Comment on "Effect of rotation on the elastic moduli of solid ^{4}He"

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This Comment is on the paper "Effect of rotation on the elastic moduli of solid ^{4}He" by T. Tsuiki, D. Takahashi, S. Murakawa,Y. Okuda, K. Kono, and K. Shirahama [Phys. Rev. B **97**, 054516 (2018)]. The authors speculate on a putative effect of the effective mass of ^{3}He impuritons on the angular momentum of solid ^{4}He under rotation. This is a crucial point in their work when trying to explain the variation of the share modulus, μ, with temperature. They refer also to my general theory of kinetics and dynamics of quasiparticles in nonstationary moving bodies subjected to time varying deformations [cf, e.g., Phys. Rep. **354**, 411 (2001)] and in particular to the effect of a strong anisotropy in the temperature dependence of the vacancy diffusion under rotation. The effect is due to Coriolis force. The authors apply by analogy the same idea to ^{3}He impuritons. However their consideration is inconsistent. They replace the bare mass of impurity in the Coriolis force by the impuriton effective mass. The role of the Coriolis force on the impuriton diffusion becomes highly exaggerated and leads to a not existing effect.

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Tsuiki *et al.* [1] have investigated the variation of share modulus, μ, of solid (h.c.p.) ^{4}He with ^{3}He impurities under rotation at different temperatures and have found a minimum of μ in the temperature region 200–500 mK. They assigned this to the role of the Coriolis force on the diffusion of ^{3}He quasiparticles (impuritons). In case of vacancions, the rotation may change the diffusion so that the temperature dependence in the radial direction is $D_{rr} \sim T^9$ while in the direction of the rotation it is $D_{zz} \sim T^{-7}$. This effect follows from the general nonlinear theory of dynamics and kinetics of quasiparticles with arbitrary dispersion relations in crystals subjected to time-varying deformations [2,3]. The theory unifies the nonlinear elasticity theory, transport theory of Boltzman type and Hamilton dynamics, and is exact in the frame of the quasipartile approach. Hamiltonian contains both the bare mass and the dispersion relation in a general form (not only in the effective mass approximation). So, it takes into account inertial effects too. As is well known, the quantity responsible for inertial effects is the *bare mass* (not the *effective mass*). A typical example is the estimation of the electron bare mass (the ratio e/m) in the *Stewart-Tolman effect* [4] (eponym given by L.D. Landau). In case of vacancies the anisotropy in the temperature dependence of D is due to the Coriolis force.

Tsuiki *et al.* applied the same idea to the ^{3}He impuritons. But they used the effective mass of the quasiparticle instead of the bare one. They state in Sec. V B: "The Coriolis force is given by $2m^*\mathbf{\Omega}\times\mathbf{v}$. We emphasize that the mass in the formula of Coriolis force is an effective mass of ^{3}He impuriton, and the velocity is the impuriton group velocity v_g, which is determined by the energy band width Δ. Since the effective mass is large ($10^4 m_3 < m^* < 10^7 m_3$), the magnitude of the Coriolis force $2m^*\Omega v$ exceeds several orders of magnitude of the gravitational and centrifugal forces."

This statement is in strong contradiction with the principle of equality of inertial and gravitational masses. Otherwise, the contribution of the effective mass of impuritons with a concentration 1 ppm and $m^* = 1.3\times10^6 m_3$ will approach the mass of the sample and its weight will increase twice. In addition, the weight will strongly depend on temperature due to the dependence of the impuritons effective mass on temperature. In order to have Coriolis force which exceeds several order of magnitude the centrifugal one the authors keep the mass in the centrifugal force unchanged (equal to the bare mass). However, it is impossible for the Coriolis and centrifugal forces to depend on different masses. The mass in the Newton equation with and without rotating is one and the same. The effective mass approximation (if workable) takes into account the role of the medium. The latter can influence the behavior (e.g., trajectory) of the particle but not the form of the force itself. When a gun shoots a shell its trajectory is affected by atmospheric conditions—rain, wind, pressure etc.—but this doesn't influence the dependence of the Coriolis force on the mass of the shell. Coriolis force depends neither on the weather nor on the crystal structure. The same is valid for centrifugal and gravitational forces.

