Magnetochiral nonreciprocity of spin wave damping in long-period structures

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The spin wave in long-periodic spiral magnetic structures demonstrates the so-called magnetochiral nonreciprocity, i.e., a noninvariance of its dispersion when it propagates parallel and antiparallel to the chiral wave vector. Motivated by recent experiments on systems with magnetochiral long-periodic orders, we studied the effect of static spatially varying magnetization structure on spin wave relaxation. A description of an intrinsic nonlocal magnetization relaxation is given within the scope of a phenomenological equation called the Landau-Lifshitz-Baryakhtar equation (V. G. Bar'yakhtar, Zh. Eksp. Teor. Fiz. **87**, 1501 (1984) [Sov. Phys. JETP **60**, 863 (1984)]) for crystals of different symmetry. Spiral magnetic structures with a helicity step due to competitive exchange interactions or due to the Dzyaloshinskii-Moriya interaction are considered, and related dissipative functions are obtained. Within the general phenomenological approach, it is demonstrated that magnetochiral nonreciprocity manifests itself in different nonlocal dampings of spin waves with opposite wave vectors, as well. More fundamentally, we show that static but spatially varying magnetization causes a finite damping of the Goldstone mode. Understanding the peculiar nonlocal relaxation of the magnetization precession in materials with spatially varying magnetic order is important for applications of such magnets in magnonics and spintronics.

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I. INTRODUCTION

Spin wave damping has been extensively studied in connection with the ferromagnetic resonance (FMR) in garnets (see, e.g., [1]). It has been established with detailed experiments that, on a phenomenological level, the intrinsic damping in magnetic dielectrics is well described by the Landau-Lifshitz-Gilbert (LLG) equation, where magnetic damping is parametrized by the Gilbert constant α_G [2,3]. The FMR provides a way to measure the damping α_G from the FMR linewidth [4], and the LLG equation is useful for numerical simulation of magnetization dynamics on the basis of the continuous-medium approximation and for interpretation of experiments. However, the research activity in spintronics with the purpose to manipulate magnetization by electric current raises a question about the role of nonlocal torque generated by the *nonuniform* magnetization ordering when an electron current flows through a spatially and temporally varying magnetization background $\mathbf{M}(\mathbf{r}, t)$. It was shown [5–8] that for systems with spatially and temporally varying magnetization $\mathbf{M}(\mathbf{r}, t)$ the dynamics equation should include not only a conventional precession term due to an effective field $\mathbf{H}_{\text{eff}}$ and a phenomenological damping term of first order in the magnetization time derivative (adiabatic torque $\sim\mathbf{M}\times\frac{\partial\mathbf{M}}{\partial t}$) but also a nonadiabatic spin torque generated by a spatially nonuniform magnetization vector $\sim\mathbf{M}\times\frac{\partial\mathbf{M}}{\partial\mathbf{r}}$.

Magnetization dynamics in materials with spatially varying magnetic structures is currently an area of considerable interest for magnonics and spintronics, where the primary goal is to create devices based on manipulation and transportation of spin current with minimal losses [9,10]. Whereas materials with inhomogeneous magnetic order have been known for a long time, some features of spin wave propagation in them still remain insufficiently investigated. In particular, a phenomenological description of the magnetization precession intrinsic damping in materials with a long-periodic magnetic order still remains not fully understood. The point is that the nonadiabatic spin torque generated by a spatially nonuniform magnetization vector affects not only the magnon energy but the magnetization precession damping as well. Thus, the magnetization dynamic equation should include not only the relaxation term α_G as a constant parameter (as it was used, e.g., in Refs. [11–15]) but the spin wave damping due to nonadiabatic effects (nonlocal torques) generated by a spatially nonuniform magnetization vector. In general, this is an example of the connection between the space geometry and dynamics, and the conversion of magnetization spatial structure into dynamical characteristics of the system is being actively discussed within the "*s-d*" model [5–8].

Recently, there has been much interest in materials with inhomogeneous magnetochiral magnetic structures. For example, in magnetic materials with competitive exchange ferromagnetic and antiferromagnetic interactions, incommensurate or amplitude-modulated magnetic structures can be preferable. Typically, incommensurate magnetic structures have periodicity that does not match the periodicity of the crystalline lattice. A classic example of a long-periodic structure is the spiral magnetic ordering. Rare-earth intermetallic compounds, where the interionic exchange interaction is the

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oscillatory Ruderman-Kittel-Kasuya-Yoshida coupling, can exhibit modulated or helical magnetic structures at finite temperature [16,17]. An example of another origin for long-periodic structures is the systems with both broken inversion symmetry and strong spin-orbit coupling. In this case, the antisymmetric component of exchange interaction, known as the Dzyaloshinskii-Moriya (DM) interaction [18], may lead to chiral long-range magnetic order. In chiral magnets, we generally get in touch with an asymmetric spin wave dispersion $\omega(\mathbf{k}) \neq \omega(-\mathbf{k})$ [19,20]. As a consequence, magnons with wave vectors $\mathbf{k}$ and $-\mathbf{k}$ possess different group velocities, giving rise to nonreciprocal magnon propagation. The bulk DM interaction is determined by the lattice symmetry, while the interfacial DM interaction can occur at coupled magnetic interfaces [11]. It is particularly strong at the interface between a ferromagnet and a normal metal having strong spin-orbit coupling [11]. The DM interaction has been studied mostly for the $B20$ structures such as MnAu, MnSi [21,22], FeCoSi [23,24], and FeGe [25,26]. Nonreciprocal magnon propagation has been experimentally demonstrated for $LiFe_5O_8$ [27], FeGe, and Co-Zn-Mn alloys [28] using spin wave spectroscopy. The nonreciprocal character of both magnon propagation and damping was detected and investigated in the noncentrosymmetric ferromagnet Cu_2OSeO_3 by employing spin wave spectroscopy [12,13,29] and in the antiferromagnet α-$Cu_2V_2O_7$ [30] by employing the magnonic Faraday effect. In these crystals, nonreciprocal magnons are due to the incompatibility between the exchange and antisymmetric DM interactions, which results in helical spin dynamics. Just recently, the nonreciprocal magnon spectrum was observed in the chiral magnet MnSi [31,32]. Asymmetry of the magnon dispersion relation and the magnon lifetime in their counterpropagating directions, due to an interfacial DM interaction, was recently directly observed in a multilayer film [14].

