Significant reduction in the thermal conductivity of Si-substituted Fe₂VAI epilayers

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We experimentally find that the substitution of Si for Al in Heusler-type $Fe₂VAL$ epilayers contributes to the significant reduction in the thermal conductivity, in which Fe₂VAl is one of the next-generation thermoelectric materials without using toxic elements. Because of the low-temperature growth of the epilayers, the Si substitution induces the decrease in the degree of *L*21 ordering, giving rise to the formation of V-Si antisite defects in the epilayers. For $Fe₂VAL_{0.57}Si_{0.43}$ epilayers, we can obtain a low thermal conductivity of ∼3.9 W/(m K), one-third less than Si-substituted Fe2VAl bulk samples [Lue *et al.*, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.75.064204) **[75](https://doi.org/10.1103/PhysRevB.75.064204)**, [064204](https://doi.org/10.1103/PhysRevB.75.064204) [\(2007\)](https://doi.org/10.1103/PhysRevB.75.064204)]. We discuss that a possible origin of the low thermal conductivity is related to the low lattice thermal conductivity due to the presence of the V-Si antisite defects.

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I. INTRODUCTION

Thermal conductivity (κ) is one of the key physical parameters in developing thermoelectric materials, which enable the conversion between thermal and electrical energy $[1-4]$. The κ value for ordinary metals and semimetals comes generally from the contributions due to electrons/holes carrier transporting heat (κ_c) and phonons traveling via the lattice (κ_{ph}) [\[1,3,5\]](#page-5-0), where κ_c can be roughly estimated from the resistivity (ρ) by using the Wiedemann-Franz law above Debye temperature [\[1,5,6\]](#page-5-0) and κ_{ph} is roughly approximated by $\kappa_{ph} = Cvl/3$ [\[5,7,8\]](#page-5-0), where *C*, *v*, and *l* are lattice specific heat, group velocity, and mean-free pass of phonons, respectively. In the field of thermoelectric conversion, low κ values are required to demonstrate high-performance thermoelectric materials and devices $[1-4]$.

In general, a low value of κ_c can be obtained by tuning the carrier concentration in thermoelectric materials [\[1\]](#page-5-0). As methods for reducing the value of κ_{ph} , on the other hand, heavy element substitutions [\[9–12\]](#page-5-0) and introducing nanostructures [\[1–4,13–18\]](#page-5-0), suppressing *v* and *l*, respectively, have been utilized. Actually, thin-film superlattice structures [\[1,17,18\]](#page-5-0), connected Si nanocrystals [\[15\]](#page-5-0), and randomly embedded quantum dots $[1,13]$ have achieved very low $\kappa_{\rm ph}$ values. The other approach is introducing atomic disorder, which can increase the mass-disorder-induced and/or anharmonic scattering of phonons, leading to the reduction of *l* [\[1,5,19\]](#page-5-0). In regard to these, the experimental demonstrations have been reported in well-known thermoelectric materials such as bismuth telluride (Bi-Te), silicon-germanium (Si-Ge), and lead-telluride (Pb-Te) compounds [\[1,20–23\]](#page-5-0).

Recently, we achieved a perfectly stoichiometric $Fe₂VAL$ epilayer by using a nonstoichiometric molecular beam epitaxy (MBE) growth technique at a low growth temperature of 350 °C [\[47\]](#page-5-0). We found that the low-temperature grown $Fe₂VAL$ epilayers have a relatively low κ value [\sim 7.5 W/(m K) [\[47\]](#page-5-0)], compared to Fe₂VAl bulk [∼28 W/(m K) [\[44\]](#page-5-0)], despite almost no reduction in *S*. In addition, because the grown $Fe₂VAL$ epilayers included the $L2_1$ -ordered structure and the κ value did not change with decrease of film thickness [\[47\]](#page-5-0), we judged that the small κ value is not caused by introducing the grain boundaries and the low-dimensional structures. From these experimental data, we speculated that the atomic-level disordered structures such as point defects in the low-temperature grown $Fe₂VAL$ epilayers result in the scattering of phonons. However, the

As a next-generation thermoelectric material to replace Bi-Te compounds from the viewpoint of not containing toxic elements, $L2_1$ -ordered Fe₂VAl [\[24\]](#page-5-0), a full-Heusler semimetallic alloy, has been reported. Many theoretical $[25-32]$ and experimental [\[33–35\]](#page-5-0) studies have so far supported the presence of a sharp pseudogap near the Fermi level (E_F) in the electronic band structure. This means that the electrical and thermoelectric properties such as electrical resistivity (ρ) and Seebeck coefficient (*S*) can be modulated by changing the electronic band structure or shifting E_F because the value of *S* in metallic systems is proportional to the gradient and reciprocal for the density of states (DOS) at E_F [\[36\]](#page-5-0). Actually, for bulk and thin-film Fe₂VAl samples, the reduction in ρ and the enhancement in |*S*| with maintaining the pseudogap structure near E_F have been demonstrated by changing the chemical composition [\[10,37–41\]](#page-5-0) or substituting constituent elements $[12, 41-46]$. As a result, the value of κ of the polycrystalline $Fe₂VTa_{0.05}Al_{0.95}$ processed by high-pressure torsion reached \sim 5 W/(m K) at 300 K [\[46\]](#page-5-0), comparable with Bi-Te compounds $\kappa \sim 1.5 \,\mathrm{W/(m\,K)}$ [\[1\]](#page-5-0).

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FIG. 1. (a) θ -2 θ XRD patterns for the Fe₂VAl_{1-*x*}Si_{*x*} epilayers. (b) ϕ -scan measurements of (202) and (111) plane for the Fe₂VAl_{1-*x*}Si_{*x*} epilayers. (c) The lattice constant of the Fe₂VAl_{1–x}Si_x epilayers estimated from the Fig. 1(a), together with those for the bulks [\[43,44\]](#page-5-0).

correlation between the atomic-level disorder and the small κ value in the Fe₂VAl Heusler alloy has not been discussed yet.

In this paper, we experimentally study the influence of the substitution of Si for Al in the MBE-grown $Fe₂VAL$ epilayers on the value of κ . Because of the low-temperature growth, we found that the Si substitution induces the decrease in the degree of $L2₁$ ordering. From the first-principles calculations, the electrical properties of the $Fe₂VAL_{1-x}Si_x$ epilayers can be understood by considering the presence of the V-Si antisite defects. For $Fe₂VAL_{0.57}Si_{0.43}$ epilayers, we can obtain a low thermal conductivity of ∼3.9 W/(m K), one-third less than Sisubstituted Fe₂VAl bulk samples $[43]$. A possible origin of the low thermal conductivity is discussed as a consequence of the low lattice thermal conductivity due to the presence of the V-Si antisite defects in the Si-substituted $Fe₂$ VAl epilayers.

