Corrections to Higgs mode masses in superfluid ^{3}He from acoustic spectroscopy

M. D. Nguyen,[*] A. M. Zimmerman, and W. P. Halperin[†]

Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA

Superfluid ^{3}He has a rich spectrum of collective modes with both massive and massless excitations. The masses of these modes can be precisely measured using acoustic spectroscopy and fit to theoretical models. Prior comparisons of the experimental results with theory did not include strong-coupling effects beyond the weak-coupling-plus BCS model, so-called nontrivial strong-coupling corrections. In this paper we utilize recent strong-coupling calculations to determine the Higgs masses and find consistency between experiments that relate them to a subdominant f-wave pairing strength.

I. INTRODUCTION

Collective modes are integral to the understanding of many-body physics because they reflect the broken symmetries for condensed phases and encode the dynamical response of a system to external forces. The dynamics of excitations in superfluid ^{3}He depend upon whether or not an energy gap exists in their corresponding energy-momentum relation, which is analogous to the mass of elementary particles [1]. In a 1985 paper, Nambu observed that the masses of bosonic collective modes of superfluid ^{3}He-B, including the analog of the Higgs boson, are related to the mass of the fermionic excitations through a sum rule [2]. Further, he speculated on the possibility that there is a hidden supersymmetry associated with the class of field theories of the Bardeen-Cooper-Schrieffer (BCS) type, including superfluid ^{3}He. Following the discovery of the Higgs boson, there has been renewed interest in the Nambu sum rule and its connection to ^{3}He-B [3], including a recent report showing that the Nambu sum rule is not exact [4]. The sum rule is violated due to mass renormalization from the polarization of the underlying fermionic vacuum as well as strong-coupling corrections to the BCS theory [4,5].

Higgs modes have been studied in several other systems including superconductors NbSe$_2$ using Raman scattering [6,7] and Nb$_{1-x}$Ti$_x$N with THz excitation [8]. The results reported here are based upon acoustic spectroscopy of two Higgs modes in superfluid ^{3}He-B [9–12]. We analyze these measurements using the theoretical calculations of the mass corrections reported in Ref. [4] to determine the fundamental interactions of the system.

Superfluidity in ^{3}He arises when the quasiparticles of the normal Fermi liquid condense into a p-wave, spin-triplet superfluid ($L = S = 1$) that can be understood from the BCS pairing theory for superconductivity [13]. The superfluid state breaks not only the U(1) symmetry of the normal liquid but also reduces the separate orbital and spin rotation symmetries to a combined SO(3)$_{L+S}$ residual symmetry in the B phase.

This complex pattern of symmetry breaking gives rise to 18 collective modes that are labeled by two quantum numbers [14]: the total angular momentum J ($= 0, 1, 2$), and the particle-hole conjugation parity c ($+, -$) [4,15]. Four of these modes are massless, Nambu-Goldstone bosons while 14 have a gap in their energy dispersion, corresponding to the Higgs masses [4,16].

The bare masses were calculated in the weak-coupling BCS theory by several authors [3,17–20]. For the gapped modes, the masses have the form

$$M_{J^c} = a_J^c(T, P)\Delta(T, P), \qquad (1)$$

where Δ is the Bogoliubov fermion mass (the energy gap of the superfluid) and a_J^c is a pressure- and temperature-dependent numerical coefficient [4]. Of particular interest are the real and imaginary squashing modes with quantum numbers and bare masses, respectively,

$$J^c = 2^+, \quad M_{2^+} = \sqrt{\frac{8}{5}}\Delta, \qquad (2)$$

$$J^c = 2^-, \quad M_{2^-} = \sqrt{\frac{12}{5}}\Delta. \qquad (3)$$

The observed masses, however, have coefficients renormalized from these bare values due to several effects including Fermi-liquid interactions and higher-order pairing interactions [5]. While the dominant pairing channel that gives rise to superfluidity in ^{3}He is p wave, a subdominant, attractive f-wave ($L = 3$) interaction is predicted by ferromagnetic, spin-fluctuation mediated pairing [21] with the interaction strength denoted by x_3^{-1}, where $x_3 = \ln T_{c3}/T_c$ and T_{c3} would be the superfluid transition temperature from f-wave pairing in the absence of p-wave pairing [22]. Nonzero values for x_3^{-1} and the Fermi-liquid interaction parameters F_2^s and F_2^a (respectively, spin-symmetric and spin-antisymmetric Landau parameters) would lead to observable shifts in the mode masses. Prior comparisons between the observed and the theoretical values of the mode masses have allowed for these two effects as well as for strong-coupling corrections to the energy gap [23] which incorporate physics beyond the weak-coupling BCS theory [12,15,24].

