Bogolon-mediated electron scattering in graphene in hybrid Bose-Fermi systems

Meng Sun,[1,2,*] K. H. A. Villegas,[1] V. M. Kovalev,[3,4] and I. G. Savenko[1,2]

[1]*Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34126, Korea*

[2]*Basic Science Program, Korea University of Science and Technology (UST), Daejeon 34113, Korea*

[3]*A. V. Rzhanov Institute of Semiconductor Physics, Siberian Branch of Russian Academy of Sciences, Novosibirsk 630090, Russia*

[4]*Department of Applied and Theoretical Physics, Novosibirsk State Technical University, Novosibirsk 630073, Russia*

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We study electron scattering in graphene in hybrid Bose-Fermi systems. We calculate the energy-dependent electron relaxation time, accounting for the processes of emission and absorption of a Bogoliubov excitation (a bogolon). Then, using the Bloch-Grüneisen approach, we find the finite-temperature resistivity of graphene and show that its principal behavior is $\sim T^4$ in the limit of low temperatures and linear at high temperatures. We show that bogolon-mediated scattering can surpass the acoustic-phonon-assisted relaxation. It can be controlled by the distance between the layers and the condensate density, giving us additional degrees of freedom and a useful tool to render electron mobility by the sample design and external pump.

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I. INTRODUCTION

Electron scattering in solid-state nanostructures plays a crucial role in their two-dimensional transport [1,2], dramatically modifying electric conductivity. Conventionally there exist two principal mechanisms of electron scattering: disorder or impurity-mediated [3,4] and lattice phonon-mediated [1] scatterings. The former processes are more pronounced at low ambient temperatures. In the case of an attracting impurity, electrons can be captured, thus the number of electrons decreases, while repulsive centers make the electron mean-free path and scattering time decrease [5–9]. With the increase of temperature, electron scattering accompanied by the emission and absorption of acoustic and optical phonons of the crystal lattice becomes more efficient [10–12] and at some point dominant.

Conventional scattering mechanisms also take place in hybrid structures of various new kinds, which are in the focus of modern research [13,14]. Hybrid systems consist of two-dimensional spatially separated layers, containing electrons in a two-dimensional electron gas (2DEG) phase and bosons, such as direct and indirect (dipolar) excitons, exciton polaritons, or the Cooper pairs in superconductors [15]. In these systems, the research is, on the one hand, devoted to high-temperature boson-mediated superconductivity [16] and other condensation phenomena in interacting structures, including the Mott phase transition from an ordered state to electron-hole plasma [17]. On the other hand, in such systems there can appear new mechanisms of scattering of fermions in the 2DEG, thus modifying the temperature dependence of the kinetic coefficients. These arguments explain the motivations to study electron transport in hybrid systems.

In this article, we show that, in hybrid Bose-Fermi systems, which consist of a spatially separated 2DEG in a graphene layer and an exciton gas, interacting via the Coulomb forces [17–19], there appears a counterpart to the phonon-mediated scattering, when the gas of bosons is condensed [20–22]. Two-dimensional condensation has been reported in various solid-state systems [23–25]. There the lattice vibrations turn out to be not the only *sound* available. In the presence of a Bose-Einstein condensate (BEC) [23], there come into play other excitations, commonly referred to as Bogoliubov quasiparticles or bogolons, which have linear dispersion at small momenta.

We ascertain that an additional principal mechanism of electron scattering appears, stemming from the interlayer electron-exciton interaction: bogolon-mediated scattering. And the difference between acoustic-phonon-related and bogolon-assisted scattering is more than just the magnitude of the sound velocity. The dependence of the bogolon-mediated resistivity of graphene on temperature is $\sim T^4$ at low temperatures and $\sim T$ in the high-temperature limit. In contrast, a precise calculation of the acoustic-phonon-mediated resistivity in graphene shows $\rho \propto T^{\alpha}$ at low temperatures with $\alpha \sim 6$ [26]. Moreover, the phonon-mediated scattering is vulnerable to the screening effects [27]. This makes a great deal of difference between the two mechanisms of scattering, and one can surpass the other.

II. SYSTEM SCHEMATIC

Let us consider a hybrid system consisting of a graphene layer, separated by a distance l from a double quantum well, containing a dipolar exciton gas, where the distance between the layers of electrons and holes is d (see Fig. 1). The electron-exciton interaction can be described by the Hamiltonian,

$$V = \int d\mathbf{r} \int d\mathbf{R} \Psi_{\mathbf{r}}^{\dagger} \Psi_{\mathbf{r}} g(\mathbf{r}-\mathbf{R}) \Phi_{\mathbf{R}}^{\dagger} \Phi_{\mathbf{R}}, \tag{1}$$

where $\Psi_{\mathbf{r}}$ and $\Phi_{\mathbf{R}}$ are the quantum field operators of electrons and excitons, correspondingly, $g(\mathbf{r}-\mathbf{R})$ is the Coulomb

*Corresponding author: sunmeg.89@gmail.com

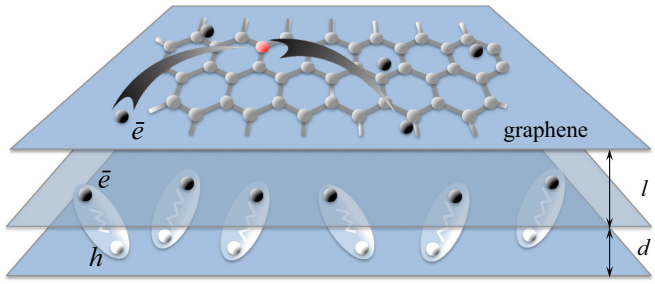

FIG. 1. System schematic. Graphene, located at a distance l from a two-dimensional dipolar exciton gas, residing in two parallel layers, which are at the distance d from each other. Particles couple via the Coulomb interaction.

interaction between an electron and an exciton, $\mathbf{r}$ is electron coordinate within the graphene plane, and $\mathbf{R}$ is the exciton center-of-mass coordinate. We will disregard the internal structure of excitons and only focus on their collective motion.