II. RESULTS

A. Epitaxial growth and structural characterization

Si-substituted Fe₂VAl (Fe₂VAl_{1-*x*}Si_{*x*}) epilayers with thicknesses of 50–150 nm were grown on $MgAl_2O_4(100)$ substrates (sample size: 1×1 cm²) by using an MBE technique with nonstoichiometric deposition [\[47–52\]](#page-5-0). As the mismatch between the lattice constant of $Fe₂VAL_{1-x}Si_x$ polycrystalline bulks $(0.576-0.572 \text{ nm})$ [\[43,44\]](#page-5-0) and a half of diagonal length of the lattice constant of MgAl₂O₄ ($1/\sqrt{2} \times 0.8083$ nm = 0.5715 nm) is less than 1%, high-quality epitaxial growth of the Fe₂VAl_{1–*x*}Si_{*x*} layers can be expected [\[47\]](#page-5-0). After loading the $MgAl₂O₄(100)$ substrates into the MBE chamber with a base pressure of $\sim 10^{-7}$ Pa, the heat treatment at 600 °C for 1 h was performed. From *in situ* reflection high-energy electron diffraction (RHEED) observations (not shown here), the good surface flatness of the $MgAl₂O₄(100)$ substrates was confirmed. Cooling the substrate temperature down to 350° C, we grew $Fe₂VAL_{1-x}Si_x$ layers by co-evaporating Fe, V, Al, and Si using Knudsen cells. Here we set the supplied atomic composition ratio of Fe : V : Al : Si to 1.8 : 1.2 : 2(1-*x*) : *x* during the growth. The chemical composition of the grown layers was estimated using the results of energy dispersive

x-ray spectroscopy (EDX). The estimated errors for each *x* are shown by error bars in Figs. $1(c)$, [3,](#page-2-0) and $4(a)$.

From the RHEED observations, good two-dimensional epitaxial growth was guaranteed for all the $Fe₂VAL_{1-x}Si_x$ layers (not shown here). After the growth, structural characterizations of the grown $Fe₂VAL_{1-x}Si_x$ epilayers were carried out. Figure 1(a) shows the θ -2 θ x-ray diffraction (XRD) patterns for the Fe₂VAl_{1-x}Si_x epilayers in $0 \le x \le 0.43$. Except for the peaks derived from the $MgAl₂O₄$ substrate, only the (200) and (400) diffraction peaks are clearly seen at 2θ values of ∼31◦ and ∼65◦, respectively, indicating the formation of (100)-oriented Fe₂VAl_{1–*x*}Si_{*x*} epilayers. The ϕ -scan measurements for various x are presented in Fig. $1(b)$. For all the epilayers, (202) diffraction peaks with fourfold symmetry are seen, indicating the in-plane crystal orientation of $Fe₂VAL100//MgAl₂O₄[110](100)$. The (111) diffraction peaks meaning the presence of the *L*21-ordered structure are also observed but the intensity of the (111) diffraction peaks for the Si-substituted epilayers is relatively decreased compared to that for the nonsubstituted epilayer. This feature implies that the substitution of Si for Al causes the decrease in the degree of $L2_1$ ordering in the $Fe₂VAL_{1-x}Si_x$ epilayers. We can also estimate the lattice constant of the $Fe₂VAL_{1-x}Si_x$ epilayers from the XRD data in Fig. $1(a)$ and summarize the lattice constant as a function of *x*, together with those for $Fe₂VAL_{1-x}Si_x$ polycrystalline bulks reported previously [\[43,44\]](#page-5-0), in Fig. 1(c). For both $Fe₂VAL_{1-x}Si_x$ epilayers and bulks, the lattice constant decreases almost linearly following the size of the atomic radius of Al and Si $(Al > Si)$ with increasing *x*. This feature indicates that the change in the lattice constant according to the conventional Vegard's law can be seen even for the low-temperature grown $Fe₂VAL_{1-x}Si_x$ epilayers.

Further structural investigations were carried out by cross-sectional transmission electron microscopy (TEM) and high-angle annular dark-field scanning transmission electron microscopy (HAADF-STEM). A typical HAADF-STEM image of the $Fe₂VAI_{0.57}Si_{0.43}$ epilayer is shown in Fig. [2.](#page-2-0) The Fe₂VAl_{0.57}Si_{0.43} epilayer on MgAl₂O₄(100) is uniformly

FIG. 2. HAADF-STEM image of $Fe₂VAL_{0.57}Si_{0.43}/MgAl₂O₄(100).$

grown although there are some dislocations in the epilayer along $[001]$, similar to the stoichiometric $Fe₂$ VAl epilayer previously shown in Ref. [\[47\]](#page-5-0). The contrast of the HAADF image in Fig. 2 is nearly homogeneous over the measured area. These features are also similar to those in the stoichiometric $Fe₂VAL$ epilayer [\[47\]](#page-5-0). Thus, although the substitution of Si for Al does not largely deteriorate the quality of the $Fe₂VAL_{1-x}Si_x$ epilayer, it can only induce the decrease in the degree of *L*21 ordering.

B. Electrical properties and electronic band structures

To understand the impact of the decrease in the degree of $L2_1$ ordering on the electronic band structure in Si substituted Fe2VAl epilayers, we first examined electrical properties with changing *x*. For evaluating transport properties, the $Fe₂VAL_{1-x}Si_x$ epilayers were patterned into Hall-bar devices with 80 \times 80 μ m² in size, as shown in the inset of Fig. 3, by conventional photolithography and Ar ion milling [\[47\]](#page-5-0). Figure 3 shows the values of ρ at 300 K as a function of x in the $Fe₂VAL_{1-x}Si_x$ epilayers, together with those in polycrystalline bulks in Refs. [\[43,44\]](#page-5-0). Here the temperature dependence of ρ for various *x* is presented in the Supplemental Material (Fig. S1) [\[53\]](#page-5-0). The value of ρ at 300 K in the epilayer is decreased with increasing *x*, together with a small change in ρ at $x =$

FIG. 3. The values of ρ at 300 K as a function of x in the $Fe₂VAL_{1-x}Si_x$ epilayers and bulks in Refs. [\[43,44\]](#page-5-0). The inset shows an optical micrograph of a fabricated Hall-bar device.

FIG. 4. Absolute value of carrier concentration of the (a) $Fe₂VAL_{1-x}Si_x$ epilayers and (b) polycrystalline $Fe₂VAL_{1-x}Si_x$ bulk [\[44\]](#page-5-0) at 300 K as a function of *x*. (c) Total density of states (DOS) of Fe2VAl0.875Si0.¹²⁵ for V-Si antisite defects (top) and *L*21-ordered (bottom) configuration. The dotted vertical lines indicate E_F .