[*]mannguyen2019@u.northwestern.edu
[†]w-halperin@northwestern.edu

However, there are also strong-coupling corrections to the coefficients a_J^c, referred to as nontrivial strong-coupling corrections. The existence of these corrections has been noted in the literature [5,27] but a complete, dynamical theory of all Fermi-liquid, f-wave, and strong-coupling effects has not yet been achieved. Koch and Wölfle noted [27] that these nontrivial corrections to a_J^c can be estimated using the strong-coupling corrections to the β parameters of the Ginzburg-Landau (GL) functional. The five β parameters β_i are the coefficients of the fourth-order invariants of the order parameter [4,28]. The addition of strong-coupling corrections to β_i and therefore a_J^c is made possible by recent advances in determining the strong-coupling interactions and their temperature dependence [29,30].

Here, we apply a simple procedure along these lines that incorporates mass renormalization due to Fermi-liquid, f-wave, and strong-coupling effects to both Δ and a_J^c. In terms of β parameters, a_J^c is given by [4]

$$a_2^+ = 2\left(\frac{\frac{1}{3}(\beta_3 + \beta_4 + \beta_5)}{\beta_B}\right)^{1/2}, \qquad (4)$$

$$a_2^- = 2\left(\frac{-\beta_1}{\beta_B}\right)^{1/2}, \qquad (5)$$

with $\beta_B = \beta_1 + \beta_2 + \frac{1}{3}(\beta_3 + \beta_4 + \beta_5)$. In the weak-coupling limit, the β parameters satisfy the relation

$$-2\beta_1 = \beta_2 = \beta_3 = \beta_4 = -\beta_5, \qquad (6)$$

which reduces a_2^+ to $\sqrt{\frac{8}{5}}$ and a_2^- to $\sqrt{\frac{12}{5}}$. The relation in Eq. (6) no longer holds when strong-coupling corrections are included. However, Eqs. (4) and (5) are still valid and can be used in conjunction with the strong-coupling β parameters. Using this procedure, we find improved agreement for the f-wave pairing strength determined from several independent experiments.

II. EXPERIMENT

Experimental signatures of the collective modes are observed with acoustic spectroscopy. Superfluid ^{3}He is able to support propagating sound at low temperature with either longitudinal [31] or transverse polarization [26,32]. Collective modes in the superfluid couple to sound for which there are both longitudinal and transverse restoring forces. The latter is a unique property of superfluid ^{3}He. This coupling allows one to probe the collective mode spectra of the 2^- and 2^+ modes. Both are acoustically active and have sharp spectroscopic signatures [12]. The phase velocity and attenuation of sound waves diverge sharply when the frequency of sound matches the energy of the mode as shown in Fig. 1, providing a determination of the mode mass. While longitudinal sound has been used to measure both modes, the 2^- mode has a much stronger coupling to longitudinal sound than does the 2^+ mode. This leads to a broad resonance for 2^- while the 2^+ mode is sharp and very well defined. Therefore, longitudinal sound measurements are only suitable for precise measurements of the 2^+ mode. Transverse sound, on the other hand, can only propagate due to an off-resonant coupling to the 2^- mode [26,32]. When the transverse sound frequency is less

than the energy of the 2^- mode, sound propagation ceases abruptly, giving a clear indication of the mode mass. Temperature, pressure, and frequency sweeps have been performed by several groups to map out the energy of the modes throughout the entire superfluid phase diagram [9–12].

For temperature and pressure sweeps, a piezoelectric transducer is driven either continuously or with pulsed excitations at one of its odd harmonics. This method was employed by Mast *et al.* [9], Giannetta *et al.* [10], and Davis *et al.* [12,25]. While this method can be used to obtain precise values for attenuation and sound velocity, the frequency range is restricted to discrete harmonics of the transducer. Complementary to this, Fraenkel *et al.* [11] employed pulsed excitation of a nonresonant ultrasound transducer to perform frequency sweeps at fixed temperature and pressure. In this case the frequency was swept through the mode and a Lorentzian absorption spectrum was observed. The frequency of the maximum absorption corresponds to the mode mass. The frequency sweep method, however, cannot extract the absolute attenuation [11].