We assume that the temperature of the system is below the critical temperature, at which the excitons become a degenerate Bose gas [28]. This temperature is given by $kT_c = \frac{2\pi\hbar^2}{m_x} n_x$, where n_x and m_x are the exciton density and effective mass, respectively. We can then use the model of a weakly interacting nonideal Bose gas and write the exciton field operators as $\Phi_{\mathbf{R}} = \sqrt{n_c} + \varphi_{\mathbf{R}}$, where n_c is the condensate density. That is, we separate the condensed and noncondensed particles. Substituting this in Eq. (1) and taking into account the selection rules, we find the electron-bogolon interaction potential,

$$V = \sqrt{n_c} \int d\mathbf{r} \Psi_{\mathbf{r}}^{\dagger} \Psi_{\mathbf{r}} \int d\mathbf{R} g(\mathbf{r} - \mathbf{R})[\varphi_{\mathbf{R}}^{\dagger} + \varphi_{\mathbf{R}}]. \quad (2)$$

Furthermore, we take the Fourier transform of the operators in Eq. (2), using

$$\varphi_{\mathbf{R}}^{\dagger} + \varphi_{\mathbf{R}} = \sum_{\mathbf{p}} e^{i\mathbf{p}\mathbf{R}}[(u_{\mathbf{p}} + v_{-\mathbf{p}})b_{\mathbf{p}} + (v_{\mathbf{p}} + u_{-\mathbf{p}})b_{-\mathbf{p}}^{\dagger}], \quad (3)$$

where $b_{\mathbf{p}}^{\dagger}$ and $b_{\mathbf{p}}$ are the creation and annihilation operators of the bogolons, and the coefficients read [29]

$$u_{\mathbf{p}}^2 = 1 + v_{\mathbf{p}}^2 = \frac{1}{2}\left(1 + \left[1 + \frac{(Ms^2)^2}{\omega_{\mathbf{p}}^2}\right]^{1/2}\right),$$
$$u_{\mathbf{p}} v_{\mathbf{p}} = -\frac{Ms^2}{2\omega_{\mathbf{p}}}. \quad (4)$$

Here M is the exciton mass, $s = \sqrt{\kappa n_c / M}$ is the sound velocity of bogolons, $\omega_k = sk(1 + k^2\xi^2)^{1/2}$ is their spectrum, $\kappa = e_0^2 d/\epsilon$ is the Fourier image of the exciton-exciton interaction strength, e_0 is the electron charge, ϵ is the dielectric function, and $\xi = \hbar/(2Ms)$ is the healing length. Combining Eqs. (2) and (3) yields

$$V = \sqrt{n_c} \sum_{\mathbf{k},\mathbf{p}} g_{\mathbf{p}}[(v_{\mathbf{p}} + u_{-\mathbf{p}})b_{-\mathbf{p}}^{\dagger} + (u_{\mathbf{p}} + v_{-\mathbf{p}})b_{\mathbf{p}}]c_{\mathbf{k}+\mathbf{p}}^{\dagger} c_{\mathbf{k}}, \quad (5)$$

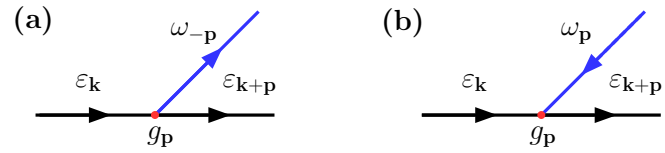

FIG. 2. Schematic of the electron scattering, mediated by (a) the bogolon emission and (b) absorption processes.

where $g_{\mathbf{k}} = e_0^2 d e^{-kl}/(2\epsilon)$ is the Fourier image of the electron-exciton interaction. The schematic of the processes (5) is presented in Fig. 2, showing scattering of an electron, mediated by an absorption or emission of a bogolon.

III. TRANSPORT OF PARTICLES

Furthermore, we use the Boltzmann transport theory [30] to calculate the resistivity of electrons in graphene, which is given by

$$\rho^{-1} = e_0^2 D(E_F) \frac{v_F^2}{2} \langle \tau \rangle, \quad (6)$$

where v_F is the Fermi velocity, E_F is the Fermi energy, and the density of states of graphene at the Fermi level reads $D(E_F) = (g_s g_v/2\pi\hbar^2) E_F v_F^2$, where $g_s = 2$ and $g_v = 2$ are the spin and valley g factors, respectively. We can write the energy-averaged relaxation time as

$$\langle \tau \rangle = \frac{\int d\varepsilon D(\varepsilon)\tau(\varepsilon)\left[-\frac{df^0(\varepsilon)}{d\varepsilon}\right]}{\int d\varepsilon D(\varepsilon)\left[-\frac{df^0(\varepsilon)}{d\varepsilon}\right]}, \quad (7)$$

where $f^0(\varepsilon) = \{\exp[(\varepsilon - \mu)/(k_B T)]\}^{-1}$ is the Fermi distribution function, μ is the chemical potential, k_B is the Boltzmann constant, and the energy-dependent inverse relaxation time reads

