Magnetic ground state, field-induced transitions, electronic structure, and optical band gap of the frustrated antiferromagnet $GeCo_2O_4$

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Systematic studies of magnetic ordering, magnetic-field-induced transitions, electronic structure, and optical properties of the frustrated spinel GeCo₂O₄ (GCO) are reported. Our results reveal that GCO orders antiferromagnetically (AFM) at $T_N = 20.4$ K but with significant short-range ferromagnetic (FM) order up to $T \sim 5 T_N$. The paramagnetic susceptibility (χ) fits the modified Curie–Weiss law, $\chi = \chi_0 + C/(T - \theta)$, with $\theta = +51$ K for 100 K < T < 800 K. The fit to high-temperature-series expansion of $\chi(T)$ yields $J_1/k_B = 14.7 \text{ K}$ as the dominant FM exchange coupling for the pyrochlore lattice of Co²⁺ spins consisting of alternate planes of Kagomé (KGM) and Triangular (TRI) spins lying perpendicular to [111] direction. From the analysis of the M-H plots at 2 K and published results, three critical fields are identified: $H_d \sim 11$ kOe due to AFM domains, $H_{C1} \approx 44$ kOe related to spin-flips and FM ordering of the TRI spins, and $H_{C2} \approx 97$ kOe related to FM ordering of the KGM spins. For $H > H_{C2}$, GCO is a forced ferromagnet with some canting of the spins. Magnetic field dependence of T_N follows the relation $T_N(H) = T_N(0) - D_1 H^2$ valid for antiferromagnets with $D_1 = 6.63 \times 10^{-10}$ K/Oe². This magnitude of $T_N(H)$ along with the temperature dependence of H_d , H_{C1} , and H_{C2} are used to construct the H-T phase diagram. From the magnitudes of the Curie constant (C) and the saturation magnetization at 2 K it is shown that Co^{2+} ions in GCO have the ground state with effective spin S = 1/2. High resolution x-ray photoelectron spectra of 2p and 3d orbitals of Co and Ge confirm the divalent and tetravalent electronic states of Co and Ge, respectively, in GCO. The energy band gap ($E_g = 3.28 \text{ eV}$) evaluated using DFT+U calculations is in good agreement with the experimental results ($E_g = 3.16 \text{ eV}$) obtained from the diffuse reflectance spectroscopy.

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I. INTRODUCTION

Recently, properties of cobalt-based spinels diluted with nonmagnetic metals such as Sn, Ge, Zn, and Ti have received considerable attention, partly because of their excellent electrochemical characteristics useful for high performance batteries in addition to their anomalous low-temperature magnetic ordering [1–11]. Among these spinels, properties and nature of magnetic ordering in GeCo₂O₄ (GCO) have received significant attention because it contains pyrochlorelike lattice structure for the magnetic Co²⁺ ions, a situation for which magnetic frustration is inherently present as first pointed out by Anderson [12] and elaborated in more recent studies on GCO [7,13-17]. Unlike the inverse ferromagnetic spinels such as Co₂MO₄ (M = Sn, Ru, Ti), GCO crystallizes in the normal spinel structure with nonmagnetic Ge4+ occupying the tetrahedral A-sites and the octahedral B-sites occupied by magnetic Co^{2+} ions leading to the structure: $(Ge^{4+})_A$ [2Co²⁺]_BO₄. Other B-site magnetic spinels in which considerable amount of magnetic frustration has been reported are MgMnO₃ [18,19], ZnFe₂O₄ [20], and AlV₂O₄ [21]. Magnetic properties and nature of magnetic ordering in GCO have been

reported using the data of temperature (T) and magnetic-field (H) dependence of the magnetization (M) [13-15,17,22,23]as well as neutron diffraction techniques [7,14,16,24,25]. These magnetic studies have reported that GCO orders antiferromagnetically (AFM) with a Néel temperature T_N varying between from 20 K and 23 K although the temperature dependence of paramagnetic susceptibility for T > 100 K when fit to the Curie-Weiss (CW) law; $\chi = C/(T - \theta)$, yielded positive $\theta = 40-80$ K, signifying that the dominant exchange coupling between Co²⁺ ions is ferromagnetic (FM). The different magnitudes of θ resulted from different temperature range used for fitting the χ versus T data. From the M versus H variations measured for $T < T_N$, peaks in (dM/dH) have been reported at $H_{\rm C1} \simeq$ 44 kOe and $H_{\rm C2} \simeq$ 97 kOe at 4 K [14–16]. However, M at 2 K is not saturated even in applied H up to 550 kOe [14]. Several studies in GCO have also reported tetragonal distortion of the cubic lattice accompanying $T_{\rm N}$ [15,17,22–25] yielding $c/a \simeq 1.001$ although a recent highresolution x-ray diffraction study by Barton et al. [23] showed that the lattice distortion occurs at $T_d \simeq 16$ K, a few degrees below $T_{\rm N}$, signifying decoupling of the structural distortion and magnetic ordering.

Summarizing the results of neutron diffraction studies in GCO reported by a number of groups [7,14,16,24,25], the following picture of spin ordering in GCO has emerged: the

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pyrochlore lattice of Co²⁺ ions consist of alternate planes of Kagomé (KGM) spins and spins on triangular (TRI) lattice, all lying in the (111) planes, with the propagation vector \overrightarrow{k} = (1/2, 1/2, 1/2) and with the likely easy direction of $[11\overline{2}]$. In applied magnetic field H = 0, the KGM and TRI spins though parallel within each plane are antiparallel to those in neighboring planes thus yielding an overall AFM ordering. At $H = H_{C1} \simeq 44$ kOe, any coupling between the KGM and TRI spins breaks down in that TRI spins become ordered ferromagnetically, whereas, the KGM spins in neighboring KGM planes remain AFM ordered. At $H = H_{C2} \simeq 97$ kOe, the spins in the neighboring KGM planes also become ordered ferromagnetically but with some canting. This canting produces the weak H-dependence of magnetization for H > H_{C2} . The magnetic frustration in GCO results from factors such as the geometrical frustration of the pyrochlore lattice of the Co²⁺ spins noted earlier along with the presence of several interlayer antiferromagnetic exchange couplings among the KGM and TRI spins [16,24–26], in addition to the dominant in-plane ferromagnetic exchange coupling.

