Universal dielectric response across a continuous metal-insulator transition

Prosenjit Haldar,[1,2,3,*] M. S. Laad,[2,3,†] and S. R. Hassan[2,3,‡]

[1]*Centre for Condensed Matter Theory, Department of Physics, Indian Institute of Sciences, Bangalore 560012, India*

[2]*Institute of Mathematical Sciences, Taramani, Chennai 600113, India*

[3]*Homi Bhabha National Institute Training School Complex, Anushakti Nagar, Mumbai 400085, India*

(Received 1 October 2018; revised manuscript received 18 March 2019; published 26 March 2019)

A wide range of disordered materials, including disordered correlated systems, show universal dielectric response (UDR), followed by a superlinear power-law increase in their optical responses over exceptionally broad frequency regimes. While extensively used in various contexts over the years, the microscopics underpinning UDR remains controversial. Here, we investigate the optical response of the simplest model of correlated fermions, the Falicov-Kimball model, across the continuous metal-insulator transition (MIT) and analyze the associated quantum criticality in detail using cluster extension of dynamical mean-field theory. Surprisingly, we find that UDR naturally emerges in the quantum critical region associated with the continuous MIT. We tie the emergence of these novel features to a many-body orthogonality catastrophe accompanying the onset of strongly correlated *electronic glassy dynamics* close to the MIT, providing a microscopic realization of Jonscher's time-honored proposal as well as a rationale for similarities in optical responses between correlated electronic matter and canonical glass formers.

DOI: 10.1103/PhysRevB.99.125147

I. INTRODUCTION

Optical conductivity has long been used to characterize elementary excitations in condensed matter. The response of matter to ac electromagnetic fields is usually encoded in the complex conductivity [1], $\sigma_{xx}(\omega) = \sigma'(\omega) + i\sigma''(\omega)$, or the complex dielectric constant $\epsilon(\omega)$, related to each other by $\sigma'(\omega) = \omega\epsilon_0\epsilon''(\omega)$, where $\epsilon''(\omega)$ quantifies the dielectric loss and ϵ_0 is the permittivity of free space. Optical studies have been especially valuable in strongly correlated electronic matter [2] and, as a particular example, have led to insights into the breakdown of traditional concepts in cuprates [3].

Such studies have also led to much progress in understanding complex charge dynamics in disordered matter. In the 1970s, the pioneering work of Jonscher [4,5] showed a universal dielectric response (UDR) of disordered quantum matter to ac electromagnetic fields, wherein $\sigma_{xx}(\omega) \simeq \omega^{\alpha}$, with $\alpha \leqslant 1$ in the subgigahertz regime. More recently, Lunkenheimer and Loidl [6] reported astonishingly similar responses in a wide class of disordered matter over a more extended energy window: among others, doped, weakly, and strongly correlated semiconductors exhibit UDR, followed by a superlinear power-law increase in $\sigma(\omega)$, bridging the gap between classical dielectric and infrared regions. This behavior is also common to dipolar and ionic liquids as well as to canonical glass formers. Very recently, materials which belong to the elusive class of spin liquids [7] were also interpreted in terms of UDR: in this case, it is possible that intrinsic disorder, arising from geometric frustration, is implicated in the emergence of UDR. This suggests involvement of a deeper, more fundamental and common element, related to the onset of a possibly intrinsic, glassy dynamics in the emergence of UDR. In the context of correlated quantum matter (such as the Mott insulator $LaTiO_3$ and $Pr_{0.65}(Ca_{0.8}Sr_{0.2})_{0.35}MnO_{0.35}$ (PCSMO) [6]), such unconventional "glassy" dynamics can emerge near the doping-induced metal-insulator transition (MIT) as a consequence of substitutional and/or intrinsic disorder due to inhomogeneous electronic phase(s) near the MIT. On the other hand, early on, Jonscher himself suggested the relevance of many-body processes akin to the seminal Anderson orthogonality catastrophe (AOC) for UDR. Thus, the link between AOC and an emergent, slow glassy dynamics underlying the electronic processes leading to UDR in disordered, interacting electronic systems remains a challenging and largely unaddressed issue for theory, to the best of our knowledge.

Motivated thereby, we investigate these issues with a careful study of the optical response of the Falicov-Kimball model (FKM). The FKM is the simplest representative model of correlated electrons on a lattice and possesses an exact solution within both dynamical mean-field theory (DMFT) [8] and its cluster extensions (CDMFT) [9,10]. Remarkably, it can be solved almost analytically, even in CDMFT [10], leading to enormous computational simplifications in transport studies [11–13]. Across a critical U, the FKM is known to undergo a $T = 0$ continuous MIT of the Hubbard band-splitting type [8].

As found earlier for transport properties, it turns out that precise computation of the optical response for the FKM within two-site cellular DMFT [14] is facilitated by the facts that (i) explicit closed-form expressions for the cluster propagators $G(\mathbf{K}, \omega)$, with $\mathbf{K} = (0, 0), (\pi, \pi)$, greatly reduce computational cost, even in CDMFT, and (ii) the cluster-resolved

*prosenjith@iisc.ac.in
†mslaad@imsc.res.in
‡shassan@imsc.res.in

irreducible particle-hole vertex functions are negligibly small and we ignore them in the Bethe-Salpeter equations for all conductivities, thanks to an almost rigorous symmetry argument [15]: upon a cluster-to-orbital mapping (which is implicit in our mapping of the two-site CDMFT to two, "S, P" channels [14]). In this "multiorbital" scenario, the irreducible vertex corrections entering the Bethe-Salpeter equation for the conductivity are still negligibly small [16]. Thus, the optical conductivity acquires a form similar to DMFT, but with an additional sum over cluster momenta (or the two S and P channels on the cluster).

