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Control of the two-electron exchange interaction in a nanowire double quantum dot

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The two-electron exchange coupling in a nanowire double quantum dot (DQD) is shown to possess Moriya's anisotropic superexchange interaction under the influence of both the Rashba and Dresselhaus spin-orbit couplings (SOCs) and a Zeeman field. We reveal the controllability of the anisotropic exchange interaction via tuning the SOC and the direction of the external magnetic field. The exchange interaction can be transformed into an isotropic Heisenberg interaction, but the uniform magnetic field becomes an effective inhomogeneous field whose measurable inhomogeneity reflects the SOC strength. Moreover, the presence of the effective inhomogeneous field gives rise to an energy-level anticrossing in the low-energy spectrum of the DQD. By fitting the analytical expression for the energy gap to the experimental spectroscopic detections [S. Nadj-Perge *et al.*, Phys. Rev. Lett. **108**, 166801 (2012)], we obtain the complete features of the SOC in an InSb nanowire DQD.

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I. INTRODUCTION

Achieving an effective manipulation of the electron spins is of essential importance in the spin-based quantum information processing (see, e.g., Refs. [1,2]). It has been shown that the double quantum dot (DQD) is experimentally convenient for implementing logical gate operations [3–6]. In this case, the two-spin manipulation can be based on the exchange interaction in a DQD [7,8]. Generally, the intrinsic exchange interaction in a system results in a specific alignment of the spins. For example, the ferromagnetic exchange interaction leads to spin polarization [9], and the Dzyaloshinskii-Moriya (DM) exchange interaction induces the spin texture [10–12], which may give rise to skyrmion excitations in magnetic crystals [13,14]. Therefore, it is of great importance to realize the tunability of the exchange interaction between electrons.

Owing to the spin-orbit coupling (SOC), the spin degree of freedom is correlated with the orbital degree of freedom for electrons [15–20]. In the absence of the SOC, the combined effects of the Coulomb interaction and the Pauli exclusion principle in the DQD give rise to the isotropic Heisenberg exchange interaction between electrons [7,8]. The presence of the SOC in semiconductor nanostructures mediates an anisotropic exchange interaction between electrons [21–24]. However, the anisotropic exchange interaction can be mapped via an unitary transformation onto an isotropic Heisenberg interaction in the absence of an external magnetic field [21,25–28]. Thus, the SOC seems only to have trivial influences on the exchange interaction. As shown here, this is not the case when an external magnetic field is present.

In the recent decade, the quasi-one-dimensional nanostructure with SOC has aroused much attention. Specifically, it can implement fast spin manipulations via an electric field [29–32]. Also, it can act as an effective spin splitter between two spin reservoirs [33,34] and offer a possible platform for searching the Majorana fermions in the superconductor-semiconductor hybrid systems [35–38]. Moreover, the SOC in a carbon nanotube plays an important role in determining spin transport properties [39,40] and facilitates unconventional superconductivity [41]. In this paper, we investigate the two-electron exchange interaction in a symmetric nanowire DQD in the presence of a strong SOC and an external Zeeman field. In the strong intradot Coulomb repulsion regime [42], the effective Hamiltonian describing the two electrons consists of a Zeeman term and a Moriya's anisotropic superexchange interaction [25,43]. This anisotropic exchange interaction depends on the SOC strength in the material and can be manipulated by regulating the direction of the external magnetic field. Furthermore, we show that when the anisotropic exchange interaction is transformed to an isotropic Heisenberg interaction, the uniform magnetic field becomes an effective inhomogeneous field, with the inhomogeneity depending on the SOC strength.