The above summary of the important results reported in literature shows that some important issues regarding the nature of magnetism in GCO are still not settled. For example, a proper interpretation of the temperature dependence of the paramagnetic susceptibility in terms of the CW law or more elaborate models and its correlation with the measured values of the magnetization for $T < T_N$ has not been made. Also, results of the magnetic studies need to be reconciled with those from the recent electron magnetic resonance (EMR) studies in single crystals of GCO by Okubo et al. [27] who reported the observation of a broad EMR line at 86 K with the g value = 5.26 for H along [111] with similar g values along the [100] and [110] directions. The line broadens out at higher T because of rapid spin-lattice relaxation and for T < $T_{\rm N}$, the observed AFMR (antiferromagnetic resonance) lines are yet to be interpreted quantitatively. The above issues are important regarding the nature of the ground state of Co²⁺ in GCO. As noted for $CoCl_2$ [28], $\beta - Co(OH)_2$ [29] and more recently by Tomiyasu et al. [25,26] in GCO, Co²⁺ ions in octahedral crystalline field with small trigonal distortion combined with the effects of spin-orbit coupling $\lambda L \cdot \vec{S}$ $(\lambda = -180 \text{ cm}^{-1})$ leads to Kramer doublet as the ground state with effective spin S = 1/2. This doublet is separated from the nearby levels with effective S = 3/2 situated at ~ 450 K and effective S = 5/2 at ~ 1000 K. Neutron diffraction measurements in GCO by Diaz et al. [13,14] reported 3.02 μ_B as the magnetic moment on Co^{2+} ion at 1.5 K and about 6.3 $\mu_B/\text{f.u.}$ as the saturation magnetization. These observations of the magnetization data in GCO for T $> T_{\rm N}$ need to be reconciled with the magnetization data for T $< T_{\rm N}$ for a proper understanding of the nature of magnetism in this system. Finally, building on the results of Hoshi et al. [15], H-T phase diagram for GCO needs to be established and magnitudes of the dominant exchange couplings need to be determined.

In this paper, we report results and analyses from our detailed investigation of the temperature and magnetic field dependence of the magnetization of a polycrystalline sample of GCO to address the above questions. XPS (x-ray photo electron spectroscopy) combined with the Rietveld analysis

of the x-ray diffraction patterns is used to show that Ge⁴⁺ indeed occupies the tetrahedral A-site whereas the electron states of cobalt occupying the B-sites is Co²⁺. Temperature dependence of paramagnetic susceptibility above $T_{\rm N} =$ 20.4 K is analyzed with the modified Curie-Weiss law including temperature-independent contribution and it is shown that neglecting the temperature independent contribution to the paramagnetic susceptibility has significant effect on the derived parameters. The magnitude of the dominant ferromagnetic exchange coupling is determined from the fit of paramagnetic susceptibility to high temperature series and significant amount of short-range magnetic order at T up to $\sim 5T_{\rm N}$ is evident from this analysis. We also show that the ground state with effective spin S = 1/2 of Co^{2+} ions can systematically explain the measured magnetization both above and below T_N and magnetic moment of Co^{2+} ions measured by neutron diffraction experiments. A new H-T phase diagram for GCO is presented based on the temperature dependence of three critical fields and magnetic field dependence of T_N . New results from optical absorption studies in GCO show that its energy band gap $E_g \simeq 3.16$ eV, in agreement with $E_g =$ 3.28 eV obtained from our DFT+U calculations included here. Detailed of these results along with their discussion and analysis are given in the following pages.

II. EXPERIMENTAL DETAILS

The polycrystalline sample of GeCo₂O₄ (GCO) with bulksize grains was synthesized by the standard solid-state reaction route using the following procedure: Stoichiometric amounts of Co₃O₄ and GeO₂ were mixed in an agate mortar and sintered in air at 1000°C. After the intermediate regrinding and pelletizing process, the sample was sintered at 1200°C for 12 h in air using a high-temperature tube furnace from Nabertherm (Germany), followed by natural cooling to room temperature. The structural characterization of the sample was done using a high-resolution XPERT-PRO diffractometer (Co- K_{α} radiation with $\lambda = 1.78901$ Å). The Rietveld refinement of the diffraction data was performed using the FullProf program. Magnetization measurements were done using a SQUID-based magnetometer MPMS from Quantum Design. For optical characterization we used spectrophotometer (Perkin Elmer Lambda-950) with diffuse reflectance accessory (DRA) covering the wavelength range of 200-800 nm. For determining the electronic states of the ions and elemental analysis, we used a high-resolution x-ray photoelectron spectroscope from Kratos Axis Ultra, Model AXIS 165 equipped with an ion-gun (EX-05) for cleaning the surface. The binding-energy resolution is 0.01 eV, while background correction was done by using the Tougard algorithm and data were fitted using the x-ray photoelectron spectroscopy software XPSFIT 4.1. The core-level binding energies were aligned with the carbon binding energy of 284.8 eV.

III. ELECTRONIC AND STRUCTURAL CHARACTERIZATION

A. X-ray photoelectron spectroscopy (XPS)

Figure 1 shows photoelectron intensity (I) versus bindingenergy (eV) spectra of GCO for the individual elements:

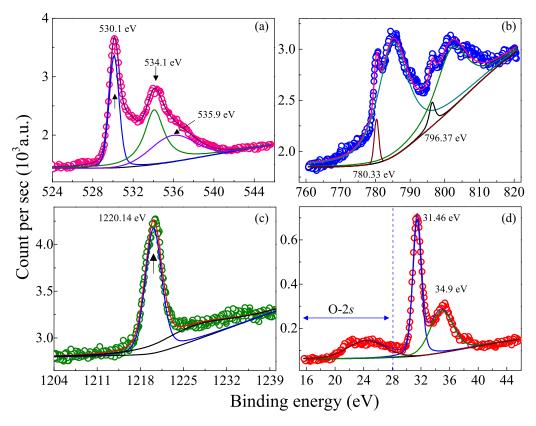


FIG. 1. X-ray-photoemission spectra of (a) O-1s, (b) Co-2p, and (c, d) Ge- $2p_{3/2}$ and Ge-3d peaks in the GeCo₂O₄ sample.