0.12 and 0.22. Whereas the decrease in ρ with increasing *x* is consistent with that in the bulks in Refs. [\[43,44\]](#page-5-0), the value of ρ in the epilayer is always larger than that in the bulk in all *x*. Considering the structural characterizations in Figs. [1](#page-1-0) and 2, we can interpret that the increase in ρ is related to the decrease in the degree of $L2₁$ ordering, which leads to the carrier scattering in the $Fe₂VAL_{1-x}Si_x$ epilayers.

Next, the absolute value of the electron/hole concentration $(|n|$ or $|p|$) versus *x* for the Fe₂VAl_{1–*x*}Si_{*x*} epilayers and the $Fe₂VAL_{1-x}Si_x$ polycrystalline bulks in Ref. [\[44\]](#page-5-0) is shown in Figs. $4(a)$ and $4(b)$, respectively. Here the value of |*n*| or |*p*| in the $Fe₂VAL_{1-x}Si_x$ epilayers was estimated from the slope of the Hall voltage versus applied magnetic field (*H*) curves in the high magnetic region ($|H| > 7$ T). In Fig. 4(a) the value of | $n|$ | or | $p|$ | is increased by increasing *x* (Si substitution) and the carrier polarity is switched from *p*-type to *n*-type. Whereas the switching of the carrier polarity at $x = 0.05$ in polycrystalline bulks in Ref. [\[44\]](#page-5-0) is already observed in Fig. 4(b), it cannot be seen in the Fe₂VAl_{1-x}Si_x epilayers in Fig. 4(a). Namely,

the switching of the carrier polarity was seen at $x = 0.05$ for the polycrystalline bulks $[44]$ while that occurs $x = 0.12{\text -}0.33$ for the MBE-grown epilayers. Taking the data in Figs. $1-3$ $1-3$ into account, we can guess that the above deviation is derived from the change in the electronic band structure, induced by the decrease in the degree of $L2_1$ ordering in $Fe₂VAL_{1-x}Si_x$ epilayers.

To interpret the electrical properties described above, we conducted first-principles calculations for Si-substituted Fe2VAl to confirm the impact of the decrease in the degree of $L2₁$ ordering on the electronic band structure, where the electronic band structure calculations for perfect and disordered structures of the Heusler-type Fe₂VAl alloys were performed using the Vienna *ab initio* Simulation Package (VASP) for the pseudopotential method [\[54–](#page-5-0)[58\]](#page-6-0). In the calculation, the generalized gradient approximation (GGA) of Perdew, Burke, and Ernzerho was used to express the exchange-correlation potential [\[59,60\]](#page-6-0), and the projector augmented wave (PAW) potentials were used for describing the valence and core elec-trons [\[61,62\]](#page-6-0). The first Brillouin zone was sampled with a 2 \times $2 \times 2k$ -point mesh. The cutoff and convergence energies were set to 500 and 10−⁵ eV, respectively. The optimized lattice parameter of each structure was determined by the relation between the total energy and lattice parameter fitting to the Birch-Murnghan equation. All calculations were performed using a large-scale parallel computer system in Research Center for Computational Science of Okazaki in Japan.

Because the structural characterizations in Figs. [1](#page-1-0) and [2](#page-2-0) imply that the Si substitution mainly induces the decrease in the degree of $L2₁$ ordering, the predominant disorder in the Fe₂VAl_{1–*x*}Si_{*x*} epilayers can be regarded as V-Si antisite defects. Thus, we focus on the electronic band structure around E_F for Si-substituted Fe₂VAl with and without the antisite defects. To calculate the effect of the decrease in the degree of $L2_1$ ordering on the electronic band structure, the band structure calculations on the Heusler $Fe₂VAL_{0.875}Si_{0.125} (Fe₆₄V₃₂Al₂₈Si₄) with and without the V-Si$ antisite defects were performed. For calculation in the dilute Si substitution system, $2 \times 2 \times 2$ supercells for the Heusleralloy unit cell $(Fe_8V_4Al_4)$ were used. As the part of the unit cell on $L2_1$ -ordered Fe₂VAl and Fe₂VAl_{0.875}Si_{0.125} with and without the V-Si antisite defects, we used the crystal structures in Figs. $S2(a) - S2(c)$ in the Supplemental Material [\[53\]](#page-5-0). The POSCAR files used for the calculations were also attached to the Supplemental Materials for references. Figure $4(c)$ shows DOS of Fe₂VAl_{0.875}Si_{0.125} ($x = 0.125$) with (top panel) and without (bottom panel) antisite defects. As described in a previous work $[44]$, for $x = 0.125$ without antisite defects, an evident pseudogap structure around E_F is reconfirmed (bottom panel). As a consequence, the line of E_F is located on the conduction band, leading to the rapid change in the carrier polarity from *p*-type to *n*-type $[44]$. For $Fe₂VAL_{0.875}Si_{0.125}$ with antisite defects, on the other hand, an in-gap state at E_F can be clearly seen (top panel). The presence of the in-gap state at E_F enables to disturb the switching of the carrier polarity, as shown in Fig. $4(b)$. This scenario is consistent with the presence of a small change in ρ between $x = 0.12$ and 0.22, shown in Fig. [3.](#page-2-0) From these considerations, the electrical properties in Figs. [3](#page-2-0) and $4(a)$ in the Si-substituted $Fe₂VAL$ epilayers can be interpreted by the impact of the

FIG. 5. Total thermal resistance as a function of layer thickness for the $Fe₂VAL_{0.57}Si_{0.43}$ epilayer.

decrease in the degree of *L*21 ordering on the electronic band structure.

C. Thermoelectric properties

To explore the correlation between the atomic-level disorder in $Fe₂VAL_{1-x}Si_x$ epilayers and thermoelectric properties, we measured the values of the Seebeck coefficient (*S*) and κ for the Fe₂VAl_{0.57}Si_{0.43} epilayers at 300 K. Here, because the $MgAl₂O₄$ substrate is an insulator with an extremely large sheet resistance ($>10^7$ Ω /sq), the measured *S* value corresponds to that of the $Fe₂VAL_{0.57}Si_{0.43}$ epilayer with a relatively small sheet resistance of (\sim 10² Ω /sq). As a result, the value of *S* is estimated to be $-64 \sim -87 \mu V/K$, as shown in the Supplemental Material (Fig. S3) [\[53\]](#page-5-0). Although the sign of *S* was changed from positive to negative by substituting Si for Al even in the $Fe₂VAL_{1-x}Si_x$ epilayers, the magnitude of *S* was relatively low compared to that for bulks (\sim 130 μ V/K) [\[44\]](#page-5-0). Thus, we experimentally judge that there is an impact of the decrease in the degree of *L*21 ordering on *S* values, as discussed in Refs. [\[63,64\]](#page-6-0).