The rest of this paper is organized as follows: In Sec. II we describe the model we study in this work and the calculation of optical conductivity using cluster DMFT formalism. In Sec. III we present our numerical results and analyze (i) Mott-like quantum criticality in the optical response using CDMFT and (ii) universal dielectric response across the MIT. We then tie the UDR to an emergent many-body orthogonality catastrophe in the FKM within our CDMFT approach. In Sec. IV we discuss our findings in the context of real materials exhibiting UDR.

II. GENERAL FORMULATION OF OPTICAL CONDUCTIVITY WITHIN CLUSTER DMFT

The Hamiltonian of the spinless FKM model is

$$H_{\rm FKM} = -t\sum_{\langle i,j\rangle}(c_i^\dagger c_j + {\rm H.c.}) - \mu\sum_i n_{i,c} + U\sum_i n_{i,d}n_{i,c}. \tag{1}$$

The Hamiltonian describes a band of dispersive fermions $(c, c^\dagger)$ interacting locally via a "Hubbard"-type interaction with dispersionless d fermions. Since $v_i = Un_{i,d}$ is a random (binary) potential in the symmetry-unbroken phases of the FKM, Eq. (1) can also be viewed (as has long been known [17]) as a model of fermions in a random binary alloy potential.

In recent work [14], we used a cluster extension of DMFT to solve FKM using the Dyson-Schwinger equation of motion technique. Remarkably, the cluster-local Green's function in two-site cluster DMFT is obtained analytically and reads

$$\hat{\mathbf{G}} = \begin{bmatrix} G_{00}(\omega) & G_{\alpha 0}(\omega) \\ G_{\alpha 0}(\omega) & G_{00}(\omega)\end{bmatrix},$$

where the matrix element $G_{ij}(\omega)$, with $i, j = 0, \alpha$, is

$$G_{ij}(\omega) = \left[\frac{1-\langle n_{0d}\rangle - \langle n_{\alpha d}\rangle + \langle n_{0d}n_{\alpha d}\rangle}{\xi_2(\omega)} + \frac{\langle n_{0d}\rangle - \langle n_{0d}n_{\alpha d}\rangle}{\xi_2(\omega)-U}\right]\left[\delta_{ij} - \frac{F_2(\omega)}{[t-\Delta_{\alpha 0}(\omega)]}(1-\delta_{ij})\right] + \left[\frac{\langle n_{\alpha d}\rangle - \langle n_{0d}n_{\alpha d}\rangle}{\xi_1(\omega)} + \frac{\langle n_{0d}n_{\alpha d}\rangle}{\xi_1(\omega)-U}\right]\left[\delta_{ij} - \frac{F_1(\omega)}{[t-\Delta_{\alpha 0}(\omega)]}(1-\delta_{ij})\right], \tag{2}$$

with $\xi_1(\omega) = [\omega - \Delta_{00}(\omega) - F_1(\omega)]$, $\xi_2(\omega) = [\omega - \Delta_{00}(\omega) - F_2(\omega)]$, and $F_1(\omega) = \frac{(t-\Delta_{\alpha 0})^2}{\omega - U - \Delta_{00}(\omega)}$, $F_2(\omega) = \frac{(t-\Delta_{\alpha 0})^2}{\omega - \Delta_{00}(\omega)}$, where the bath function $\hat{\mathbf{\Delta}}(\omega)$ is related to the local Green's function through a suitable self-consistency condition. The self-energy is given as

$$\hat{\mathbf{\Sigma}}(\omega) = \hat{\mathcal{G}}_0^{-1}(\omega) - \hat{\mathbf{G}}^{-1}(\omega), \tag{3}$$

where $\hat{\mathcal{G}}_0(\omega)$ is the Weiss Green's function, $\hat{\mathcal{G}}_0(\omega) = (\omega + \mu)1 - \hat{\mathbf{\Delta}}(\omega)$. We use the algorithm described in Ref. [14] to find the local Green's function and self-energy. In the symmetric basis (cluster momentum basis) we can write $G_S = (G_{00} + G_{\alpha 0})$ and $G_P = (G_{00} - G_{\alpha 0})$, where S and P are even and odd orbitals, respectively.

The optical conductivity is evaluated using the Kubo-Greenwood formalism. In the near absence of vertex corrections, only the bare bubble, composed from the CDMFT propagators, contributes. The explicit form of the optical conductivity within cluster DMFT then reads

$$\sigma'(\omega) = \sigma_0 \sum_{\mathbf{K}\in[S,P]} \int_{-\infty}^{\infty} d\epsilon\, v^2(\epsilon)\rho_0^{\mathbf{K}}(\epsilon) \times \int_{-\infty}^{\infty} d\tilde{\omega} A_{\mathbf{K}}(\epsilon,\tilde{\omega})A_{\mathbf{K}}(\epsilon,\tilde{\omega}+\omega)\frac{f(\tilde{\omega})-f(\tilde{\omega}+\omega)}{\omega}, \tag{4}$$

with

$$A_{\mathbf{K}}(\epsilon,\omega) = {\rm Im}\left[\frac{1}{\omega+\mu-\epsilon-\Sigma_{\mathbf{K}}(\omega)}\right]. \tag{5}$$

Here, $\rho_0^{\mathbf{K}}(\epsilon)$ is the noninteracting spectral function of the even and odd orbitals, and $f(\omega)$ is the Fermi distribution. This simplification allows a comprehensive study of the optical response of the FKM within CDMFT, which we now describe.

III. RESULTS AND DISCUSSION

We consider the Bethe lattice with the half bandwidth of the conduction electron (c fermions) as unity ($2t = 1$). We define the short-range order parameter $f_{0\alpha}$ as $f_{0\alpha} = \langle n_{0d}n_{\alpha d}\rangle - \langle n_{0d}\rangle\langle n_{\alpha d}\rangle$.

