Entangling distant spin qubits via a magnetic domain wall

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The scalability of quantum networks based on solid-state spin qubits is hampered by the short range of natural spin-spin interactions. Here, we propose a scheme to entangle distant spin qubits via the soft modes of an antiferromagnetic domain wall (DW). As spin qubits, we focus on quantum impurities (QIs) placed in the vicinity of an insulating antiferromagnetic thin film. The low-energy modes harbored by the DW are embedded in the antiferromagnetic bulk, whose intrinsic spin-wave dynamics have a gap that can exceed the THz range. By setting the QI frequency and the temperature well within the bulk gap, we focus on the dipolar interaction between the QI and two soft modes localized at the DW. One is a stringlike mode associated with transverse displacements of the DW position, while the dynamics of the other, corresponding to planar rotations of the Néel order parameter, constitute a spin superfluid. By choosing the geometry in which the QI does not couple to the string mode, we use an external magnetic field to control the gap of the spin superfluid and the qubit-qubit coupling it engenders. We suggest that a tunable micron-range coherent coupling between qubits can be established using common antiferromagnetic materials.

Introduction. The discovery of quantum-impurity (QI) model systems, such as NV color centers [1] and spin qubits in silicon [2], which show long coherence times and can be efficiently initialized and read out [3,4], has stimulated a vast interest within the field of quantum computing [5–8]. Direct coherent coupling between single NV centers has been already observed by several groups [9–11]. However, due to its dipolar nature, such spin-spin coupling extends only up to tens of nanometers. This distance requirement for their interaction limits the implementation of large-scale quantum entanglement schemes, where the ability of addressing each qubit individually must also be preserved. To circumvent this drawback, numerous proposals for the coherent coupling of atomistic qubits revolve around hybrid quantum devices, where distant qubits interact indirectly via, e.g., mechanical resonators [12,13], superconducting flux qubits [14], photons [15], metallic gates [16] or spin waves [17].

While the interaction between NV centers and spin waves has recently allowed one to probe a range of magnetic phenomena with high spatiotemporal resolution [18], hybrid architectures relying on magnetic insulators as building blocks remain relatively unexplored. In Ref. [17], spin waves in microfabricated ferromagnetic waveguides have been proposed to mediate long-distance coupling between spin qubits. In this Rapid Communication, we instead suggest to employ low-energy excitations associated with extended spin textures, such as domain walls, in an otherwise homogeneous magnetic background.

Specifically, we consider an antiferromagnetic insulating film with uniaxial anisotropy, which supports an extended domain wall (DW), as depicted in Fig. 1(a). The antiferromagnetic DW harbors two types of Goldstone modes associated with its real- and spin-space dynamics [19]. These are respectively related to the zero modes associated with the DW displacement Y and the azimuthal angle Φ of the

Néel order parameter therein. Two QI spins, placed above the DW, interact dipolarly with the collective spin modes of the antiferromagnetic system, whose dispersion relations are shown in Fig. 1(b). By setting the QI resonance frequency within the bulk gap, which is of the order of 0.1–1 THz in many of the common antiferromagnetic materials [20], we can focus on the excitations endowed by the DW for both qubit decoherence and qubit-qubit coupling. The string mode is a translational-symmetry restoring Goldstone mode that cannot be easily gapped (unless we pin the DW position), which may pose problems for controlling the relative importance of the coherent coupling and the decoherence [19]. We show, however, that in the appropriate geometry, the string mode decouples from the spin qubits, at the leading order. Therefore, the $U(1)$-symmetry restoring Goldstone mode, whose dynamics realizes spin superfluidity [21], is left to control both the effective qubit-qubit coupling and qubit decoherence. These can thus be tuned via an in-plane magnetic field, which opens a gap in the spin-superfluid spectrum.