Next, the cross-plane κ value in the Fe₂VAl_{0.57}Si_{0.43} epilayers was measured using the 2ω method [\[4,15\]](#page-5-0), where the thermoreflectance signal was fitted by the one-dimensional heat dissipation model using the literature value [\[65,66\]](#page-6-0). For measuring a thermal resistance, Au films were deposited on the epilayer surfaces. It should be noted that the sheet resistance of the Au films (0.1–0.7 Ω /sq) was three orders of magnitude smaller than that of the $Fe₂VAL_{0.57}Si_{0.43}$ epilayers (\geq 100 Ω /sq). Thus, the Joule heating in the Fe₂VAl_{0.57}Si_{0.43} epilayers can be negligible. The details of the measurements for Fe2VAl epilayers were reported in our previous study [\[47\]](#page-5-0). To obtain reliable κ values, we evaluated the layerthickness dependence of the thermal resistance, as shown in Fig. 5. Because we observe a linear relationship between the thermal resistance and layer thickness, we can assume that the obtained thermal resistance is a series of a layer thermal resistance and an interfacial thermal resistance [\[47\]](#page-5-0). According to the reciprocal of the slope, the κ value at 300 K

	Epitaxial film (this work) $Fe2VAL0.57Si0.43$	Epitaxial film $[47]$ Fe ₂ VAL	Polycrystalline bulk [43] $Fe2VAL0.6Si0.4$	Polycrystalline bulk [43] Fe ₂ VAL
Ordering	$L2_1$ (w/ V-Si anti-site defects)	$L2_1$	$L2_1$	$L2_1$
ρ ($\mu \Omega$ cm)	4.7×10^{2}	\sim 1.4 \times 10 ³	\sim 2.9 \times 10 ²	$\sim 1.0 \times 10^3$
κ_c [W/(mK)]	\sim 1.6	~ 0.53	\sim 2.6	~ 0.73
$\kappa_{\rm ph}$ [W/(m K)]	\sim 2.3	~ 7.0	\sim 11	\sim 23

TABLE I. Comparison of ρ , κ_c , and κ_{ph} at 300 K among this work, Ref. [\[43\]](#page-5-0), and Ref. [\[47\]](#page-5-0).

can be estimated, as shown in Fig. [5.](#page-3-0) Note that a low κ value of \sim 3.9 ± 1.4 W/(m K), smaller than bulk and thin-film samples of polycrystalline Fe₂VAl-based alloys with heavy element substitution and/or small grain sizes [\[10,16\]](#page-5-0), is obtained. The low value of \sim 3.9 ± 1.4 W/(m K) is comparable to that of bulk Ru₂NbGa alloys [\sim 5.0 W/(m K)] [\[67\]](#page-6-0). Since the *κ* value did not depend on the layer thickness for the $Fe₂VAl_{0.57}Si_{0.43}$ epilayer, the low κ value was not caused by introducing the low-dimensional structures [\[64\]](#page-6-0). Thus, we should discuss the correlation between the decrease in the degree of *L*21 ordering in Fe₂VAl_{1–*x*}Si_{*x*} epilayers and the significant reduction in κ later.

We finally comment on the thermoelectric performance of the Fe₂VAl_{1–*x*}Si_{*x*} epilayers. The dimensionless figure of merit $[ZT = S^2T/(\rho \kappa)]$ at 300 K for $x = 0.43$ can be estimated to be 0.067 ∼ 0.11, which is lower than the highest *ZT* value of ∼0.25 (at 300 K) in one of the Fe₂VAl-based alloys, i.e., the Fe₂VTa_{0.05}Al_{0.95} bulk [\[46\]](#page-5-0). The low *ZT* value is attributed to the presence of the in-gap state at E_F , disturbing the enhancement in $|S|$. Thus, we should further study how to simultaneously demonstrate the reduction in κ and the enhancement in |*S*| even in the epilayers.

III. DISCUSSION

To discuss a possible origin of the low κ value [\sim 3.9 \pm $1.4 W/(m K)$] for the Fe₂VAl_{0.57}Si_{0.43} epilayer, we summarize the correlation among $L2_1$ ordering, ρ , κ_c , and κ_{ph} in Table I for the Fe₂VAl_{0.57}Si_{0.43} epilayer and Fe₂VAl_{0.6}Si_{0.4} polycrystalline bulks in Ref. [\[43\]](#page-5-0), together with those for the $Fe₂VAL$ epilayer in Ref. [\[47\]](#page-5-0) and the $Fe₂VAL$ polycrystalline bulk in Ref. [\[43\]](#page-5-0). Here, the value of κ_c was evaluated using the Wiedemann-Franz law, $\kappa_c \rho / T = L_0$, where L_0 is the Lorenz number (= 2.45×10^{-8} WΩK⁻²) [\[43,44\]](#page-5-0). As consequences, the order of the κ_c value in the Fe₂VAl_{0.57}Si_{0.43} epilayer corresponds to that in the $Fe₂VAL_{0.6}Si_{0.4}$ polycrys-talline bulk in Ref. [\[43\]](#page-5-0). On the other hand, the κ_{ph} value in the $Fe₂VAL_{0.57}Si_{0.43}$ epilayer is about one-fourth of that in the polycrystalline bulk [\[43\]](#page-5-0). This feature means that the low total κ of the Fe₂VAl_{0.57}Si_{0.43} epilayer originates mainly from the very low value of κ_{ph} [∼2.3 W/(m K)] at 300 K. Because the tendency of the decrease in κ_{ph} with substituting Si for Al in the epilayers is similar to that in the polycrystalline bulks in Ref. [\[43\]](#page-5-0), we can roughly discuss that the phonon scattering is mainly due to the point defects, as discussed in Ref. [\[43\]](#page-5-0).

Judging from the experimental and theoretical investigations in Figs. [1](#page-1-0)[–4,](#page-2-0) we can understand that the significant reduction in κ of the Si-substituted Fe₂VAl epilayer in Fig. [5](#page-3-0) is given by the decrease in the degree of *L*21 ordering, particularly, due to the presence of V-Si antisite defects. Namely,

we can regard the very low κ_{ph} in the Si-substituted Fe₂VAl epilayers as a consequence of the presence of V-Si antisite defects in Fe2VAl-based Heusler alloys.