A. Quantum criticality near the MIT

We exhibit the real part of the optical conductivity near and across the MIT ($1.6 \leqslant U \leqslant 2.0$), computed from Eq. (4) as a function of U for (a) the completely disordered case (short-range order parameter $f_{0\alpha} = 0$ in our earlier work [14]) in the top panel of Fig. 1 and (b) the short-range ordered case ($f_{0\alpha} \neq 0$) in Fig. 2. Several features stand out clearly: in case (a), $\sigma'(\omega)$ shows an incoherent low-energy bump centered at $\omega = 0$, whose weight decreases continuously as the MIT

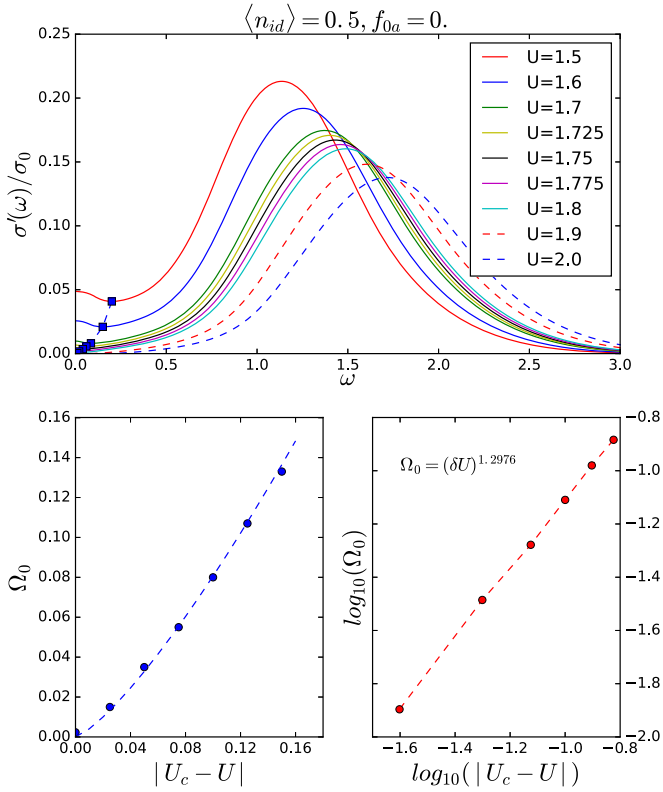

FIG. 1. Optical conductivity of the completely random ($f_{0\alpha} = 0$) FKM within two-site CDMFT, showing its evolution with U at temperature $T \to 0$. The MIT occurs at $U_c = 1.8$. Blue symbols show how an emergent scale $\Omega_0(U)$, associated with a smooth crossover between metallic and insulating states, collapses at the Mott transition ($U = 1.8$) as $(\delta U)^{\nu}$ with $\nu = 1.29$, close to 4/3 (see text).

is approached (at $U = 1.8$). It is important to note that (i) there is no low-energy Drude component in $\sigma'(\omega)$ since the CDMFT propagators have no pole structure [14] and (ii) as expected, low-energy spectral weight is continuously transferred from the bad-metallic and midinfrared (MIR) regions to high energies $O(U)$ across the MIT. This is characteristic of a correlation-driven MIT, and the continuous depletion of low-energy weight is a consequence of the continuous MIT in the FKM driven by increasing U. In Fig 2, we exhibit the effect of "antiferromagnetic alloy" (AF-A) short-range order (SRO). Apart from the fact that the MIT now occurs at $(U) \simeq 1.35$ [14], the above features seem to be reproduced in this case as well. Looking more closely, however, we see marked changes in the low- and midinfrared energy range: the bad-metallic bump centered at $\omega = 0$ is suppressed by SRO, and $\sigma'(\omega)$ rises faster with ω in the MIR, showing the emergence of a low-energy pseudogap. These changes are to be expected since AF-A SRO reduces the effective kinetic energy and increases the effective U, leading to a reduction of the low-energy spectral weight and a low-energy pseudogap in optics.

A closer look at Fig. 1 reveals very interesting features. We uncover a crossover scale $\Omega_0(U)$, separating "metallic" and "insulatorlike" behaviors in $\sigma'(\omega)$. As expected, it collapses at the MIT: interestingly, we find $\Omega_0(\delta U) \simeq (\delta U)^{1.29}$,

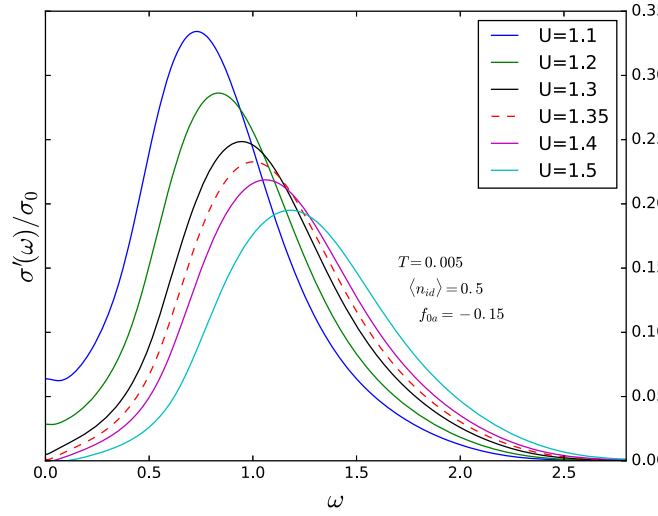

FIG. 2. The real part of the optical conductivity of the FKM with antiferromagnetic short-range order ($f_{0\alpha} = -0.15$) within two-site CDMFT close to the MIT ($1 < U < 1.5$). The critical curve at which the MIT occurs corresponds to $(U)_c = 1.35$ (red dashed line).