The nonreciprocity leads to differences in propagation of spin waves with the same energy but opposite wave vectors, namely, to differences in amplitude, phase and group velocities, the wave refractive index, etc. Understanding the specifics is important for many applications of such materials in magnonics and spintronics. For example, the nature of the magnetization precession relaxation and of the associated spin wave damping is highly important for both fundamental physics and the efficiency of any nanomagnetic devices. These and other features are fundamental for the creation of functional elements of quantum information processing based on the ideas of magnonics and magnetic spintronics [9,10,33–36]. So recent research activity in magnonics to manipulate quasiparticles associated with the magnetization dynamics, known as spin waves or magnons, renewed interest in the specifics of magnetization precession in magnetic dielectric materials with nonuniform ground-state configurations of both the magnetization and effective magnetic fields.

In this paper, we theoretically investigate the peculiarities of the magnetization precession damping in materials with a long-periodic magnetic order. To establish the dominant/specific intrinsic damping mechanisms for these systems with nonlocal torque, we used the general phenomenological approach to relaxation phenomena in magnetic materials developed previously in Refs. [37,38] (see also [39]). Here, we have generalized this theory to systems with a long-periodic magnetization order. It is shown that the presence of a preferential direction in the magnetic structure not only leads to a difference in the phase velocities of volume spin waves with given energy but also manifests itself in different linewidths of these spin waves with opposite wave vector directions. The magnetochiral nonreciprocity of dynamic and damping characteristics of long-periodic magnetic structures should be taken into account when magnetic properties of these materials are being predicted and tailored.

II. DISSIPATIVE FUNCTION OF LONG-PERIODIC MAGNETS

As is known, the general relation leading to the LLG equation is the torque equation of the form

$$\frac{\partial \mathbf{M}}{\partial t} = g\mathbf{M} \times \frac{\delta F}{\delta \mathbf{M}} + \mathbf{R}, \tag{1}$$

where F is the total energy of the system, M_0 is the saturation magnetization, g stands for the g factor, and $\mathbf{R}$ is the relaxation term. In a standard procedure to solve Eq. (1), one first needs to solve the corresponding static equations obtained by setting the time derivatives to zero and thereby to find the static spatial distribution of the magnetization. In general (e.g., as in the case of a long-periodic spiral structure; see examples below), the static distribution of equilibrium magnetization is nonuniform in both its value and direction. With the static solution at hand, the dynamical problem is solved to find the temporal evolution of the magnetization value and direction producing its certain perturbation. In the standard LLG model [2,3] the relaxation term is written as $\alpha_G \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}$. But one should pay attention to the fact that the standard LLG is a phenomenological equation utilized only within the local approximation for torques and it is not suitable for studies including nonadiabatic effects (i.e., nonlocal torques; see the reviews [7,8] for a more complete discussion). The equation for magnetization dynamics in the case of nonadiabatic spin torque generated by a spatially nonuniform magnetization usually is written as [5–8]

$$\frac{\partial \mathbf{M}}{\partial t} = -g\mathbf{M} \times \mathbf{H}^{\text{eff}} + \alpha_G \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} + \mathbf{T}, \tag{2}$$

where $\mathbf{T}$ stands for a nonadiabatic spin torque and $\mathbf{H}^{\text{eff}}$ is the effective magnetic field. But this approach is based just on general physical considerations and does not have a generalized physical theory.

Reference [37] provided the basic principles of building a general theory of the description of relaxation processes in magnets. Let us recall the main distinct features of the Landau-Lifshitz-Baryakhtar (LLBar) approach [37,38] to construct a phenomenological dissipative function. In the LLBar approach, to analyze Eq. (1) at this step, the parameter to characterize the quasiequilibrium state is chosen to be the effective magnetic field $\mathbf{H}^{\text{eff}}(\mathbf{r}, t)$, not the magnetization $\mathbf{M}(\mathbf{r}, t)$. Indeed, the effective field is more convenient than the magnetization because it is zero in the equilibrium state and small for all actual nonequilibrium states close to the ground state. The effect of relaxation terms is that, at each moment of time, the magnetization direction relaxes

towards the instantaneous direction of effective magnetic field, while the magnetization value relaxes towards that prescribed by the instantaneous longitudinal effective magnetic field. Thus, within the LLBar framework, the relaxation term $\mathbf{R}$ should be considered a functional of $\mathbf{H}^{\text{eff}}(\mathbf{r},t)$, too. Namely, $\mathbf{R}=\delta Q/\delta\mathbf{H}^{\text{eff}}$, where $Q=\int q(\mathbf{H}^{\text{eff}})dV$ is the total dissipative function of the system. For states close to the ground state, one can expand the relaxation term $\mathbf{R}(\mathbf{H}^{\text{eff}})$ in a power series in $\mathbf{H}^{\text{eff}}(\mathbf{r},t)$. Naturally, the dissipative function density $q(\mathbf{H}^{\text{eff}})$ must take into account the symmetry and conservation laws for magnetization. As a result, this should be constructed as a functional of the effective magnetic field in the form

$$q(\mathbf{H}^{\text{eff}})=\frac{1}{2}\lambda^{r}_{ik}H^{\text{eff}}_{i}H^{\text{eff}}_{k}+\frac{1}{2}\lambda^{ex}_{ik}\frac{\partial\mathbf{H}^{\text{eff}}}{\partial x_i}\frac{\partial\mathbf{H}^{\text{eff}}}{\partial x_k}+\cdots, \tag{3}$$

where the parameters λ^{r}_{ik}, λ^{ex}_{ik}, etc., are the relativistic and exchange relaxation damping constants and generally are tensors constructed according to the crystalline symmetry.

Traditionally, the intrinsic Gilbert damping is considered to have a relativistic origin, and as shown from first-principles calculations [40], it does arise from the spin-orbit coupling. Phenomenologically, the Gilbert damping is local; that is, the damping due to the nonuniform magnetization dynamics is ignored [3]. Taking into account the exchange terms in the dissipative function density $q(\mathbf{H}^{\text{eff}})$ and $\sim\lambda^{\text{ex}}_{ik}$, we describe the *nonlocal* damping due to the nonuniform effective field [41]. Thus, in the LLBar approach [37,38], the torques generated by the *nonuniform* magnetization background (nonadiabatic/nonlocal torque) and by the time-variable magnetization $\mathbf{M}(\mathbf{r},t)$ (adiabatic/local torque) are naturally taken into account on equal footing.

