Reply to "Comment on 'Effect of rotation on the elastic moduli of solid ^{4}He' "

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We respond to the Comment by Pushkarov. We find that, in solid ^{4}He, ^{3}He impurity atoms that pin dislocation segments are not impuritons, but randomly walking particles. The interpretation for the rotation effect, which was based on the impuriton model, does have to be totally reconsidered, taking into account the motion of randomly walking ^{3}He in the strong elastic potential near the core of edge dislocations.

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The elastic anomaly of solid ^{4}He discovered by Day and Beamish has been elucidated well as an effect of ^{3}He impurity atoms to the dynamics of dislocation network [1]. We have found that the elastic anomaly, an increase in shear modulus below 0.2 K, is suppressed when the solid ^{4}He sample is rotated unidirectionally up to 4 rad/s [2]. Since the dynamics of the dislocation network seems to be hardly influenced by sample rotation with several rad/s, we have concluded that the pinning probability of ^{3}He impurities are suppressed several orders of magnitude by rotation [2]. Past studies suggested that ^{3}He impurity atoms behave as so-called impuritons, a Bloch-like wave in the periodic potential of a solid ^{4}He matrix. We have presented a possible mechanism based on the impuriton picture that the circular motion of ^{3}He impuritons by Coriolis force can decrease the probability of pinning of ^{3}He on dislocation segments [2]. Pushkarov criticizes our interpretation concerning the Coriolis force acting on ^{3}He impuritons [3]. The claims are summarized as follows:

(1) We proposed that the Coriolis force exerted to ^{3}He impuritons can be enhanced by effective mass, but no enhancement occurs with gravitational and centrifugal forces. This is in contradiction with equality of inertial and gravitational masses. (2) The quantity responsible for inertial effects of ^{3}He impuritons in solid ^{4}He is not the effective mass, but the bare mass of ^{3}He. Therefore, the enhancement of Coriolis force proposed by us does not occur. (3) Pushkarov proposed in previous theoretical work that diffusion of vacancies in solid ^{4}He should show strong spatial anisotropy when the solid ^{4}He sample is rotated. We mentioned this theoretical result as a possible example of rotation effect on defect dynamics in solid ^{4}He. Pushkarov has applied the theory to the case of ^{3}He impurity and concluded that the rotation effect does not exist in our current experimental condition.

We agree with the claims: The equivalence principle should hold even in the effective band mass of impuritons. As to claim (2), we point out a counterexperiment that showed that the macroscopic motion of a neutron in a Si crystal is determined by the band effective mass of the neutron [4]. However, this experiment has also confirmed that the equivalence principle holds between gravitational and inertial effective masses. Since the effective mass of ^{3}He impuritons can reach 10^6 times the bare mass, the enhancement of ^{3}He mass would lead to an unrealistic spatial distribution in solid ^{4}He samples. Within the constraint of equivalence principle, it is difficult to conclude that the Coriolis force surpasses other forces. We therefore abandon the idea that the motion of ^{3}He is governed by the impuriton effective mass. In the last claim (3), Pushkarov develops his theory to the case of ^{3}He impuritons, and concludes that the rotation effect is absent in our experimental condition. We also agree with this conclusion, although we did not apply this theory to our experimental result in our publication [2].

Although the elasticity of solid ^{4}He is dominated by interaction between ^{3}He impurity and dislocation segment, the interaction was not theoretically studied at all. We find that, from a simple discussion shown below, the impuriton picture is inappropriate in discussing pinning of ^{3}He to dislocations because of the strong elastic interaction between dislocation and ^{3}He impurity.

Under external potential $U(r)$, the state of ^{3}He impurity is determined by the magnitude relation between impuriton bandwidth Δ and the change in potential when impurity moves from a site to a neighbor site, $a|\nabla U(r)|$, where a is a lattice constant [5]. The impuriton picture is valid at position r_0 only if the condition $\Delta > a|\nabla U(r)|_{r=r_0}$ is satisfied [5]. On the other hand, if $\Delta < a|\nabla U(r)|_{r=r_0}$, i.e., external force is large or bandwidth is small, the impuriton picture is not justified. Landesman, Winter, and Yamashita showed that in this case ^{3}He can move from site to site by random walk [6]. Since ^{3}He atoms are attracted to the dislocation core by elastic interaction, the impuriton picture breaks down near a dislocation segment. We schematically explain the situation.

Suppose a straight edge dislocation in the z direction as shown in Figs. 1(a) and 1(b). The origin and xy axes are taken in the figure. The ^{3}He dislocation elastic interaction, which is caused by the volume difference of a ^{3}He atom in the solid ^{4}He, is given by the following formula [7]:

$$W(x, y) = \frac{4}{3}\left(\frac{1+\nu}{1-\nu}\right)\frac{\mu b \epsilon r_4^3 y}{x^2 + y^2 + r_0'^2}, \tag{1}$$