quite close to $\nu = 4/3$ found in earlier work [11]. This also motivates us to investigate underlying quantum criticality in the optical response. In Fig. 3, we show that $\log_{10}[\sigma_c/\sigma'(\omega)]$ plotted versus $\omega/\Omega_0(U)$ (the latter is taken from Fig. 1) indeed reveals clean quantum-critical scaling: the insulating (I) and metallic (M) data fall on two master curves, and the beautiful mirror symmetry relating the two testifies to the unambiguous manifestation of the "Mott" quantum critical point (QCP) in the optical response. Further, we also find that $\Omega_0(\delta U) \simeq c|\delta U|^{\eta}$, with $\eta = 1.3 \simeq 4/3$, in excellent agreement with both Fig. 1 and our previous study. Using our earlier result $\xi \simeq (U - U_c)^{-\nu}$, with $\nu = 4/3$ and $z = 1$, we thus expect that $\sigma'(\omega)/\sigma_c$ should scale as $y = |U - U_c|/U_c\omega = 1/\omega\xi^{1/z\nu}$, i.e.,

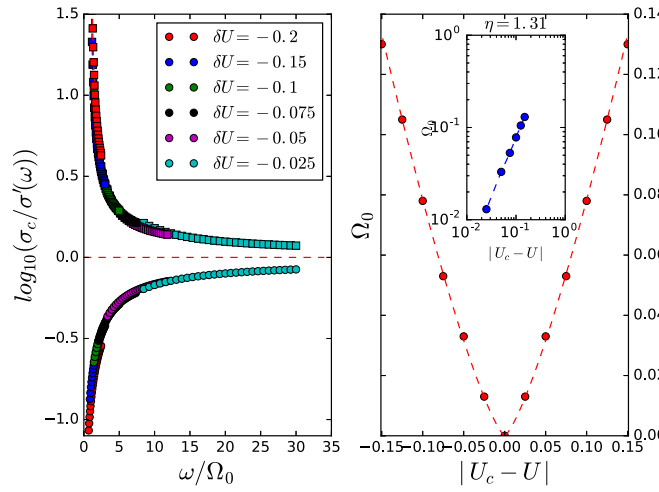

FIG. 3. Clean quantum critical scaling of the optical conductivity across the Mott QCP, as shown by the fact that $\log_{10}[\sigma_c/\sigma'(\omega)]$ versus $\omega/\Omega_0(U)$ for the metal and insulator phases falls on two universal "master" curves. σ_c is the optical conductivity at the critical U, i.e., the separatrix. We estimate $\Omega_0(\delta U) \simeq (\delta U)^{\eta}$, with $\eta = 1.31$, in very good agreement with $\nu = 4/3$ from earlier work [11].

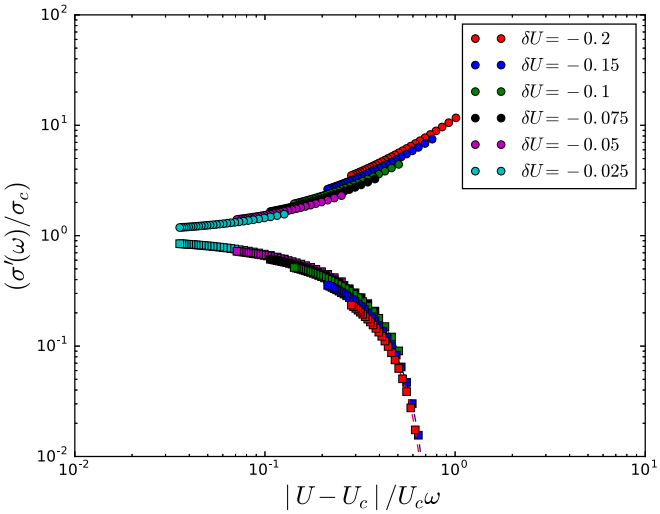

FIG. 4. Clean quantum criticality as revealed in the scaling behavior of the CDMFT optical conductivity, $\sigma'(\omega)/\sigma_c$ versus $y = |U - U_c|/U_c\omega$. Since the localization length $\xi \simeq |U - U_c|^{-4/3}$ (see text), this implies that $\sigma'(\omega)/\sigma_0 = F(\omega\xi^{1/z\nu})$. This is a manifestation of the time-temperature superposition principle [5] following from Jonscher's UDR.

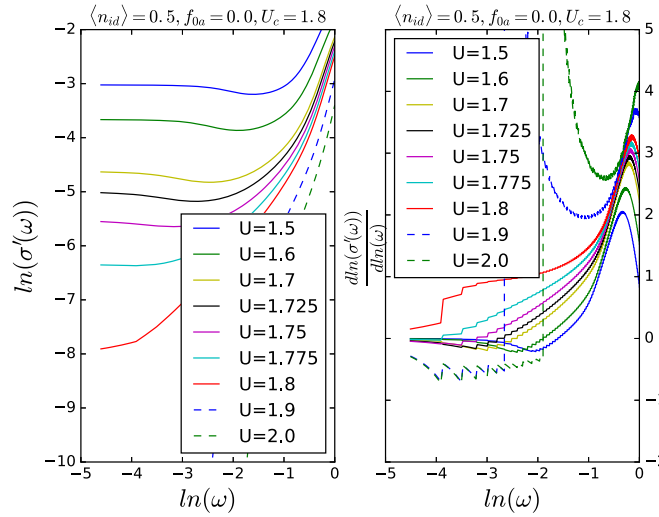

FIG. 5. Real part of the optical conductivity versus frequency ω, plotted on a log-log scale to facilitate direct comparison with data for disordered and correlated electronic systems from Lunkenheimer and Loidl [6]. Very good agreement is clearly seen. More importantly, the crossover from the dc limit to UDR around $\ln(\omega) \simeq -3$ close to the Mott QCP (for $U \simeq 1.8$) is also revealed, showing that UDR emerges in the quantum critical region associated with the continuous MIT.