Under the effects of the SOC, there is an energy-level anticrossing, corresponding to the singlet-triplet splitting, in the low-energy part of the two-electron spectrum of the DQD [44–46], which is induced by the inhomogeneity of an effective magnetic field in a symmetric nanowire DQD. More interestingly, based on the effective magnetic field we obtain an analytical expression for the singlet-triplet splitting. By fitting the analytical formula to the experimental curve in

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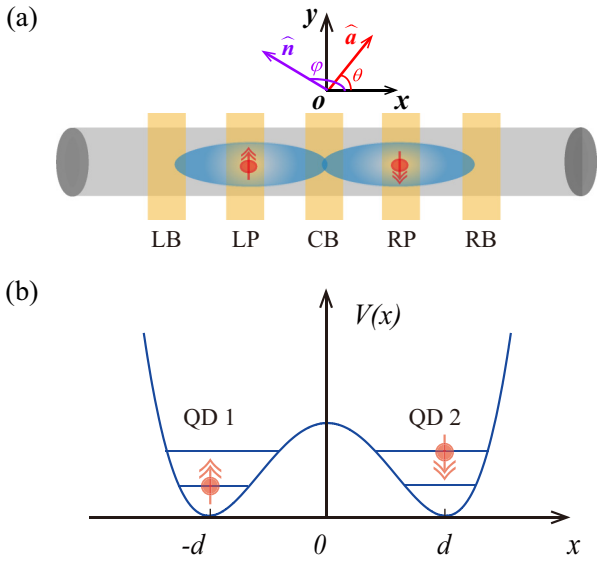

FIG. 1. (a) Schematic diagram of the nanowire double quantum dot (DQD) with a strong spin-orbit coupling (SOC). The gates LB, CB, and RB define the barriers that form the DQD, and the plunger gates LP and RP control the electron occupancy of the individual QDs. The SOC vector $\hat{\mathbf{a}} = (\cos\theta, \sin\theta, 0)$ and the magnetic-field direction $\hat{\mathbf{n}} = (\cos\varphi, \sin\varphi, 0)$ are shown in the x-y plane. (b) The confinement potential along the wire direction (the x axis). Each dot contains only a single electron, and because of the SOC the localized eigenstates of the QDs are quasispin states.

Ref. [46], we extract the strength and direction of the SOC in an InSb nanowire DQD. The consistency between our theoretical results and the experimental analyses verifies our theory. Moreover, our new results reveal that the spectroscopy measurements in the presence of an external Zeeman field can identify *separately* the Rashba and Dresselhaus parts of the SOC. For the existing experiments, the latter is very small for symmetry reasons. However, one can use other crystals, with other symmetry directions, and then both terms may appear [47,48]. To implement quantum computing with large-SOC semiconductors, Bonesteel *et al.* demonstrated the elimination of the first-order SOC by tailoring the exchange coupling between two coupled spins [49], and then they exploited the passive switching of the exchange coupling to construct quantum logic gates [50,51]. Instead, combined with the proposals for realizing universal quantum logic gates in Ref. [52], the controllability of the anisotropic exchange interaction which we find offers an *active* way to implement quantum computing with a strong-SOC system.

II. THE EFFECTIVE HAMILTONIAN OF THE NANOWIRE DQD

As shown in Fig. 1(a), we consider a semiconductor nanowire DQD with a strong SOC. For simplicity, the two QDs defined by the local gate electrodes are identical. The electron occupancy of each QD can be adjusted by changing the voltages on the electrodes [53,54]. Let the x axis be the direction along the nanowire. The axial confinement of the DQD can then be modeled as a double-well potential. As we show below, the details of this double-well potential are not important, since the coefficients of the effective Hamiltonian can actually be determined by fitting the experiments (see Sec. III). To obtain the explicit analytical form of the effective exchange interaction, we approximate the potential as a quartic function $V(x) = V_0[(x/d)^2 - 1]^2$, with $2d$ being the interdot distance [see Fig. 1(b)].

In the presence of an external magnetic field applied in the x-y plane, $\mathbf{B} = B(\cos\varphi, \sin\varphi, 0) \equiv B\hat{\mathbf{n}}$, the Hamiltonian of an electron confined in the DQD reads [55,56]

$$H_0(x) = \frac{p^2}{2m_e} + V(x) + \alpha_{\rm R}\sigma^y p + \alpha_{\rm D}\sigma^x p + \frac{g\mu_B B}{2}\sigma^n, \tag{1}$$

