Scaling theory of a quantum ratchet

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The asymmetric responses of the system between the external force of the right and left directions are called "nonreciprocal." There are many examples of nonreciprocal responses, such as the rectification by the *p*-*n* junction. However, the quantum-mechanical wave does not distinguish between the right and the left directions as long as the time-reversal symmetry is intact, and it is a highly nontrivial issue how the nonreciprocal nature originates in quantum systems. Here we demonstrate by the quantum ratchet model, i.e., a quantum particle in an asymmetric periodic potential, that the dissipation characterized by a dimensionless coupling constant α plays an essential role for nonlinear nonreciprocal response. The temperature (T) dependence of the second-order nonlinear mobility μ_2 is found to be $\mu_2 \sim T^{(6/\alpha)-4}$ for $\alpha < 1$, and $\mu_2 \sim T^{2(\alpha-1)}$ for $\alpha > 1$, respectively, where $\alpha_c = 1$ is the critical point of the localization-delocalization transition, i.e., Schmid transition. On the other hand, μ_2 shows the behavior $\mu_2 \sim T^{-11/4}$ in the high-temperature limit. Therefore, μ_2 shows the nonmonotonous temperature dependence corresponding to the classical-quantum crossover. The generic scaling form of the velocity v as a function of the external field F and temperature T is also discussed. These findings are relevant to the heavy atoms in metals, resistive superconductors with vortices and Josephson junction system and will pave a way to control the nonreciprocal responses.

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I. INTRODUCTION

Chirality is one of the most basic subjects in whole sciences including physics, chemistry, and biology [1,2]. Most of the focus is on the symmetry of the static structures of molecules and organs, etc. However, once the motion or flow of particles is considered, the distinction between the right and the left directions of the quantum dynamics is a highly nontrivial issue even when the system lacks the inversion and mirror symmetries, i.e., chiral. Classical dynamics of a particle under an asymmetric potential has been a deeply studied topic in wide fields of science since Feynman conceived the idea of the Brownian ratchet [3]. Researches range from a molecular motor [4,5], colloid dynamics [6], and optically trapped molecule [7] to a drop of mercury [8].

Quantum effects on the particle dynamics under the nonreciprocal periodic potential $V(x)$ is one of the most fundamental problems in condensed-matter physics. Without the dissipation, the engenstates of this problem are given by the Bloch wave functions characterized by the crystal momentum k and the eigenenergy $\varepsilon_n(k)$ with n being the band index. Neglecting the spin degrees of freedom, $\varepsilon_n(k)$ is symmetric between k and $-k$, i.e., $\varepsilon_n(k) = \varepsilon_n(-k)$ as far as $V(x)$ is real, i.e., Hermitian. Therefore, the transport phenomena are symmetric between the right and the left directions as long as the many-body interaction is neglected [9]. This is in sharp contrast to the daily experience, which is governed by classical mechanics that it is more difficult to climb up the steeper slope compared with the gentle one. Especially, the role of friction is important; even at the classical dynamics, the time-reversal symmetry and energy conservation law prohibit the difference between the motions to the right and the left directions. Therefore, an important question is how the dissipation brings about the nonreciprocal transport of a quantum particle.

Dynamics of a quantum Brownian particle in the periodic potential with dissipation has been the subject of intensive studies for a long term [10]. The formulation of the quantum dissipation in terms of the coupling to harmonic bath by Caldeira-Leggett gives a way to handle this problem in the path-integral formalism [11,12], and the real-time formalism to calculate the influence integral is often used to calculated the mobility [13]. Using these methods combined with the renormalization-group (RG) analysis, the quantum phase transition is discovered at the critical value of the dimensionless friction α, which separates the extended ground state at $\alpha < \alpha_c = 1$ and the localized one at $\alpha > \alpha_c = 1$ [14–21]. As far as the the linear mobility μ_1 is concerned, it approaches a finite value of $\mu_1 \propto 1/\alpha$ when $\alpha < 1$, whereas μ_1 vanishes as $\mu_1 \sim T^{2(\alpha-1)}$ when $\alpha > 1$ in the limit $T \to 0$. This transition can be regarded as that from quantum-to-classical dynamics as the friction α increases. Therefore, it is interesting to see how this transition affects the nonreciprocal dynamics of the quantum particle in the asymmetric potential.