Recently, the value of κ_{ph} in disordered alloys such as Si_xGe_{1-x} [\[19\]](#page-5-0) and PbTe_{1–*x*}Se_{*x*} [\[23\]](#page-5-0) was argued by firstprinciples calculations. In alloy systems, the effective phonon scattering rate $(\frac{1}{\tau})$ is defined as the sum of a term describing harmonic scattering due to mass disorder $(\frac{1}{\tau_h})$ and a term describing anharmonic scattering $(\frac{1}{\tau_{\text{ah}}})$ in the following Matthiessen's rule [\[19,23\]](#page-5-0), $\frac{1}{\tau} = \frac{1}{\tau_h} + \frac{1}{\tau_{ah}}$. In a previous work on the polycrystalline bulks [\[43,44\]](#page-5-0), for $\frac{1}{\tau_h}$, the impacts of the mass disorder and the lattice imperfections such as vacancies were regarded as a possible origin of the reduction in the value of $\kappa_{\rm ph}$ by the Si substitution. From Table I, however, the value of $\kappa_{\rm ph}$ in the epitaxial layers is further lowered, which is less than one fourth of that in Ref. $[43]$ and is the lowest value in Fe2VAl-based Heusler alloys. To understand the very low $\kappa_{\rm ph}$ value, the contribution of $\frac{1}{\tau_{\rm ah}}$ to $\frac{1}{\tau}$ should also be considered in Fe₂VAl-based Heusler alloys as well as half-Heusler alloys [\[68\]](#page-6-0).

IV. CONCLUSION

We experimentally found that the substitution of Si for Al in $Fe₂VAL$ epilayers markedly affects the reduction in the thermal conductivity. From the measurements of electrical properties and the electronic band structures obtained by the first-principles calculations, we understood that the Si substitution for Al induced the decrease in the degree of *L*21 ordering, giving rise to the formation of V-Si antisite defects in the epilayers. For $Fe₂VAL_{0.57}Si_{0.43}$ epilayers, we demonstrated a low thermal conductivity of \sim 3.9 W/(m K), one-third less than Si-substituted Fe₂VAl bulk samples $[43]$. We discussed that a possible origin of the low thermal conductivity is related to the very low lattice thermal conductivity due to the presence of the V-Si antisite defects. We propose that, in addition to the mass disorder effect on the phonon scattering, it is required to consider the impact of the anharmonic scattering in $Fe₂VAL$ based Heusler alloys with the presence of V-Si antisite defects.