By definition, the effective magnetic field $\mathbf{H}^{\text{eff}}(\mathbf{r},t)$ is the system's total energy F variation with respect to magnetization, taken with opposite sign, and it can be generally written as

$$\begin{aligned}\mathbf{H}^{\text{eff}}&=-\frac{\delta F}{\delta\mathbf{M}}=-\frac{\partial F}{\partial\mathbf{M}}+\frac{\partial}{\partial x_i}\frac{\partial F}{\partial\frac{\partial\mathbf{M}}{\partial x_i}}\\&\quad-\frac{\partial^2}{\partial x_i^2}\frac{\partial F}{\partial\frac{\partial^2\mathbf{M}}{\partial x_i^2}}+\cdots+(-1)^{n+1}\frac{\partial^n}{\partial x_i^n}\frac{\partial F}{\partial\frac{\partial^n\mathbf{M}}{\partial x_i^n}}.\end{aligned} \tag{4}$$

The expression for quasiequilibrium total energy (thermodynamic potential) of a magnet in the state with magnetization $\mathbf{M}(\mathbf{r},t)$ at a given temperature is determined by integrating the density of total magnetic thermodynamic potential over the crystal volume V:

$$F=\int f\big(\mathbf{M},\partial\mathbf{M}/\partial\mathbf{x}_i\big)dV, \tag{5}$$

where $f(\mathbf{M},\partial\mathbf{M}/\partial\mathbf{x}_i)$ is the total energy density for a magnet with a given (e.g., spiral) magnetic structure.

Note the LLBar approach is based on the general phenomenological principles of constructing the dissipative function in Refs. [42,43]. Namely, the dissipative function must be developed according to the same principles as the quasiequilibrium thermodynamic potential and must include terms of the same nature as the total energy. It should be noted that the constants included in the terms of the same nature have the same origin. Actually, it could be concluded that relaxation constants are proportional to corresponding constants from the energy [37,38] (see also [39]).

For a system with a given dynamic symmetry, one must also take into account the corresponding conservation laws, for example, those of total spin for the exchange symmetry or of total spin projection on a selected axis for a purely uniaxial symmetry. Such an approach was used earlier to describe the relaxation motion of different solitons, domain walls, and Bloch points [41,44,45]. In particular, it was shown that the lifetime and propagation length of short-wavelength magnons in the presence of nonlocal damping could be much smaller than those given by the LLG equation [41].

Below we consider, as an example, a system in which a spiral type of magnetic order is realized. For definiteness, we will assume that the magnetic structure under consideration is a long-periodic modulation of a ferromagnetic state.

A. Exchange spiral

If the symmetry of a crystal does not allow the Lifshitz invariants, which are linear spatial derivatives, the competition between positive and negative exchange interactions in a magnet is responsible for the appearance of the magnetization order called the exchange spiral (see, e.g., Ref. [16]). The density of the total magnetic energy of the exchange spiral in the case of a uniaxial crystal and in the absence of external magnetic field can be written as

$$\begin{aligned}f\big(\mathbf{M},\partial\mathbf{M}/\partial\mathbf{x}_i\big)&=\frac{\big(\mathbf{M}^2-M_0^2\big)^2}{8\chi_{\|}M_0^2}+\frac{1}{2}\alpha\frac{\partial\mathbf{M}}{\partial z}\frac{\partial\mathbf{M}}{\partial z}\\&\quad+\frac{1}{2}\alpha_{\perp}\left(\frac{\partial\mathbf{M}}{\partial x}\frac{\partial\mathbf{M}}{\partial x}+\frac{\partial\mathbf{M}}{\partial y}\frac{\partial\mathbf{M}}{\partial y}\right)\\&\quad+\frac{1}{2}\gamma\frac{\partial^2\mathbf{M}}{\partial z^2}\frac{\partial^2\mathbf{M}}{\partial z^2}-\frac{1}{2}K_1M_z^2-\frac{1}{4}K_2M_z^4.\end{aligned} \tag{6}$$

Here, M_0 stands for the saturation magnetization, and the first term in this expression is the energy of the homogeneous exchange interaction, which is characterized by a local longitudinal magnetic susceptibility $\chi_{\|}$ [37,38]. The anisotropy energy is described by the last two terms with the corresponding uniaxial magnetic anisotropy constants K_1 and K_2. The second, third, and fourth terms in Eq. (6) describe the inhomogeneous exchange interaction, which is responsible for the spiral structure formation. We will suppose that the corresponding inhomogeneous exchange constants satisfy the conditions $\alpha<0,\alpha_{\perp}>0,\gamma>0$. This leads to an inhomogeneity only in the direction of the Oz axis. In such magnets, the following modulated magnetic structures can be realized: a "simple spiral" (SS) and a "longitudinal spin wave". In both cases the spiral wave vector is $\mathbf{k}_0=(0,0,k_0=\sqrt{|\alpha|/2\gamma})$.

For the case of the exchange spiral structure with the energy (6), the dissipative function takes the form

$$\begin{aligned}q(\mathbf{H}^{\text{eff}})&=\frac{1}{2}\lambda^{r}_{ik}H^{\text{eff}}_{i}H^{\text{eff}}_{k}+\frac{1}{2}\lambda^{\text{ex}}_{ik}\frac{\partial\mathbf{H}^{\text{eff}}}{\partial x_i}\frac{\partial\mathbf{H}^{\text{eff}}}{\partial x_k}\\&\quad+\frac{1}{2}\lambda^{exs}_{ik}\frac{\partial^2\mathbf{H}^{\text{eff}}}{\partial x_i^2}\frac{\partial^2\mathbf{H}^{\text{eff}}}{\partial x_i^2},\end{aligned} \tag{7}$$

where λ^{r}_{ik}, $\lambda^{\rm ex}_{ik}$, and λ^{exs}_{ik} are tensors which characterize the dissipative processes of relativistic (λ^{r}_{ik}) and exchange ($\lambda^{\rm ex}_{ik}$, λ^{exs}_{ik}) natures. It is important to note that nonzero components of these tensors are determined by the crystalline symmetry [37,38]. In the case of energy represented by expression (6), they will have the form $\lambda^{r}_{ik} = {\rm diag}(\lambda^{r}_{11}, \lambda^{r}_{11}, 0)$, $\lambda^{\rm ex}_{ik} = {\rm diag}(\lambda^{\rm ex}_{11}, \lambda^{\rm ex}_{11}, \lambda^{\rm ex}_{33})$, $\lambda^{exs}_{ik} = {\rm diag}(0, 0, \lambda^{exs}_{33})$. As was said above, relaxation constants $\lambda^{\rm ex}_{11}$ and $\lambda^{\rm ex}_{33}$ must be proportional α and $\alpha_{\perp}$, respectively, and λ^{exs}_{33} must be proportional to γ. Note that if there are high-order derivatives in the system's energy, we must take into account magnetization derivatives in the dissipation function up to this order, too. Thus, from Eq. (7), it is easy to obtain the relaxation term in the form

$$\mathbf{R} = \lambda^{r}_{ik}\mathbf{H}^{\rm eff} - \lambda^{\rm ex}_{ik}\frac{\partial^2 \mathbf{H}^{\rm eff}}{\partial x_i \partial x_k} + \lambda^{exs}_{ik}\frac{\partial^4 \mathbf{H}^{\rm eff}}{\partial x_i^2 \partial x_k^2}. \tag{8}$$

We see that the presence of higher-order derivatives in the dissipative function (7) leads to the appearance of the higher-order spatial derivatives in the relaxation term of the LL equation as well. These terms represent both the adiabatic torque generated by the magnetization variation in time, generally considered the intrinsic Gilbert damping [the first term in Eq. (8)], and the nonadiabatic torque generated by the magnetization variation in space [last two terms in Eq. (8)].