Experimentally, the quantum ratchet effects in semiconductor heterostructure with artificial asymmetric gating [22], Josephson junction array [23], and φ Josephson junction [24] are reported.

Recently, the vortex flow resistance in a noncentrosymmetric superconductor is shown to express a large directional dichroism at the low temperature [25]. The classical dissipative dynamics of a point particle in the presence the asymmetric pinning potential is investigating as a

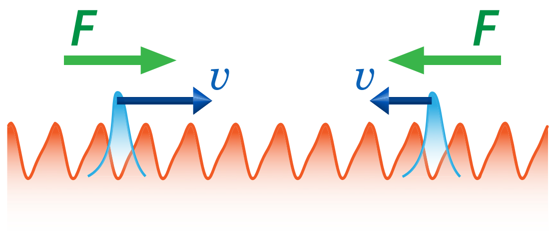

FIG. 1. Schematic of the present system. The particle wave packet under the ratchet potential is driven by the external force F resulting in a nonreciprocal velocity; $|v(-F)| \neq |v(F)|$.

candidate model [26], however, the low-temperature behavior is not addressed where the quantum tunneling plays a vital role.

In this paper, we study the quantum dynamics of the particle in an asymmetric periodic potential with Ohmic dissipation (see Fig. 1). The form of the potential is, for example, taken as $V(x) = V_1 \cos(2\pi \frac{x}{a}) + V_2 \sin(4\pi \frac{x}{a})$, which breaks the inversion symmetry $x \to -x$. This model describes the quantum ratchet, and several earlier works addressed this problem [27–35]. The instanton approach in the strong-coupling limit has been employed in Refs. [28–30] where the nonmonotonous temperature dependence of the nonlinear mobility μ_2 has been obtained due to the crossover from temperature-assisted transition to quantum tunneling. Here, the coherence between the tunneling events has been neglected, which eventually becomes important in the low-temperature limit. Scheidl-Vinokur [32] and Peguiron-Grifoni [34,35] employed the weak-coupling perturbation theory with respect to the potential $V(x)$ and obtained the lowest-order expression for the second-order mobility $\mu_2 \propto V_1^2 V_2$ and the rectified velocity $v(F) + v(-F) \propto V_1^2 V_2$, respectively, in terms of the integral over the two time variables t_1 and t_2. However they have not carefully examined the detailed temperature dependence especially at low temperatures.

II. CALCULATION OF STEADY-STATE VELOCITY

Here, we rederive the general expression of the steady-state velocity as a function of external force F in the presence of the dissipation and the general form of asymmetric corrugation $V(x)$ in a perturbative way. This perturbation theory is justified for $\alpha < 1$ where the potential is irrelevant. We will discuss the other case $\alpha > 1$ later. The general formula for steady velocity $v(F)$ enables us to investigate the detailed temperature scaling for arbitrary order mobility μ_n. The dissipation is handled in terms of the Feynman-Vernon's influence functional technique [13] where the infinite set of harmonic oscillators with Ohmic spectral density $J(\omega) = \eta\omega$ is coupled bilinearly to the quantum-mechanical point particle and integrated out. The lowest-order perturbative expansion with respect to $V(x)$ allows us to compute the velocity and the mobility in the long-time limit in the real-time expression for the general strength of the dissipation, temperature T, and the external force F. Since the derivation is tedious and just a straightforward generalization of earlier works [17,32–35], the details are given in the Supplemental Material (SM) [36], and we here show only the final expression. Another approach to derive the same expression is also given in SM [36]. Throughout this paper, we set $\hbar = k_B = 1$.

The zeroth order in V gives $v^{(0)} = F/\eta$, and the first-order correction is zero. In the order of V^2, the modification to velocity is [17,32–35]

$$v^{(2)} = -\frac{2}{\eta}\int_0^\infty dt \sum_k k|V_k|^2 \sin\left[\frac{F}{\eta}kt\right] \times \sin\left[\frac{1}{\pi\eta}k^2 Q_1(t)\right] \exp\left[-\frac{1}{\pi\eta}k^2 Q_2(t)\right]. \quad (1)$$

Here V_k is the Fourier component of the periodic potential $V(x)$ with k being the integer multiple of $2\pi/a$. Q_1 and Q_2 are [12]

$$Q_1(t) = \int_0^\infty d\omega \frac{J(\omega)}{\eta\omega^2} \sin(\omega t) f(\omega/\gamma), \quad (2)$$

$$Q_2(t) = \int_0^\infty d\omega \frac{J(\omega)}{\eta\omega^2}[1 - \cos(\omega t)] \coth\left(\frac{\omega}{2T}\right) f(\omega/\gamma). \quad (3)$$