that $\sigma'(\omega)/\sigma_c = F(\omega\xi^{1/z\nu})$. This is indeed adequately borne out in Fig. 4 for both M and I phases. This is a manifestation of the "time-temperature superposition principle" [5], expressible as a scaling law, $\frac{\sigma'(\omega)}{\sigma_c} = F(\frac{\omega}{\Omega_0})$, with F being a T-independent scaling function and $\Omega_0(U)$ being a scaling parameter corresponding to the onset of conductivity dispersion, precisely as found here. The variation of Ω_0 with U reflects the nontrivial interplay between itinerancy (hopping) and Mott-like localization in the FKM. In analogy with the parameter $T_0(\delta U) \simeq c(\delta U)^{z\nu}$ for the dc transport criticality [11], Ω_0 also scales like $(\delta U)^{z\nu}$. Finally, the fact that $\Omega_0(\delta U) \simeq (\delta U)^{z\nu}$ in *both* the M and I phases reflects the fact, alluded to in earlier work [11,18], that the basic electronic processes governing the I phase are also relevant deep into the M phase.

B. Universal dielectric response

Having shown a Mott-like quasilocal quantum criticality, we now turn to the UDR near the MIT. Since the FKM is isomorphic to the binary-alloy Anderson disorder problem, we inquire how CDMFT performs in the context of the remarkable universality in the dielectric response in disordered quantum matter alluded to before [6]. In Figs. 5 and 6, we show $\ln\sigma'(\omega)$ and the dielectric loss, $\ln[\sigma'(\omega)/\omega]$ versus $\ln(\omega)$, as functions of U to facilitate meaningful comparison with data of Lunkenheimer and Loidl. It is indeed quite remarkable that *all* the basic features reported for disordered quantum matter are faithfully reproduced by our CDMFT calculation. Specifically, (i) for $1.5 < U < 1.8$, a "dc" conductivity regime at the lowest energy (up to 10^{-4}–10^{-3}) smoothly goes over to a sublinear-in-ω regime (UDR, in the region 10^{-2}–10^{-1}) followed by a superlinear-in-ω regime (around 10^{-1}), connecting up smoothly with the "boson" peak. These regimes are especially visible around $U = 1.8$, precisely where the MIT occurs. (ii) Moreover, corroborating behavior is also clearly seen in Fig. 6, where we exhibit the dielectric loss function vs ω on a log-log scale. It is clearly seen that a shallow minimum separates the UDR and superlinear regimes at approximately $\ln(\omega) = -0.8$ (in the meV region) in the very bad metallic state close to the MIT. This is in excellent agreement with results for both $LaTiO_3$ and PCSMO [6]. Moreover, the energy dependence of the optical conductivity also seems to be in good qualitative agreement with data when we compare our results with Figs. 1, 2, and 3

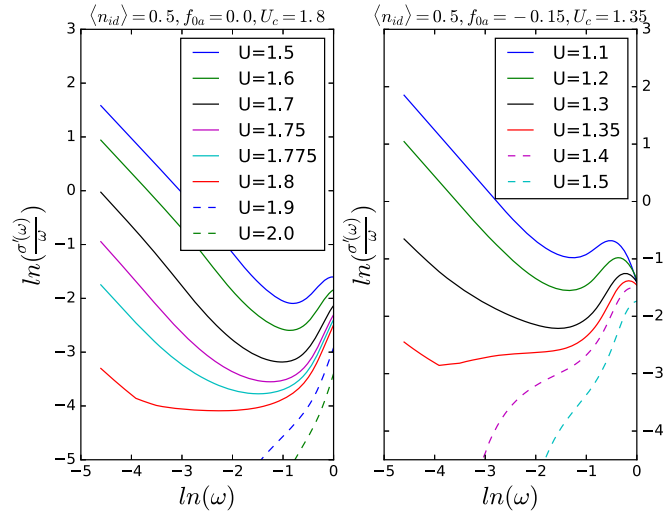

FIG. 6. The dielectric loss $\mathrm{Re}\sigma_{xx}(\omega)/\omega$, plotted versus ω on a log-log scale to facilitate direct comparison with Lunkenheimer and Loidl [6]. In excellent agreement with data on correlated and disordered systems in Ref. [5], a shallow minimum separates the UDR from a superlinear power-law regime around $\ln\omega \simeq -1, -2$ as U increases from 1.5 up to 1.8, the critical value for the Mott QCP.

of Lunkenheimer and Loidl. In the right panel of Fig. 6, we also show that short-range spatial correlations do not qualitatively modify these conclusions, attesting to their robustness against finite short-range order. Finally, precisely at the MIT (red curves in Fig. 6), we unearth a very interesting feature: $\mathrm{Im}\epsilon(0,\omega) \simeq \omega^{-\eta}$, with $\eta = 0.8, 0.75$ for $f_{0\alpha} = 0.0, -0.15$. Hence, apart from a "dielectric phase angle" $\cot(\pi\eta/2)$, the real part of the dielectric constant also varies like $\omega^{-\eta}$ as the MIT is approached from the metallic side. Remarkably, this is a concrete manifestation of the dielectric (or polarization) catastrophe that is expected to occur at a QCP associated with a continuous MIT.