(i) O-1s, (ii) Co-2p, (iii) Ge- $2p_{3/2}$, and (iv) Ge- $3d_{5/2}$ corelevel photoelectrons. All these spectra were calibrated by selecting the binding energy of carbon C-1s orbital (located at $E_C = 284.8 \text{ eV}$) as an internal reference. The O-1s spectrum is resolved into three Gaussian-Lorentzian peaks centered at 530.1, 534.1, and 535.9 eV [as shown by arrow marks in Fig. 1(a)] [30-32]. The origin of the most intense peak at 530.1 eV is associated with the bonding between metal and lattice oxygen, in the present case it is Ge-O and Co-O [31,32]. While the second-highest intense peak appears at 534.1 eV, which is associated with the surface-absorbed oxygen [32]. The additional broad peak at 535.9 eV is mainly associated with the excess oxygen present in the system [32]. The asymmetric behavior observed in O-1s core-level spectrum is mainly due to the presence of oxygen vacancies and different atomic environment faced by the O²⁻ anions at the Wyckoff positions 32e (0.2378, 0.2378, 0.2378) [30,32]. The Co-2p core-level XPS spectrum is deconvoluted into two major peaks and two broad satellite peaks centered at 780.33 eV $(\text{Co-}2p_{3/2})$, 796.37 eV $(\text{Co-}2p_{1/2})$ and 785.03 eV, 802.01 eV, respectively (the full width of half maximum, FWHM ~ 10 eV for the satellite peak). The binding energy separation between the doublet $\Delta E_{\text{Co}}(2p_{1/2}-2p_{3/2})$ is 16.04 eV signifying the divalent oxidation state of octahedrally coordinated "Co" inside the GeCo₂O₄ [33,34]. It is notable that the higher intensities of satellite peaks as compared to the doublets characterize the loss in the system. The electronic state of Ge was analyzed by considering only the Ge- $2p_{3/2}$ core-level region, since Ge-2p exhibits significantly high spin-orbit splitting $\Delta E_{\rm Ge}(2p_{1/2}-2p_{3/2})\sim 31.1$ eV [35–38]. Figure 1(c) shows the Ge-2 $p_{3/2}$ core-level photoelectron spectrum. This spectrum comprises a main peak and a satellite peak centered at 1220.14 and 1226.56 eV, respectively, suggesting the presence of tetravalent oxidation state of Ge [36–38]. To further confirm this, we have analyzed the high-energy Ge-3d x-ray photoelectron spectrum shown in Fig. 1(d). Here, the peak profile of Ge-3d is resolved into two peaks located at 31.46 and 34.89 eV associated with the binding energies of Ge⁰⁺ (Ge-3d) and Ge⁴⁺, respectively, and the binding energy separation $\Delta \sim 3.43$ eV confirming the tetravalent oxidation state of germanium ion. An additional photoelectron peak was observed at 24.35 eV, which is originating due to the emission of O-2s core-level photoelectrons [39]. For determining the exact location (either tetrahedral-A or octahedral-B sites) of the cations and site occupancies we performed a detailed crystal structure study presented below [37,38].

B. Structural characterization

Figure 2 shows the x-ray diffraction (XRD) patterns recorded at room temperature for the GCO sample along with the corresponding Rietveld refinement data. The spectra confirm the cubic spinel structure of GCO (space group Fd3m) without evidence of any impurity phase. The Rietveld refinement was done by considering two cases: Ge^{4+} occupying the (a) tetrahedral A-sites and (b) octahedral B-sites. The red hollow symbols shown in Fig. 2 represent the experimental data, and the black solid lines are the simulated XRD patterns. The blue line is the difference between the experimental and simulated patterns with the vertical straight lines representing the position of the Bragg peaks. The goodness of the fit (σ) of the Rietveld refinement is 2.8 when Ge^{4+} occupies the

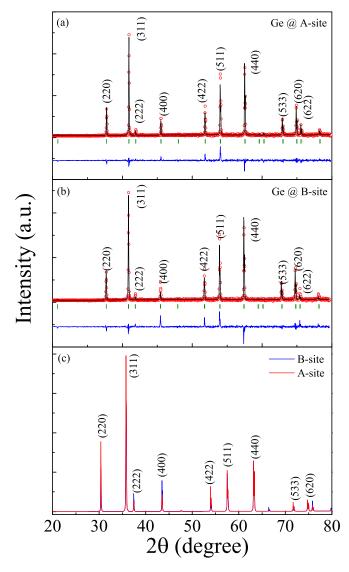


FIG. 2. Room temperature x-ray diffraction data of $GeCo_2O_4$ along with its Rietveld refinement patterns: (a) Ge-placed at tetrahedral-A site and (b) Ge at the octahedral B-site. Panel (c) shows the simulated x-ray diffraction (XRD) patterns with the red color pattern for Ge located at tetrahedral-A site and blue color pattern for Ge-situated at octahedral B-site.

tetrahedral A-site and $\sigma \sim 6.8$ when it resides in the octahedral B-site, indicating that Ge⁴⁺ occupies the tetrahedral A-site (normal spinel) instead of B-site (inverse spinel) in GCO. This is different from SnCo₂O₄ and TiCo₂O₄ which exhibit inverse spinel structure in that the nonmagnetic elements Sn and Ti occupy the B sites. The Wyckoff positions for O²⁻, Co²⁺, and Ge^{4+} are 32e(0.2378, 0.2378, 0.2378), 16c(0, 0, 0), and8b(3/8, 3/8, 3/8), respectively, if GCO crystallizes in normal spinel structure. However, if GeCo₂O₄ crystallizes in inverse spinel structure, then two Co²⁺ Wyckoff positions locate at (i) 16c(0, 0, 0) and (ii) 8b(3/8, 3/8, 3/8); and Ge^{4+} occupies the Wyckoff position 16c(0, 0, 0). Considering these two different site occupancies we have simulated the XRD patterns for GeCo₂O₄, Fig. 2(c), where the pattern generated for the normal (inverse) spinel configuration is shown in red (blue) solid line. From these simulated patterns it is noted that if

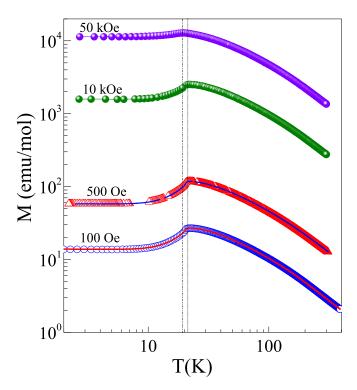


FIG. 3. Temperature dependence of magnetization, M(T), of $GeCo_2O_4$ measured with different applied DC-magnetic fields under the zero-field-cooled (symbols) and field-cooled (solid lines) conditions. Log scale is used for temperatures to highlight the data at the lower temperatures.

Ge⁴⁺ occupies the octahedral B-sites, then peak intensities of the (222) and (400) lines should be higher as compared to the Ge⁴⁺ located at A-sites. It is noted that our experimental data of intensities match well with the simulated pattern generated for 8b Wyckoff position (A-site) occupied by Ge⁴⁺ ions. Hence, these comparisons rule out the possibility of Ge⁴⁺ occupying the octahedral B-sites in GCO, which is usually observed in its sister compounds of SnCo₂O₄, TiCo₂O₄, and RuCo₂O₄ [40]. In addition, from the Rietveld refinement we have evaluated the bond-angle Co-O-Co (81.1°) and bond-length Co-O (2.96 Å), which are consistent with our density functional theory calculations (89.7° and 2.12 Å) presented later (Sec. V).