Davis *et al.* used lanthanum diluted cerium magnesium nitrate (LCMN) susceptibility thermometry precise to within $30\ \mu$K that can be calibrated using fixed points from the Greywall melting-curve temperature scale [33]. Giannetta *et al.* used the Helsinki scale [34] which is referenced to a different superfluid transition temperature than the more precise Greywall scale. For our analysis, we rescaled the temperatures reported by Ref. [10] to the Greywall melting-curve scale. Fraenkel *et al.* expressed temperature dependence in terms of the normal-fluid fraction ρ_n/ρ which we converted to reduced temperature T/T_c by interpolating data from Ref. [35].

III. STRONG-COUPLING AND f-WAVE CORRECTIONS

The experimental results indicate that a_2^+ is 10%–15% smaller (depending upon pressure) than the bare value of $\sqrt{\frac{8}{5}}$. On the other hand, a_2^- is only 1%–4% larger than its bare value of $\sqrt{\frac{12}{5}}$. Therefore, Fermi-liquid, f-wave, and strong-coupling effects must be included to obtain a consistent understanding of the mode masses.

The mode masses M_{2^+} and M_{2^-}, including renormalization due to Fermi-liquid and f-wave interactions, were first calculated by Sauls and Serene [4,5] in the weak-coupling limit. We combine their result with Eqs. (4) and (5) to obtain

$$0 = M_{2^+}^2 - \left[4\left(\frac{\frac{1}{3}(\beta_3 + \beta_4 + \beta_5)}{\beta_B}\right)\Delta^2\right]$$
$$+ \lambda(M_{2^+}, T)\left(M_{2^+}^2 - 4\Delta^2\right)\left(\frac{2}{25}F_2^a + x_3^{-1}\left(\frac{M_{2^+}}{2\Delta}\right)^2\right) \quad (7)$$

$$0 = M_{2^-}^2 - \left[4\left(\frac{-\beta_1}{\beta_B}\right)\Delta^2\right]$$
$$+ \lambda(M_{2^-}, T)\left(M_{2^-}^2 - 4\Delta^2\right)\left(\frac{3}{25}F_2^s + x_3^{-1}\left(\frac{M_{2^-}}{2\Delta}\right)^2\right), \quad (8)$$