B. Exchange-relativistic spiral

The competition between exchange and exchange-relativistic anisotropic interactions is another cause for the appearance of a spiral structure in magnets. In this case, there are terms in the system's energy that are linear in magnetization derivatives, conventionally written as $\sim \mathbf{M}rot\mathbf{M}$ and describing the so-called DM interaction in the absence of an inversion center in the crystal [18,46]. The magnetic energy density of crystal now acquires linear derivatives and reads

$$\begin{aligned} f\left(\mathbf{M}, \partial\mathbf{M}/\partial\mathbf{x}_i\right) = {} & \frac{\left(\mathbf{M}^2 - M_0^2\right)^2}{8\chi_{||}M_0^2} + \frac{1}{2}\alpha\frac{\partial \mathbf{M}}{\partial x_i}\frac{\partial \mathbf{M}}{\partial x_i} + D\mathbf{M}rot\mathbf{M} \\ & - \frac{1}{2}K_1\left(M_x^2M_y^2 + M_x^2M_z^2 + M_y^2M_z^2\right) \\ & - \frac{1}{2}K_2M_x^2M_y^2M_z^2. \end{aligned} \tag{9}$$

At a strong enough DM interaction D, the presence of the Lifshitz invariant $\mathbf{M}rot\mathbf{M}$ leads to the appearance of a long-wave modulation state, the so-called relativistic spiral. In this case the modulated magnetic structure of the SS type is realized, with the spiral wave vector $\mathbf{k}_0 = (0, 0, k_0 = D/\alpha)$. Here, the constant of inhomogeneous exchange interaction α is positive $\alpha > 0$, while the constant D which characterizes the DM interaction magnitude can have any sign [18,47].

The density of the dissipative function of a magnet with a spiral magnetic structure of exchange-relativistic origin takes the form

$$\begin{aligned} q(\mathbf{H}^{\rm eff}) = {} & \frac{1}{2}\lambda^{r}_{ik}H_i^{\rm eff}H_k^{\rm eff} + \lambda^{nr}_{ik}\mathbf{H}^{\rm eff}rot\mathbf{H}^{\rm eff} \\ & + \frac{1}{2}\lambda^{\rm ex}_{ik}\frac{\partial \mathbf{H}^{\rm eff}}{\partial x_i}\frac{\partial \mathbf{H}^{\rm eff}}{\partial x_k}. \end{aligned} \tag{10}$$

Again, in accordance with the expression for magnetic energy density, the tensors in the first and second terms must obey the crystalline symmetry. Particularly, for a cubic crystal we have $\lambda^{r}_{ik} = {\rm diag}(\lambda^{r}_{11}, \lambda^{r}_{11}, \lambda^{r}_{11})$, $\lambda^{\rm ex}_{ik} = {\rm diag}(\lambda^{\rm ex}_{11}, \lambda^{\rm ex}_{11}, \lambda^{\rm ex}_{11})$. The tensor λ^{nr}_{ik} describes inhomogeneous relativistic interaction. Its components are also chosen based on the corresponding term in the energy density (9). In the simplest case, when D is a constant, the tensor components will be $\lambda^{nr}_{ik} = {\rm diag}(\lambda^{nr}_{11}, \lambda^{nr}_{11}, \lambda^{nr}_{11})$, and λ^{nr}_{11} must be proportional D. However, it is possible [18,47] that only some of the $\mathbf{M}rot\mathbf{M}$ components should be taken into account in the energy (9). In this case, only the corresponding components in the tensor λ^{nr}_{ik} are nonzero.

In accordance with Eq. (10), the relaxation term in the LL equation for the case of relativistic spiral reads

$$\mathbf{R} = \lambda^{r}_{ik}\mathbf{H}^{\rm eff} + 2\lambda^{nr}_{ik}rot\mathbf{H}^{\rm eff} - \lambda^{\rm ex}_{ik}\frac{\partial^2 \mathbf{H}^{\rm eff}}{\partial x_i \partial x_k}. \tag{11}$$

Equation (11) reveals directly that the damping term includes the local damping (adiabatic torque $\sim \mathbf{H}_{\rm eff}$) and nonlocal torque/damping generated by a spatially nonuniform magnetic order (nonadiabatic contributions $\sim rot\,\mathbf{H}^{\rm eff}$ and $\sim \frac{\partial^2 \mathbf{H}^{\rm eff}}{\partial x_i \partial x_k}$). Hence, the DM interaction modifies the magnon damping by the term $\sim rot\,\mathbf{H}^{\rm eff}$, which now contains linear derivatives and thus is antisymmetric in the wave vector. As a result, the presence of a preferential direction in the system will also be manifested in different dampings of spin waves with opposite wave vectors along the selected direction. From the relaxation term expression for the exchange spiral, Eq. (8), the spin wave damping nonreciprocity is not so directly evident, but it is present there. We illustrate the obtained results with simple examples.

III. SPIN WAVE DAMPING

A. Exchange-relativistic spiral

We start with the consideration of relativistic spiral structures due to the DM interaction which is among topical issues today [11,12,14,15,47–49]. To simplify cumbersome formulas for the dispersion law and damping of spin waves, we will neglect fourth-order terms in magnetization in the energy, Eq. (9). Propagation of a spin wave is considered along the Oz axis of the magnet, so that the direction of the magnetic oscillation wave vector $\mathbf{k} = (0, 0, k)$ coincides with the direction of the spiral wave vector $\mathbf{k}_0 = (0, 0, k_0)$.

From the group-theoretical interpretation of the functional (9) for a cubic crystal, the energy density reads

$$\begin{aligned} & f(\mathbf{M}, \partial\mathbf{M}/\partial\mathbf{x}_i) \\ & = r\left(M_+M_- + M_z^2\right) + i\frac{D}{2}\left(\frac{\partial M_+}{\partial z}M_- - \frac{\partial M_-}{\partial z}M_+\right) \\ & \quad + \frac{\alpha}{2}\left[\frac{\partial M_+}{\partial z}\frac{\partial M_-}{\partial z} + \left(\frac{\partial M_z}{\partial z}\right)^2\right], \end{aligned} \tag{12}$$

where $r = -1/4\chi_{||}$ and the components of the magnetization vector are presented in the form $M_{\pm} = M_x \pm iM_y$, M_z. Then,

the dissipative function density q is

$$q = 2i\lambda_{11}^{nr}\left(\frac{\partial H_-^{\rm eff}}{\partial z}H_+^{\rm eff} - \frac{\partial H_+^{\rm eff}}{\partial z}H_-^{\rm eff}\right) + \frac{1}{2}\lambda_{33}^{\rm ex}\left[\frac{\partial H_+^{\rm eff}}{\partial z}\frac{\partial H_-^{\rm eff}}{\partial z} + \left(\frac{\partial H_z^{\rm eff}}{\partial z}\right)^2\right]. \quad (13)$$

Here, the relativistic part of the dissipative function is omitted because the anisotropy terms in energy are neglected.