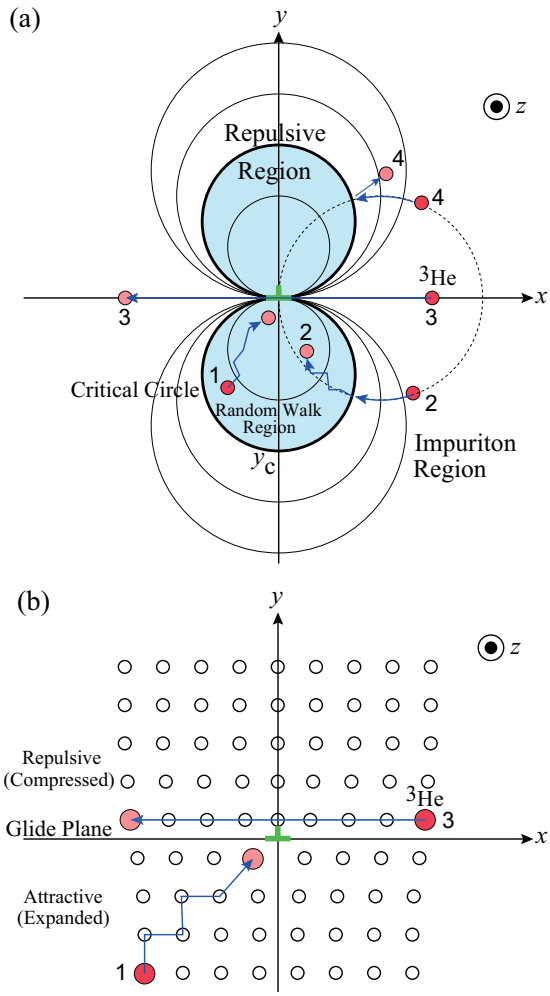

FIG. 1. (a) Solid circles show equipotential lines of the elastic interaction Eq. (1) between a ^{3}He atom and a straight edge dislocation located at the origin (shown as a mark $\perp$) [7]. Bold solid circles show the "critical circles" at which $\Delta = a|\nabla W(x, y)|$ [5]. The impuriton picture is valid only outside the circles, while inside them (light-blue region) ^{3}He atoms can move only by random walk [6]. Red and orange circles schematically indicate ^{3}He atoms, with numbers 1–4. Atom 1 can approach the dislocation core by random walk. Atom 2 propagates on a potential gradient shown by dashed circle, and is converted to randomly walking atom. Atom 3 propagates along the glide plane as an impuriton, and passes through the dislocation core. Atom 4 propagates as an impuriton in the repulsive region, and is reflected at the critical circle. (b) The dislocation core region on atomic scale. The numbering of ^{3}He atoms has the same meaning as (a). Atom 1 is trapped at the most expanded lattice site after random walk. Atom 3 runs on the glide plane and passes through the core.

where ν is Poisson's ratio ($\sim$0.33), μ is the shear modulus ($\sim$15 MPa), and b is the Burgers vector. When the radii of ^{4}He and ^{3}He atoms are given by r_4 and r_3 respectively, ϵ is defined as $\epsilon = (r_3 - r_4)/r_4$. $r_0' \sim 2b/3$ is a correction for violation of Hooke's law at the dislocation core. Iwasa and Suzuki [8] estimated the depth of the potential at the core of an edge dislocation to be about 0.6 K, which is very close to the value of the pinning energy of ^{3}He, 0.7 K, which was determined from recent elastic measurements [1]. Therefore, Eq. (1) is valid for solid ^{4}He.

Equipotential lines of this interaction are shown in Fig. 1(a). ^{3}He is attracted by dislocation in the lower half region, while in the upper half one the interaction is repulsive. The equipotential lines are circles bunched at the origin (dislocation core). Taking $x = 0$, we can estimate the "critical diameter" of the circle y_c, at which the relation $\Delta = a|\nabla W(x = 0, y)|_{y=y_c}$ holds. For our two solid samples with pressures $P = 3.6$ and 5.4 MPa, the diameter y_c is estimated to be about 3×10^{-7} and 1×10^{-6} m, respectively. These values are 1/10–1/30 of the average length of dislocation segments, i.e., separation of segments, $L_N \sim 1 \times 10^{-5}$ m.

The impuriton picture is valid only outside the "critical circles" of diameter y_c, which are shown by light blue circles (the circle in the upper half plane can also be the critical circle). As shown in Fig. 1(b), ^{3}He atoms are trapped at the sites where the lattice is most expanded. We easily see that ^{3}He cannot approach the dislocation core as an impuriton, because $a|\nabla W(x, y)|$ exceeds Δ inside the critical circle. ^{3}He can approach the trapping site only as a randomly walking particle, which is shown by an illustration number 1 in Figs. 1(a) and 1(b). The impuriton propagating outside the critical circle (shown by the number 2) can be converted to the randomly walking particle. When ^{3}He moves along the gliding plane as shown by an atom number 3 in Figs. 1(a) and 1(b), it can behave as an impuriton. However, since there is no potential gradient in the direction of the glide plane, the impuriton cannot be trapped to the dislocation but passes through the dislocation core region. Eventually, there is no possibility that impuritons are directly trapped by dislocations. Note that ^{3}He impuritons in the repulsive region are reflected at the critical circle, as shown by an atom number 4 in Fig. 1(a).

In real solid ^{4}He, the critical circles will exist around the edge and mixed dislocation segments in a three-dimensional dislocation network, but not around screw dislocations, which do not trap ^{3}He because of no volume expansion. The random walk of ^{3}He will be assisted by phonons and potential-energy fluctuations caused by movement of dislocations.

To conclude, ^{3}He atoms that pin dislocation segments are not impuritons, but randomly walking particles. Therefore, the interpretation for the rotation effect found by us [2] has to be totally reconsidered taking into account the motion of randomly walking ^{3}He in the strong elastic potential near-edge dislocation core.

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