$$\frac{1}{\tau(\varepsilon)} = \sum_{\mathbf{k}'} (1 - \cos\theta_{\mathbf{k}\mathbf{k}'}) W_{\mathbf{k}\mathbf{k}'} \frac{1 - f^0(\varepsilon')}{1 - f^0(\varepsilon)}, \quad (8)$$

where $\theta_{\mathbf{k}\mathbf{k}'}$ is the scattering angle between $\mathbf{k}$ and $\mathbf{k}'$, $\varepsilon = \hbar v_F |\mathbf{k}|$ is the dispersion of graphene, and $W_{\mathbf{k}\mathbf{k}'}$ is the probability of transition from an initial electron state $\mathbf{k}$ to the final state $\mathbf{k}'$, given by

$$W_{\mathbf{k}\mathbf{k}'} = \frac{2\pi}{\hbar} \sum_{\mathbf{q}} |C_{\mathbf{q}}|^2 \Delta(\varepsilon, \varepsilon'). \quad (9)$$

Here $C_{\mathbf{q}}$ is the scattering matrix element, and

$$\Delta(\varepsilon, \varepsilon') = N_q \delta(\varepsilon - \varepsilon' + \hbar\omega_q) + (N_q + 1)\delta(\varepsilon - \varepsilon' - \hbar\omega_q), \quad (10)$$

where $N_q = \{\exp[\hbar\omega_q/(k_B T)] - 1\}^{-1}$ is the Bose distribution function. Summing up, the energy-dependent relaxation time reads

$$\frac{1}{\tau(\varepsilon)} = \frac{e_0^4 d^2 n_c}{8\pi\epsilon^2\hbar} \int d\mathbf{k}'(1 - \cos_{\mathbf{k}\mathbf{k}'}) \int d\mathbf{q} e^{-2|q|l} |u_{\mathbf{q}} + v_{\mathbf{q}}|^2 \times \frac{1 - f^0(\varepsilon')}{1 - f^0(\varepsilon)} \Delta(\varepsilon, \varepsilon')\delta(\mathbf{q} - \mathbf{k} + \mathbf{k}'). \quad (11)$$

Using Eq. (4) and assuming linear dispersion of bogolons $\omega_{\mathbf{q}} = s|\mathbf{q}|$ (which is legitimate at $q \ll \xi^{-1}$), we find

$$\frac{1}{\tau(\varepsilon)} = \sum_{n=1,2} \frac{e_0^4 d^2 n_c}{8\pi\epsilon^2\hbar^3 v_F^2} \int_0^{2\pi} d\theta \varepsilon_n (1-\cos\theta) \frac{1-f^0(\varepsilon_n)}{1-f^0(\varepsilon)} \times e^{-2l\lambda} \left(\sqrt{1+\frac{M^2 s^2}{\hbar^2\lambda^2}} - \frac{Ms}{\hbar\lambda} \right) \frac{N_\lambda + \delta_{n,2}}{|F_n'(\varepsilon_n)|}, \tag{12}$$

where $\lambda \equiv |\mathbf{k}-\mathbf{k}'| = (k^2 + k'^2 - 2kk'\cos\theta)^{1/2}$, thus it is a function of k, k', and θ; ε_n are two roots of the equation $F_{1,2}(\varepsilon') = \varepsilon - \varepsilon' \pm \hbar\omega_\lambda = 0$, $F_n'(\varepsilon')$ is its first derivative, and $\delta_{n,2}$ is the Kronecker delta. Specifically, $n = 2$ corresponds to the bogolon emission process. Substituting Eq. (12) in the average lifetime (7), we can numerically calculate the conductivity (6). However, let us first analytically consider the limiting cases of high and low temperatures.

IV. HIGH-TEMPERATURE LIMIT

Let us analyze Eq. (12) and find the principal dependence of conductivity on T at high temperatures, $T_{BG} \ll T \ll E_F/k_B$, where we denote the Bloch-Grüneisen temperature as $T_{BG} = 2\hbar s k_F/k_B$. Since $T \gg T_{BG}$, we have $\hbar\omega_{\mathbf{q}} \ll k_B T$. In this case, the Bose-Einstein distribution can be approximated as $N_q \sim k_B T/\hbar\omega_{\mathbf{q}}$, and $\Delta(\varepsilon,\varepsilon') = (2k_B T/k_B T)\,\delta(\varepsilon-\varepsilon')$. Then we find the energy-dependent relaxation time,

$$\frac{1}{\tau(\varepsilon)} = \frac{e_0^4 d^2 k_B T}{8\pi^2\epsilon^2\hbar^2 v_F^2} \int_0^{2\pi} d\theta (1-\cos\theta)\varepsilon \times e^{-\lambda l} \left(\sqrt{\frac{1}{s^2\lambda^2} + \frac{M^2}{\hbar^2\lambda^4}} - \frac{M}{\hbar\lambda} \right). \tag{13}$$

One should notice that the integral in Eq. (13) is temperature independent. Under the limit $T \ll E_F/k_B$, the contribution from the Fermi energy in Eq. (7) is dominant. This gives us $\langle\tau\rangle \approx \tau(E_F) \sim T^{-1}$. Substituting this expression into Eq. (6), we find that the resistivity depends linearly on the temperature, as in the case of phonon-assisted relaxation [2]. Indeed the temperature should still be smaller than the exciton condensation temperature. Otherwise, bogolon-mediated relaxation does not exist.