In this Rapid Communication, we address two distinct but related problems. First, we consider a spin qubit to couple to a single quantized spin-superfluid mode of a ∼ micron-long DW. We show that the associated cooperativity can be large, suggesting that two distant spin qubits can be in principle coupled coherently via a single superfluid mode. Second, we look at the interplay between the qubit decoherence and the qubit-qubit interaction provided by the continuum of modes in an infinite DW. We find that the spin-superfluid mode can mediate a two-qubit gate with an operation rate of the order of tens of kHz, when the qubits are placed at the distance of a micron from each other. The gate-operational rate is found to be larger (by a factor ∼10^2) than the QI decoherence rate due to the spin-superfluid noise.

Main results. We consider two QI's with spin-1/2 $\mathbf{S}_i$ (for $i = 1, 2$) with resonance frequency ω, placed at a distance
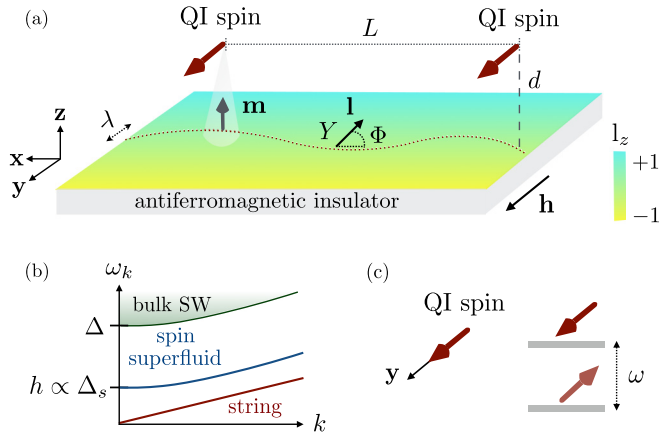
FIG. 1. (a) The proposed hybrid quantum system: An antiferromagnetic film harbors a DW of width λ along the x axis. This soliton interpolates between the ground states with $l_z = \pm 1$ at $y \to \mp\infty$, where $\mathbf{l}$ is the Néel order parameter. The x-dependent variables (Y, Φ) identify, respectively, the y position and the azimuthal angle of the order parameter at the DW center. Two QI spins, placed at a height d above the DW and distance L from each other, interact magnetostatically with the film's spin density $\mathbf{m}$. The axial-symmetry breaking magnetic field $\mathbf{h}$ is applied along the y direction. (b) Dispersions of the collective spin modes in the film. Dark green line: Bulk spin-wave (SW) dispersion with energy gap Δ. Blue line: Dispersion of the spin-superfluid mode in the presence of the magnetic field $\mathbf{h} = h\mathbf{y}$, which opens a gap $\Delta_s \propto h$. Red line: String sound mode. (c) The QI spin 1/2 is quantized along the y axis, with the level splitting of $\hbar\omega$.

L from each other and at a height d above the domain wall, as shown in Figs. 1(a) and 1(c). The QI spin at a position $\vec{r}_i$ couples to the stray field $\mathbf{B}(\vec{r}_i) = \gamma \int d\vec{r} \mathcal{D}(\vec{r}, \vec{r}_i)\mathbf{m}(\vec{r})$ generated by the antiferromagnetic spin density $\mathbf{m}(\vec{r})$ via the Zeeman interaction. Here, γ ($\tilde{\gamma}$) is the gyromagnetic ratio of the magnetic film (QI spin) and $\mathcal{D}$ the tensorial magnetostatic Green's function [22,23]. Starting from the Lagrangian of a bipartite antiferromagnetic film with uniaxial anisotropy along the $\mathbf{z}$ axis, one can derive the Hamiltonian of each DW mode by using the collective coordinate approach, i.e., focusing on the dynamics of the DW position Y and of the azimuthal angle Φ of the Néel order parameter therein [19].