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- [1] G. J. Snyder and E. S. Toberer, [Nature Mater.](https://doi.org/10.1038/nmat2090) **[7](https://doi.org/10.1038/nmat2090)**, [105](https://doi.org/10.1038/nmat2090) [\(2008\)](https://doi.org/10.1038/nmat2090).
- [2] J-F. Li, W.-S. Liu, L.-D. Zhao, and M. Zhou, [NPG Asia Mater.](https://doi.org/10.1038/asiamat.2010.138) **[2](https://doi.org/10.1038/asiamat.2010.138)**, [152](https://doi.org/10.1038/asiamat.2010.138) [\(2010\)](https://doi.org/10.1038/asiamat.2010.138).
- [3] X. Zhang and L.-D. Zhao, [J. Materiomics](https://doi.org/10.1016/j.jmat.2015.01.001) **[1](https://doi.org/10.1016/j.jmat.2015.01.001)**, [92](https://doi.org/10.1016/j.jmat.2015.01.001) [\(2015\)](https://doi.org/10.1016/j.jmat.2015.01.001).
- [4] Y. Nakamura, [Sci. Technol. Adv. Mater.](https://doi.org/10.1080/14686996.2017.1413918) **[19](https://doi.org/10.1080/14686996.2017.1413918)**, [31](https://doi.org/10.1080/14686996.2017.1413918) [\(2017\)](https://doi.org/10.1080/14686996.2017.1413918).
- [5] C. Wan, Y. Wang, N. Wang, W. Norimatsu, M. Kusunoki, and K. Koumoto, [Sci. Technol. Adv. Mater.](https://doi.org/10.1088/1468-6996/11/4/044306) **[11](https://doi.org/10.1088/1468-6996/11/4/044306)**, [044306](https://doi.org/10.1088/1468-6996/11/4/044306) [\(2010\)](https://doi.org/10.1088/1468-6996/11/4/044306).
- [6] A. Bejan and A. D. Kraus, *Heat Transfer Handbook* (Wiley, New York, 2003), p. 1338.
- [7] S. Stackhouse and L. Stixrude, [Rev. Mineral. Geochem.](https://doi.org/10.2138/rmg.2010.71.12) **[71](https://doi.org/10.2138/rmg.2010.71.12)**, [253](https://doi.org/10.2138/rmg.2010.71.12) [\(2010\)](https://doi.org/10.2138/rmg.2010.71.12).
- [8] J. S. Dugdale and D. K. C. MacDonald, [Phys. Rev.](https://doi.org/10.1103/PhysRev.98.1751) **[98](https://doi.org/10.1103/PhysRev.98.1751)**, [1751](https://doi.org/10.1103/PhysRev.98.1751) [\(1955\)](https://doi.org/10.1103/PhysRev.98.1751).
- [9] Y. Wang, Y. Sui, J. Cheng, X. Wang, J. Miao, Z. Liu, Z. Qian, and W. Su, [J. Alloys Compd.](https://doi.org/10.1016/j.jallcom.2006.10.047) **[448](https://doi.org/10.1016/j.jallcom.2006.10.047)**, [1](https://doi.org/10.1016/j.jallcom.2006.10.047) [\(2008\)](https://doi.org/10.1016/j.jallcom.2006.10.047).
- [10] T. Sugiura and Y. Nishino, [J. Jpn. Inst. Met.](https://doi.org/10.2320/jinstmet.73.846) **[73](https://doi.org/10.2320/jinstmet.73.846)**, [846](https://doi.org/10.2320/jinstmet.73.846) [\(2009\)](https://doi.org/10.2320/jinstmet.73.846).
- [11] M. Mikami, Y. Kinemuchi, K. Ozaki, Y. Terazawa, and T. Takeuchi, [J. Appl. Phys.](https://doi.org/10.1063/1.4710990) **[111](https://doi.org/10.1063/1.4710990)**, [093710](https://doi.org/10.1063/1.4710990) [\(2012\)](https://doi.org/10.1063/1.4710990).
- [12] T. Takeuchi, Y. Terazawa, Y. Furuta, A. Yamamoto, and M. Mikami, [J. Electron. Mater.](https://doi.org/10.1007/s11664-013-2532-0) **[42](https://doi.org/10.1007/s11664-013-2532-0)**, [2084](https://doi.org/10.1007/s11664-013-2532-0) [\(2013\)](https://doi.org/10.1007/s11664-013-2532-0).
- [13] W. Kim, J. Zide, A. Gossard, D. Klenov, S. Stemmer, A. Shakouri, and A. Majumdar, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.96.045901) **[96](https://doi.org/10.1103/PhysRevLett.96.045901)**, [045901](https://doi.org/10.1103/PhysRevLett.96.045901) [\(2006\)](https://doi.org/10.1103/PhysRevLett.96.045901).
- [14] B. Poudel, Q. Hao, Y. Ma, Y. Lan, A. Minnich, B. Yu, X. Yan, D. Wang, A. Muto, D. Vashaee, X. Chen, J. Liu, M. S. Dresselhaus, G. Chen, and Z. Ren, [Science](https://doi.org/10.1126/science.1156446) **[320](https://doi.org/10.1126/science.1156446)**, [634](https://doi.org/10.1126/science.1156446) [\(2008\)](https://doi.org/10.1126/science.1156446).
- [15] Y. Nakamura, M. Isogawa, T. Ueda, S. Yamasaka, H. Matsui, J. Kikkawa, S. Ikeuchi, T. Oyake, T. Hori, J. Shiomi, and A. Sakai, [Nano Energy](https://doi.org/10.1016/j.nanoen.2014.11.029) **[12](https://doi.org/10.1016/j.nanoen.2014.11.029)**, [845](https://doi.org/10.1016/j.nanoen.2014.11.029) [\(2015\)](https://doi.org/10.1016/j.nanoen.2014.11.029).
- [16] N. Fukatani, Y. Kurosaki, S. Yabuuchi, A. Nishide, and J. Hayakawa, [Appl. Phys. Lett.](https://doi.org/10.1063/1.5012106) **[112](https://doi.org/10.1063/1.5012106)**, [033902](https://doi.org/10.1063/1.5012106) [\(2018\)](https://doi.org/10.1063/1.5012106).
- [17] H. Beyer, J. Nurnus, H. Böttner, A. Lambrecht, E. Wagner, and G. Bauer, [Physica E](https://doi.org/10.1016/S1386-9477(02)00246-1) **[13](https://doi.org/10.1016/S1386-9477(02)00246-1)**, [965](https://doi.org/10.1016/S1386-9477(02)00246-1) [\(2002\)](https://doi.org/10.1016/S1386-9477(02)00246-1).
- [18] J. C. Caylor, K. Coonley, J. Stuart, T. Colpitts, and R. Venkatasubramanian, [Appl. Phys. Lett.](https://doi.org/10.1063/1.1992662) **[87](https://doi.org/10.1063/1.1992662)**, [023105](https://doi.org/10.1063/1.1992662) [\(2005\)](https://doi.org/10.1063/1.1992662).
- [19] [J. Garg, N. Bonini, B. Kozinsky, and N. Marzari,](https://doi.org/10.1103/PhysRevLett.106.045901) Phys. Rev. Lett. **[106](https://doi.org/10.1103/PhysRevLett.106.045901)**, [045901](https://doi.org/10.1103/PhysRevLett.106.045901) [\(2011\)](https://doi.org/10.1103/PhysRevLett.106.045901).
- [20] D. A. Wright, [Nature \(London\)](https://doi.org/10.1038/181834a0) **[181](https://doi.org/10.1038/181834a0)**, [834](https://doi.org/10.1038/181834a0) [\(1958\)](https://doi.org/10.1038/181834a0).
- [21] B. Abeles, [Phys. Rev.](https://doi.org/10.1103/PhysRev.131.1906) **[131](https://doi.org/10.1103/PhysRev.131.1906)**, [1906](https://doi.org/10.1103/PhysRev.131.1906) [\(1963\)](https://doi.org/10.1103/PhysRev.131.1906).
- [22] R. Cheaito, J. C. Duda, T. E. Beechem, K. Hattar, J. F. Ihlefeld, D. L. Medlin, M. A. Rodriguez, M. J. Campion, E. S. Piekos, and P. E. Hopkins, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.109.195901) **[109](https://doi.org/10.1103/PhysRevLett.109.195901)**, [195901](https://doi.org/10.1103/PhysRevLett.109.195901) [\(2012\)](https://doi.org/10.1103/PhysRevLett.109.195901).
- [23] Z. Tian, J. Garg, K. Esfarjani, T. Shiga, J. Shiomi, and G. Chen, [Phys. Rev. B.](https://doi.org/10.1103/PhysRevB.85.184303) **[85](https://doi.org/10.1103/PhysRevB.85.184303)**, [184303](https://doi.org/10.1103/PhysRevB.85.184303) [\(2012\)](https://doi.org/10.1103/PhysRevB.85.184303).
- [24] Y. Nishino, M. Kato, S. Asano, K. Soda, M. Hayasaki, and U. Mizutani, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.79.1909) **[79](https://doi.org/10.1103/PhysRevLett.79.1909)**, [1909](https://doi.org/10.1103/PhysRevLett.79.1909) [\(1997\)](https://doi.org/10.1103/PhysRevLett.79.1909).
- [25] G. A. Botton, Y. Nishino, and C. J. Humphreys, [Intermetallics](https://doi.org/10.1016/S0966-9795(00)00043-1) **[8](https://doi.org/10.1016/S0966-9795(00)00043-1)**, [1209](https://doi.org/10.1016/S0966-9795(00)00043-1) [\(2000\)](https://doi.org/10.1016/S0966-9795(00)00043-1).
- [26] D. J. Singh and I. I. Mazin, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.57.14352) **[57](https://doi.org/10.1103/PhysRevB.57.14352)**, [14352](https://doi.org/10.1103/PhysRevB.57.14352) [\(1998\)](https://doi.org/10.1103/PhysRevB.57.14352).
- [27] R. Weht and W. E. Pickett, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.58.6855) **[58](https://doi.org/10.1103/PhysRevB.58.6855)**, [6855](https://doi.org/10.1103/PhysRevB.58.6855) [\(1998\)](https://doi.org/10.1103/PhysRevB.58.6855).
- [28] M. Weinert and R. E. Watson, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.58.9732) **[58](https://doi.org/10.1103/PhysRevB.58.9732)**, [9732](https://doi.org/10.1103/PhysRevB.58.9732) [\(1998\)](https://doi.org/10.1103/PhysRevB.58.9732).
- [29] [A. Bansil, S. Kaprzyk, P. E. Mijnarends, and J. Toboła,](https://doi.org/10.1103/PhysRevB.60.13396) Phys. Rev. B **[60](https://doi.org/10.1103/PhysRevB.60.13396)**, [13396](https://doi.org/10.1103/PhysRevB.60.13396) [\(1999\)](https://doi.org/10.1103/PhysRevB.60.13396).