There is important misunderstanding with the effect of anisotropy in the temperature dependence of the vacancy diffusion due to rotation. The authors apply this effect without taking into account the difference between impuritons and vacancions, as well as in the physical conditions. They write in Sec. II C: "This dramatic anisotropy of defecton diffusion *is essentially the same conclusion as our consideration* focusing on circular motion of impuritons by Coriolis force which is discussed in Secs. V and VI."

I would be glad if my theory can help in solving such a fundamental problem. But it must be done without braking fundamental physical principles and in the region of its validity.

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The authors incorrectly suppose (Sec. II C) that my theory and Fokker-Plank equation refer to zero-point vacancies. It is valid for thermal vacancies delocalized at low temperatures. The anisotropy in the diffusion temperature dependence appears as a result of a Coriolis term in the Fokker-Plank equation. The idea to exploit this effect in case of ^{3}He impuritons is very attractive. However, the conditions for impuritons are different. The vacancion energy bandwidth Δ_v may be of the order of several degrees, and hence $T < \Delta_v$. Only low energy levels are populated and the effective mass approximation works. The impuriton bandwidth in the experiment considered is $\Delta \leqslant 10^{-5}$K and $T \gg \Delta$. This means that the impuriton population in the band is uniform and the concept of effective mass doesn't work. The numbers of impuritons with positive and negative effective masses are equal.

It is important that the Fokker-Plank equation [3] derived for vacancies is valid when the temperature is small compared to the vacancion bandwidth, $T < \Delta_v$, while the situation considered in the paper is the opposite. In fact, the physical conditions of the experiment do not allow observation of any temperature dependence of the impuriton diffusion. The impuriton concentration is $x = 0.3$ ppm and the angular velocity is about $\Omega = 4$ rad/sec. Two polycrystalline samples are employed corresponding to pressures (molar volumes) 3.6 MPa ($V_m = 20.27$ cm^3/mole) and 5.4 MPa ($V_m = 19.44$ cm^3/mole).

Near the melting point at the lowest pressure $V_m \approx 21$ cm^3/mole and the impuriton bandwidth is $\Delta \approx 10^{-4}$ K [5]. It decreases with increasing pressure, so $\Delta \leqslant 10^{-5}$ K for the samples used. As in any complex lattice with two atoms per lattice cell, there are two impuriton branches (acoustical and optical) [5] in h.c.p. helium. The exchange integral J is of the order of $J \approx 10^{-6}$ K $\approx 10^{-29}$ J or less. The impuriton effective mass and velocity are $m^* = \hbar^2/2Ja^2 \geqslant 10^6\, m_3$ and $v \approx a\Delta/\hbar \approx 10^{-3}$ m/s [5], respectively, with an interatomic distance $a \approx 0.36$ nm. In an ideal periodic crystal, the diffusion coefficient depends on temperature due to defecton-phonon scattering. If $T \gg \Delta$ (as in the case considered), then

$$D_T \approx as\left(\frac{\Delta}{\theta_p}\right)^2\left(\frac{\theta_p}{T}\right)^9, \quad \theta_p = \frac{\hbar s}{2a}$$

with s for the Debye sound velocity. If $T \to 0$ then $D \to \infty$ due to infinite impuriton mean free path in the absence of phonons. Then the scattering on other impuritons or lattice deformations comes into effect. In the gas-kinetic approximation, the diffusion is inversely proportional to concentration x and the scattering cross section σ (in units a^2):

$$D_0 \approx Ja^2/x\sigma, \quad \sigma \sim 10^2.$$

The total diffusion coefficient is obtained using the Matthiessen rule:

$$D^{-1} = D_0^{-1} + D_T^{-1}.$$

At $0.1 < T < 1$ K the contribution of D_T is negligible. This means that there is no temperature dependence and of course no anisotropic one.

Let me note that the above consideration is valid for monocrystals with a not too large number of imperfections. The conditions of the quasiparticle approach must be fulfilled. But whatever happens, the Coriolis force will keep its form.

The effect of rotation on the elastic moduli of solid ^{4}He has not found a reasonable theoretical explanation. The considerations based on the Coriolis force and its putative dependence on the impuriton effective mass are in contradiction with the principle of equality of gravitational and inertial masses and are not acceptable. The impuriton diffusion at the experimental conditions does not depend on temperature, so no anisotropy in its temperature dependence can occur. The paper contains interesting experimental results, however they are still waiting for consistent theoretical interpretation.

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