V. LOW-TEMPERATURE LIMIT

To investigate the principal T dependence of resistivity at low temperatures, we use the Bloch-Grüneisen formalism, described in Refs. [31,32]. We start from the Boltzmann equation

$$e_0\mathbf{E}\cdot\frac{\partial f}{\hbar\partial\mathbf{p}} = I\{f\}, \tag{14}$$

where f is the electron distribution, $\mathbf{p}$ is the wave vector ($p \equiv |\mathbf{p}|$), $\mathbf{E}$ is the perturbing electric field, and $I\{f\}$ is the collision integral (see Appendix A for the explicit form of I and other details of the derivation). For relatively weak electric fields, f can be expanded as

$$f = f^0(\varepsilon_p) - \left(-\frac{\partial f^0}{\partial\varepsilon_p}\right) f_{\mathbf{p}}^{(1)}, \tag{15}$$

where the correction $f_{\mathbf{p}}^{(1)}$ has the dimensionality of energy. Without loss of generality, we put the electric field to direct along the x axis and use the ansatz

$$f_{\mathbf{p}}^{(1)} = v_F \frac{e_0 E_x p_x}{k_F} \tau(\varepsilon_p). \tag{16}$$

After some algebra, we find the resistivity in the form

$$\rho \propto \frac{1}{\tau_0} = \frac{\hbar\xi_I^2}{8\pi^2 k_F M} \frac{1}{k_B T} \int_0^\infty dq q^4 e^{-2ql} q(\Gamma_- - \Gamma_+)_{k_F} \times N_q(1+N_q), \tag{17}$$

where τ_0 is an effective scattering time, $\xi_I = e_0^2 d\sqrt{n_c}/2\epsilon$,

$$\Gamma_\pm = \frac{2|v_F k_F \pm sq|(2v_F s k_F - v_F^2 q)}{\hbar v_F^3 k_F q\sqrt{\pm 4k_F s v_F q + 4k_F^2 v_F^2 - v_F^2 q^2}}, \tag{18}$$

and the subscript k_F in the expression $(\Gamma_- - \Gamma_+)_{k_F}$ in Eq. (17) means that all the electron wave vectors p are to be substituted by k_F there.

For temperatures much lower than the Bloch-Grüneisen temperature, we find the following expression:

$$\frac{1}{\tau_0} = \frac{I_0 \xi_I^2 k_F^2}{4\pi^2\hbar\alpha^4 v_F^2 M} \left(\frac{k_B T}{E_F}\right)^4, \tag{19}$$

where $I_0 \approx 26.2$ is a dimensionless factor. In terms of the resistivity,

$$\rho = \frac{\pi\hbar^2}{e_0^2 E_F} \frac{1}{\tau_0} = (1.0\times 10^6\ \Omega)\left(\frac{k_B T}{E_F}\right)^4. \tag{20}$$

In this esteem, we used a dimensionless parameter $\tilde{l} = l k_B T/(\hbar s)$ which is determined by the interlayer distance l, the sound velocity s (which is in turn determined by the condensate density), and temperature; and we used the condition $\tilde{l} \ll 1$ to get an analytical dependence at low T.

For temperatures far less than the room temperature ($k_B T_R \approx 26$ meV) we have $\tilde{l} \ll 1$. If $T \ll T_{BG}$, where $T_{BG} \ll E_F/k_B$ since $s \ll v_F$, we find the precise form of what we mean by *low temperatures*: $k_B T/E_F < 10^{-2}$. For typical $E_F \sim 10^{-1}$ eV, this gives $T_{BG} = 183$ K and $T < 18$ K (for the particular range of distances between the layers l up to 50 nm).

Onwards, it is interesting and instructive to compare the formula (20) rewritten in a different form,

$$\frac{1}{\tau_0} = \frac{5 I_0 e_0^6}{8\pi^2\epsilon^2 v_F^2} \frac{n_c d}{M} \frac{1}{E_F k_F} \left(\frac{k_B T}{\hbar s}\right)^4, \tag{21}$$

with the phonon-mediated-scattering case [2]

$$\frac{1}{\tilde{\tau}_0} = \frac{D^2 4! \zeta(4)}{2\pi\rho_m v_{ph}} \frac{1}{E_F k_F} \left(\frac{k_B T}{\hbar v_{ph}}\right)^4, \tag{22}$$

where ρ_m is the density of graphene, ζ is the Riemann zeta function. We see that both the inverse times have the same T dependence at low temperatures with the phonon velocity

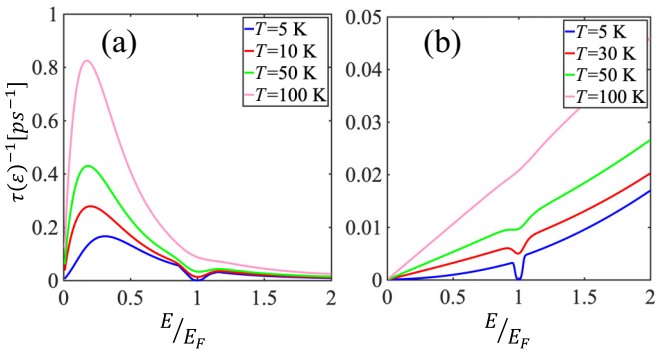

FIG. 3. Energy-dependent inverse relaxation time of electrons for the (a) single-bogolon (emission and absorption) processes and (b) phonon-assisted processes for different temperatures; $n_c = 10^{11}$ cm^{-2} and thus $s \approx 7 \times 10^6$ cm/s.