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- [30] S. Bandaru and P. Jund, [Phys. Status Solidi B](https://doi.org/10.1002/pssb.201600441) **[254](https://doi.org/10.1002/pssb.201600441)**, [1600441](https://doi.org/10.1002/pssb.201600441) [\(2017\)](https://doi.org/10.1002/pssb.201600441).
- [31] [G. Y. Guo, G. A. Botton, and Y. Nishino,](https://doi.org/10.1088/0953-8984/10/8/002) J. Phys.: Condens. Matter **[10](https://doi.org/10.1088/0953-8984/10/8/002)**, [L119](https://doi.org/10.1088/0953-8984/10/8/002) [\(1998\)](https://doi.org/10.1088/0953-8984/10/8/002).
- [32] Y. Nishino, [Mater. Trans.](https://doi.org/10.2320/matertrans.42.902) **[42](https://doi.org/10.2320/matertrans.42.902)**, [902](https://doi.org/10.2320/matertrans.42.902) [\(2001\)](https://doi.org/10.2320/matertrans.42.902).
- [33] H. Okamura, J. Kawahara, T. Nanba, S. Kimura, K. Soda, U. Mizutani, Y. Nishino, M. Kato, I. Shimoyama, H. Miura, K. [Fukui, K. Nakagawa, H. Nakagawa, and T. Kinoshita,](https://doi.org/10.1103/PhysRevLett.84.3674) Phys. Rev. Lett. **[84](https://doi.org/10.1103/PhysRevLett.84.3674)**, [3674](https://doi.org/10.1103/PhysRevLett.84.3674) [\(2000\)](https://doi.org/10.1103/PhysRevLett.84.3674).
- [34] C. S. Lue and Joseph H. Ross, Jr., [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.61.9863) **[61](https://doi.org/10.1103/PhysRevB.61.9863)**, [9863](https://doi.org/10.1103/PhysRevB.61.9863) [\(2000\)](https://doi.org/10.1103/PhysRevB.61.9863).
- [35] K. Soda, T. Mizutani, O. Yoshimoto, S. Yagi, U. Mizutani, H. Sumi, Y. Nishino, Y. Yamada, T. Yokoya, S. Shin, A. Sekiyama, and S. Suga, [J. Synchrotron Rad.](https://doi.org/10.1107/S0909049502009214) **[9](https://doi.org/10.1107/S0909049502009214)**, [233](https://doi.org/10.1107/S0909049502009214) [\(2002\)](https://doi.org/10.1107/S0909049502009214).
- [36] N. F. Mott and H. Jones, *The Theory of the Properties of Metals* (Clarendon Press, Oxford, 1936).
- [37] C. S. Lue and Y.-K. Kuo, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.66.085121) **[66](https://doi.org/10.1103/PhysRevB.66.085121)**, [085121](https://doi.org/10.1103/PhysRevB.66.085121) [\(2002\)](https://doi.org/10.1103/PhysRevB.66.085121).
- [38] [M. Mikami, M. Inukai, H. Miyazaki, and Y. Nishino,](https://doi.org/10.1007/s11664-015-3999-7) J. Electron. Mater. **[45](https://doi.org/10.1007/s11664-015-3999-7)**, [1284](https://doi.org/10.1007/s11664-015-3999-7) [\(2016\)](https://doi.org/10.1007/s11664-015-3999-7).
- [39] Y. Nishino, H. Kato, M. Kato, and U. Mizutani, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.63.233303) **[63](https://doi.org/10.1103/PhysRevB.63.233303)**, [233303](https://doi.org/10.1103/PhysRevB.63.233303) [\(2001\)](https://doi.org/10.1103/PhysRevB.63.233303).
- [40] [H. Miyazaki, S. Tanaka, N. Ide, K. Soda, and Y. Nishino,](https://doi.org/10.1088/2053-1591/1/1/015901) Mater. Res. Express. **[1](https://doi.org/10.1088/2053-1591/1/1/015901)**, [015901](https://doi.org/10.1088/2053-1591/1/1/015901) [\(2014\)](https://doi.org/10.1088/2053-1591/1/1/015901).
- [41] Y. Nishino and Y. Tamada, [J. Appl. Phys.](https://doi.org/10.1063/1.4869395) **[115](https://doi.org/10.1063/1.4869395)**, [123707](https://doi.org/10.1063/1.4869395) [\(2014\)](https://doi.org/10.1063/1.4869395).
- [42] K. Renard, A. Mori, Y. Yamada, S. Tanaka, H. Miyazaki, and Y. Nishino, [J. Appl. Phys.](https://doi.org/10.1063/1.4861419) **[115](https://doi.org/10.1063/1.4861419)**, [033707](https://doi.org/10.1063/1.4861419) [\(2014\)](https://doi.org/10.1063/1.4861419).
- [43] [C. S. Lue, C. F. Chen, J. Y. Lin, Y. T. Yu, and Y. K. Kuo,](https://doi.org/10.1103/PhysRevB.75.064204) *Phys.* Rev. B **[75](https://doi.org/10.1103/PhysRevB.75.064204)**, [064204](https://doi.org/10.1103/PhysRevB.75.064204) [\(2007\)](https://doi.org/10.1103/PhysRevB.75.064204).
- [44] Y. Nishino, S. Deguchi, and U. Mizutani, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.74.115115) **[74](https://doi.org/10.1103/PhysRevB.74.115115)**, [115115](https://doi.org/10.1103/PhysRevB.74.115115) [\(2006\)](https://doi.org/10.1103/PhysRevB.74.115115).
- [45] M. Vasundhara, V. Srinivas, and V. V. Rao, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.77.224415) **[77](https://doi.org/10.1103/PhysRevB.77.224415)**, [224415](https://doi.org/10.1103/PhysRevB.77.224415) [\(2008\)](https://doi.org/10.1103/PhysRevB.77.224415).
- [46] S. Masuda, K. Tsuchiya, J. Qiang, H. Miyazaki, and Y. Nishino, [J. Appl. Phys.](https://doi.org/10.1063/1.5034390) **[124](https://doi.org/10.1063/1.5034390)**, [035106](https://doi.org/10.1063/1.5034390) [\(2018\)](https://doi.org/10.1063/1.5034390).
- [47] S. Yamada, K. Kudo, R. Okuhata, J. Chikada, Y. Nakamura, and K. Hamaya, [Appl. Phys. Express](https://doi.org/10.7567/APEX.10.115802) **[10](https://doi.org/10.7567/APEX.10.115802)**, [115802](https://doi.org/10.7567/APEX.10.115802) [\(2017\)](https://doi.org/10.7567/APEX.10.115802).
- [48] K. Hamaya, H. Itoh, O. Nakatsuka, K. Ueda, K. Yamamoto, M. Itakura, T. Taniyama, T. Ono, and M. Miyao, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.102.137204) **[102](https://doi.org/10.1103/PhysRevLett.102.137204)**, [137204](https://doi.org/10.1103/PhysRevLett.102.137204) [\(2009\)](https://doi.org/10.1103/PhysRevLett.102.137204).