γ, being η divided by the particle mass M, is the characteristic frequency scale in the present system. f is the appropriate soft cutoff function. Here we take $f(\omega/\gamma) = e^{-\omega/\gamma}$. This result is the same as the one from Peguiron-Grifoni [34,35] and reduces to the result of Scheidl-Vinokur [32] in the small F limit and to the result of Fisher-Zwerger [17] if we take only $k = \pm\frac{2\pi}{a}$. Note here that, as the effect of the asymmetry of the potential $V(x)$ is missing in this formula, this results in nothing to do with the ratchet effect, therefore, $v^{(2)}$ is the odd function of F. To clarify the low-temperature behavior of $v^{(2)}$, the asymptotic forms of Q_1 and Q_2 for $t, T^{-1} \gg \gamma^{-1}$ are important,

$$Q_1(t) = \tan^{-1}(\gamma t) \to \text{const.}, \quad (4)$$

$$Q_2(t) = \log\left([1+(\gamma t)^2]^{1/2}\left|\frac{\Gamma\left(1+\frac{T}{\gamma}\right)}{\Gamma\left(1+\frac{T}{\gamma}+iTt\right)}\right|^2\right) \to \log(\gamma t) + \log\left(\frac{\sinh(\pi T t)}{\pi T t}\right) \quad (5)$$

with $\Gamma(\cdot)$ being the Γ function. From these asymptotic behaviors, when expanded in F, the nth order term of $v^{(2)}$ scales in the leading order as

$$v^{(2)} \sim T^{(2/\alpha)-1-n} F^n, \quad (6)$$

in the order of F^n with n being odd integers. Here the widely used dimensionless dissipation strength is

$$\alpha = \frac{\eta a^2}{2\pi}. \quad (7)$$

In the third order of V's where the quantum ratchet effect appears, we similarly have [17,32–35]

$$v^{(3)} = \frac{4}{\eta}\int_0^\infty dt_1 \int_0^\infty dt_2 \sum_{\substack{k_1,k_2,k_3 \\ k_1+k_2+k_3=0}} k_1 \left\{ \mathrm{Re}[V_{k_1}V_{k_2}V_{k_3}] \sin\left[\frac{F}{\eta}(k_1t_1 - k_3t_2)\right] + \mathrm{Im}[V_{k_1}V_{k_2}V_{k_3}]\left[\cos\left(\frac{F}{\eta}(k_1t_1-k_3t_2)\right) - 1\right]\right\}$$
$$\times \exp\left[\frac{1}{\pi\eta}[k_1k_2Q_2(t_1) + k_2k_3Q_2(t_2) + k_3k_1Q_2(t_1+t_2)]\right] \sin\left[\frac{1}{\pi\eta}k_1k_2Q_1(t_1)\right] \sin\left[\frac{1}{\pi\eta}[k_2k_3Q_1(t_2) + k_3k_1Q_1(t_1+t_2)]\right]. \tag{8}$$

This result reduces to the result of Scheidl-Vinokur [32] in the order of F^2 and reproduces the result of Peguiron-Grifoni for the rectified velocity $v(F) + v(-F)$ in the presence of up to the second-harmonic potential; $k = \pm\frac{2\pi}{a}, \pm\frac{4\pi}{a}$ [34,35]. Although the expression is rather complex, we can see the behavior in the low-temperature limit by the power counting of the integrand using the asymptotic forms as follows. We see from Eq. (5) that the exponential of $-Q_2(t)$ function gives us a power of t and the large-t cutoff of the form $\exp[-\pi T t]$ at finite temperature. Thus we are allowed to count the power at zero temperature and cutoff the integral domain $[0, T^{-1}]$ to see the T dependence at low temperatures.