This wide-ranging agreement with data is quite remarkable and begs a microscopic clarification in terms of basic electronic processes at work near the MIT. A phenomenological way is to posit that the universality is linked to glassiness [6] as follows: (i) first, our finding [11] of $\nu = 4/3$ (and $z = 1$) is characteristic of percolative transport that is naturally expected to arise in glassy systems. (ii) It has also been shown [19] (for the disorder problem) that electronic glassy behavior *precedes* an insulating phase. Thanks to the mapping between the FKM and a binary-alloy "disorder" problem, we also expect an *intrinsic* electronic glassy phase near the continuous MIT in the FKM. This suggests a close link between UDR and the onset of electronic glassy dynamics near the continuous MIT in the FKM.

On a more microscopic level, it is *very* interesting that Jonscher himself [5] hinted at the relevance of many-body processes at the heart of UDR. In particular, an explicit parallel with the seminal x-ray edgelike physics was already (phenomenologically) invoked to account for such features. It is indeed remarkable that such x-ray edge physics naturally falls out in the DMFT *and* CDMFT solutions of the FKM in high D [20]: in DMFT, the mobile (c-fermion) propagator exhibits a pseudogapped metal going over to an insulator across a critical $U = U_c$, while the d-fermion propagator exhibits an "x-ray edge" singular behavior linked to the seminal Anderson orthogonality catastrophe (AOC). In our CDMFT [14], we find similar behavior: (i) a clear correlation-induced pseudogapped metal without Landau quasiparticles going over to an insulator at $U_c = 1.8$, with an anomalous self-energy $\mathrm{Im}\Sigma^{(c)}_{\mathrm{loc}}(\omega) \simeq |\omega|^{1/3}$ and density of states $\rho_c(\omega) \simeq |\omega|^{1/3}$ at the Mott QCP and (ii) unusual power-law behavior of the dynamical charge susceptibility near U_c, as well as, most importantly, anomalous energy dependence of the d-fermion propagator at long times. In CDMFT, these features arise precisely from the fact that (a) the dispersive c fermions interact with the $d_\pm = (d_1 \pm d_2)/\sqrt{2}$ dispersionless fermion modes at the *intracluster* level via U, while (b) the c fermions do *not* hybridize at all with the $d_\pm$ localized mode at the single-fermion level. Physically, the origin of the AOC is that the dynamical screening arising from strong intracluster interactions in the FKM is nontrivially affected by the hopping motion of the carriers: since the $c_\pm$ fermions do *not* hybridize with the $d_\pm$-fermion modes at the one-fermion level [there is no term of the form $V(c^\dagger_\pm d_\pm + \mathrm{H.c.})$ on the cluster], this screening is nontrivial and arises from the "slow" reaction of the $d_\pm$ modes to the "sudden" jumps of the $c_\pm$ fermions (the latter occurs on a much shorter timescale of $\hbar/t$ in the FKM). Due to the *local* symmetry implied by $[n_{i,d}, H_{\mathrm{FKM}}] = 0$ at each i, a hopping carrier experiences a sudden change in the local potential on the cluster (from zero to U and vice versa while hopping), now in the sense of a "sudden local quench." The rigorous absence of c-d-fermion one-particle mixing in the FKM implies the lack of heavy-particle recoil in the "two-impurity" cluster problem, leading to "Kondo destruction" and generation of an AOC in our two-site CDMFT as above, *in precise agreement with Jonscher's original suggestion*. We present the origin of the many-body AOC using an analytical approach in the following section.

C. Cluster orthogonality catastrophe in two-site CDMFT for the FKM

Here, we present an analytic argument that exposes the venerated orthogonality catastrophe in our CDMFT approach. It turns out that this is most conveniently done by using the underlying two-impurity problem of our CDMFT, which we now describe.

The two-impurity FKM is

$$H_{\mathrm{FKM}} = t(c^\dagger_0 c_\alpha + \mathrm{H.c.}) + \sum_k \epsilon_k c^\dagger_k c_k + t\sum_{k,i=0,\alpha}(e^{ik\cdot R_i} c^\dagger_i c_k + \mathrm{H.c.}) + U\sum_{i=0,\alpha} n_{i,c} n_{i,d}. \tag{6}$$

Introducing the bonding-antibonding fermions, $c_\pm = (c_0 \pm c_\alpha)/\sqrt{2}$, $d_\pm = (d_0 \pm d_\alpha)/\sqrt{2}$. Then, $H_{\mathrm{FKM}} = H_{12} + H_{\mathrm{coupl}} + H_{\mathrm{band}}$, with

$$H_{12} = t(n_{c,+} - n_{c,-}) + \frac{U}{2}\sum_{a,a'=\pm}(n_{c,a} n_{d,a'} + c^\dagger_a c_{a'} d^\dagger_a d_{a'}), \tag{7}$$

$$H_{\mathrm{coupl}} = \sqrt{2}t\sum_k [\cos(ka/2) c^\dagger_+ c_k + i\sin(ka/2) c^\dagger_- c_k + \mathrm{H.c.}], \tag{8}$$

and

$$H_{\mathrm{band}} = \sum_k \epsilon_k c^\dagger_k c_k. \tag{9}$$

This "cluster-to-orbital" mapping exposes the novel structure of the cluster-local correlations at the Mott QCP. Specifically, we observe that while the $d_\pm$ fermions interact with the $c_\pm$ fermions via U, they do not hybridize with each other at the one-fermion level [i.e., there is no term of the form $V_\pm(c^\dagger_\pm d_\pm + \mathrm{H.c.})$ in the FKM]. Thus, the two-impurity FKM maps rigorously onto a cluster version of the classic problem of recoilless, "infinite-mass" $d_\pm$ scatterers in a "Fermi sea" of the $c_\pm$, i.e., to the cluster version of the venerated x-ray edge problem. One now directly reads off that the spectral function of the $d_\pm$ fermions is infrared singular with an interaction-dependent power-law behavior [21]:

$$\rho_{d_\pm}(\omega) \simeq \frac{\theta(\omega)}{|\omega|^{1-\eta_\pm}}, \tag{10}$$

where $\eta_\pm = (\delta_\pm/\pi)^2$ and $\pi\delta_\pm = \tan^{-1}[U\rho_{c_\pm}(0)\pi]$ is the scattering phase shift. There will be an additional contribution to the scattering phase shift arising from the term

$\frac{U}{2}\sum_{a,a'} c_a^\dagger c_{a'} d_a^\dagger d_{a'}$, but this will not qualitatively change the power-law behavior above. This many-body orthogonality catastrophe will carry over into the self-consistently embedded two-site CDMFT solution of the FKM.