where m_e is the effective electron mass, $p = -i\hbar\partial/\partial x$, $\alpha_{\rm R}$ ($\alpha_{\rm D}$) is the Rashba (Dresselhaus) SOC strength, g is the effective Landé factor, μ_B is the Bohr magneton, and $\sigma^n \equiv \hat{\boldsymbol{\sigma}}\cdot\hat{\mathbf{n}}$, with the Pauli matrices $\hat{\boldsymbol{\sigma}} = (\sigma_x, \sigma_y, \sigma_z)$. Conveniently, by defining a new Pauli matrix $\sigma^{\rm a} \equiv \hat{\boldsymbol{\sigma}}\cdot\hat{\mathbf{a}}$, where the SOC vector is $\hat{\mathbf{a}} = (\cos\theta, \sin\theta, 0)$ with the angle $\theta = \operatorname{arccot}(\alpha_{\rm D}/\alpha_{\rm R})$, the SOC terms can be rewritten in a compact form, i.e., $\alpha_{\rm R}\sigma^y p + \alpha_{\rm D}\sigma^x p = \alpha\sigma^{\rm a} p$, where $\alpha = \sqrt{\alpha_{\rm D}^2 + \alpha_{\rm R}^2}$.

Usually, the electronic eigenstates of a single QD in the coexistence of the Zeeman and SOC terms are analytically obtained by perturbative approaches [17,18]. In the context of strong SOC, it is optimal to perform an exact analysis of the SOC terms and treat the Zeeman term as a perturbation as long as the Zeeman splitting is much smaller than the orbital splitting, i.e., $g\mu_B B \ll \hbar\omega$, where $\omega = \sqrt{8V_0/(d^2 m_e)}$. Meanwhile, in the case of small Zeeman splitting, only the lowest approximate Zeeman sublevels in each dot are kept to facilitate the study of the low-energy dynamics of the electron, i.e., the Hund-Mulliken approximation. Let $|\Phi_j^+\rangle$ and $|\Phi_j^-\rangle$ ($j = 1, 2$) denote the two lowest Zeeman sublevels of each QD. In general, the localized eigenstates of the different dots are not orthogonal due to the nonzero overlaps among them. Nevertheless, based on these four localized eigenstates, orthonormal basis states $|\Phi_{j\Uparrow}\rangle$ and $|\Phi_{j\Downarrow}\rangle$ ($j = 1, 2$) can be constructed via the Schmidt orthogonalization [57]. Expanding the electron field operator in terms of the orthonormal basis states, $\Psi_e(x) = \sum_{j=1,2;\sigma=\Uparrow,\Downarrow} c_{j\sigma}|\Phi_{j\sigma}\rangle$, we can write the second-quantization form of the Hamiltonian $H_0(x)$ in Eq. (1) as $H_0 = \int dx \Psi_e^\dagger(x) H_0(x) \Psi_e(x) = \sum_{j=1,2}\sum_\sigma \varepsilon_{j\sigma} c_{j\sigma}^\dagger c_{j\sigma} + \sum_\sigma (t_\sigma c_{1\sigma}^\dagger c_{2\sigma} + t'_\sigma c_{1\sigma}^\dagger c_{2\bar{\sigma}} + \text{H.c.})$, where $c^\dagger$ (c) is the electron creation (annihilation) operator, $\varepsilon_{j\sigma}$ represents the single-electron energy in each QD, while t_σ and t'_σ represent, respectively, the spin-conserved and spin-flipped tunnelings between the two QDs (see Supplemental Material in Ref. [57] for their explicit expressions).

When there are two electrons confined in the nanowire DQD, keeping only the leading Coulomb-interaction terms, we can reduce the second-quantized Coulomb Hamiltonian $H_{\rm c}$ to $H_{\rm c} = \frac{U'}{2}\sum_{j\neq j'}\sum_{\sigma\sigma'} n_{j\sigma} n_{j'\sigma'} + \frac{U}{2}\sum_j\sum_\sigma n_{j\sigma} n_{j\bar{\sigma}}$, where $n_{j\sigma} = c_{j\sigma}^\dagger c_{j\sigma}$ is the particle number operator, and U (U') denotes the intradot (interdot) Coulomb repulsion. The total Hamiltonian of the system is $H = H_0 + H_{\rm c}$.