The dominant contribution to the integral originates from $(k_1, k_2, k_3) = \pm\frac{2\pi}{a}(1, 1, -2)$ and its permutations. By means of the polar coordinate (r, θ), the integral is $F^n \int r\,dr\, r^{n-(6/\alpha)} \sim T^{(6/\alpha)-2-n}F^n$. On the other hand, if we fix one of the variables, say t_1, the integral behaves as $F^n \int dt_2 t_2^{n-(2/\alpha)} \sim T^{(2/\alpha)-1-n}F^n$ (see Fig. 2). Although the latter contribution seems to dominate the former one at low temperatures for $\alpha < 4$, the closer inspection shows that the summation over k_1, k_2, and k_3 causes an exact cancellation of these leading-order contributions. The proof of this cancellation is given in the SM [36], and numerical calculations support this cancellation up to 12 digits in double-precision calculations. Thus, the low-temperature exponent is governed by the subleading contributions,

$$v^{(3)} \sim T^{(6/\alpha)-2-n}F^n, \tag{9}$$

in the order of F^n with n being a positive integer.

The numerical evaluation of second-order mobility $\mu_2^{(3)}$ which is given by the expansion of Eq. (8) with respect to F depicted in Figs. 3(a) and 3(b) clearly show temperature dependence as described by Eq. (9) at low temperatures. For $0 < \alpha < 3/2$, μ_2 turn to decrease as decreasing temperature around $T = T^* \sim \gamma$. This is a peculiar behavior of the present system which can be captured in real experiments. For $\alpha > 1$, the potential is a relevant operator, and therefore, the perturbative expansion with respect to the potential diverges towards the low temperatures. In this case, the system is in the localized phase, and therefore, $\mu_2^{(3)}$ must vanish at the zero temperature. This indicates the existence of another crossover temperature T^{**}, which can be lower than T^* when the potential is weak enough. In the view point of the RG analysis, the potential V scales as $V(\Lambda) = V(\Lambda_0)(\Lambda/\Lambda_0)^{(1/\alpha)-1}$ for the high-energy cutoff Λ [17]. The cutoff is truncated at $\Lambda \sim T$

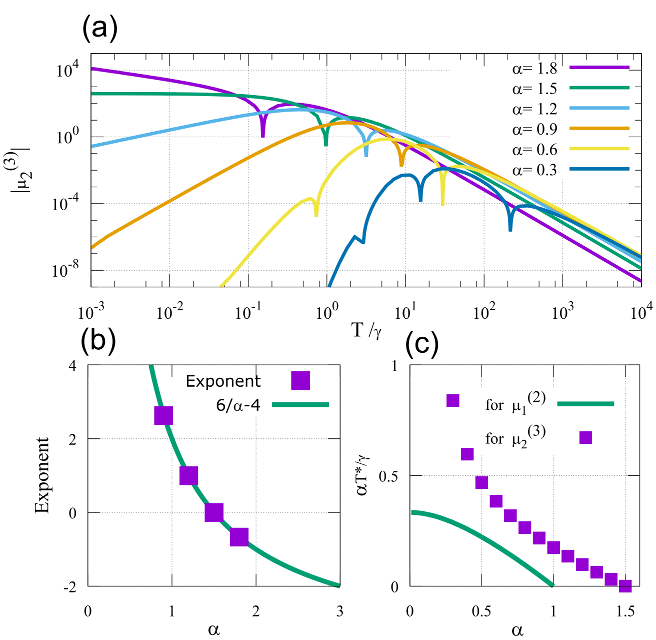

FIG. 3. Temperature dependence of the second-order mobility $\mu_2^{(3)}$. (a) The second-order mobility $\mu_2^{(3)}$ is evaluated from Eq. (8) for the asymmetric potential $V(x) = V_1\cos(2\pi x/a) + V_2\sin(4\pi x/a)$ with $V_2 = V_1/4$ for each value of α. There are two power-law regions with different exponents; $(6/\alpha) - 4$ for the low-temperature region and $-11/4$ for the high-temperature region. The quantum-to-classical crossover region with sign changes (cusps) in between is also seen. For $\alpha > 1$, as the perturbative expansion with respect to the potential fails and the system goes to the localized phase, $\mu_2^{(3)}$ vanishes at zero temperature. Therefore, there must be another crossover point T^{**} at low temperatures where the perturbative treatment breaks down. (b) The low-temperature power-law exponent of $\mu_2^{(3)}$ which clearly follows the asymptotic form $\mu_2^{(3)} \propto T^{(6/\alpha)-4}$. (c) The crossover temperature T^* defined by peak positions in (a). The green line is that for $\mu_1^{(2)}$ evaluated from Eq. (1).