IV. MAGNETIC PROPERTIES

A. Temperature dependence of magnetic susceptibility

Temperature dependence of the magnetization (M) measured in the temperature range of 2 K to 400 K in applied H=100 Oe, 500 Oe, 10 kOe, and 50 kOe, under the zero-field-cooled (ZFC) and field-cooled (FC) protocols is plotted in Fig. 3. The peaks in M near 20 K are associated with the Néel temperature T_N ; more accurate determination of T_N and its H dependence is presented later. To analyze the temperature dependence of $\chi = M/H$ we have incorporated the data of Diaz *et al.* [13] for T up to 800 K and these combined data of χ versus T are shown in Fig. 4(a). We first analyzed these data of χ versus T in terms of Curie-law given by $\chi = C/T$ with

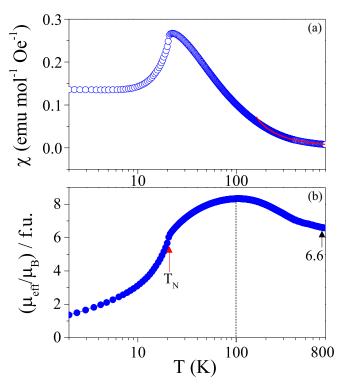


FIG. 4. (a) Temperature variation of magnetic susceptibility, $\chi(T)$, measured for $H_{\rm DC}=100$ Oe. Log scale for temperature scale is used to show details of the low-temperature variations. (b) Temperature variation of the ratio of effective magnetic moment $\mu_{\rm eff}$ and Bohr magneton $\mu_{\rm B}$ determined using the Curie law.

 $C = N_A \mu^2 / 3k_B$ (N_A = Avogadro's number, μ = magnetic moment, k_B = Boltzmann constant). Note that the measured M and χ in units of emu/g is multiplied by the molecular weight of GCO (M.W. = 254.50 mol/g) to obtain M and χ in molar units. From the Curie-law, effective $(\mu/\mu_B)^2$ = $3k_B\chi T/N_A\mu_B^2$ can be determined. Following analysis for β -Co(OH)₂ [29], the plot of (μ/μ_B) per formula unit (f.u.) of GCO against temperature depicted in Fig. 3(b) shows that with decrease in temperature, (μ/μ_B) increases peaking near 100 K and then decreases. If there were no exchange coupling among Co^{2+} ions, then (μ/μ_B) should be temperature independent. The increase in (μ/μ_B) from 800 to 100 K shows ferromagnetic (FM) coupling, whereas decrease of (μ/μ_B) for T < 100 K is indicative of onset of antiferromagnetic (AFM) coupling. The inflexion in the (μ/μ_B) versus T curve near 20 K is due to T_N . It is noted that the peak in (μ/μ_B) near 100 K in Fig. 4(b) is like the broad peak near 100 K in C_P/T versus T data of Lashley et al. [22] and it represents considerable amount of short-range FM ordering up to $5T_N$.

The data of χ versus T for $T > T_N$ is often done using the modified Curie-Weiss (CW) law given by

$$\chi = \chi_0 + C/(T - \theta). \tag{1}$$

Here χ_0 represents contributions from diamagnetic susceptibility which is present in all materials and Van Vleck susceptibility if applicable [41]. Although magnitude of χ_0 may be comparatively negligible for low temperatures, its inclusion in the high temperature data can be quite important for accurate

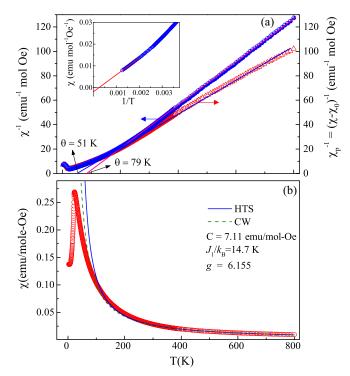


FIG. 5. (a) Inset shows the plot of magnetic susceptibility χ ($H=1000\mathrm{e}$) of $\mathrm{GeCo_2O_4}$ vs. inverse temperature to determine temperature independent term $\chi_o=-1.95\times 10^{-3}$ emu mol $^{-1}\mathrm{Oe^{-1}}$ from the y intercept. Other plots are temperature variations of $\chi_p^{-1}(=\chi-\chi_o)^{-1}$ after correcting for χ_o and that of $\chi^{-1}(T)$ without correcting for χ_o with the solid lines as fits to the Curie-Weiss law. (b) Temperature variation of magnetic susceptibility with the solid line as fit to the high temperature series (HTS) expression [Eq. (2)] and the dashed line as fit to the Curie-Weiss law [Eq. (1)].

determination of the parameters C and θ . To experimentally determine χ_0 , the data of χ versus (1/T) is plotted in the inset of Fig. 5(a) and $\chi_0 = \chi$ in the limit of $1/T \longrightarrow 0$ where the contribution of $C/(T-\theta)$ should go to zero. This analysis yields $\chi_0 = -1.95 \times 10^{-3}$ emu mol⁻¹Oe⁻¹. Using this, the plot of $1/\chi_P = 1/(\chi - \chi_0)$ versus T is made to determine C and θ . In Fig. 5(a), we show the plots of χ_P^{-1} versus T and χ^{-1} versus T. The plot of χ_P^{-1} versus T yields a straight line for T > 100 K yielding $\theta = 51$ K, whereas χ^{-1} versus Tplot yields a straight line for T > 170 K with $\theta = 79$ K. The magnitudes of θ , C and calculated (μ/μ_B) from C and other derived quantities are listed in Table I. This analysis shows that the magnitudes of θ and C are affected by χ_0 Since χ_0 was ignored in all previous publications on GCO, the magnitudes of C and (μ/μ_B) determined here are somewhat different and in our view more accurate.