Interestingly, we thus find that the orthogonality catastrophe and the accompanying breakdown of adiabatic continuity also hold for the case of spatially separated recoilless scatterers on the length scale of the cluster. Using a different approach, this aspect was also studied previously [22]. Thus, incorporation of intersite correlations between spatially separated scattering centers does not qualitatively modify the orthogonality catastrophe, an interesting result in itself. In modern parlance, this means that the vanishing fidelity as well as the anomalous long-time behavior of the Loschmidt echo, both manifestations of the orthogonality catastrophe [23], also holds for spatially separated, sudden local quenches, a result that may have more widespread applications.

Thus, the classic orthogonality catastrophe, arising from the sudden local but spatially correlated quenches in our two-impurity model, is a genuine feature in our CDMFT. This also provides a concrete microscopic rationalization linking the Jonscher UDR to this many-body effect, as conjectured early on by Jonscher himself.

IV. DISCUSSION AND CONCLUSION

Our findings can profitably be utilized to interpret a wider range of data on dielectric responses of a wide range of disordered electronic matter, e.g., disordered semiconductors, doped Mott insulators, *p-n* junction devices [5], etc. In reality, the optical response at low energy will now be $\mathrm{Re}\sigma_{xx}(\omega) = \sigma_{dc} + \sigma_0\omega^\alpha$, with $0 < \alpha < 1$. This directly implies that $\mathrm{Im}\sigma_{xx}(\omega) = \tan(\pi\alpha/2)\sigma_0\omega^\alpha + \omega\epsilon_0\epsilon_\infty$, with σ_{dc} being the dc conductivity and ϵ_∞ being the bare dielectric constant. The corresponding (dynamic) capacitance and impedance read $C(\omega) = C_\infty + (\sigma_0/2\pi)\tan(\pi\alpha/2)\omega^{\alpha-1}$ and $Z^*(\omega) \simeq [i\omega\epsilon^*(\omega)]^{-1}$. Along with causality (Kramers-Krönig relations), UDR implies that the real and imaginary parts of the dielectric function (thus of the susceptibility) are related to each other by a dielectric phase angle, $\chi'(\omega)/\chi''(\omega) = \cot(\pi\alpha/2)$ [5], *independent* of ω, in stark contrast to Debye-like relaxation, where this ratio equals $\omega\tau$. Such forms have been widely used to analyze data in detail for a wide range of disordered matter [4,5,24] for a long time. Within CDMFT, our findings provide a microscopic rationale for use of these relations.

Theoretically, it is very interesting that such features appear near a correlation-driven MIT in the FKM since this is a band-splitting-type Mott (rather than pure Anderson localization in a disorder model or a first-order Mott transition in the pure *Hubbard* model) transition. It supports views [11,25] that the disorder problem at strong coupling, where $k_F l \simeq O(1)$, is characterized by a different Mott-like quantum criticality, a view nicely supported by our earlier finding [11] of $\beta(g) \simeq \ln(g)$ instead of $\beta(g) \simeq (d-2) - 1/g$ even deep in the (bad) metallic phase. This is not unreasonable, as it has long been known [26] that the coherent potential approximation, the best mean-field theory for the Anderson disorder problem, is equivalent to the Hubbard III band-splitting view of the Mott transition (the latter becomes exact for the FKM in $d = \infty$ [27]).

(a) As concrete examples on the materials front, we note that various aspects of manganite physics have also been successfully modeled by an effective FKM, where the *c* fermions represent effectively spinless fermions (due to strong Hund's coupling) strongly scattered by a disordered "liquid" of effectively localized Jahn-Teller polarons [28]. In this context, it is also interesting to notice that a field-induced percolative MIT has also long been known to occur in manganites [29]. Thus, our model can serve as the simplest effective model for PCSMO [6]. Application to $LaTiO_3$ would require using a full Hubbard model very close to half filling, where *intrinsic* disorder due to inhomogeneous phases near the MIT would generally be expected to be relevant. (b) It is also very interesting that related features, namely, (i) the non-Landau quasiparticle (Drude) "strange" but *infrared singular* power-law optical response and (ii) anomalous optical phase angle, characterize the strange metallicity in near-optimally doped cuprates [3]. One scenario, based on the hidden-Fermi-liquid idea, posits that an *inverse* orthogonality catastrophe also underlies [30] such non-Landau Fermi-liquid observations in cuprates. However, the microscopics in this case involves momentum-selective Mott physics within CDMFT [31], and such novel responses could be linked to coexisting nodal ("itinerant") and Mott-localized antinodal states. While more work is certainly needed to cleanly show such features in the cuprate context, it is out of the scope of our present study.

Thus, the central message of our work is that nonperturbative *dynamical* effects of strong intrinsic scattering in the FKM lead to the onset of a many-body AOC. We find that it is this specific aspect that is at the heart of the "universal" dielectric response.

[1] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Harcourt, New York, 1976).