The equation of motion of the magnetic moment in the component form is

$$\begin{aligned}\frac{\partial M_+}{\partial t} &= ig\left(M_+H_z^{\rm eff} - 2M_zH_-^{\rm eff}\right) + R_+,\\ \frac{\partial M_-}{\partial t} &= -ig\left(M_-H_z^{\rm eff} - 2M_zH_+^{\rm eff}\right) + R_-,\\ \frac{\partial M_z}{\partial t} &= ig\left(M_-H_-^{\rm eff} - M_+H_+^{\rm eff}\right) + R_z.\end{aligned} \quad (14)$$

In the system (14), the effective magnetic field $\mathbf{H}^{\rm eff}$ can be easily found from the energy, Eq. (12), using expression (4), and the relaxation term $\mathbf{R}$ can be obtained similarly from Eq. (13). To calculate the frequencies of spin waves, we will consider small deviations from the equilibrium values of magnetic moment $\mathbf{M}(\mathbf{r},t) = \mathbf{M}_0(\mathbf{r}) + \mathbf{m}(\mathbf{r},t)$ and also represent them in terms of the Fourier components $\mathbf{m}(\mathbf{r},t) \sim \exp[-i(\omega t - \mathbf{kr})]$. We have

$$\begin{aligned}M_+ &= M_{0+} + m_+ = M_0e^{ik_0z} + m_1e^{ik_0z}e^{-i(\omega t-\mathbf{kr})},\\ M_- &= M_{0-} + m_- = M_0e^{-ik_0z} + m_2e^{-ik_0z}e^{i(\omega t-\mathbf{kr})},\\ M_z &= 0 + m_z = m_3e^{-i(\omega t-\mathbf{kr})}.\end{aligned} \quad (15)$$

Here, $\mathbf{k}$ and ω are the wave vector and the angular frequency of spin waves, respectively, and M_0 stands for the saturation magnetization. The factor $\exp(\pm ik_0z)$ in the components of magnetization vector describes a long-periodic spiral structure with the modulation period $2\pi/k_0$. After substituting Eqs. (15) into Eqs. (14) and their linearization, we obtain the system of equations

$$\begin{aligned}i\omega m_1 &= L_1Z_1m_1 + igM_0\Omega_1m_3,\\ i\omega m_2 &= L_2Z_2m_2 - igM_0\Omega_1m_3,\\ i2\omega m_3 &= igM_0\Omega_2m_1 - igM_0\Omega_3m_2 + 2L_3Z_3m_3,\end{aligned} \quad (16)$$

where the quantities $\Omega_1 = [2Dk_0 + \alpha(k^2 - k_0^2)]$, $\Omega_2 = k[-2D + \alpha(k + 2k_0)]$, and $\Omega_3 = k[2D + \alpha(k - 2k_0)]$ determine the spin wave frequencies, while $Z_1 = [2r - 2D(k + k_0) + \alpha(k + k_0)^2]$, $Z_2 = [2r + 2D(k - k_0) + \alpha(k - k_0)^2]$, $Z_3 = (2r + \alpha k^2)$, $L_1 = [-2\lambda_{11}^{nr}(k + k_0) + \lambda_{33}^{\rm ex}(k + k_0)^2]$, $L_2 = [2\lambda_{11}^{nr}(k - k_0) + \lambda_{33}^{\rm ex}(k - k_0)^2]$, and $L_3 = \lambda_{33}^{\rm ex}k^2$ characterize the spin wave damping. The spin wave dispersion relation can be obtained from the system of equations (16), and its general form can be written as

$$g^2M_0^2\Omega_1\Omega_2(i\omega - L_1Z_1) - (i\omega - L_2Z_2)(2\omega^2 + 2i\omega - g^2M_0^2\Omega_1\Omega_2 - 2L_1L_3Z_1Z_3) = 0. \quad (17)$$

Within a linear approximation in the relaxation tensor, the dispersion law can be represented in the form

$$\omega_{sw} = \pm\Omega + i\Gamma, \quad (18)$$

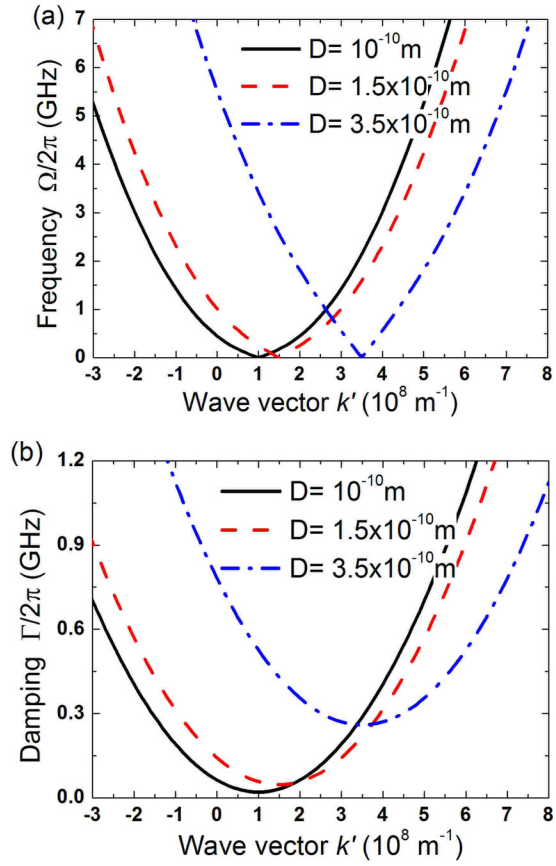

FIG. 1. (a) Dispersion of the spin wave frequency and (b) spin wave damping in the exchange-relativistic spirals. The dispersion curves were computed for various values of the DM interaction.

where the spin wave frequency Ω is

$$\Omega = \Omega(k, k_0) = gM_0\sqrt{\Omega_1(\Omega_2 + \Omega_3)}, \quad (18a)$$

and the spin wave damping Γ reads

$$\Gamma = \Gamma(k, k_0) = -\frac{1}{2}\left(\frac{\Omega_2}{\Omega_2 + \Omega_3}L_1Z_1 + \frac{\Omega_3}{\Omega_2 + \Omega_3}L_2Z_2 + L_3Z_3\right). \quad (18b)$$

The quadratic terms in the damping parameter omitted here describe the decrease in spin wave frequency; they are cumbersome but small.