The above analysis yields $C/f.u. = N_A \mu^2/3k_B$. Since C is proportional to μ^2 and GCO contains two Co^{2+} ions/f.u., then $\mu^2/f.u. = \mu_1^2 + \mu_2^2$, where $\mu_1 = \mu_2 = \mu(Co^{2+})$ in this case. So $\mu/Co^{2+} = (\mu/f.u.)/\sqrt{2}$. This distinction has also not been stressed in other publications. In Table I, we have listed the magnitudes of C, θ and other derived parameters based on both the χ_P^{-1} versus T and χ^{-1} versus T fits. Using $\mu^2 = g^2S(S+1)$, the magnitudes of derived g for both S=3/2 and effective S=1/2 are also listed along with

Parameters	χ_P^{-1} vs. T fit	$\chi^{-1} \text{ vs. } T \text{ fit}$ 5.78 ± 0.03	
C (emu/mol K)	7.11 ± 0.04		
θ (K)	51	79	
$\mu/f.u.$	$7.538 \ \mu_B \pm 0.023 \ \mu_B$	$6.796 \ \mu_B \pm 0.014 \ \mu_B$ $4.80 \ \mu_B \pm 0.01 \ \mu_B$ 2.483 ± 0.004 5.548 ± 0.012	
μ/Co^{2+}	$5.330 \ \mu_B \pm 0.016 \ \mu_B$		
g/Co^{2+} for $S = 3/2$	2.758 ± 0.002		
g/Co^{2+} for $S = 1/2$	6.1546 ± 0.019		
$\mu_z = gS/\text{Co}^{2+}$, for $S = 3/2$	$4.13~\mu_B$	$3.72~\mu_B$	
$\mu_z = gS/\text{Co}^{2+}$, for $S = 1/2$	$3.077~\mu_B$	$2.776~\mu_B$	
$M_S = 2N_A gS$ for $S = 3/2$	46,208 emu/mol	41,550 emu/mol	
$M_S = 2N_A gS$ for $S = 1/2$	34,372 emu/mol	31,002 emu/mol	

TABLE I. Parameters obtained from the Curie-Weiss fitting of magnetic susceptibility versus temperature data.

the magnetic moment $\mu_z = gS$. Later, these magnitudes are compared with experimental parameters.

B. Paramagnetic susceptibility, high temperature series, and exchange constant

We next fit the data of χ_P versus T to the high temperature series (HTS) of a spin S = 1/2 Heisenberg system for a pyrochlore lattice which is valid for GCO. This HTS is written as [42,43]

$$\chi_P = \frac{C}{T} \sum_{n=0}^{\infty} C_n \left(\frac{-2J_1}{k_B T} \right)^n . \tag{2}$$

Using the information given in Ref. [42], we determined $C_0 = 1, C_1 = 3/2, C_2 = 3/2, C_3 = -1.0625, C_4 = 0.664062,$ $C_5 = -0.60624$, $C_6 = 0.65778$, $C_7 = -0.49058$, and $C_8 = -0.60624$ 0.187472. Here J_1 is the exchange interaction in the Heisenberg exchange Hamiltonian $-2J_1\overrightarrow{S_1}.\overrightarrow{S_2}$ so that a positive J_1 represents ferromagnetic coupling. The first two terms of series of Eq. (2) can be written in the CW form with $\theta = 3J_1/k_B$. Using $\theta = 51$ K yields $J_1/k_B = 17$ K as the dominant ferromagnetic interaction. Using all eight terms in the series and C = 7.11 emu K/mol Oe and g = 6.155 listed in Table I yields a pretty decent fit to the χ_P versus T data in the 100 to 800 K range [see Fig. 5(b)]. The fit to the CW law is also shown in Fig. 5(b). The fit to HTS is only a slight improvement over the fit to the CW law with $J_1/k_B = 14.7$ K determined from HTS compared to $J_1/k_B = 17$ K evaluated from the CW fit. The fit of the data to HTS for T < 100 Kis poor because of the onset of the weaker AFM exchange coupling in this range as noted earlier. Since HTS for more than one exchange constant is not available, no information on the AFM exchange constants can be determined from this analysis.

C. Saturation magnetization and magnetic ground state

As noted in the Introduction, the ground state of Co^{2+} ions in GCO is expected to have an effective spin S=1/2. So, the low-temperature experimental results of saturation magnetization and magnetic moment per Co^{2+} ion should be interpreted on that basis. Neutron diffraction measurements in GCO by Diaz *et al.* [13,14] reported 3.02 μ_B as the magnetic moment on Co^{2+} at 1.5 K and about 6.3 $\mu_B/\text{f.u.}$ as the saturation magnetization. The magnitude of 6.3 $\mu_B/\text{f.u.}$

as the saturation magnetization M_S leads to $M_S = 35,184$ emu/mol which is in excellent agreement with the calculated $M_S = 34{,}372 \text{ emu/mol using } S = 1/2 \text{ as the ground state.}$ Considering S = 3/2 as the ground state would yield the calculated $M_S = 46,208$ emu/mol which is over 30% larger than the measured value $M_S = 35{,}184 \text{ emu/mol}$. Using g =6.1546 determined for S = 1/2 from the Curie constant C (see Table I), $\mu_z = gS = 3.077 \ \mu_B \text{ per Co}^{2+} \text{ ion is determined in}$ good agreement with the experimental results of Diaz et al. [14]. One may question the magnitude of g = 6.1546 derived for the S = 1/2 state using the fit to the CW law. However, g values up to 6.6 have been reported for Co^{2+} ions [29,44]. Also, EMR measurements reported by Okubo et al. [27] in GCO reported a broad line for $T > T_N$ with g = 5.26 for H along [111] as noted in the Introduction. Lashley et al. [22] in their fitting of χ^{-1} versus T data above T_N without including the exchange interaction determined g = 6.6 for the ground state. These considerations of the measured magnetic moment and magnetization for $T \ll T_N$ and their agreement with the calculated values assuming S = 1/2 as the ground state show that the magnetization data above $T_{\rm N}$ and below $T_{\rm N}$ in GCO can be reconciled with a single set of parameters. We consider this to be an important contribution of this work.

D. Néel temperature and its magnetic field dependence:

In Fig. 4, the temperature dependence of the $\chi(FC)$ measured in H = 100 Oe, 500 Oe, 10 kOe, and 50 kOe showed a peak in $\chi(FC)$ near $T_P \simeq 22$ K with slight decrease in T_P with increase in H. Although T_P is often associated with onset of AFM ordering, the position of the Néel temperature $T_{\rm N}$ in antiferromagnets is more accurately determined by the peak in $\partial(\chi T)/\partial T$ since χT represents the magnetic energy and so specific heat $C_p \propto \partial(\chi T)/\partial T$ [45]. In Fig. 6, the plot of $\partial(\chi T)/\partial T$ versus T is shown giving $T_N = 20.4$ K, 20.3 K, 20.0 K, and 18.0 K for H = 100 Oe, 500 Oe, 10 kOe, and 50 kOe, respectively. Our observed $T_{\rm N} = 20.4$ K for H = 100 Oe is in good agreement with $T_{\rm N} = 20.6$ K reported by Lashley et al. [22], the peak position in the specific heat in zero applied field. Lashley et al. [22] also reported the shift in the peak position of the C_P versus T data in applied fields up to H = 140 kOe. The variation of T_N versus H is fitted to the molecular-field-based