[2] M. Imada, A. Fujimori, and Y. Tokura, Metal-insulator transitions, Rev. Mod. Phys. **70**, 1039 (1998).

[3] D. van der Marel, H. J. A. Molegraaf, J. Zaanen, Z. Nussinov, F. Carbone, A. Damascelli, H. Eisaki, M. Greven, P. H. Kes, and M. Li, Quantum critical behaviour in a high-T_c superconductor, Nature (London) **425**, 271 (2003).

[4] A. K. Jonscher, The universal dielectric response, Nature (London) **267**, 673 (1977).

[5] A. K. Jonscher, Dielectric relaxation in solids, J. Phys. D **32**, R57 (1999).

[6] P. Lunkenheimer and A. Loidl, Response of Disordered Matter to Electromagnetic Fields, Phys. Rev. Lett. **91**, 207601 (2003).

[7] M. Dressel and A. Pustogow, Electrodynamics of quantum spin liquids, J. Phys.: Condens. Matter **30**, 203001 (2018).

[8] J. K. Freericks and V. Zlatić, Exact dynamical mean-field theory of the Falicov-Kimball model, Rev. Mod. Phys. **75**, 1333 (2003).

[9] M. Jarrell and H. R. Krishnamurthy, Systematic and causal corrections to the coherent potential approximation, Phys. Rev. B **63**, 125102 (2001).

[10] M. S Laad and M. van den Bossche, Non-local effects in the fermion dynamical mean-field framework; application to the two-dimensional Falicov-Kimball model, J. Phys.: Condens. Matter **12**, 2209 (2000).

[11] P. Haldar, M. S. Laad, and S. R. Hassan, Quantum critical transport at a continuous metal-insulator transition, Phys. Rev. B **94**, 081115(R) (2016).

[12] P. Haldar, M. S. Laad, S. R. Hassan, M. Chand, and P. Raychaudhuri, Quantum critical magnetotransport at a continuous metal-insulator transition, Phys. Rev. B **96**, 155113 (2017).

[13] P. Haldar, M. S. Laad, and S. R. Hassan, Thermal transport across a continuous metal-insulator transition, Phys. Rev. B **96**, 125137 (2017).

[14] P. Haldar, M. S. Laad, and S. R. Hassan, Real-space cluster dynamical mean-field approach to the Falicov-Kimball model: An alloy-analogy approach, Phys. Rev. B **95**, 125116 (2017).

[15] K. Haule and G. Kotliar, Optical conductivity and kinetic energy of the superconducting state: A cluster dynamical mean field study, Europhys. Lett. **77**, 27007 (2007).

[16] J. M. Tomczak and S. Biermann, Optical properties of correlated materials: Generalized Peierls approach and its application to VO_2, Phys. Rev. B **80**, 085117 (2009).

[17] R. Łyżwa and Z. Domański, Falicov-Kimball model and its relation to the Hubbard model: Studies on clusters, Phys. Rev. B **50**, 11381 (1994).

[18] H. Terletska, J. Vučičević, D. Tanasković, and V. Dobrosavljević, Quantum Critical Transport Near the Mott Transition, Phys. Rev. Lett. **107**, 026401 (2011).

[19] A. A. Pastor and V. Dobrosavljević, Melting of the Electron Glass, Phys. Rev. Lett. **83**, 4642 (1999).

[20] V Janis, A lattice model for x-ray emission and absorption: The edge singularity, J. Phys.: Condens. Matter **5**, L425 (1993).

[21] P. Nozières and C. T. De Dominicis, Singularities in the x-ray absorption and emission of metals. III. One-body theory exact solution, Phys. Rev. **178**, 1097 (1969).

[22] Y. Gefen, R. Berkovits, I. V. Lerner, and B. L. Altshuler, Anderson orthogonality catastrophe in disordered systems, Phys. Rev. B **65**, 081106(R) (2002).

[23] A. Silva, Statistics of the Work Done on a Quantum Critical System by Quenching a Control Parameter, Phys. Rev. Lett. **101**, 120603 (2008).

[24] P. Lunkenheimer, S. Krohns, F. Gemander, W. W. Schmahl, and A. Loidl, Dielectric characterization of a nonlinear optical material, Sci. Rep. **4**, 6020 (2014).

[25] V. Dobrosavljević, E. Abrahams, E. Miranda, and S. Chakravarty, Scaling Theory of Two-Dimensional Metal-Insulator Transitions, Phys. Rev. Lett. **79**, 455 (1997).

[26] B. Velický, Theory of electronic transport in disordered binary alloys: Coherent-potential approximation, Phys. Rev. **184**, 614 (1969).

[27] P. G. J. van Dongen and D. Vollhardt, Exact Mean-Field Hamiltonian for Fermionic Lattice Models in High Dimensions, Phys. Rev. Lett. **65**, 1663 (1990).

[28] T. V. Ramakrishnan, H. R. Krishnamurthy, S. R. Hassan, and G. V. Pai, Theory of Insulator Metal Transition and Colossal Magnetoresistance in Doped Manganites, Phys. Rev. Lett. **92**, 157203 (2004).

[29] M. Fäth, S. Freisem, A. A. Menovsky, Y. Tomioka, J. Aarts, and J. A. Mydosh, Spatially inhomogeneous metal-insulator transition in doped manganites, Science **285**, 1540 (1999).

[30] P. W. Anderson, The strange metal is a projected Fermi liquid with edge singularities, Nat. Phys. **2**, 626 (2006).

[31] M. Ferrero, P. S. Cornaglia, L. De Leo, O. Parcollet, G. Kotliar, and A. Georges, Pseudogap opening and formation of Fermi arcs as an orbital-selective Mott transition in momentum space, Phys. Rev. B **80**, 064501 (2009).