Here we focus on the strong intradot Coulomb repulsion regime, i.e., $U - U' \gg |t_\sigma|, |t'_\sigma|$, which means that each QD can only be occupied by one electron. The effective Hamiltonian describing the two electrons can be simplified to [57]

$$H_{\rm eff} = \Delta_z\left(S_1^z + S_2^z\right) + J\mathbf{S}_1\cdot\mathbf{S}_2 + \mathbf{D}\cdot\mathbf{S}_1\times\mathbf{S}_2 + \mathbf{S}_1\overleftrightarrow{\Gamma}\mathbf{S}_2, \tag{2}$$

where $\mathbf{S}_j = (1/2)\sum_{\sigma,\sigma'=\Uparrow,\Downarrow} c^\dagger_{j\sigma}\hat{\boldsymbol{\sigma}}_{\sigma\sigma'}c_{j\sigma'}$, $j = 1, 2$, are the pseudospin operators defined by the orthonormal basis, and $\Delta_z = g\mu_B Bf$ is the SOC-modified Zeeman splitting [18], with

$$f \equiv \sqrt{\cos^2(\varphi-\theta) + e^{-2x_0^2/x_{\rm so}^2}\sin^2(\varphi-\theta)}, \tag{3}$$

where $x_0 \equiv \sqrt{\hbar/(m_e\omega)}$ is the "Bohr" radius of the QDs, and $x_{\rm so} \equiv \hbar/(m_e\alpha)$ the spin-orbit length. The SOC-dependent exchange coupling strengths in Eq. (2) are

$$\begin{aligned} J &= J_0\cos^2(2d/x_{\rm so}), \quad \mathbf{D} = J_0\sin(4d/x_{\rm so})\hat{v},\\ \overleftrightarrow{\Gamma} &= J_0\sin^2(2d/x_{\rm so})(2\hat{v}\hat{v} - 1), \end{aligned} \tag{4}$$

with the bare exchange coupling strength $J_0 = 4(|t_\Uparrow|^2 + |t'_\Uparrow|^2)/(U - U')$. The corresponding DM unit vector is

$$\hat{v} = \mathbf{e}_z\cos\gamma - \mathbf{e}_x\sin\gamma, \tag{5}$$

where $\mathbf{e}_\xi$, $\xi = x,\ y,\ z$, represent the unit vectors in the three-dimensional space of pseudospins and γ is given by

$$\gamma = \arccos\left[\cos(\varphi-\theta)/f\right], \tag{6}$$

with $\varphi - \theta$ being the angle between the SOC direction $\mathbf{a}$ and the applied magnetic field, and with f given in Eq. (3). Interestingly, the two-electron exchange coupling in Eq. (2) is shown to possess Moriya's anisotropic superexchange interaction [25,26]. Also, it should be noted that the specific form of the confinement potential between the two QDs does not alter the exchange-interaction structure in Eq. (2), but it affects the magnitude of J_0. In practice, for a realistic nanowire DQD, the value of J_0 can be detected experimentally (see below).

III. THE ROLE OF THE ZEEMAN TERM

In the absence of the external magnetic field, the Hamiltonian describing an electron in the DQD reads $H_0'(x) = p^2/(2m_e) + V(x) + \alpha_{\rm R}\sigma^y p + \alpha_{\rm D}\sigma^x p$. This Hamiltonian possesses time-reversal symmetry, i.e., $(i\sigma^y K)H_0'(i\sigma^y K)^{-1} = H_0'$, where K is the complex conjugate operator, so that there are degenerate states in this case (Kramers pairs). In the presence of nonzero magnetic field, we can also obtain the effective Hamiltonian of the two electrons in the DQD. However, because of the Kramers degeneracy in the present case, the direction of the anisotropic exchange interaction is not determined, and can be chosen arbitrarily. Here we assume that the DM vector $\hat{v}$ takes the same form as Eq. (5) to facilitate the study, and the effective Hamiltonian of the two electrons is $H'_{\rm eff} = J\mathbf{S}_1\cdot\mathbf{S}_2 + \mathbf{D}\cdot\mathbf{S}_1\times\mathbf{S}_2 + \mathbf{S}_1\overleftrightarrow{\Gamma}\mathbf{S}_2$, with the parameters given in Eq. (4).