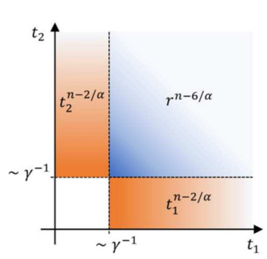

FIG. 2. Asymptotic behavior of the integrand. Asymptotic behavior of the integrand of nth order expansion with respect to F of Eq. (8) in each region on the t_1-t_2 plane. As the leading-order contributions from the orange regions cancel out among the terms, the blue region determines the temperature behaviors.

at finite temperature, therefore, we can estimate the crossover temperature as $V(\Lambda_0)(T^{**}/\Lambda_0)^{(1/\alpha)-1} \sim T^{**}$.

The higher crossover temperature deduced from the peaks of Fig. 3(a) is shown in Fig. 3(c) together with that for the linear mobility $\mu_1^{(2)}$ evaluated from Eq. (1). The crossover temperature for $\mu_2^{(3)}$ is always larger than that for $\mu_1^{(2)}$ but is comparable. Thus we can conclude that the crossover observed in $\mu_2^{(3)}$ is the quantum-to-classical crossover as known for $\mu_1^{(2)}$. Note that the peaks in $\mu_2^{(3)}$ for small α are not clear due to many sign changes in the crossover region.

This low-temperature dependence is in contrast to the saturating behavior discussed in Ref. [32] where a nontrivial approximation is made in the evaluation of Q_2, which fails to capture the quantitative behavior of $\mu_2^{(3)}$. For the higher temperature, $\mu_2^{(3)}$ decreases equally irrespective of α as $\mu_2^{(3)} \sim T^{-11/4}$ whose derivation is given in the SM [36]. This value is slightly different from $T^{-17/6}$ obtained in Ref. [32]. This discrepancy is due to the difference of the choice of cutoff function $f(\omega/\gamma)$ as discussed in the SM [36]. In the intermediate temperature, the crossoverlike behavior and some sign changes are observed as pointed out by Ref. [32].

III. SCALING FORMS

For $\alpha < 1$, the perturbative treatment of the potential V's is appropriate. And the leading-order terms lead to the scaling form in the low-temperature limit as

$$\begin{aligned} v &= \frac{F}{\eta} - F^{(2/\alpha)-1} f_o^<(F/T) - F^{(6/\alpha)-2} f_e^<(F/T) \\ &= \frac{F}{\eta} - T^{(2/\alpha)-1} g_o^<(F/T) - T^{(6/\alpha)-2} g_e^<(F/T), \end{aligned} \quad (10)$$

where $f_o^<, g_o^<$ are odd functions whereas $f_e^<, g_e^<$ are even. The basis of this scaling is that the velocity vanished in the limit $F \to 0$, which is given by the integral region of the large time variable $t \gtrsim 1/F$. Note that only the asymptotic behavior of the integrand at the large time variable determines the scaling behavior for the velocity v itself, whereas the expression for the coefficient of F_n for the velocity v does not appear so. Therefore, the divergence of the nonlinear mobility as $T \to 0$ does not mean the divergence of v, but the functional form becomes nonanalytic at the zero-temperature $T = 0$. In Eq. (10), the functions $g_o^<, g_e^<$ are an analytic functions of their argument F/T since the perturbative expansion is always possible when $F \ll T$ whereas $f_o^<, f_e^<$ are not. Trivially, they are related by $f_i^<(\eta) = \eta^{1-(2/\alpha)} g_i^<(\eta)$ with $i \in \{e, o\}$. The role of the nonreciprocal potential, i.e., V_2 is to introduce the even component $g_e^<$. One can easily see that the second-order nonlinear mobility μ_2 scales as $\mu_2 \sim T^{(6/\alpha)-4}$. Furthermore, the generic odd (even) nonlinear mobility of the nth order scales as $\mu_n \sim T^{(2/\alpha)-n-1}$ ($\mu_n \sim T^{(6/\alpha)-2-n}$) for $\alpha < 1$, and it diverges when $2/(n+1) < \alpha < 1$ [$6/(n+2) < \alpha < 1$], whereas it vanished otherwise in the limit $T \to 0$. Note here that the I-V relation of the Tomonaga-Luttinger liquid (TLL) system under the weak asymmetric potential $I \sim V^{6g-2}$ with g being the Tomonaga-Luttinger's interaction parameter is shown in Ref. [37] which is analogous to the $f_e^<$ term in Eq. (10). There are many well-known similarities between the present system and the TLL system [19,20], and some of them are exemplified in the SM [36].