where $\lambda(M_{J^c}, T)$ is the frequency- and temperature-dependent superfluid response function, evaluated at the mode mass [26]. This expression can be used in both the weak- and strong-coupling limits with appropriate values of the β parameters
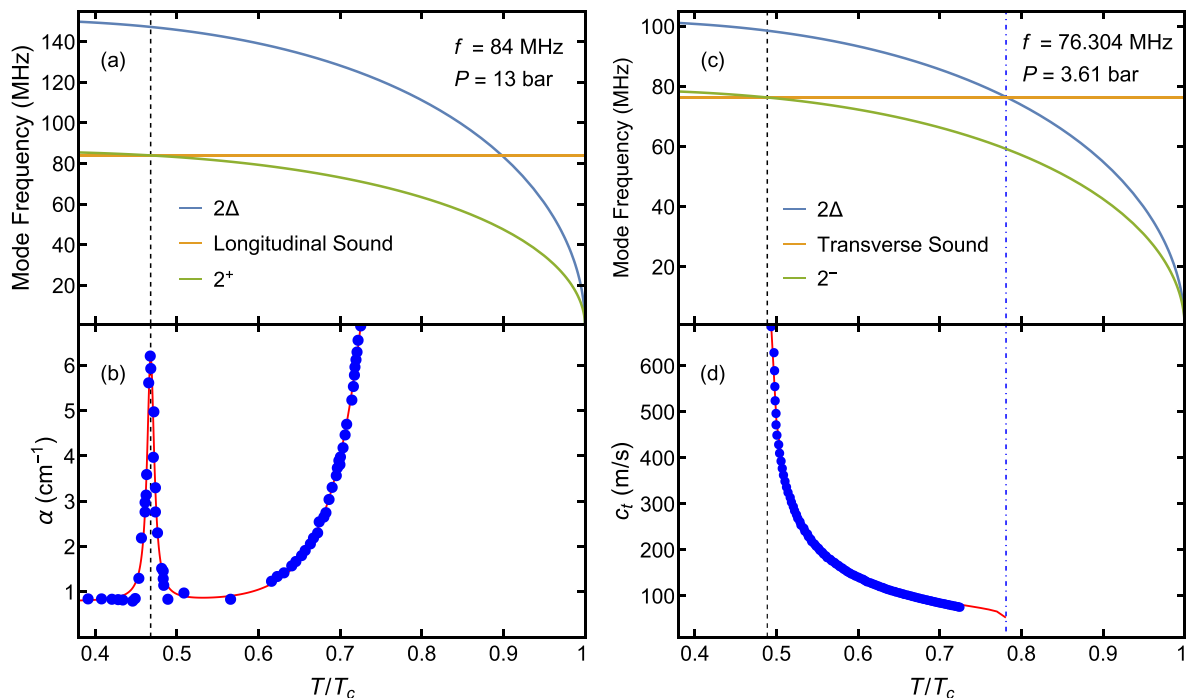
FIG. 1. Spectroscopic signatures of the 2^+ mode at 13 bars and 84 MHz (left) and of the 2^- mode at 3.61 bars and 76.304 MHz (right). The blue trace in the top two panels shows the superfluid energy gap Δ opening up as temperature is lowered while the green trace shows the corresponding increase in mode masses. For (a) and (b), longitudinal sound is excited at a fixed frequency of 84 MHz, represented by the orange trace. When the sound frequency crosses the mode energy, denoted by the vertical dashed line, a sharp resonance in the acoustic attenuation α is observed, indicating the value of the 2^+ mode mass, as seen in (b) (blue circles are data from Ref. [9], and the red line is a phenomenological model). For (c) and (d), transverse sound is excited at a fixed frequency of 76.304 MHz. Transverse sound can only propagate in the region between the black dashed and blue dotted-dashed lines, where its frequency is between 2Δ and the 2^- mode. At the crossing, the transverse phase velocity c_t diverges, indicating the value of the 2^- mode mass (blue circles are taken from data [25]; the red line is from the theory [26]).

inside the square brackets. The f-wave interaction parameter x_3^{-1} can be determined using Eqs. (7) and (8) with sufficiently precise measurements of the masses and Fermi-liquid parameters. Independent measurements of these parameters give values for F_2^s ranging from -0.2 at 0 bar to $+0.6$ at 30 bars while F_2^a is negative for all pressures, ranging from -0.9 at 0 bar to -0.1 at 30 bars [15]. Positive values for Fermi-liquid or f-wave interactions increase the mass while negative values decrease the mass (see Ref. [4], Fig. 3).

The gap used in our calculation is determined from Rainer and Serene's weak-coupling-plus model that extends the BCS theory to include strong-coupling interactions [15,36]. This model is believed to accurately represent the energy gap, limited by the accuracy of measurements of the heat capacity jump [15]. Masuhara et al. [37] performed direct measurements of the energy gap using acoustic spectroscopy where they concluded that the weak-coupling-plus model overestimates the energy gap by 2%–4%. However, what they believed to be the 2Δ pair-breaking edge was likely another collective mode with mass just below 2Δ [38]. This new mode has mass between 1.97Δ and 1.99Δ, which, if incorrectly interpreted as the 2Δ pair-breaking edge, could lead to an erroneous conclusion. Other determinations of the gap have been performed by measuring quasiparticle damping [39]. The Fermi-liquid parameters $F_2^{a,s}$ have uncertainties discussed

elsewhere [15]. The accuracy of the temperature scale, the value of the energy gap, and the Fermi-liquid parameters are the dominant sources of experimental uncertainty in our analysis.

Davis et al. calculated x_3^{-1} in the weak-coupling limit from their transverse acoustic measurement of the 2^- mode mass. In this limit, the term in square brackets for Eq. (8) reduces to $\frac{12}{5}\Delta^2$ and the results are shown in Fig. 2(a). To make a comparison with the 2^+ mode, we have used the data from Fraenkel et al., Giannetta et al., and Mast et al. to calculate x_3^{-1} in the weak-coupling limit, where the term in square brackets for Eq. (7) reduces to $\frac{8}{5}\Delta^2$. As seen in Fig. 2(a), there is significant disagreement between the x_3^{-1} inferred from these four experiments. We find that this discrepancy is removed by incorporating strong-coupling corrections to the coefficients a_j^c.