Via a unitary transformation, the above anisotropic exchange Hamiltonian $H'_{\rm eff}$ can be mapped onto an isotropic Heisenberg Hamiltonian [21,25], $\widetilde{H}'_{\rm eff} = J_0\widetilde{\mathbf{S}}_1\cdot\widetilde{\mathbf{S}}_2$, where the spin operators $\widetilde{\mathbf{S}}_1$ and $\widetilde{\mathbf{S}}_2$ are obtained by rotating $\mathbf{S}_1$ and $\mathbf{S}_2$ around the DM vector $\hat{v}$ with angles $-\vartheta$ and ϑ, respectively,

$$\begin{aligned} \widetilde{\mathbf{S}}_1 &\equiv \exp(i\vartheta\hat{v}\cdot\hat{\mathbf{S}})\mathbf{S}_1\exp(-i\vartheta\hat{v}\cdot\hat{\mathbf{S}}),\\ \widetilde{\mathbf{S}}_2 &\equiv \exp(-i\vartheta\hat{v}\cdot\hat{\mathbf{S}})\mathbf{S}_2\exp(i\vartheta\hat{v}\cdot\hat{\mathbf{S}}), \end{aligned} \tag{7}$$

where $\vartheta = 2d/x_{\rm so}$ and $\hat{\mathbf{S}} = (1/2)\hat{\boldsymbol{\sigma}}$.

When an external Zeeman field is applied, time-reversal symmetry is broken and the two-electron Hamiltonian takes the form of Eq. (2). If we still perform the rotation for Eq. (2) as done above, the rotated Hamiltonian becomes $\widetilde{H}_{\rm eff} = \widetilde{H}_0 + \Delta\widetilde{H}$, with

$$\begin{aligned} \widetilde{H}_0 &= J_0\widetilde{\mathbf{S}}_1\cdot\widetilde{\mathbf{S}}_2 + \Delta_z\frac{\mathbf{B}_1+\mathbf{B}_2}{2B}\cdot(\widetilde{\mathbf{S}}_1+\widetilde{\mathbf{S}}_2),\\ \Delta\widetilde{H} &= \Delta_z\frac{\mathbf{B}_1-\mathbf{B}_2}{2B}\cdot(\widetilde{\mathbf{S}}_1-\widetilde{\mathbf{S}}_2). \end{aligned} \tag{8}$$

The uniform external magnetic field now becomes an effective *inhomogeneous* magnetic field, with the local effective magnetic fields in the two QDs given by $\mathbf{B}_1 = B(\beta_x, \beta_y, \beta_z)$ and $\mathbf{B}_2 = B(\beta_x, -\beta_y, \beta_z)$, where $\beta_x = (\cos\vartheta - 1)\sin\gamma\cos\gamma$, $\beta_y = \sin\vartheta\sin\gamma$, and $\beta_z = \cos^2\gamma + \sin^2\gamma\cos\vartheta$. It is easy to find that the eigenstates of $\widetilde{H}_0$ are the singlet and triplet states $|{\rm S}_0\rangle = (1/\sqrt{2})(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$, $|{\rm T}_0\rangle = (1/\sqrt{2})(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, $|{\rm T}_-\rangle = |\downarrow\downarrow\rangle$, and $|{\rm T}_+\rangle = |\uparrow\uparrow\rangle$, with the spin direction determined by the average field direction $\hat{\mathbf{w}} \equiv (\mathbf{B}_1 + \mathbf{B}_2)/(2B)$: $\hat{\mathbf{w}}\cdot\hat{\boldsymbol{\sigma}}|{\rm T}_\pm\rangle = \pm\sqrt{1-\beta_y^2}|{\rm T}_\pm\rangle$. However, $\Delta\widetilde{H}$ can lead to the mixing of the triplet and singlet states. Expanding the Hamiltonian $\widetilde{H}_{\rm eff}$ in the subspace spanned by $|{\rm T}_+\rangle$, $|{\rm T}_0\rangle$, $|{\rm T}_-\rangle$, and $|{\rm S}_0\rangle$, we obtain