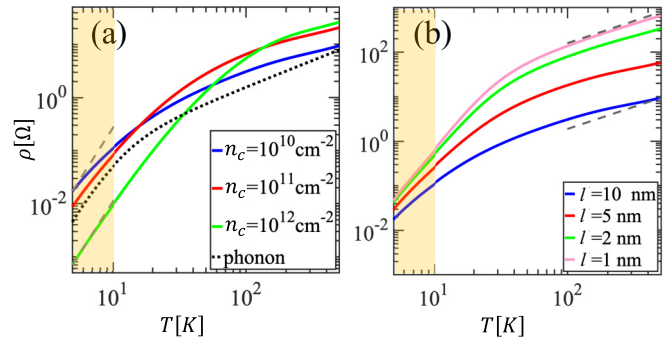

FIG. 4. Bogolon-mediated resistivity of graphene as a function of temperature for different densities of particles in the condensate n_c at (a) $l = 10$ nm and for different interlayer distances l at (b) $n_c = 10^{10}$ cm^{-2}. The dashed gray lines stand for the low- and high-temperature analytics, indicating the $\sim T^4$ and $\sim T$ behavior, respectively. The black dotted line in panel (a) shows the phonon-mediated resistivity for comparison. The yellow-shaded regions highlight the temperature regime in which condensation of indirect excitons in GaAs structures was experimentally reported.

v_{ph} replaced by the sound speed s in the bogolon-mediated scattering case.

This is not the end of the story yet. The result presented in Ref. [2] [and Eq. (22)] assumes that the dominant contribution to the scattering comes from the longitudinal acoustic phonons. A more recent study [26] shows that the transverse acoustic phonons dominate at low temperatures. As a result, the resistivity obeys the power law $\rho_{ph} \propto T^{\alpha}$ with $\alpha \sim 6$, even in the absence of screening [27], which can additionally impair the impact of the phonon-related scattering. The screening in the case of the hybrid system is a nontrivial issue, which requires separate consideration. We only note that here the screening can likely be disregarded for certain l. Thus we conclude that, at low temperatures, the T dependence of resistivity due to bogolon-mediated scattering events is fundamentally different from the phonon case. Since the bogolons have a smaller temperature exponent T^4 than phonons, we envisage the former to dominate (at $T \ll T_{BG}$).

It should be emphasized that we do not have to put $\tilde{l} \ll 1$. However, the general case does not allow for analytical extraction of the temperature dependence of resistivity out of the integration, thus requiring a numerical approach.

VI. NUMERICAL TREATMENT

To build the plots, we use Eqs. (6), (7), and (12) and parameters typical for GaAs-based structures: $\epsilon = 12.5\epsilon_0$, where ϵ_0 is a vacuum permittivity, $M = 0.52m_0$, where m_0 is the free-electron mass, $d = 10$ nm, $l = 10$ nm, and $v_F = 10^8$ cm/s [33,34].

Figure 3 shows the inverse energy-dependent relaxation time as a function of energy for different temperatures. We thus compare the bogolon-mediated scattering with the acoustic-phonon-assisted relaxation under the conditions [35]. There are some similarities between the bogolon- and phonon-mediated processes. In both cases, the inverse lifetime grows with the increase of temperature due to the increase of the number of fermions and bosons (bogolons or phonons) in the system. We also observe low-temperature dips at the Fermi energy, which are due to the sharpening of the Fermi surface.

Nevertheless there is a conceptual difference between the two principal channels of scattering, tracing its origin to the mechanisms of electron-phonon and electron-bogolon interaction. The former is stemming from the crystal lattice deformation-potential theory, while electron-bogolon interaction has an electric nature, and the matrix element contains the Coulomb interaction term.

Figure 4 demonstrates the behavior of the graphene resistivity as a function of temperature for different condensate densities and interlayer spacings. We also compare it with the phonon-mediated resistivity. All the curves show $\sim T^4$ dependence at low temperatures and $\sim T$ at high temperature. Thus the principal behavior of resistivity is deceptively similar to the phonon-assisted case, reported in Ref. [2]. In the case of bogolons, different n_c affect the sound velocity, and the Bloch-Grüneisen temperature changes correspondingly: $T_{BG} \approx 54$, 190, and 540 K for the densities $n_c = 10^{10}$, 10^{11}, and 10^{12} cm^{-2}, respectively. That is why we have a better agreement between the numerical results and T^4 analytics in the high-density regime. Furthermore, Fig. 4(b) shows that, by decreasing l, we can increase the strength of the Coulomb interaction and then the resistivity of graphene increases.

We note that we used the parameters of a GaAs-based material. Indirect excitons there only condense at temperatures less than 10 K (yellow regions in Fig. 4). However, higher T_c, where we predict linear temperature dependence of resistivity, might be achieved in other materials or systems. For example, the critical temperature for the degenerate exciton Bose gas can possibly reach $\sim$100 K in MoS_2 [28]. Another example is exciton polaritons, where quasicondensation was reported even at room temperature [36].

VII. CONCLUSION

We have studied the finite-temperature electron conductivity in graphene, coupled with a two-dimensional dipolar exciton gas via the Coulomb interaction. We have calculated the energy-dependent relaxation time of electrons, accompanied by the emission and absorption of a Bogoliubov excitation.

We have further calculated the resistivity of graphene in this hybrid Bose-Fermi system and showed that bogolon-mediated scattering not only gives a significant correction to the phonon-assisted relaxation but it prevails, given specific system geometry and temperatures. We believe the reported results can be used to design new types of graphene-based hybrid systems.

