magnetization: $\mathbf{M}_0 \parallel \mathbf{e}_x$. The field $\mathbf{H}_S(x) = (H_S(x), 0, 0)$ with $H_S(x)$ is determined from the equation

$$\text{div} \mathbf{B}_S(x) = 0:$$

$$\partial_x H_S(x) = -4\pi \partial_x M_S(x). \quad (41)$$

Thus we obtain

$$H_S(x) = -4\pi M_S(x). \quad (42)$$

The equation $\nabla \times \mathbf{H}_S(x) = (4\pi/c)\mathbf{j}$ then yields

$$\mathbf{j} = 0. \quad (43)$$

The magnetic induction \mathbf{B} does not depend on x and equals zero in both the S and F films.

IV. CONCLUSIONS

We have studied the spatial dependence of the Meissner currents $j_S(x)$ and magnetic induction $B_S(x)$ that spontaneously arise in an S/F structure even in the absence of an external field. The fields $B_S(x)$ and $H_S(x)$ originate due to the orbital and spin polarization effects and contain long-range and short-range components [see Eq. (1)]. The amplitude of the short-range component $B_{s-r}(0)$ is due to the inverse proximity effect and is much larger than $B_{l-r}(0)$. On the other hand, the amplitude of the long-range component $B_{l-r}(0, J)$ is caused by both the Meissner currents and the spin polarization and depends on the magnitude of the exchange energy J in the F film. It changes sign at $J \sim \epsilon_{bF} = D_F/(R_b\sigma_F d_F)$, being negative for $J \gg \epsilon_{bF}$ and positive for $J < \epsilon_{bF}$. Note that at large J the field B_S and H_S are small because both spin polarization [see Eq. (18)] and orbital effects are small. In Ref. [36] the inverse proximity effect was neglected, and therefore only the orbital contribution $B_{\text{orb}}(0) = -4\pi M_0 \theta_F^2/2$ was obtained, where $\theta_F^2 = (d_F/\lambda_F)^2 \ll 1$. However, as explained above, by decreasing the exchange energy J , the inverse proximity effect prevails, leads to a finite magnetic moment $M_S \neq 0$, and leads to a change in the sign of $B_{l-r}(0)$. In such a case its magnitude clearly exceeds the value of $|B_{\text{orb}}(0)|$.

Moreover, the results for the vector potential $A(x)$, and hence for the current density $j(x)$, depend crucially on the inverse proximity effect [case (b) in Sec. III A] and are qualitatively different from the case in which this effect is neglected (case (a) in Sec. III A and Ref. [36]). In particular, we find that the Meissner current density j_S changes sign in the S region if the induced magnetization is taken into account.

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APPENDIX A: SOLUTION OF THE USADEL EQUATION IN THE S-F STRUCTURE

The Usadel equations have the form

$$-D_S \partial_x (\hat{g}_S \partial_x \hat{g}_S) + \omega[\tau_3, \hat{g}_S] + \Delta[\tau_1, \hat{g}_S] = 0, \quad (A1)$$

$$-D_F \partial_x (\hat{g}_F \partial_x \hat{g}_F) + \omega[\tau_3, \hat{g}_F] + iJ[\tau_3 \sigma_3, \hat{g}_F] = 0 \quad (A2)$$

for the S and F films, respectively. Equations (A1) and (A2) are complemented by the normalization relation

$$(\hat{g}\hat{g}) = \hat{1} \quad (\text{A3})$$

and the boundary conditions [41]

$$(\hat{g}\partial_x\hat{g})_F = \kappa_{bF}[\hat{g}_S, \hat{g}_F], \quad (\hat{g}\partial_x\hat{g})_S = \kappa_{bS}[\hat{g}_S, \hat{g}_F], \quad (\text{A4})$$

where $\kappa_{bF(S)} = (R_b\sigma_{F(S)})^{-1}$ and R_b is the interface resistance per unit area.

The linearized Eq. (A1) is

$$-\partial_{xx}^2\delta\hat{g}_S + \kappa_\omega^2\delta\hat{g}_S = 2\delta\Delta\omega(\omega\tau_1 - \Delta\tau_3)/D_S, \quad (\text{A5})$$

where $\kappa_\omega^2 = 2\sqrt{\omega^2 + \Delta^2}/D_S$. We used the relation

$$\delta\hat{g}_S\hat{g}_S + \hat{g}_S\delta\hat{g}_S = 0, \quad (\text{A6})$$

which follows from the normalization condition, Eq. (A3).

The induced magnetization is determined by the component $\delta g_{S3} = \text{Tr}(\tau_3\hat{g}_S)/4$ [see Eq. (6)]. We multiply Eq. (A5) by $\tau_3\sigma_3$ and calculate the trace. We find the solution

$$g_{S3}(x) = g_{S3}(0) \exp(x\kappa_\omega). \quad (\text{A7})$$

The integration constant is found from the boundary condition (A4), which yields

$$\partial_x g_{S3}(x)|_{x=0} = 2\kappa_{Sb}F_S[F_S g_{F3} - G_S f_{F3}], \quad (\text{A8})$$

where $f_{F3} = \epsilon_{bF}F_S\text{Im}(1/\tilde{\zeta}_{\omega+})$ and $g_{F3} = \text{Im}(\tilde{\omega}_+/\tilde{\zeta}_{\omega+})$. The function $\tilde{\zeta}_{\omega+}$ is defined in Eq. (10).

We obtain for $g_{S3}(x)$

$$\begin{aligned} g_{S3}(x, \omega) &= \frac{2\kappa_{bS}}{\kappa} F_S[F_S g_{F3} - G_S f_{F3}] \exp(x\kappa_\omega) \\ &\equiv g_{33}^{(S)}(0, \omega) \exp(x\kappa_\omega). \end{aligned} \quad (\text{A9})$$

One can see that this correction $\delta\hat{g}_S$ is small if the condition

$$\kappa_{bS}\xi_S \ll 1 \quad (\text{A10})$$

is fulfilled, that is, $R_{Sb} \gg \rho_S\xi_S$.

APPENDIX B: THE MAGNETIC FIELD

For completeness we show in this Appendix the expressions for the magnetic induction $B(x) = \partial_x A(x)$ and magnetic field $H(x) = B(x) - 4\pi M_F$ which can be obtained from Eqs. (27) and (29):

$$B_F(x) \cong a_0 \frac{x}{\lambda_F^2} + (h_0 + 4\pi M_0) \left(1 + \frac{x^2}{2\lambda_F^2}\right), \quad (\text{B1})$$

$$H_F(x) \cong a_0 \frac{x}{\lambda_F^2} + h_0 \left(1 + \frac{x^2}{2\lambda_F^2}\right) + 4\pi M_0 \frac{x^2}{2\lambda_F^2}, \quad (\text{B2})$$

$$\begin{aligned} B_S(x) &= \frac{a_S}{\lambda_S} \frac{\sinh[(x + d_S)/\lambda_S]}{\cosh\theta_S} + 4\pi M_S(x) \\ &\quad - 4\pi \sum_{\omega \geq 0} m_S(\omega) \delta_S \exp(x\kappa_\omega), \end{aligned} \quad (\text{B3})$$

$$H_S(x) = \frac{a_S}{\lambda_S} \frac{\sinh[(x + d_S)/\lambda_S]}{\cosh\theta_S} - 4\pi \sum_{\omega \geq 0} m_S(\omega) \delta_S^2 \exp(x\kappa_\omega), \quad (\text{B4})$$

where $\delta_S = 1/\lambda_S\kappa_\omega$.

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