An external magnetic field $\mathbf{h} = h\mathbf{y}$ sets the QI quantization axis along the y direction and enforces a Bloch domain wall configuration, i.e., $\Phi = 0$. As discussed in detail later, these choices lead to vanishing coupling between each QI spin and the DW string mode. Moreover, the magnetic field opens up a gap $\Delta_s = \gamma h$ in the spin-superfluid dispersion, as shown in Fig. 1(b). Here, we take the spin-superfluid gap to be much smaller than the spin-wave bulk gap. Thus, at QI resonance frequencies comparable with the spin-superfluid gap, we can neglect the QI coupling with bulk spin waves and focus on its interaction with the spin-superfluid mode.

For a DW of length ℓ, we focus on the coupling between a QI spin and a single spin-superfluid mode. The latter can be quantized in terms of the magnon creation (annihilation) operator $a_k^\dagger$ (a_k) with dispersion $\hbar\omega_k$. The interaction Hamiltonian

becomes

$$\mathcal{H}_{\text{int}} = g\sigma^+ a_k + \text{H.c.}, \tag{1}$$

where $\sigma^\pm = \sigma_{\tilde{x}} \pm i\sigma_{\tilde{y}}$, with $\sigma_{\tilde{\alpha}}$ being the α Pauli matrix in the QI spin reference frame. Note that, in deriving Eq. (1), we have assumed $g \ll \omega \simeq \omega_k$. The cooperativity associated to Eq. (1) can be defined as $C = g^2 \tau_s T_2$ [13], where T_2 is the intrinsic QI dephasing time and τ_s the DW-mode relaxation time. We find the coupling g as

$$g = \frac{\hbar\gamma\tilde{\gamma}}{2}\sqrt{\frac{\lambda\chi\hbar\omega_k\left[\mathcal{D}_{xz}^2(k,d) + \mathcal{D}_{zz}^2(k,d)\right]}{2\ell}}, \tag{2}$$

where χ is the static uniform transverse spin susceptibility and λ the DW width. Here, $\mathcal{D}_{\alpha\beta}(k,d)$ is the one-dimensional Fourier transform of the magnetostatic Green's function $\mathcal{D}_{\alpha\beta}(\vec{r}, \vec{r}_i)$ at the QI position $\vec{r}_i = (0, 0, d)$. This function decreases rapidly as function of the distance d and it is maximized for $k \sim 1/d$.

For a DW of infinite length, we focus on the coupling between the QI spins and the continuum of the spin-superfluid mode. The strength of the effective qubit-qubit coupling and the single-qubit decoherence are parametrized, respectively, by the real (χ') and imaginary (χ'') part of the spin-superfluid dynamical transverse spin susceptibility $\chi_{zz}(k, \omega)$. As discussed below, we find the coupling Hamiltonian as

$$\mathcal{H}_c = \int \frac{dk}{2\pi} f(k, d)\chi'_{zz}(k, \omega)\cos(kL)\sigma_1^+\sigma_2^- + \text{H.c.}, \tag{3}$$

with $f(k, d) = [D_{zz}^2(k, d) + D_{xz}^2(k, d)](\gamma\tilde{\gamma})^2/16$. In our geometry, the stray field associated with the spin-superfluid mode is transverse to the QI quantization axis. Thus, while there is no QI dephasing due to purely longitudinal coupling, the spin-superfluid mode gives rise to QI relaxation processes. The associated QI relaxation rate reads as

$$T_1^{-1}(\omega) = \coth(\beta\hbar\omega/2)\int \frac{dk}{\pi} f(k, d)\chi''_{zz}(k, \omega). \tag{4}$$

Equation (4) accounts for processes in which the creation or annihilation of a magnon gives rise to a QI transition between its spin states and vice versa. For $\omega < \Delta_s$ (or $\omega < \Delta_s - 1/\tau_s$, when accounting for magnetic damping), the magnon spectral density vanishes and the relaxation rate (4) is minimized. On the other hand, the real part of the spin susceptibility decays exponentially on the length scale $\ell_s = c/\sqrt{\Delta_s^2 - \omega^2}$, i.e., $\chi'_{zz}(x) \propto \ell_s e^{-x/\ell_s}$. Thus, to maximize the ratio between the effective qubit-qubit coupling and the single-qubit decoherence, one needs to set the QI frequency just below the gap, i.e., $\omega \lesssim \Delta_s$ (or $\omega \lesssim \Delta_s - 1/\tau_s$).