Strong-coupling corrections lead to a pressure and temperature dependence for the terms in square brackets in Eqs. (7) and (8). The pressure dependences of the β_i have been calculated by Wiman [30] using a microscopic model of quasiparticle scattering along with normal-state Fermi-liquid data and measurements of the specific heat jump at T_c. Independently, Choi et al. [29] calculated the pressure dependence of β_i from measurements in superfluid ^{3}He. Wiman et al. [30,40] showed that the temperature dependence of the β_i can be
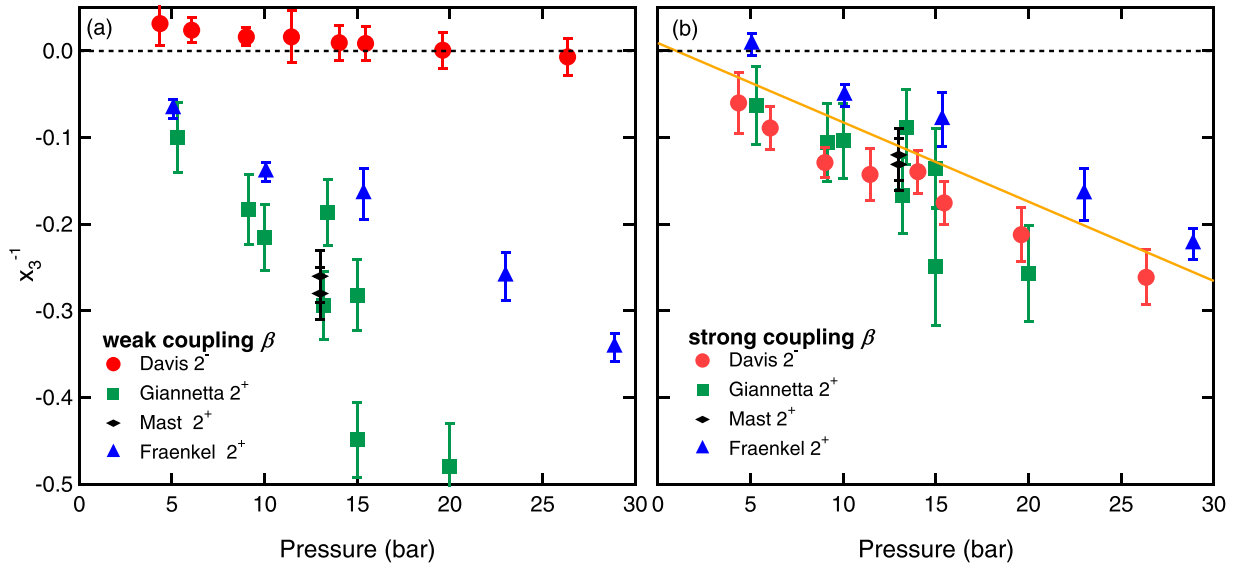
FIG. 2. f-wave interaction strength x_3^{-1} calculated using data from Davis *et al.* [25], Fraenkel *et al.* [11], Giannetta *et al.* [10], and Mast *et al.* [9]. The results in the left panel were calculated from Eqs. (7) and (8) using the weak-coupling limits of a_J^c. The right panel incorporates strong-coupling corrections, which brings all four experiments into better agreement. A linear fit (orange solid line) through the entire data set, weighted by uncertainties, yields a pressure dependence for the f-wave interaction strength that is reasonably consistent with all experiments.

taken to be proportional to T/T_c allowing them to extend the applicability of the GL theory to lower temperatures, based in part on Serene and Rainer's strong-coupling theory [23]. The pressure- and temperature-dependent strong-coupling β-parameters are given by

$$\beta_i(T, P) = \beta_i^{\mathrm{WC}} + \frac{T}{T_c} \Delta \beta_i(P), \qquad (9)$$

where β_i^{WC} is the weak-coupling value and $\Delta \beta_i(P)$ is the pressure-dependent deviation away from the weak-coupling values at T_c reported in Ref. [29]. Temperature scaling of the β parameters has also been used in studies of superfluid ^{3}He in silica aerogel [29,41,42]. This linear temperature scaling weakens strong-coupling effects as temperature is lowered. However, at sufficiently low temperatures, estimated to be approximately $0.3T_c$, the linear dependence is expected to break down [43]. The overall uncertainty in our analysis is indicated by error bars.