Note that the dispersion law (18) is obtained in the intrinsic (rotated) frame of the spiral. To find the spectrum of spin waves in the laboratory frame, one needs to do the transition $k \longrightarrow k' - k_0$. In this case, the Goldstone mode arises in the spectrum at $k' = k_0$. This Goldstone mode is associated with the violation of the ground-state translational symmetry in the direction of the $\mathbf{k}_0$ vector (for more details see, e.g., Ref. [16]).

The results of numerical calculations of the spin wave spectrum of a relativistic spiral in the laboratory coordinate system are shown in Fig. 1, where the dispersion of spin wave frequency [Fig. 1(a)] and spin wave damping [Fig. 1(b)] are shown for different values of the DM interaction. For the material parameters to have such a structure occur we used [12,14,15,49] $g \approx 2 \times 10^7\,{\rm Oe^{-1}\,s^{-1}}$, $M_0 \approx 10^4\,{\rm Oe}$, $\alpha \approx 10^{-18}\,{\rm m^2}$ (corresponding to $A \approx 10^{-11}\,{\rm J/m}$), $D \approx 10^{-10}\,{\rm m}$

($\approx 10^{-3}$ J/m^2), $\chi_{||} \approx M_0\mu_B/k_BT_C$, and $T_C \approx 300$ K; k_B and μ_B are the Boltzmann constant and the Bohr magneton, respectively. The relaxation constants are roughly estimated for the case of small damping. Stasinopoulos *et al.* found for the insulating chiral magnet Cu_2OSeO_3 a remarkably small Gilbert damping of about 10^{-4} [13]. For our effective damping parameter we used the values $\lambda_{11}^{nr} = 10^{-4}$ m and $\lambda_{33}^{\rm ex} = 10^{-10}$ m^2.

As one can see in Fig. 1(a), if $k' \longrightarrow k_0$, the frequency of the spin waves goes to zero, and in the vicinity of this wave vector the energy of the spin wave exhibits almost linear momentum dependence. But, as Fig. 1(b) shows, due to the DM interaction, at $k' = k_0 (= D/\alpha)$ the spin wave relaxation stays finite, and the spin wave frequency ω_{sw} becomes fully imaginary. Hence, there is a nonzero spin wave damping of the nonreciprocal magnons even for the Goldstone mode. Namely, when $k' \longrightarrow k_0$ the spin wave damping is a function of the spiral wave vector $\mathbf{k}_0$ modulus:

$$\Gamma_{k'\longrightarrow k_0} \longrightarrow -\frac{1}{2}\left(2r - 2Dk_0 + \alpha k_0^2\right)\left(-2\lambda_{11}^{nr}k_0 + \lambda_{33}^{\rm ex}k_0^2\right) = \frac{D}{2\alpha}\left(\frac{1}{2\chi_{||}} + \frac{D^2}{\alpha}\right)\left(\lambda_{33}^{\rm ex}\frac{D}{\alpha} - 2\lambda_{11}^{nr}\right). \quad (19)$$

This result is sufficiently general and robust against the details of the magnon scattering mechanism. Since the anisotropy energy of relativistic nature (fourth-order terms in magnetization) was not taken into account in expression (12) for the total energy, the corresponding relativistic part was omitted also in the dissipative function (13). If we take into account the relativistic part of the dissipation, we obtain an additional term, $(\frac{1}{2\chi_{||}} + \frac{D^2}{\alpha})(\lambda_{11}^r)$, that corresponds to a standard LLG-like part of the damping.

B. Exchange spiral

Let us now investigate the spin wave spectrum for the SS structure that is realized if $K_1 < 0$ in Eq. (6). As above, the propagation of spin waves will be considered along the chiral axis. Assuming the same simplifications as for the relativistic spiral structure and using the $M_\pm$ and M_z variables, the energy density for a system with exchange spiral structure reads

$$f(\mathbf{M}, \partial\mathbf{M}/\partial\mathbf{x}_i) = r\left(M_+M_- + M_z^2\right) + \frac{1}{2}\alpha\left[\frac{\partial M_+}{\partial z}\frac{\partial M_-}{\partial z} + \left(\frac{\partial M_z}{\partial z}\right)^2\right] + \frac{1}{2}\gamma\left[\frac{\partial^2 M_+}{\partial z^2}\frac{\partial^2 M_-}{\partial z^2} + \left(\frac{\partial^2 M_z}{\partial z^2}\right)^2\right] - \frac{1}{2}K_1M_z^2, \quad (20)$$

and the dissipative function density q is

$$q = \frac{1}{2}\lambda_{11}^r H_+^{\rm eff}H_-^{\rm eff} + \frac{1}{2}\lambda_{33}^{\rm ex}\left[\frac{\partial H_+^{\rm eff}}{\partial z}\frac{\partial H_-^{\rm eff}}{\partial z} + \left(\frac{\partial H_z^{\rm eff}}{\partial z}\right)^2\right] + \frac{1}{2}\lambda_{33}^{exs}\left[\frac{\partial^2 H_+^{\rm eff}}{\partial z^2}\frac{\partial^2 H_-^{\rm eff}}{\partial z^2} + \left(\frac{\partial^2 H_z^{\rm eff}}{\partial z^2}\right)^2\right]. \quad (21)$$

The effective magnetic field $\mathbf{H}^{\rm eff}$ can easily be found from the energy (20) using expression (4), and the relaxation term $\mathbf{R}$ can be obtained similarly from Eq. (8). After substituting them into Eq. (14) and using approximation (15), we can calculate the dispersion law of spin waves. It also can be presented as Eq. (18) and dispersion laws (18a) and (18b), but with different quantities. Now we have $\Omega_1 = [-K_1 + \alpha(k^2 - k_0^2) + \gamma(k^4 - k_0^4)]$, $\Omega_2 = k(k + 2k_0)[\alpha + \gamma(2k_0^2 + 2k_0k + k^2)]$, $\Omega_3(k_0) = \Omega_2(-k_0)$, $Z_1 = [2r + \alpha(k + k_0)^2 + \gamma(k + k_0)^4]$, $Z_2(k_0) = Z_1(-k_0)$, $Z_3 = (2r - K_1 + \alpha k^2 + \gamma k^4)$, $L_1 = [\lambda_{11}^r + \lambda_{33}^{\rm ex}(k + k_0)^2 + \lambda_{33}^{exs}(k + k_0)^4]$, $L_2(k_0) = L_1(-k_0)$, and $L_3 = \lambda_{33}^{\rm ex}k^2 + \lambda_{33}^{exs}k^4$. Equation (18a) gives a well-known result for the frequency of spin waves in a spiral ferromagnet [16] when the damping is neglected. A formal transition to a uniaxial ferromagnet with an "easy-plane" anisotropy occurs when the wave vector of the spiral structure $\mathbf{k}_0$ goes to zero. In this case the dispersion law (18a) transforms into the previously obtained results [37,38].