Antiferromagnetic system. At temperatures far below the Néel temperature, we can describe the low-energy long-wavelength dynamics of a bipartite antiferromagnet in terms of the directional Néel order parameter field $\mathbf{l}(\vec{r}, t)$, with $|\mathbf{l}| = 1$. The Lagrangian of an isotropic cubic antiferromagnet with exchange stiffness A and uniaxial anisotropy K can be written as

$$\mathcal{L}[\mathbf{l}, \dot{\mathbf{l}}] = \frac{\chi}{2}\int d\vec{r}(\dot{\mathbf{l}} + \gamma\mathbf{l} \times \mathbf{h} \times \mathbf{l})^2 - \mathcal{H}[\mathbf{l}],$$

with

$$\mathcal{H}[\mathbf{l}] = A(\vec{\nabla}\mathbf{l})^2 + K|\mathbf{z} \times \mathbf{l}|^2. \quad (5)$$

Varying Eq. (5) with respect to **m** leads to the constitutive relation $\mathbf{m} = \chi\,\mathbf{l} \times (\partial_t\mathbf{l} - \gamma\,\mathbf{l} \times \mathbf{h})$ [24]. Dissipation can be introduced by means of the Rayleigh function $\mathcal{R}[\mathbf{l}] = \alpha s \int d\vec{r}(\partial_t\mathbf{l})^2/2$, where α is the Gilbert damping constant and s the saturated spin density of both sublattices. The model (5) admits also a solution for a static domain wall of width $\lambda = \sqrt{A/K}$. For boundary conditions of the form $l_z(y \to \pm\infty) = \pm 1$ and using the parametrization $\mathbf{l} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$, the DW solution is given by

$$\cos\theta(\vec{r}) = \tanh\frac{y - Y}{\lambda}, \quad \phi(\vec{r}) = \Phi. \quad (6)$$

By plugging the DW solution (6) into Eq. (5) and promoting the azimuthal angle to a dynamical field $\Phi(x, t)$, we obtain the Lagrangian of the spin-superfluid mode as

$$\mathcal{L}[\Phi, \dot{\Phi}] = \lambda \int dx[\chi\dot{\Phi}^2 - A(\partial_x\Phi)^2 + \chi(\Delta_s\Phi)^2]. \quad (7)$$

Following the standard canonical quantization of a harmonic oscillator, Eq. (7) can be quantized in terms of the magnon operator a_k ($a_k^\dagger$) with dispersion $\hbar\omega_k = \sqrt{(ck)^2 + \Delta_s^2}$, where $c = \sqrt{A/\chi}$. From the Lagrangian (5) and the Rayleigh function, we can derive the transverse spin susceptibility of the spin-superfluid mode as

$$\chi_{zz}(k, \omega) = \frac{2\lambda\chi\omega^2}{\omega_k^2 - \omega^2 - i\alpha s\omega/\chi} + 2\lambda\chi. \quad (8)$$

Noise. The interaction between a QI spin and the antiferromagnetic spin density can be generally recast in the form

$$\mathcal{H} = \sigma^+ \otimes X + \sigma_{\tilde{z}} \otimes Z + \text{H.c.}, \quad (9)$$

where X and Z are fluctuating fields coupling, respectively, transversely and longitudinally to the QI quantization axis. The relaxation, T_1^{-1}, and dephasing, T_2^{-1}, rates of each qubit can be written as [25]

$$T_1^{-1} = \hbar^{-2}S_Y(\omega), \quad T_2^{-1} = \tfrac{1}{2}T_1^{-1} + \hbar^{-2}S_X(0). \quad (10)$$