- [49] S. Yamada, K. Tanikawa, S. Oki, M. Kawano, M. Miyao, and K. Hamaya, [Appl. Phys. Lett.](https://doi.org/10.1063/1.4893608) **[105](https://doi.org/10.1063/1.4893608)**, [071601](https://doi.org/10.1063/1.4893608) [\(2014\)](https://doi.org/10.1063/1.4893608).
- [50] Y. Fujita, M. Yamada, M. Tsukahara, T. Oka, S. Yamada, T. Kanashima, K. Sawano, and K. Hamaya, [Phys. Rev. Applied](https://doi.org/10.1103/PhysRevApplied.8.014007) **[8](https://doi.org/10.1103/PhysRevApplied.8.014007)**, [014007](https://doi.org/10.1103/PhysRevApplied.8.014007) [\(2017\)](https://doi.org/10.1103/PhysRevApplied.8.014007).
- [51] K. Arima, F. Kuroda, S. Yamada, T. Fukushima, T. Oguchi, and K. Hamaya, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.97.054427) **[97](https://doi.org/10.1103/PhysRevB.97.054427)**, [054427](https://doi.org/10.1103/PhysRevB.97.054427) [\(2018\)](https://doi.org/10.1103/PhysRevB.97.054427).
- [52] K. Kudo, S. Yamada, J. Chikada, Y. Shimanuki, Y. Nakamura, and K. Hamaya, [Jpn. J. Appl. Phys.](https://doi.org/10.7567/JJAP.57.040306) **[57](https://doi.org/10.7567/JJAP.57.040306)**, [040306](https://doi.org/10.7567/JJAP.57.040306) [\(2018\)](https://doi.org/10.7567/JJAP.57.040306).
- [53] See Supplemental Material at [http://link.aps.org/supplemental/](http://link.aps.org/supplemental/10.1103/PhysRevB.99.054201) 10.1103/PhysRevB.99.054201 for the electrical and thermoelectric properties of the $Fe₂VAL_{1-x}Si_x$ films.
- [54] G. Kresse and J. Hafner, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.47.558) **[47](https://doi.org/10.1103/PhysRevB.47.558)**, [558\(R\)](https://doi.org/10.1103/PhysRevB.47.558) [\(1993\)](https://doi.org/10.1103/PhysRevB.47.558).
- [55] G. Kresse and J. Hafner, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.48.13115) **[48](https://doi.org/10.1103/PhysRevB.48.13115)**, [13115](https://doi.org/10.1103/PhysRevB.48.13115) [\(1993\)](https://doi.org/10.1103/PhysRevB.48.13115).
- [56] G. Kresse and J. Hafner, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.49.14251) **[49](https://doi.org/10.1103/PhysRevB.49.14251)**, [14251](https://doi.org/10.1103/PhysRevB.49.14251) [\(1994\)](https://doi.org/10.1103/PhysRevB.49.14251).
- [57] G. Kresse and J. Furthmüller, [Comput. Mater. Sci.](https://doi.org/10.1016/0927-0256(96)00008-0) **[6](https://doi.org/10.1016/0927-0256(96)00008-0)**, [15](https://doi.org/10.1016/0927-0256(96)00008-0) [\(1996\)](https://doi.org/10.1016/0927-0256(96)00008-0).
- [58] G. Kresse and J. Furthmüller, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.54.11169) **[54](https://doi.org/10.1103/PhysRevB.54.11169)**, [11169](https://doi.org/10.1103/PhysRevB.54.11169) [\(1996\)](https://doi.org/10.1103/PhysRevB.54.11169).
- [59] J. P. Perdew, K. Burke, and M. Ernzerhof, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.77.3865) **[77](https://doi.org/10.1103/PhysRevLett.77.3865)**, [3865](https://doi.org/10.1103/PhysRevLett.77.3865) [\(1996\)](https://doi.org/10.1103/PhysRevLett.77.3865).
- [60] J. P. Perdew, K. Burke, and M. Ernzerhof, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.78.1396) **[78](https://doi.org/10.1103/PhysRevLett.78.1396)**, [1396](https://doi.org/10.1103/PhysRevLett.78.1396) [\(1997\)](https://doi.org/10.1103/PhysRevLett.78.1396).
- [61] P. E. Blöchl, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.50.17953) **[50](https://doi.org/10.1103/PhysRevB.50.17953)**, [17953](https://doi.org/10.1103/PhysRevB.50.17953) [\(1994\)](https://doi.org/10.1103/PhysRevB.50.17953).
- [62] G. Kresse and D. Joubert, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.59.1758) **[59](https://doi.org/10.1103/PhysRevB.59.1758)**, [1758](https://doi.org/10.1103/PhysRevB.59.1758) [\(1999\)](https://doi.org/10.1103/PhysRevB.59.1758).
- [63] S. Hiroi, M. Mikami, and T. Takeuchi, [Mater. Trans.](https://doi.org/10.2320/matertrans.E-M2016824) **[57](https://doi.org/10.2320/matertrans.E-M2016824)**, [1628](https://doi.org/10.2320/matertrans.E-M2016824) [\(2016\)](https://doi.org/10.2320/matertrans.E-M2016824).
- [64] Y. Furuta, K. Kato, T. Miyawaki, H. Asano, and T. Takeuchi, [J. Electron. Mater.](https://doi.org/10.1007/s11664-014-2994-8) **[43](https://doi.org/10.1007/s11664-014-2994-8)**, [2157](https://doi.org/10.1007/s11664-014-2994-8) [\(2014\)](https://doi.org/10.1007/s11664-014-2994-8).
- [65] St. Burghartz and B. Schulz, [J. Nucl. Mater.](https://doi.org/10.1016/0022-3115(94)90996-2) **[212-215](https://doi.org/10.1016/0022-3115(94)90996-2)**, [1065](https://doi.org/10.1016/0022-3115(94)90996-2) [\(1994\)](https://doi.org/10.1016/0022-3115(94)90996-2).
- [66] [Y. Kawaharada, K. Kurosaki, and S. Yamanaka,](https://doi.org/10.1016/S0925-8388(02)01132-5) J. Alloy. Compd. **[352](https://doi.org/10.1016/S0925-8388(02)01132-5)**, [48](https://doi.org/10.1016/S0925-8388(02)01132-5) [\(2003\)](https://doi.org/10.1016/S0925-8388(02)01132-5).
- [67] C. N. Kuo, H. W. Lee, C.-M. Wei, Y. H. Lin, Y. K. Kuo, and C. S. Lue, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.94.205116) **[94](https://doi.org/10.1103/PhysRevB.94.205116)**, [205116](https://doi.org/10.1103/PhysRevB.94.205116) [\(2016\)](https://doi.org/10.1103/PhysRevB.94.205116).
- [68] J. Shiomi, K. Esfarjani, and G. Chen, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.84.104302) **[84](https://doi.org/10.1103/PhysRevB.84.104302)**, [104302](https://doi.org/10.1103/PhysRevB.84.104302) [\(2011\)](https://doi.org/10.1103/PhysRevB.84.104302).