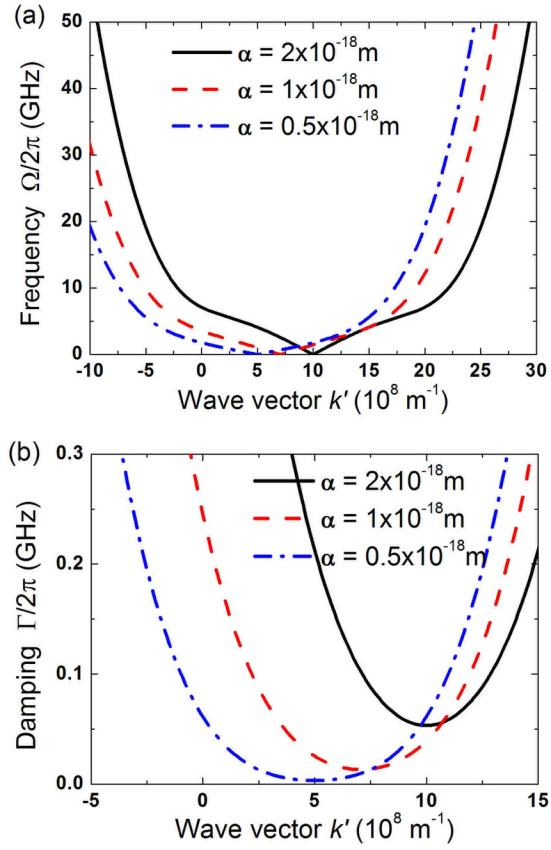

FIG. 2. (a) Dispersion of the spin wave frequency and (b) spin wave damping in the exchange spirals. The dispersion curves were computed for various values of the inhomogeneous exchange interaction.

The results of numerical calculations of the spin wave spectrum in the exchange spiral in the laboratory coordinate system are shown in Fig. 2, where the dispersion of the spin wave frequency [Fig. 2(a)] and spin wave damping [Fig. 2(b)] are shown for different values of the exchange parameter. (The relaxation constants are roughly estimated for the case of small damping in a magnet with $T_C \approx 360$ K and $\lambda_{11}^r = 10^{-6}$, $\lambda_{33}^{\rm ex} = 10^{-15}$ m^2, and $\lambda_{33}^{exs} = 10^{-30}$ m^4.) As in the case of a relativistic spiral, the spin wave frequency goes to zero at

the point $k' = k_0$ ($= \sqrt{-\alpha/2\gamma}$), while the spin wave damping remains finite when $k' \longrightarrow k_0$. Now we have in the limit

$$\Gamma_{k' \longrightarrow k_0} \longrightarrow -\frac{1}{2}\left(2r + \alpha k_0^2 + \gamma k_0^4\right)\left(\lambda_{11}^r + \lambda_{33}^{\mathrm{ex}} k_0^2 + \lambda_{33}^{exs} k_0^4\right) = \frac{1}{4}\left(\frac{1}{\chi_{||}} + \frac{\alpha^2}{2\gamma}\right)\left(\lambda_{11}^r - \lambda_{33}^{\mathrm{ex}}\frac{\alpha}{2\gamma} + \lambda_{33}^{exs}\frac{\alpha^2}{4\gamma^2}\right). \tag{22}$$

From expression (22), one can see that the spin wave damping consists of the standard LLG-like part $\Gamma_{LLG} \sim \frac{\alpha^2}{\gamma}\lambda_{11}^r$ (for the exchange-relativistic spiral, $\Gamma_{LLG} \sim \frac{D^2}{\alpha}\lambda_{11}^r$). This part takes into account the nonreciprocity of the damping but only partially. Our results show that there are additional nonadiabatic parts (due to nonlocal torques) $\Gamma_{NL} \sim (\frac{1}{\chi_{||}} + \frac{\alpha^2}{2\gamma})(-\lambda_{33}^{\mathrm{ex}}\frac{\alpha}{2\gamma} + \lambda_{33}^{exs}\frac{\alpha^2}{4\gamma^2})$ and (19). The contribution of these nonadiabatic parts is important for the nonzero wave vector k' and substantially increases for short spin waves (large k' values).

Since the frequency (18a) of spin waves for $k' \longrightarrow k_0$ tends to zero, it also follows from (19) and (22) that, in magnets with a modulated magnetic structure, spin waves with a wave vector equal to $\mathbf{k}_0$ are absolutely damped and cannot propagate. This situation is similar to that found for the case of longitudinal oscillations in ferromagnets [37–39]. From the system of Eqs. (16), the frequency ω_m of longitudinal oscillations of the magnetic moment absolute value in the equilibrium state can also be obtained. In the linear approximation in relaxation tensors, it equals the spin wave damping: $\omega_m = i\Gamma$ [see Eq. (18b)]. Thus, the oscillation frequency of the magnetic moment absolute value is purely imaginary. This fact indicates a rapid aperiodic alignment of magnetization along its equilibrium direction [37,38].

An important general feature of the magnetization dynamics damping in long-periodic structures comes from the existence of the term $1/\chi_{||}$ (this term characterizes the homogeneous exchange energy) in expressions (18b), (19), and (22). The longitudinal magnetic susceptibility describes the homogeneous exchange interaction, and its order of magnitude is $\chi_{||} \sim M_0\mu_B/J_{exc}$, where J_{exc} is the exchange integral. Thus, there is an exchange amplification of both the transversal spin wave damping and the frequency of longitudinal magnetization oscillations. It is determined in this case by the anisotropy of exchange interaction (different inhomogeneous exchange constants for different directions). This result differs from those obtained in earlier works [37–39] where the exchange interaction was assumed to be isotropic and an amplification was obtained only for longitudinal magnetization oscillations.

IV. DISCUSSION AND CONCLUSION

During the past decade spin waves, as elementary excitations in nanostructured magnetic materials, have been the subject of many experimental and theoretical studies. Now, it has become clear that increasing the lifetime and the stability of magnons, primarily determined by the relaxation processes, is of crucial importance for their applications. The standard LLG equation is useful in numerical simulations of spin dynamics. However, one should pay attention to the fact that it is a phenomenological equation utilized only within a local approximation of torques and is not suitable for studies of nonlocal effects in systems with noncollinear magnetic structure. Recently, it was shown within the phenomenological description of the LLG equation that in noncollinear magnetic systems (e.g., magnetic vortices, skyrmions, etc.) the effective damping parameter α_{eff} is larger than the Gilbert damping due to the noncollinear spin arrangement [50].