With the strong-coupling corrections that have been presented here, a_2^+ decreases with pressure while a_2^- increases. When combined with the F_2^a and F_2^s corrections, we obtain a consistent determination of x_3^{-1} from independent experiments on two different order parameter collective modes, as seen in Fig. 2(b). The left-hand panel shows the x_3^{-1} determinations using the weak-coupling values for a_J^c while the right-hand panel uses the strong-coupling β_i, bringing the four experiments into better agreement. While the f-wave interaction parameter only has a pressure dependence, its effect on the masses also has a temperature dependence inherited from the superfluid response function $\lambda(M, T)$. This implicit dependence leads to measurements at the same pressure being shifted by different amounts.

To within experimental uncertainty, the four experiments find x_3^{-1} varying consistently from close to zero at low pressures to -0.25 at high pressures. A linear fit through the

entire data set, weighted by uncertainties, yields a pressure dependence for x_3^{-1},

$$x_3^{-1}(P) = 0.0091 - 0.0092 \, P/(\text{bar}). \qquad (10)$$

The f-wave interaction parameter is negative throughout the phase diagram, indicating an attractive pairing interaction in this channel, consistent with pairing mediated by ferromagnetic spin fluctuations. The magnitude of x_3^{-1} can be used to calculate the instability temperature at which liquid ^{3}He would undergo a superfluid transition with f-wave Cooper pairs, if the p-wave channel did not exist. For the present values of x_3^{-1} at high pressure, this temperature is 90 μK at 34 bars. It is also noteworthy that x_3^{-1} is close to zero at zero pressure.

Superfluid ^{3}He and superconductors are usually investigated assuming a single pairing channel. However, the possibility of pairing in multiple angular momentum channels has been predicted to exist in certain high-temperature superconductors [44,45]. Here, we find evidence that the dynamics of superfluid ^{3}He are indeed best modeled by a pairing potential with multiple angular momentum channels.

IV. CONCLUSION

This strong-coupling analysis of the Higgs masses of the $J = 2$ collective modes in superfluid ^{3}He is significantly different from the earlier work which only accounts for strong-coupling corrections to the energy gap. Our work incorporates nontrivial strong coupling using the pressure- and temperature-dependent β parameters of the time-dependent GL theory. We find that the measurements of the collective modes from transverse and longitudinal acoustics are

sufficiently accurate that it is possible to extract the pairing interaction in the f-wave channel. Observations from two different collective modes, $J^c = 2^+$ and 2^-, indicate that f-wave pairing is attractive and is stronger with increasing pressure, consistent with spin-fluctuation mediated pairing. The consistency between the results suggests that the theory of the collective modes, and correspondingly the Higgs masses, is now well established.

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[2] Y. Nambu, Physica D **15**, 147 (1985).

[3] G. E. Volovik and M. A. Zubkov, J. Low Temp. Phys. **175**, 486 (2014).

[4] J. A. Sauls and T. Mizushima, Phys. Rev. B **95**, 094515 (2017).

[5] J. A. Sauls and J. W. Serene, Phys. Rev. B **23**, 4798 (1981).

[6] R. Sooryakumar and M. V. Klein, Phys. Rev. Lett. **45**, 660 (1980).

[7] P. B. Littlewood and C. M. Varma, Phys. Rev. B **26**, 4883 (1982).

[8] R. Matsunaga, N. Tsuji, H. Fujita, A. Sugioka, K. Makise, Y. Uzawa, H. Terai, Z. Wang, H. Aoki, and R. Shimano, Science **345**, 1145 (2014).

[9] D. B. Mast, B. K. Sarma, J. R. Owers-Bradley, I. D. Calder, J. B. Ketterson, and W. P. Halperin, Phys. Rev. Lett. **45**, 266 (1980).

[10] R. W. Giannetta, A. Ahonen, E. Polturak, J. Saunders, E. K. Zeise, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. **45**, 262 (1980).

[11] P. N. Fraenkel, R. Keolian, and J. D. Reppy, Phys. Rev. Lett. **62**, 1126 (1989).

[12] J. P. Davis, H. Choi, J. Pollanen, and W. P. Halperin, Phys. Rev. Lett. **97**, 115301 (2006).