Here, $S_A(\omega) = \int dt\,e^{-i\omega t}\langle\{A^+(t), A(0)\}\rangle$ is the power spectrum of the operator A and $\langle\cdots\rangle$ stands for the equilibrium (thermal) average. The power spectrum $S_{X(Y)}(\omega)$ can be expressed in terms of the Fourier transform of the spin-spin correlator $C_{\alpha\beta}(\vec{r}_i, \vec{r}_j; t) = \langle\{m_\alpha(\vec{r}_i, t), m_\beta(\vec{r}_j, 0)\}\rangle$, with $\alpha, \beta = x, y, z$, through the magnetic field that the spin-density fluctuations induce. Invoking the fluctuation-dissipation theorem [26], we can write $C_{\alpha\beta}(k, \omega) = \coth(\beta\hbar\omega/2)\chi''_{\alpha\beta}(k, \omega)$, where $\beta = 1/k_BT$, with k_B being the Boltzmann constant and T the temperature.

For the isotropic bulk, the spin-susceptibility tensor is diagonal, with $\chi_{xx} = \chi_{yy}$ [27,28]. The response χ''_{xx} (χ''_{zz}) stems from spin fluctuations transverse (longitudinal) to the **z** axis, i.e., to the equilibrium orientation of the Néel order parameter in the bulk. As discussed in Ref. [28], transverse fluctuations

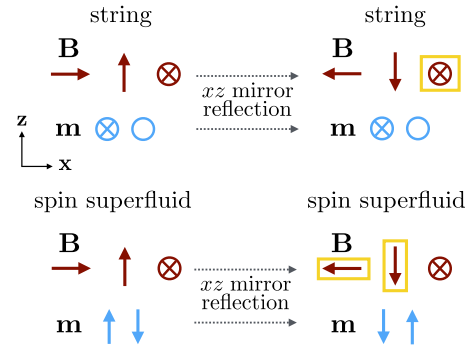
FIG. 2. Stray field **B** produced by a Bloch domain wall. The yellow contour highlights the stray-field components (red arrows) allowed by symmetry. Upper side: The spin density **m** (blue arrow) associated with the string-mode dynamics is invariant under mirror reflections through the xz plane. The associated stray field is allowed to have only a y component. Lower side: The spin density **m** (blue arrow) produced by the spin-superfluid dynamics flips sign under mirror reflections through the xz plane. Therefore, the allowed stray-field components are oriented along the **x** and **z** axes.

of the spin density correspond to one-magnon processes, i.e., the creation or annihilation of a magnon. The associated relaxation rate is proportional to the magnon spectral density at the QI resonance frequency. The latter is vanishing for $\omega \lesssim \Delta - 1/\tau_s$, with Δ being the spin-wave bulk gap. Thus, by tuning the QI frequency, one has $T_1^{-1} = 0$. Furthermore, the imaginary part of the bulk transverse spin susceptibility scales as $\chi''_{xx}(\omega) \propto \omega^3$ for $\omega \to 0$ [29], leading to a vanishing dephasing rate, i.e., $T_2^{-1} = 0$.

The bulk longitudinal spin fluctuations correspond instead to two-magnon processes. The associated QI decoherence rate reflects the likelihood of magnons scattering with energy gain (loss) equal to the QI frequency; it is thus maximized at zero frequency, to then decrease with increasing QI frequency [19]. However, magnons freeze out as the temperature drops below the spin-wave gap Δ and, by setting the temperature far below the bulk spin-wave gap, $S_X(0)$ can be neglected.