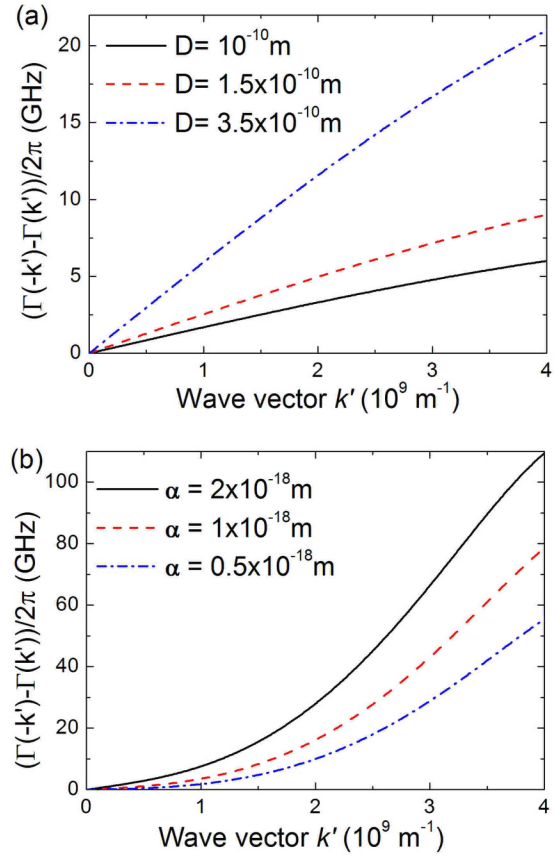

FIG. 3. The chiral-induced damping difference $\Delta\Gamma = \Gamma(-k) - \Gamma(+k)$ of different counterpropagating spin waves for (a) the exchange-relativistic and (b) exchange spirals.

In this paper, a general method was given to find the relaxation term in the magnetization equation of motion for magnetic materials with magnetochiral structure. It was shown that the presence of a preferential direction in the magnetic structure not only leads to a difference in the phase velocities of volume spin waves with given energy and opposite wave vectors but also manifests itself in different dampings of these waves. In Fig. 3, the chiral-induced damping difference $\Delta\Gamma = \Gamma(-\mathbf{k}) - \Gamma(\mathbf{k})$ of different counterpropagating spin waves is shown for exchange-relativistic [Fig. 3(a)] and exchange [Fig. 3(b)] spirals. Figure 3 directly reveals that, in the long-wave limit, the damping for spin waves propagating against the spiral direction is faster than for those propagating along spiral direction.

Damping nonreciprocity is physically conditioned by the competition between different magnetic interactions. In the case of exchange-relativistic long-periodic magnetic structures, the isotropic exchange interaction α stabilizes the collinear ferromagnetic spin state, whereas the DM

interaction D favors helical magnetochirality, and their ratio D/α determines the incommensurate wave vector of the helical modulation. Because of the competition between these two terms, spin fluctuations of the dynamical state are not around the static spin structure, in particular, resulting in a nonreciprocal magnon damping. In the case of the exchange spiral, there is a competition between the exchange interactions stabilizing the ferromagnetic and antiferromagnetic orders. Again, due to competition between different magnetic orders, spin fluctuations of the dynamical state are not around the static spin structure, which results in different spin wave damping of nonreciprocal magnons. This feature of magnetization dynamics in materials with spatially varying local magnetic structures is sufficiently general and robust against the detailed magnon scattering mechanisms. In particular, this causes finite damping even of the Goldstone mode. These are the main results of our analysis.

Just recently, the asymmetry of the magnon dispersion relation and the magnon lifetime with respect to their counterpropagating directions was observed in multilayer films [14]. The nonreciprocal character of magnon propagation caused by the bulk crystallographic structure has been detected and investigated in a noncentrosymmetric ferromagnet [12,13,29] and in a such an antiferromagnet [30]. The authors analyzed the obtained results in the framework of the standard LLG equation where the relaxation term reads $\mathbf{R} = \alpha_G \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}$ and the Gilbert damping α_G is constant. In linear approximation by the parameter α_G, it was found that spin wave damping is $\Gamma \approx \alpha_G \Omega(k, k_0)$. Thus, in the presence of the DM interaction, the spin wave linewidth is modified by a term $\sim\omega_{DM}$, which is antisymmetric in the wave vector. This, indeed, accounts for the experimentally observed asymmetric linewidth [12–14,29,30], but only partially. In particular, within this approach, the Goldstone mode damping becomes zero at $k' = k_0$ [see Figs. 1(a) and 2(a) for spin wave frequency dispersion $\Omega(k, k_0)$], in qualitative disagreement with the results obtained using the relaxation term (8) or (11) [see Figs. 1(b) and 2(b)]. Thus, while some peculiarities of spin wave dynamics are reproduced within the standard LLG framework, it does not take into account all the magnetochiral nonreciprocity effects in magnetization dynamics. To get them, the dissipative function should be constructed with the same principles as the quasiequilibrium thermodynamical potential and has to contain terms with the same nature as the total energy. Thus, it should be noted that the phenomenological models obtained in this work may be applicable to all similar cases (DM interaction or inhomogeneous exchange interaction).

In conclusion, the interrelation between space-time geometry and dynamics may be the most fundamental concept in modern physics. From this aspect, the meaning of chirality is to convert the spatial structure to the dynamical properties of matter [8,51]. In this paper, within the LLBar phenomenological description of relaxation phenomena in magnetic materials, a dissipative function was constructed for crystals with different symmetry with magnetochiral nonreciprocity. It was shown that the existence of a distinguished spatial direction in the static magnetic structure not only leads to an "energy" nonreciprocity but manifests itself in different intrinsic nonlocal dampings of volume spin waves for a given wave vector but with opposite directions, $\Gamma(\mathbf{k}) \neq \Gamma(-\mathbf{k})$. This feature of magnetization dynamics in materials with spatially varying local magnetic structures is general and does not depend on the details of magnon scattering. The "damping" nonreciprocity for magnetic dynamics should be taken into account in numerical simulations of spin waves dynamics in such systems (e.g., by incorporating the LLBar equation into the Object Oriented Micromagnetic Framework (OOMMF) code [52]). Experimental analysis of the spin wave dynamics should include these specifics too. We expect that the present results may have practical applications in the field of magnonics, and they could be applied for other types of periodically modulated structures [53]. In particular, these features of spin wave dynamics pave a way for the design of efficient spin wave diodes based on crystallographic symmetry and long-periodic magnetic structures.

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