[13] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3* (Taylor & Francis, London, 1990).

[14] The third quantum number m corresponding to the angular momentum projection counts the degeneracy in zero magnetic field.

[15] W. P. Halperin and E. Varoquaux, in *Helium Three*, edited by W. P. Halperin and L. P. Pitaevskii, Modern Problems in Condensed Matter Sciences Vol. 26 (Elsevier, Amsterdam, 1990), Chap. 7, p. 353.

[16] V. V. Zavjalov, S. Autti, V. B. Eltsov, P. J. Heikkinen, and G. E. Volovik, Nat. Commun. **7**, 10294 EP (2016).

[17] Y. A. Vdovin, in *Application of Methods of Quantum Field Theory to Problems of Many Particles*, edited by A. I. Alekseyeva (Gos Atom Isdat, Moscow, 1963), p. 94.

[18] K. Nagai, Prog. Theor. Phys. **54**, 1 (1975).

[19] K. Maki, J. Low Temp. Phys. **24**, 755 (1976).

[20] P. Wölfle, Physica B+C **90**, 96 (1977).

[21] D. Fay and A. Layzer, Phys. Rev. Lett. **20**, 187 (1968).

[22] J. A. Sauls, Phys. Rev. B **34**, 4861 (1986).

[23] J. Serene and D. Rainer, Phys. Rep. **101**, 221 (1983).

[24] C. A. Collett, J. Pollanen, J. I. A. Li, W. J. Gannon, and W. P. Halperin, Phys. Rev. B **87**, 024502 (2013).

[25] J. P. Davis, Transverse sound spectroscopy of excited Cooper pair states in superfluid ^{3}He, Ph.D. thesis, Northwestern University, 2008.

[26] G. F. Moores and J. A. Sauls, J. Low Temp. Phys. **91**, 13 (1993).

[27] V. E. Koch and P. Wölfle, Phys. Rev. Lett. **46**, 486 (1981).

[28] N. D. Mermin and C. Stare, Phys. Rev. Lett. **30**, 1135 (1973).

[29] H. Choi, J. P. Davis, J. Pollanen, T. M. Haard, and W. P. Halperin, Phys. Rev. B **75**, 174503 (2007).

[30] J. J. Wiman, Quantitative superfluid helium-3 from confinement to bulk, Ph.D. thesis, Northwestern University, 2018.

[31] K. Maki, J. Low Temp. Phys. **16**, 465 (1974).

[32] Y. Lee, T. M. Haard, W. P. Halperin, and J. A. Sauls, Nature (London) **400**, 431 (1999).

[33] D. S. Greywall, Phys. Rev. B **33**, 7520 (1986).

[34] T. A. Alvesalo, T. Haavasoja, M. T. Manninen, and A. T. Soinne, Phys. Rev. Lett. **44**, 1076 (1980).

[35] J. M. Parpia, D. G. Wildes, J. Saunders, E. K. Zeise, J. D. Reppy, and R. C. Richardson, J. Low Temp. Phys. **61**, 337 (1985).

[36] D. Rainer and J. W. Serene, Phys. Rev. B **13**, 4745 (1976).

[37] N. Masuhara, B. C. Watson, and M. W. Meisel, Phys. Rev. Lett. **85**, 2537 (2000).

[38] J. P. Davis, J. Pollanen, H. Choi, J. A. Sauls, and W. P. Halperin, Nat. Phys. **4**, 571 (2008).

[39] I. A. Todoschenko, H. Alles, A. Babkin, A. Y. Parshin, and V. Tsepelin, J. Low Temp. Phys. **126**, 1449 (2002).

[40] J. J. Wiman and J. A. Sauls, Phys. Rev. B **92**, 144515 (2015).

[41] G. Gervais, K. Yawata, N. Mulders, and W. P. Halperin, Phys. Rev. B **66**, 054528 (2002).

[42] J. A. Sauls and P. Sharma, Phys. Rev. B **68**, 224502 (2003).

[43] J. A. Sauls (private communication).

[44] D. Rainer, H. Burkhardt, M. Fogelström, and J. Sauls, J. Phys. Chem. Solids **59**, 2040 (1998).

[45] Q.-H. Wang, Z. D. Wang, and Q. Li, Phys. Rev. B **60**, 15364 (1999).