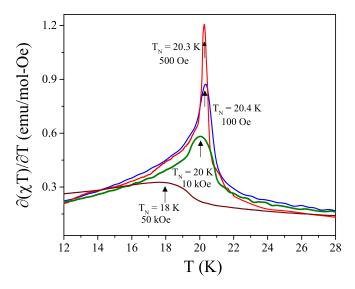


FIG. 6. Temperature derivative of the product of magnetic susceptibility χ and temperature T of the $GeCo_2O_4$ sample is plotted as a function of temperature for different applied DC-magnetic fields. The peaks marked by arrows define the Néel temperature T_N .

theoretical equation [46,47]

$$T_N(H) = T_N(0) - D_1 H^2,$$

 $D_1 = g^2 \mu_B^2 (2S^2 + 2S + 1)/40k_B^2 T_N(0).$ (3)

The plot of $T_{\rm N}$ versus H^2 is shown in Fig. 7 in which we have also included the data reported by Lashley *et al.* [22]. The expected linear variation is observed with $D_1 = 6.6 \times 10^{-10}$ K/Oe². Using g=6.1546 and S = 1/2 determined earlier for GCO (Table I) yields $D_1 = 5.24 \times 10^{-10}$ K/Oe², in fair agreement with the experimental $D_1 = 6.6 \times 10^{-10}$ K/Oe². A similar discrepancy between the

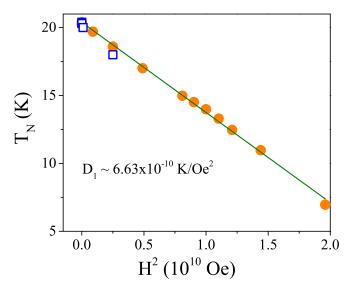


FIG. 7. Variation of the Néel temperatures T_N versus H^2 of GCO plotted to test the equation $T_N(H) = T_N(0) - D_1H^2$. The solid line is the linear fit to the experimental data points obtained from $C_P(T)$ vs. T data of Lashley *et al.* [22] (solid circles) and from the present work of $d(\chi T)/dT$ vs. T data of Fig. 6 (open squares).

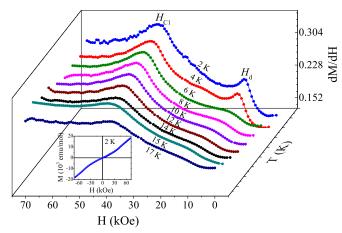


FIG. 8. Plots of computed dM/dH versus H for $GeCo_2O_4$ determined from the data of isothermal magnetization curves at different temperatures such as the curve shown in the inset measured at 2 K. The positions of the peaks determine the critical fields H_{C1} and H_d used in the H-T phase diagram of Fig. 9.

experimental and measured D_1 has been reported in antiferromagnets MnF₂ [47] and Er₂O₃ [46], and it has been attributed to the molecular field approximation used in driving Eq. (3). Using D_1 = 6.6 ×10⁻¹⁰ K/Oe² yields H_C =176 kOe as the magnetic field for which $T_N(H) \rightarrow 0$ K. This information on H_C is used in the following section.

E. Magnetic-field-induced transitions and *H-T* phase diagram

The variation of M with H up to 70 kOe recorded at 2 K is shown in the inset of Fig. 8. From such isotherms of M versus H data at different temperatures, dM/dH versus H was computed and these plots are shown in Fig. 8. From the peak positions in dM/dH, two transitions are evident, one at $H_d \simeq 11$ kOe and the other at $H_{C1} \simeq 44$ kOe. Temperature dependence of H_d and H_{C1} are displayed in Fig. 9, from which it is evident that the intensity of the peak showing H_d weakens rapidly with increasing temperature, whereas the peak corresponding to H_{C1} eventually shifts to lower H on approach to $T_{\rm N} \simeq 20.4$ K. Several other groups [13,14,24] have also reported M versus H data in GCO for $T \ll T_N$ for H up to 550 kOe and have shown the transition at $H_{C1} \simeq 44$ kOe and $H_{C2} \simeq 97$ kOe. Since our measurements are limited to H up to 70 kOe, we did not observe $H_{C2} \simeq 97$ kOe. The recent measurements of Fabréges et al. [24] who employed neutron and x-ray diffraction in single crystals of GCO are particularly noteworthy in understanding the nature of the H-induced phase transitions in H up to 100 kOe. Fabréges et al. [24] also reported a broad transition near $H_d \simeq 15$ kOe which they associated with domain reorientation in addition to the transitions at H_{C1} and H_{C2} . Below we summarize the nature of three transitions at H_d , H_{C1} , and H_{C2} and present H-T phase diagram for GCO, not reported before.

In the pyrochlore structure of Co^{2+} spins in GCO, the spins lying in the (111) planes, consist of Kagomé (KGM) spins separated by spins in the triangular (TRI) planes as first noted by Anderson [12]. In H=0 and $T\ll T_N$, spins in the neighboring KGM and TRI planes are antiparallel to each other but parallel within each plane. This provides

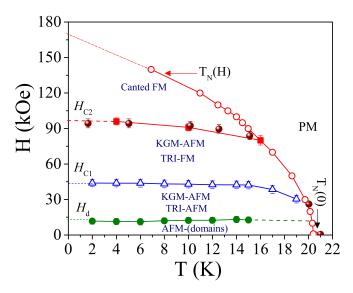


FIG. 9. The variation of the critical fields H_d , H_{C1} , and H_{C2} with temperature (T) obtained from the M-H data. Data for H_{C1} and H_d is from the present work whereas the data for H_{C2} is from Refs. [13,15]. The open circles represent the magnetic field variation of T_N shown in Fig. 7 with the solid line marked $T_N(H)$ representing the Eq. $T_N(H) = T_N(0) - D_1 H^2$. Other lines connecting the data points are drawn for visual clarity. See text for discussion of the nature of magnetic ordering in the different regions of $H < H_d$, $H_d < H < H_{C1}$, $H_{C1} < H < H_{C2}$, and $H > H_{C2}$.

overall AFM ordering although spins within each KGM and TRI planes are parallel to each other and lying in the (111) planes. Although direction of ordering within (111) planes is not uniquely defined, it is likely along either the $[11\overline{2}]$ or [110] directions, like that reported in cubic antiferromagnet MnO [48]. Since there are three equivalent such directions, this leads to formation of S domains. The weak transitions observed near $H_d \sim 11$ kOe is due to elimination of these domains. Following Fabréges et al. [24], the transition at $H_{C1} \simeq 44$ kOe represents the alignment of the TRI magnetic moments along the applied H but the spins in the neighboring KGM planes are still antiparallel. Finally, at $H = H_{C2} \simeq 97$ kOe, the spins in the KGM planes also flip to become nearly parallel to the applied H, applied along [11 $\overline{2}$] directions, although some spins canting remains. This spin canting results in nonzero susceptibility and nonsaturation even at H = 550 kOe. In summary, the transition at H_{C1} and H_{C2} are spin-flip transitions likely because the magnetic anisotropy in GCO is very large. Based on the above discussion, the H-T phase diagram for GCO is represented in Fig. 9. The boundary separating the paramagnetic (PM) phase is based on Eq. (3) describing the variation of T_N with applied H.