$$\widetilde{H}_{\rm eff} = \begin{pmatrix} \frac{J_0}{4} + \Delta'_z & 0 & 0 & -i\frac{\sqrt{2}}{2}\beta_y\Delta_z \\ 0 & \frac{J_0}{4} & 0 & 0 \\ 0 & 0 & \frac{J_0}{4} - \Delta'_z & -i\frac{\sqrt{2}}{2}\beta_y\Delta_z \\ i\frac{\sqrt{2}}{2}\beta_y\Delta_z & 0 & i\frac{\sqrt{2}}{2}\beta_y\Delta_z & -\frac{3J_0}{4} \end{pmatrix}, \tag{9}$$

with $\Delta'_z = \sqrt{1-\beta_y^2}\Delta_z$.

Below we show that the effect of the SOC can be reflected by the inhomogeneity of the effective magnetic field,

$$\Delta B \equiv \frac{|\mathbf{B}_1 - \mathbf{B}_2|}{2B} = |\beta_y|. \tag{10}$$

When there is no difference between the two local effective fields, i.e., $\Delta B = 0$, obviously the eigenstates of the Hamiltonian $\widetilde{H}_{\rm eff}$ are the singlet and triplet states $|\Phi_1\rangle = |{\rm T}_+\rangle$, $|\Phi_2\rangle = |{\rm T}_0\rangle$, $|\Phi_3\rangle = |{\rm T}_-\rangle$, and $|\Phi_4\rangle = |{\rm S}_0\rangle$. Note, though, that there is an energy-level crossing between the singlet and triplet states, i.e., $|\Phi_3\rangle$ and $|\Phi_4\rangle$, at a critical magnetic field B_0, where the Zeeman splitting is $\Delta_z = J_0$ (see Fig. 2). However, in the case of two different local effective fields, i.e., $\Delta B \neq 0$, the energy-level crossing is avoided around the critical magnetic field and there is an anticrossing between these two levels (see Fig. 2). For a nanowire DQD with the spin-orbit length $x_{\rm so} \gg d, x_0$, the singlet-triplet splitting at the anticrossing point can be analytically written as $2\Delta_{\rm SO}^{\rm DD} = \sqrt{2}J_0\Delta B$.

To fit the model with the real system, the parameters of the nanowire DQD used in the following calculations are taken

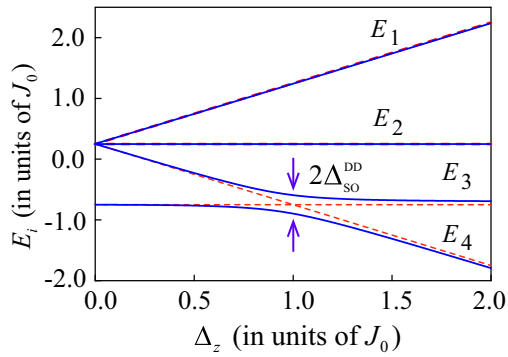

FIG. 2. The energy spectrum of the nanowire DQD versus the Zeeman-field splitting Δ_z for different values of the inhomogeneity ΔB. The dashed (red) curves represent the case when $\Delta B = 0$. The solid (blue) curves are for $\Delta B = 0.21$, where $2\Delta_{SO}^{DD}$ denotes the energy gap between the levels $|\Phi_3\rangle$ and $|\Phi_4\rangle$ at the anticrossing point when $\Delta_z \simeq J_0$.

from Ref. [46], with $2d = 50$ nm and $x_0 \simeq 30$ nm. Moreover, it was found that $g \simeq 33$ [61] and $B_0 \simeq 13.3$ mT for the InSb nanowire DQD. From these experimentally detected values, based on $\Delta_z = J_0$, we can deduce the bare exchange coupling strength $J_0 \simeq 24.6\ \mu$eV. The specific value of the energy splitting Δ_{SO}^{DD} depends on the magnetic-field direction and the SOC in the nanowire, as explained below.