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APPENDIX: DERIVATION OF RESISTIVITY IN LOW-TEMPERATURE LIMIT BY BLOCH-GRÜNEISEN APPROACH

In this Appendix we derive the low-temperature T dependence of the conductivity of graphene in the hybrid Bose-Fermi system using the Bloch-Grüneisen approach. We start from the Boltzmann equation

$$e_0\mathbf{E}\cdot\frac{\partial f}{\hbar\partial\mathbf{p}} = I\{f\}, \tag{A1}$$

where $\mathbf{p}$ is the wave vector and we will use $p \equiv |\mathbf{p}|$, $\mathbf{E}$ is the perturbing electric field, and f is the distribution function. The scattering integral is given by

$$I\{f\} = -\frac{1}{\hbar}\int\frac{d\mathbf{q}d\mathbf{p}'}{(2\pi)^2}|M_q|^2[N_q f_p(1-f_{p'})\delta(\varepsilon_p-\varepsilon_{p'}+\hbar\omega_q)\delta(\mathbf{p}-\mathbf{p}'+\mathbf{q}) + (N_q+1)f_p(1-f_{p'})\delta(\varepsilon_p-\varepsilon_{p'}-\hbar\omega_q)\delta(\mathbf{p}-\mathbf{p}'-\mathbf{q})$$
$$+N_q f_{p'}(1-f_p)\delta(\varepsilon_{p'}-\varepsilon_p+\hbar\omega_q)\delta(\mathbf{p}'-\mathbf{p}+\mathbf{q}) + (N_q+1)f_{p'}(1-f_p)\delta(\varepsilon_{p'}-\varepsilon_p-\hbar\omega_q)\delta(\mathbf{p}'-\mathbf{p}-\mathbf{q})]. \tag{A2}$$

Note that writing this integral we set the length of the sample L equal to one. In performing a dimensionality analysis, one should include the length squared, so that $I\{f\}$ has a dimension of inverse time, as it should.

For small enough electric fields, the electron distribution is not substantially different from the equilibrium Fermi distribution, thus it can be presented in the form

$$f = f^0(\varepsilon_p) - \left(-\frac{\partial f^0}{\partial\varepsilon_p}\right)f^{(1)}_{\mathbf{p}}, \tag{A3}$$

where f^0 is the equilibrium Fermi-Dirac distribution and $f^{(1)}_{\mathbf{p}}$ has a dimensionality of energy. Following the steps of the derivation reported in Ref. [32], we rewrite

$$e_0\mathbf{E}\cdot\frac{\partial f}{\hbar\partial\mathbf{p}} = v_F\frac{e_0\mathbf{E}\cdot\mathbf{p}}{|\mathbf{p}|}\frac{\partial f^0}{\partial\varepsilon_p} = I\{f^{(1)}_{\mathbf{p}}\}, \tag{A4}$$

$$I\{f^{(1)}_{\mathbf{p}}\} = -\frac{1}{\hbar}\int\frac{d\mathbf{q}d\mathbf{p}'}{(2\pi)^2}|M_q|^2\frac{1}{\hbar}\frac{\partial N_q}{\partial\omega_q}[f^0(\varepsilon_p)-f^0(\varepsilon_{p'})]\left(f^{(1)}_{\mathbf{p}}-f^{(1)}_{\mathbf{p}'}\right)\left[\delta(\varepsilon_p-\varepsilon_{p'}-\hbar\omega_q)\delta(\mathbf{p}-\mathbf{p}'-\mathbf{q})\right.$$
$$\left.-\delta(\varepsilon_p-\varepsilon_{p'}+\hbar\omega_q)\delta(\mathbf{p}-\mathbf{p}'+\mathbf{q})\right], \tag{A5}$$

where

$$\frac{\partial N_q}{\partial\omega_q} = -\frac{\hbar}{k_BT}N_q(1+N_q).$$

Furthermore, we integrate over the electron wave vector $\mathbf{p}'$ and find

$$v_F\frac{e_0\mathbf{E}\cdot\mathbf{p}}{p}\frac{\partial f^0}{\partial\varepsilon_p} = -\frac{1}{\hbar}\int\frac{d\mathbf{q}}{(2\pi)^2}|M_q|^2\frac{1}{\hbar}\frac{\partial N_q}{\partial\omega_q}[f^0(\varepsilon_p)-f^0(\varepsilon_p-\hbar\omega_q)]\left(f^{(1)}_{\mathbf{p}}-f^{(1)}_{\mathbf{p}-\mathbf{q}}\right)\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}-\mathbf{q}}-\hbar\omega_{\mathbf{q}})$$
$$+\frac{1}{\hbar}\int\frac{d\mathbf{q}}{(2\pi)^2}|M_q|^2\frac{1}{\hbar}\frac{\partial N_q}{\partial\omega_q}[f^0(\varepsilon_p)-f^0(\varepsilon_p+\hbar\omega_q)]\left(f^{(1)}_{\mathbf{p}}-f^{(1)}_{\mathbf{p}+\mathbf{q}}\right)\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}+\mathbf{q}}+\hbar\omega_{\mathbf{q}}). \tag{A6}$$