V. OPTICAL PROPERTIES

In this section we report our studies on the optical properties of GCO, which have not been reported in the literature till now. These measurements were performed using the diffuse reflectance spectroscopy in the UV visible and near IR range. In addition, we interpret our experimental results with our theoretically calculated density of states and

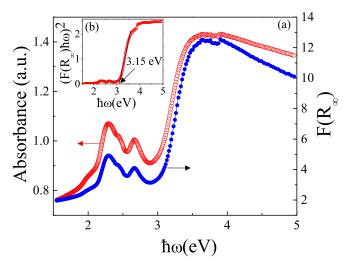


FIG. 10. (a) The variation of optical absorbance versus photon energy $(\hbar\omega)$ (shown on left-hand side scale) and Kubelka-Munk (K-M) function versus $\hbar\omega$ (shown on right-hand side scale) for GCO; (b) The inset plot shows $[F(R_\infty)\hbar\omega]^2$ versus $\hbar\omega$. The dotted line represents extrapolation of the linear region of the curve providing the optical energy band gap, E_g .

the band-structure using density functional theory DFT+U(U being the Columbic potential). We have employed the density functional theory (DFT) based calculations [49,50] using the projector augmented-wave (PAW) method [51,52] as implemented in Vienna ab initio simulation package (VASP) [53–55]. Perdew-Burke-Ernzerhof (PBE) parameterized generalized gradient approximation (GGA) for exchange correlation functional was employed [56]. The simulations were performed using 650 eV as the kinetic energy cut off of plane wave basis and Monkhorst-Pack of $8 \times 8 \times 8$ k-grid mesh. The calculations are carried out using 14 atoms (2 formula units of the spinel primitive cell). The effects of electron localization were addressed by the approach of Dudarev et al. [57]. The Hunds coupling parameter, J is considered to be 0 eV and the Coulomb parameter U is considered to be 2 eV for Co and 0 eV for Ge. The electronic self-consistency is continued until the energy convergence is of the order of 10⁻⁷ eV. Structural relaxations are performed until residual forces on each atom converge to less than 10^{-4} eV/Å.

To determine the optical band gap E_g from the Kubelka-Munk (KM) analysis of the experimental data [58,59], we employed KM equation $[F(R_{\infty})\hbar\omega]^2 = \alpha(\hbar\omega - E_g)$, where R_{∞} is the ratio of the reflectance of the sample and $F(R_{\infty})$ is the KM function [58]. In the above equation, $\hbar\omega$ is the energy of the single photon and α is absorption coefficient. The left-hand scale of Fig. 10(a) shows the intensity of optical absorbance of GCO versus the photon energy $(\hbar\omega)$ (eV) and the variation of $F(R_{\infty})$ as a function of photon energy is shown in the right-hand scale of Fig. 10(b). To determine the band gap, the function $[F(R_{\infty})\hbar\omega]^2$ versus $\hbar\omega$ is plotted and the extrapolation of the band tail (as shown by the dotted lines) intercepts the photon energy axis at $\hbar\omega = 3.156$ eV, which corresponds to the direct band gap of the system and is in good agreement with the theoretical calculations $E_g \sim 3.28$ eV. Two additional subbands are observed at 2.29 and 2.66 eV

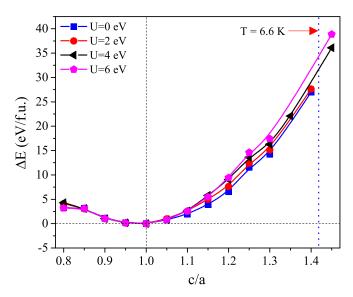


FIG. 11. The variation of total free energy as a function of tetragonal distortion (c/a) for GCO for different values of U.

(Fig. 10) below the main absorption band (3.63 eV), which may be associated to the charge transition between $\text{Co}^{2+}(e_g^{\uparrow}) \rightarrow \text{Co}^{2+}(t_{2g}^{\downarrow})$ and $\text{Co}^{2+}(t_{2g}^{\downarrow}) \rightarrow \text{Co}^{2+}(e_g^{\downarrow})$, respectively. The main absorption peak arises due to charge transfer between O $(2p) \rightarrow \text{Co}^{2+}(e_g^{\downarrow})$ [60].

As noted earlier, several publications have reported cubic to tetragonal distortion in GCO at low temperature (T < 16 K) [15,17,22-25]. To take account of this we have performed the DFT calculations to compute the total energies per formula unit after distorting the cubic structure, i.e., c is fixed at 8.32 Å and a is varied between 10.4 Å and 5.546 Å. In Fig. 11, the total energy is plotted as a function of c/a ratio for different values of U(=0, 2, 4, and 6 eV). The experimental observation at T = 6.6 K is shown by blue dotted line in the Fig. 11. The DFT results show the cubic structure is more stable than the tetragonal structure for all the values of U. Hence, we carried out all the electronic structure calculations considering the cubic structure of GCO. Generally, in the DFT calculations the coulomb interaction parameter U is used to take account of the valence electron interactions [61]. The free parameter U has been chosen in such a way to match the experimental observations. To investigate the effect of U on the electronic structure we performed a detailed computation of the density of states of GCO system by varying U between 0 and 6 eV. Figures 12(a)12(c) show the density of states plots for U = 0, 2, 4 eV, respectively. For all the cases we find that the spin-up and spin-down density of states are nearly equal and exhibiting an antiferromagnetic configuration which is consistent with our experimental observations. For U = 0 eV [Fig. 12(a)], the Fermi level falls below the maximum of the valence band indicating metallic nature of GCO in contradiction to the experimental observations. However, for the finite U values the system displays semiconductor/insulating behavior and the energy band gap gradually increases while increasing the U values. Figure 12(b) shows the density of states of GCO calculated using U = 2.0 eV for Co ions. Close examination

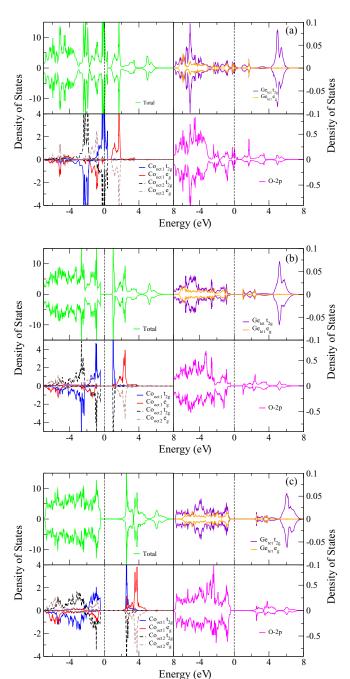


FIG. 12. Total and atom-projected electronic density of states of GCO is plotted as function of energy for (a) U=0 eV, (b) U=2 eV, and (c) U=4 eV for octahedral Co²⁺ ions.