IV. DEPENDENCE ON THE MAGNETIC-FIELD DIRECTION

It is known that the magnetic-field direction plays an important role in observing the SOC effects in QDs [62–65]. Below we study the influence of the magnetic-field direction on the two-electron exchange interaction in the nanowire DQD.

From the analytical expressions for the DM vector $\hat{v}$ in Eq. (5) and the angle γ in Eq. (6), it is easy to find that the exchange anisotropy direction in the DQD can be manipulated by regulating the direction of the magnetic field. In Fig. 3(a), the components of the DM vector $\hat{v}$ versus the angle $\varphi - \theta$ are plotted. For example, when the magnetic-field direction is perpendicular to the SOC direction, i.e., $\varphi - \theta = 90^\circ$ or 270°,

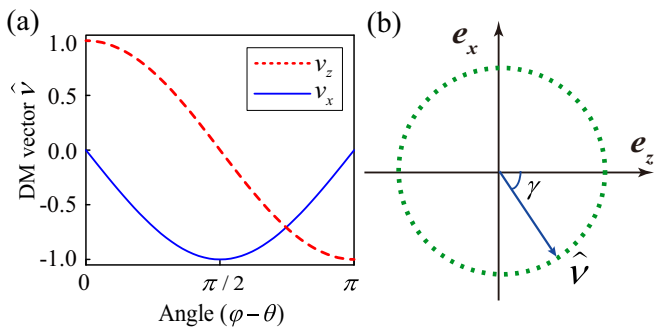

FIG. 3. (a) Components of the DM vector $\hat{v}$ as a function of the angle $\varphi - \theta$, for $x_0 = 30$ nm and $x_{so} = 200$ nm. (b) Schematic diagram of the DM vector $\hat{v}$ in the $\mathbf{e}_x - \mathbf{e}_z$ plane, with the azimuthal angle $\gamma = \arccos[\cos(\varphi - \theta)/f]$.

the DM interaction points along the $-\mathbf{e}_x$ or $\mathbf{e}_x$ direction. If the two directions are parallel, i.e., $\varphi - \theta = 0^\circ$ or 180°, the DM interaction is along the $\mathbf{e}_z$ or $-\mathbf{e}_z$ direction. However, in this case, the SOC seems to have a trivial contribution to the anisotropic exchange interaction because the effective magnetic field is homogeneous ($\Delta B = 0$). In short, we can continuously rotate the DM vector in the $\mathbf{e}_x - \mathbf{e}_z$ plane by just varying the magnetic-field direction, as shown in Fig. 3(b).

Furthermore, based on the rotated $\widetilde{H}_{\rm eff}$ and for the SOC-dependent factor $f \simeq 1$, we obtain a specific relationship between the anisotropic energy gap Δ_{SO}^{DD} and the magnetic-field direction angle φ at the anticrossing point,

$$\Delta_{SO}^{DD} = \frac{\sqrt{2}}{2} J_0 \sin(2d/x_{so})|\sin(\varphi - \theta)|. \tag{11}$$

Interestingly, the cosine dependence of the energy gap Δ_{SO}^{DD} on the magnetic-field direction angle φ has indeed been detected experimentally, cf. Fig. 4(i) in Ref. [46], i.e.,

$$\Delta_{SO}^{DD} = \Delta_{SO}|\cos(\varphi - \varphi_0)|, \tag{12}$$

with the fitting parameters $\Delta_{SO} \simeq 5.2\ \mu$eV and $\varphi_0 \simeq 1^\circ$, where Δ_{SO} is the maximal SOC-induced energy gap when the magnetic-field direction is perpendicular to the SOC, and φ_0 is the offset angle resulting from the coexistence of the Rashba and Dresselhaus SOCs in the nanowire (see below). By comparing the experimental fitting function with the analytical expression of Δ_{SO}^{DD} in Eq. (11), we can obtain the formulas for the spin-orbit length x_{so} and the spin-orbit angle θ,

$$x_{so} = \frac{2d}{\arcsin(\sqrt{2}\Delta_{SO}/J_0)}, \qquad \theta = \frac{\pi}{2} + \varphi_0. \tag{13}$$

Using the specific values of the fitting parameters and the bare exchange coupling strength, we find the magnitudes of the spin-orbit parameters, $x_{so} = 165$ nm, and $\theta_{ex} = 91^\circ$ [66]. The agreement between the experimental observations and the theoretical results is shown in Fig. 4.