For $d \gg \lambda$, we can relate the spin susceptibility of the magnetic film to the one associated with the DW modes as $\chi_{\alpha\beta}(\vec{r}_i, \vec{r}_j; \omega) = \chi_{\alpha\beta}(|x_i - x_j|, \omega)\delta(y_i)\delta(y_j)$. In a Bloch DW, the order parameter lies along the **x** axis. Thus, according to the constraint $\mathbf{m} \cdot \mathbf{l} = 0$, there is no finite spin-density component along the x direction. A finite out-of-plane spin density (per unit of length) is engendered by the spin-superfluid dynamics, while string-mode fluctuations give rise a spin density (per unit of length) along the **y** axis. In linear response, the (one-magnon) longitudinal and transverse spin fluctuations do not interfere and can be considered separately [19]. Hence, the relevant response functions are the yy and zz components of the imaginary part of the (one-magnon) spin susceptibility. Since $\chi''_{yy}(\omega)$, $\chi''_{zz}(\omega) \propto \omega^3$ for $\omega \to 0$, we can set $S_X(0) = 0$ [29]. As it can be deduced from the symmetry argument illustrated in Fig. 2, the stray field generated by the string mode is parallel to **y** axis. Thus, when the QI quantization axis is oriented along the y direction, we have $T_1^{-1} = 0$, and, consequently, $T_2^{-1} = 0$. Instead, for the spin-superfluid mode,

the associated stray-field components are oriented along the **x** and **z** axes, as shown in Fig. 2. The corresponding relaxation rate is given by Eq. (4). Since the spin-superfluid dynamics give rise to spin fluctuations transverse to the equilibrium orientation of the order parameter, the QI relaxation rate can be minimized by tuning the QI frequency below the spin-superfluid gap.

Qubit-qubit coupling. Assuming the QI coupling to the antiferromagnetic spin density to be much smaller than the QI resonance frequency, we can derive the effective qubit-qubit interaction by applying the lowest-order Schrieffer-Wolff transformation [30] to the interaction Hamiltonian

$$\mathcal{H}_{\text{int}} = -\frac{\hbar\tilde{\gamma}}{2} \sum_{i=1,2} \boldsymbol{\sigma}_i \cdot \mathbf{B}(\vec{r}_i). \tag{11}$$

The resulting single-qubit terms such as $J\sigma_1^+\sigma_1^+$ vanish in the spin-1/2 subspace, while terms of the type $J\sigma_1^+\sigma_1^-$ can be reabsorbed in the definition of the QI frequency [31]. Terms acting in the subspace $(|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle)$, e.g., $J\sigma_1^+\sigma_2^+$, can reduce the gate fidelity; however, as already discussed in Ref. [16], we can neglect them as long as $J \ll \omega$. Focusing on the spin-superfluid mode, which does not couple longitudinally to the QIs, no terms involving the operator $\sigma_{\bar{z},1(2)}$ appear. These considerations lead to the effective qubit-qubit coupling Hamiltonian (3).

A controlled-NOT and arbitrary one-qubit gates suffice for defining a universal set of gates. For a NV center, the initialization and read-out of the spin state can be performed optically, while single-qubit operations can be carried out by locally applying resonant microwave fields. A controlled-NOT gate can be decomposed into two iSWAP gates. By rewriting Eq. (3) as

$$\mathcal{H}_c = J(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-), \tag{12}$$

an iSWAP gate can be implemented as $U_{\text{iSWAP}} = \exp(-i\mathcal{H}_c t_J/\hbar)$, with $t_J = \pi/4J$ [32].

The qubit-qubit interaction mediated by the transverse and longitudinal bulk spin waves can be neglected when the temperature and the QI frequency lie much below the bulk spin-wave gap. For any QI frequency, instead, the string mode mediates a Ruderman-Kittel-Kasuya-Yosida (RKKY)-like interaction between the qubits, which can be found as

$$\mathcal{H}_c = \left(\frac{\gamma\tilde{\gamma}}{2}\right)^2 \int \frac{dk}{2\pi} \chi'_{yy}(k,0)\cos(kL)\sigma_{\bar{z},1}\sigma_{\bar{z},2}. \tag{13}$$

In our model, the real part of the static spin susceptibility decays very rapidly, i.e., $\chi'_{yy}(x) \propto \delta(x)$. Accounting for a finite exchange stiffness A_s associated with the spin-density field, which translates into adding a term $\propto A_s(\vec{\nabla}\mathbf{m})^2$ to Eq. (5), would introduce a finite decay length $\lambda_s = \sqrt{A_s/K}$. However, as this length scale is atomistically short, within a Heisenberg model for the antiferromagnet, it can be taken to be much shorter than the characteristic length scale ℓ_s that controls the strength of the qubit-qubit coupling mediated by the spin superfluid (3).