of these plots reveals the electronic states at $E \sim -1.39 \text{ eV}$ and -2.62 eV pertaining to e_g^{\uparrow} and t_{2g}^{\downarrow} , respectively. However, t_{2g}^{\uparrow} states are localized at the top of the valence band maximum ($\sim -0.88 \text{ eV}$) and the conduction minimum ($\sim 1.02 \text{ eV}$). Interestingly, both the octahedral Co ions are willing to compensate each other contribution and yielding a stable antiferromagnetic structure of GeCo_2O_4 . The contribution from tetrahedral Ge^{4+} is negligible to the total magnitude of density of states. All the optical transitions observed in GCO are listed in Table II together with those determined from the

TABLE II. The list of optical transitions and their positions obtained from the experimental results and DFT+U-based theoretical calculations.

Transitions	U (eV)	Type	Theoretical energy (eV)	Experimental energy (eV)
$Co^{2+}(t_{2g}^{\uparrow\downarrow}) \rightarrow Co^{2+}(t_{2g}^{\uparrow\downarrow})$	1.0	d-d	1.14	1.95
	2.0		1.96	
	3.0		2.72	
	4.0		3.49	
$\operatorname{Co}^{2+}(e_g^{\uparrow}) \to \operatorname{Co}^{2+}(t_{2g}^{\uparrow})$	1.0	d- d	1.76	2.29
	2.0		2.25	
	3.0		2.72	
	4.0		3.49	
$\operatorname{Co}^{2+}(e_g^{\downarrow}) \to \operatorname{Co}^{2+}(t_{2g}^{\downarrow})$	1.0	d- d	1.95	2.67
	2.0		2.38	
	3.0		2.79	
	4.0		3.89	
$O(2p) \to Co^{2+}(e_g^{\uparrow\downarrow})$	1.0	p-d	2.45	3.16
	2.0	-	3.26	
	3.0		3.89	
	4.0		4.48	

DFT+U calculations and interpreted using the band structure and density of states of GCO (Figs. 12(a)-12(c) and 13). In what follows we discuss the changes occurring in the density of states as U is increased from U=0-6 eV as shown in Figs. 12(a)-12(c). For U=2 eV, the t_{2g}^{\uparrow} states of octahedral Co^{2+} are localized both at the top of the valence band and bottom of the conduction band. However, for higher U values (≥ 4 eV) the t_{2g}^{\uparrow} states in the conduction band dominates over the valence band. Similar features are noticeable in case of t_{2g}^{\downarrow} states. For the lower U values, the splitting in the t_{2g}^{\downarrow} states are visible in the valence band, which gradually diminishes and gets delocalized near the Fermi level for higher U values (≥ 4 eV). Upon increasing U all the states (t_{2g}^{\uparrow} , t_{2g}^{\uparrow}) move toward higher energies, where the shift in the t_{2g}^{\downarrow} states are more significant as compared to the others.

To interpret the optical measurements, we use the density of states data to predict the possible optical transitions for dif-

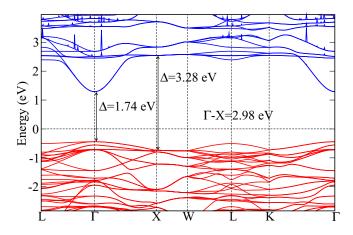


FIG. 13. The band structure of GCO is plotted with the symmetry points in the reciprocal lattice for U = 4.0 eV for Co^{2+} ions.

ferent values of U ($U=1-4~{\rm eV}$), which are listed in Table II along with those values obtained from the experiments. We note that the theoretical values of the optical transitions for $U=2.0~{\rm eV}$ match quite well with the experimentally observed transitions, whereas the optical transitions corresponding to $U\geqslant 3~{\rm eV}$ appeared to be higher than the experimental results. Moreover, the energy band gap of GCO calculated using the Coulombic interaction $U=2.0~{\rm eV}$ for Co-ions yields at Γ - Γ and X-X points are 1.39 and 2.60 eV, respectively, whereas the indirect band gap at Γ -X occurs at 2.28 eV. It is noted that our calculations for $U=4.0~{\rm eV}$ (for the Co ions) give the direct X-X band gap of 3.28 eV (Fig. 13), which is in good agreement with our experimentally obtained optical band gap of 3.156 eV (Fig. 10).

VI. SUMMARY AND CONCLUSIONS

The significant results presented here include the following: (i) The χ_P versus T data above T_N and saturation magnetization below T_N in GCO have been interpreted using the same set of parameters for Co^{2+} ions, namely effective spin S = 1/2 and g = 6.155; (ii) these parameters also explain the measured magnetic moment = $3.02 \mu_B$ per Co²⁺ reported by Diaz et al. using neutron diffraction; (iii) our analysis shows that neglecting the temperature independent term in magnetic susceptibility makes a significant difference on the determined parameters; (iv) fitting the magnetic susceptibility data to HTS, the magnitude of the dominant ferromagnetic exchange constant $J_1/k_B = 14.7$ K has been determined; (v) the plot of $\mu_{\rm eff}$ / $\mu_{\rm B}$ in Fig. 4(b) shows the presence of short-range 2D ferromagnetic order near 100 K, in agreement with the broad peak in C_p/T versus T data of Lashley *et al.* [22]; (vi) Electronic states of Ge^{4+} and Ge^{2+} in Ge^{2+} determined from the XPS studies and Rietveld refinement of the XRD patterns establishes Ge⁴⁺ and Co²⁺ occupying the tetrahedral and octahedral sites respectively; and (vii) a direct energy band gap $E_g = 3.156$ eV in GCO is determined

from the diffuse reflectance spectroscopy which is in good agreement with those obtained from DFT+U calculations ($E_g = 3.28 \text{ eV}$). The remaining issues for complete understanding of the magnetic properties of GCO include the determination of the important interlayer AFM exchange constants, interpretation of the AFMR modes reported by Okubo *et al.* [27], and theoretical interpretation of the critical fields of Fig. 9 in terms of the anisotropy and exchange constants in a manner e.g. reported for the quasi-2D, spin 1/2 antiferromagnet copper formate tetrahydrate [62]. However, as noted earlier, GCO has two types of spins (KGM and TRI) and several different exchange couplings among them making this task quite challenging.

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