For the spin-orbit length in an InSb nanowire DQD, the discrepancy between the theoretical result $x_{so} = 165$ nm and the experimental estimate in Ref. [46], $l_{so} = 230$ nm, mainly originates from the different quantitative methods. In the experiment, the spin-orbit length was quantified using an

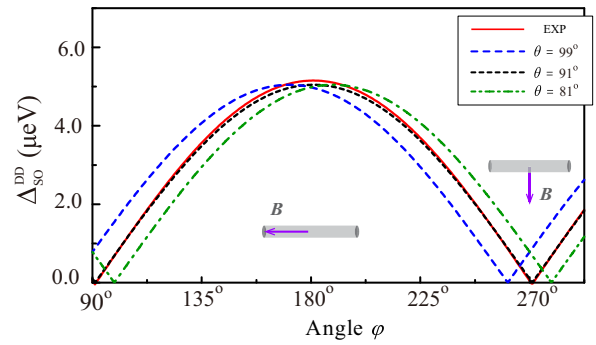

FIG. 4. The magnitude of the energy gap Δ_{SO}^{DD} as a function of the magnetic-field direction angle φ. The solid (red) line corresponds to the experimental data in Ref. [46]. The other three lines represent the theoretical results based on Eq. (11) under different values of θ, with $J_0 \simeq 24.6\ \mu$eV, 2d = 50 nm, and $x_{so} = 165$ nm.

approximation method [67], $(2d/l_{so}) \approx (\Delta_{SO}/J_0)$, and then from Eq. (13) the ratio of l_{so} to x_{so} can be identified as $l_{so}/x_{so} \simeq \sqrt{2}$. Using the definition of the spin-orbit angle $\theta \equiv \text{arccot}(\alpha_D/\alpha_R)$, in the absence of the Dresselhaus SOC, i.e., $\theta = 90^\circ$ or 270°, it follows from Eq. (11) that the magnitude of Δ_{SO}^{DD} reaches its maximal (minimal) value when the magnetic field is parallel (perpendicular) to the nanowire axis. Obviously, it is the presence of a small Dresselhaus SOC that gives rise to $\theta_{ex} = 91^\circ$. The absolutely dominant role of the Rashba SOC, which would imply $\theta_{ex} = 90^\circ$, was predicted by the symmetry analysis of the nanowire DQD in Ref. [46]. Our fit, which does give a small Dresselhaus contribution, must result from deviations of the finite sample from the ideal crystal [68]. However, the consistency between our theoretical results (Fig. 4) and the experimental detection [46] validates the controllability of the exchange interaction in the nanowire DQD by varying the direction of the external magnetic field.

V. CONCLUSIONS

We have studied the two-electron anisotropic exchange interaction in a nanowire DQD under the influence of a strong SOC and a Zeeman field. As in the case of zero magnetic field, the exchange interaction can be mapped onto an isotropic Heisenberg interaction, but the uniform external magnetic field becomes then an effective inhomogeneous field and the inhomogeneity of this effective magnetic field reflects the SOC strength. Also, we reveal the controllability of the anisotropic exchange interaction by tuning the direction of the external magnetic field and obtain an analytical expression for the dependence of the singlet-triplet splitting on the magnetic-field direction, as detected in an InSb nanowire DQD [46]. Our theory provides a tool to explore the novel properties of the exchange interaction in a nanowire DQD under the strong SOC and also offers a complete method to experimentally detect separately the Rashba and Dresselhaus SOCs in the nanowire.

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[67] See the Supplemental Material in Ref. [46] for the estimation method in determining the spin-orbit length l_{so}.

[68] In the actual InSb nanowire, the $\sim 1^\circ$ deviation of the spin-orbit angle θ_{ex} from 90° is most likely due to uncertainties present in the measurements. The Dresselhaus SOC may also result from the finiteness of the sample, due to the influence of its boundaries or the misalignment of the lattice field.