Estimate. As a QI prototype we consider a NV center, i.e., a spin triplet with an intrinsic dephasing time exceeding $T_2 \sim 100$ ms at cryogenic temperatures [4]. By tuning the magnetic field, we can isolate a subsystem of the spin triplet and treat a NV center as an effective two-level system. Writing $A = J_H S^2$ and $\chi = \hbar^2/8J_H S^2 a^2$ [33], with J_H being the Heisenberg exchange, S the spin, and a the lattice constant, we set $S \approx 1$ and $a \approx 5$ Å. We take $\gamma = \tilde{\gamma} \approx 2\mu_B/\hbar$, with μ_B being the Bohr magneton, $\Delta_s \approx 1$ GHz, $d \approx 20$ nm, and $\lambda \approx 5$ nm. For $\ell \sim 1$ μm, we obtain, for a magnon mode with $k \sim 1/d$, a coupling strength $g \sim 10$ kHz. We note that the latter, for a given DW width, does not depend on the exchange stiffness. From the Landau-Lifshitz-Gilbert (LLG) phenomenology [19], we have $\tau_s = 2\chi/s\alpha$, where α is the Gilbert damping, which we set to $\alpha \sim 10^{-4}$. For $J_H \sim$ 0.1–1 THz, we find a cooperativity $C \sim 10$–100 [34], much higher than the one associated with, e.g., hybrid devices based on NV centers and mechanical resonators [13]. When the QI couples to the continuum spin-superfluid mode, to minimize the one-magnon noise one needs to set the QI frequency below the magnon continuum, i.e., $\omega < \Delta_s - 1/\tau_s$, but not too far from it, in order to still obtain a sizable coupling. Plugging Eq. (8) into Eq. (3) and setting, e.g., $\Delta_s - \omega \sim$ 1 MHz, we find, for $J_H \sim 0.1$ THz, an operation rate $t_J^{-1} \sim$ 10 kHz for $L \sim 1$ μm. The QI relaxation rate induced by the spin-superfluid noise, which can obtained by integrating numerically Eq. (4), is of order of tens of Hz at $T = 100$ mK, i.e., two orders of magnitude smaller than the operation rate. Setting $J_H \sim 1$ THz decreases the operation rate to $t_J^{-1} \sim$ 1 kHz, but it leads to a ratio between the latter and the QI relaxation rate of the order $\sim 10^4$.

Discussion. Recently, strongly localized quantized magnetic solitons with nonlinear features have been proposed as carriers of quantum information [35]. Here, changing the perspective, we focus on the soft bosonic modes of extended domain walls to mediate coupling between magnetic qubits that are extrinsic to the antiferromagnetic medium. Specifically, we show that the spin-superfluid mode harbored by an antiferromagnetic DW can mediate a tunable coherent coupling between spin qubits separated on a micron scale, i.e., a distance larger than, e.g., the one required to address NV centers separately [36]. We propose a universal set of gates that can be switched on and off via an external magnetic field. Our approach relies on a tunable one-dimensional waveguide along naturally occurring domain walls in easy-axis antiferromagnets, thus avoiding a need for microfabricated structures [17]. Moreover, our proposal opens up prospects for using multiple NV centers to investigate how quantum correlation and entanglement propagate through a magnetic material.

Future works should more systematically address the role of quenched disorder, the effects of higher-order magnon processes associated with the DW dynamics, and the decoherence from other sources. These may include the phononic background and dynamic spin impurities in the magnetic medium (which go beyond the Gilbert damping phenomenology of collective dissipation).

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