Let the electric field be directed along the x axis. Then we can use the correction function in the form

$$f^{(1)}_{\mathbf{p}} = v_F\frac{e_0E_xp_x}{k_F}\tau(\varepsilon_p), \tag{A7}$$

where k_F is the Fermi wave vector and $\tau(\varepsilon_p)$ is the relaxation time. We have

$$\frac{p_x}{p}\frac{\partial f^0}{\partial\varepsilon_p} = -\frac{1}{\hbar}\int\frac{d\mathbf{q}}{(2\pi)^2}|M_q|^2\frac{1}{\hbar}\frac{\partial N_q}{\partial\omega_q}[f^0(\varepsilon_p)-f^0(\varepsilon_p-\hbar\omega_q)]\left[\frac{p_x}{k_F}\tau(\varepsilon_p)-\frac{p_x-q_x}{k_F}\tau(\varepsilon_p-\hbar\omega_q)\right]\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}-\mathbf{q}}-\hbar\omega_{\mathbf{q}})$$
$$+\frac{1}{\hbar}\int\frac{d\mathbf{q}}{(2\pi)^2}|M_q|^2\frac{1}{\hbar}\frac{\partial N_q}{\partial\omega_q}[f^0(\varepsilon_p)-f^0(\varepsilon_p+\hbar\omega_q)]\left[\frac{p_x}{k_F}\tau(\varepsilon_p)-\frac{p_x+q_x}{k_F}\tau(\varepsilon_p+\hbar\omega_q)\right]\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}+\mathbf{q}}+\hbar\omega_{\mathbf{q}}). \tag{A8}$$

Assuming that the relaxation time is constant [31] $\tau = \tau_0$, we find

$$\frac{k_F}{p} p_x \frac{\partial f^0}{\partial \varepsilon_p} = -\frac{\tau_0}{\hbar} \int \frac{d\mathbf{q}}{(2\pi)^2} q_x |M_q|^2 \frac{1}{\hbar} \frac{\partial N_q}{\partial \omega_q} [f^0(\varepsilon_p) - f^0(\varepsilon_p - \hbar\omega_q)] \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{q}} - \hbar\omega_{\mathbf{q}}) + \frac{\tau_0}{\hbar} \int \frac{d\mathbf{q}}{(2\pi)^2} q_x |M_q|^2 \frac{1}{\hbar} \frac{\partial N_q}{\partial \omega_q} [f^0(\varepsilon_p) - f^0(\varepsilon_p + \hbar\omega_q)] \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}+\mathbf{q}} + \hbar\omega_{\mathbf{q}}). \tag{A9}$$

Let us denote the angle between the vectors $\mathbf{p}$ and $\mathbf{q}$ as φ and the angle between the vectors $\mathbf{p}$ and $\mathbf{E}$ as β. Then $q_x = q\cos(\varphi + \beta)$ and $p_x = p\cos\beta$. Integrating over ϕ, we find

$$\int_0^{2\pi} d\phi \cos(\phi + \beta) \delta(a - \sqrt{b^2 \pm c^2 \cos\phi}) = \pm \frac{4|a|(b^2 - a^2)\Theta[c^4 - (b^2 - a^2)^2]}{c^2\sqrt{c^4 - (b^2 - a^2)^2}} \cos\beta, \tag{A10}$$

where $\Theta[x]$ is the Heaviside step function, $a = \hbar(v_F p - sq)$, $b^2 = \hbar^2 v_F^2 (p^2 + q^2)$, and $c^2 = 2\hbar^2 v_F^2 pq$. To derive Eq. (A10), we denoted a new variable $x = \cos\phi$. This implied $d\phi = \mp dx[1 - x^2]^{-1/2}$, where the $-(+)$ case is for $0 \leqslant \phi < \pi$ ($\pi \leqslant \phi < 2\pi$).

After integrating over the angle ϕ, we can integrate Eq. (A9) over $\xi_p = \varepsilon_p - \mu$, using

$$\int_{-\infty}^{\infty} d\xi_p [f^0(\varepsilon_p) - f^0(\varepsilon_p \pm \hbar\omega_q)] = \mp\hbar\omega_q, \qquad \int_{-\infty}^{\infty} d\xi_p \frac{\partial f^0}{\partial \varepsilon_p} = -1, \tag{A11}$$

and putting all electron wave vectors to be $p = k_F$.

The resistivity is inversely proportional to the scattering time,

$$\rho \propto \frac{1}{\tau_0} = \frac{\hbar\xi_I^2}{8\pi^2 k_F M} \frac{1}{kT} \int_0^{\infty} dq q^4 e^{-2ql} q \times (\Gamma_- - \Gamma_+)_{k_F} N_q(1 + N_q), \tag{A12}$$

where we introduced $\xi_I = e_0^2 d\sqrt{n_c}/2\epsilon$ and

$$\Gamma_\pm = \frac{4|a_\pm|(a_\pm^2 - b^2)\Theta[c^4 - (a_\pm^2 - b^2)^2]}{c^2\sqrt{c^4 - (a_\pm^2 - b^2)^2}}. \tag{A13}$$

The subscript k_F in the expression $(\Gamma_- - \Gamma_+)_{k_F}$ in Eq. (A12) means that all the electron wave vectors p are to be substituted by the Fermi value k_F.

We now introduce a new dimensionless variable

$$u = \frac{\hbar sq}{k_B T} \tag{A14}$$

in Eq. (A12) and obtain

$$\frac{1}{\tau_0} = \frac{\xi_I^2}{8\pi^2 k_F M s} \left(\frac{k_B T}{\hbar s}\right)^4 \int_0^{\infty} du \frac{u^4 e^{(1-2\tilde{l})u}}{(e^u - 1)^2} \times (\Gamma_- - \Gamma_+)_{k_F}, \tag{A15}$$

where

$$\tilde{l} = \frac{l k_B T}{\hbar s} \sim \frac{k_B T}{10\ \text{meV}} \tag{A16}$$

and we used $s = 10^5$ m/s and $l = 5.0 \times 10^{-8}$ m/s. Note that the room temperature is $k_B T_R \sim 26$ meV, so that for temperatures far less than the room temperature we have $\tilde{l} \ll 1$. Hence we can replace

$$e^{(1-2\tilde{l})u} \to e^u. \tag{A17}$$

Let us now look at the argument of the Heaviside theta function in Eq. (A13). It can be simplified as

$$-\left(v_F^2 - s^2\right)q^2 \pm 4k_F s v_F q + 4k_F^2 v_F^2. \tag{A18}$$

This expression is positive for

$$0 \leqslant q < \frac{2k_F v_F}{v_F \mp s} \approx 2k_F, \tag{A19}$$

or

$$0 \leqslant u < \frac{T_{BG}}{T} \equiv \Lambda, \tag{A20}$$

where $T_{BG} = 2\hbar s k_F / k_B$ is the Bloch-Grüneisen temperature for bogolons.