From the viewpoint of the RG, V_1 is irrelevant for $\alpha < 1$, whereas it becomes relevant for $\alpha > 1$. Similarly, V_2 is irrelevant for $\alpha < 4$ and becomes relevant for $\alpha > 4$. Naively, this might lead to the critical α being 4 for the nonreciprocal mobility. However, the RG procedure generates the composite operator V_1V_2, which includes $\sin(2\pi\frac{x}{a})$, which has the same scaling dimension as V_1. This fact is reflected in each term of the double-time integral where the dominant contribution comes from the region where one of t_1 and t_2 is finite, and the asymptotic behavior is basically given by the one-dimensional integral over time. However, the combination of $\cos(2\pi x/a)$ and $\sin(2\pi x/a)$ simply shifts the potential leaving the inversion symmetry intact. This is the reason why the cancellation occurs among the leading-order terms $\propto T^{(2/\alpha)-1-n}$ in $v^{(3)}$.

Now we turn to the case of $\alpha > 1$, where V's are relevant and scale to larger values [14]. In this case, the tunneling t between the potential barrier is the irrelevant operator, and the perturbation theory in t should be employed [19,20]. The question is how the asymmetry of the potential enters the problem. For this purpose, let us consider the tilting of the potential under the external field F. Due to the asymmetry of the potential, the change in the potential barrier linear in F exists, which results in the F dependence of t, i.e., $t(F) = t + \gamma F$. This $t(F)$ is used for the calculation of v in the lowest perturbation, which results in

$$v = t(F)^2 F^{2\alpha-1} f_o^>(F/T) = t(F)^2 T^{2\alpha-1} g_o^>(F/T), \quad (11)$$

where $g_o^>(F/T)$ is the odd function of its argument, i.e., it contains only the odd order term in the Tailor expansion. Therefore, the second-order nonlinear mobility μ_2 scales with $T^{2(\alpha-1)}$ similar to the linear mobility μ_1 and goes to 0 as $T \to 0$.

For the check of the scaling form Eq. (11) also in the strong-coupling regime where potential terms are relevant operators, we calculated a temperature dependence of the linear and the third-order mobility in the perturbation in t. As shown in detail in the SM [36], by the perturbation with respect to the tunneling amplitude, they precisely follow the expected power law as Eq. (11).

IV. DISCUSSION

Lastly, we comment on the array of the resistively shunted Josephson junction model, which is a direct generalization of the present system to higher dimensions. This model, composed of the superconducting islands connected by Josephson couplings with the symmetric cosine potential and the shunting Ohmic dissipation, is a promising candidate to explain the low-temperature behavior of the thin film of granular superconductors [38,39]. It is shown that the model shows a quantum phase transition between coherent (superconducting) and disordered (normal) states at $\alpha = h/(4e^2R) = 1/z_0$ where R is the shunting resistance and z_0 is the half of the coordination number of the lattice of islands [38]. If we introduce a asymmetric potential to the Josephson phase, the nonlinear transport coefficients of the system should follow the present scaling form. One difference is that the current in the Josephson array acts as a tilting to the potential whereas the resulting time derivative of the Josephson phase is the voltage drop, therefore nonlinear resistivity, instead of mobility, follows the scaling given in the present paper. Another difference is the absence of the voltage drop for $z_0\alpha > 1$ due to the superconductivity. Thus we can conclude that nth order resistivity with odd (even) n scales as $R_n \sim T^{2/(z_0\alpha)-n-1}$ ($R_n \sim T^{6/(z_0\alpha)-n-2}$) and diverges when $2/(n+1) < z_0\alpha < 1$ [$6/(n+2) < z_0\alpha < 1$] at zero temperature.

To summarize, we have studied the role of dissipation in the nonreciprocal transport of the quantum particle in the asymmetric periodic potential, i.e., the quantum Ratchet model. We have derived the general expression of the steady-state velocity v for the general value of the dissipation α, force F, temperature T, and shape of the periodic potential $V(x)$ and found different scalings behavior at low temperatures depending on the even and odd powers of F. These results can be applied to various situations, such as the asymmetric Josephson junction array, motion of heavy atoms in the noncentrosymmetric crystal, and vortex motion in noncentrosymmetric superconductors.

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