Let us consider the case when the temperature $T \ll T_{BG}$, which specifically means $\Lambda > 10$. This inequality gives us the precise form of what we mean by *low temperature*: $k_B T / E_F < 10^{-2}$. For typical $E_F \sim 10^{-1}$ eV, which gives $T_{BG} = 183$ K and $T < 18$ K (for the particular distance between the layers $l > 50$ nm). It should be mentioned that the condition $\tilde{l} \ll 1$ is not a requirement. However, the general case does not allow for analytical extraction of temperature out of the integral, since we come up with the term $\sim \exp[l k_B T u/(\hbar s)]$ under the integration.

For large u (or q), the factors $\Gamma_\pm$ in Eq. (A15) approach constant values. In the mean time, the term $u^4 \exp(-u)$ rapidly goes to zero for $u > 10$. Therefore we can remove the theta function in Eq. (A13) and this incurs only a small (imaginary) error. Doing this and also using $v_F^2 - s^2 \sim v_F^2$, Eq. (A13) now becomes

$$\Gamma_\pm = \frac{2|v_F k_F \pm sq|\left(2v_F s k_F - v_F^2 q\right)}{\hbar v_F^3 k_F q \sqrt{\pm 4k_F s v_F q + 4k_F^2 v_F^2 - v_F^2 q^2}}. \tag{A21}$$

The expression in the numerator above can be rewritten as

$$2v_F s k_F - v_F^2 q = 2v_F s k_F - v_F^2 \frac{k_B T}{\hbar s} u = \frac{v_F k_B T}{\hbar s}(s\Lambda - v_F u). \tag{A22}$$

Here Λ does depend on T, as was defined in Eq. (A20). However, for $T \ll T_{BG}$, due to the factor $\exp(-u)$ we can simply replace $\Lambda \sim 10$ (or greater) without significantly affecting the result. We introduce the ratio of velocities $\alpha = s/v_F$ and, for simplicity, we choose Λ such that $\alpha\Lambda = 1$ as long as $\Lambda > 10$

to get

$$2v_F sk_F - v_F^2 q = \frac{v_F k_B T}{\alpha\hbar}(1-u). \tag{A23}$$

We find

$$\frac{1}{\tau_0} = \frac{10 v_F \xi_I^2}{8\pi^2 k_F M s}\left(\frac{k_B T}{\hbar s}\right)^5 \times \int_0^\infty du \frac{u^4 e^u (1-u)}{(e^u-1)^2}(\gamma_- - \gamma_+)_{k_F}, \tag{A24}$$

where

$$\gamma_\pm = \frac{2|v_F k_F \pm sq|}{\hbar v_F^3 k_F q\sqrt{\pm 4k_F s v_F q + 4k_F^2 v_F^2 - v_F^2 q^2}}. \tag{A25}$$

We can rewrite the numerator as

$$|v_F k_F \pm sq| = \frac{k_B T}{\hbar}\left|\frac{\Lambda}{2\alpha} \pm u\right|. \tag{A26}$$

Finally, the term under the square root in (A25) can be written as

$$\pm 4k_F s v_F q + 4k_F^2 v_F^2 - v_F^2 q^2 \sim -v_F^2(q-2k_F)(q+2k_F) = -\frac{v_F^2 k_B^2 T^2}{\hbar^2 s^2}(u-\Lambda)(u+\Lambda). \tag{A27}$$

Summing up, we find

$$\frac{1}{\tau_0} = \frac{I_0 \xi_I^2 k_F^2}{4\pi^2\hbar\alpha^4 v_F^2 M}\left(\frac{k_B T}{E_F}\right)^4 = \frac{5 I_0 e_0^2}{8\pi^2\hbar^3 v_F}\frac{M}{n_c E_F^2}(k_B T)^4, \tag{A28}$$

where I_0 is a dimensionless integral, which can be found numerically,

$$I_0 = \int_0^\infty du \frac{u^3(1-u)e^u}{(e^u-1)^2\sqrt{100-u^2}} \times\left(\left|\frac{\Lambda}{2\alpha} - u\right| - \left|\frac{\Lambda}{2\alpha} + u\right|\right) \approx 26.2. \tag{A29}$$

This gives

$$\frac{1}{\tau_0} = (1.4\times 10^{16}\mathrm{s}^{-1})\left(\frac{k_B T}{E_F}\right)^4. \tag{A30}$$

In terms of the resistivity,

$$\rho = \frac{\pi\hbar^2}{e_0^2 E_F}\frac{1}{\tau_0} = (1.0\times 10^6\ \Omega)\left(\frac{k_B T}{E_F}\right)^4. \tag{A31}$$

Thus we conclude, that at low temperatures $T \ll T_{BG}$, the resistivity is $\propto T^4$.

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