

Spin-orbit coupling effects in the quantum Hall regime probed by electron spin resonanceA. V. Shchepetilnikov,^{1,*} D. D. Frolov,¹ Yu. A. Nefyodov,¹ I. V. Kukushkin,¹ L. Tiemann,² C. Reichl,³
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The spin resonance of two-dimensional (2D) electrons confined in a high-quality 4.5-nm AlAs quantum well was studied in the regime of the integer quantum Hall effect. The electron g -factor extracted from the magnetic field position of the spin resonance at a fixed microwave frequency demonstrated a strong nonlinear dependence on the magnetic field with discontinuities around even filling factors. The value of the g -factor tended to increase with a decrease of the magnetic field around each odd filling. Furthermore, the g -factor at the exactly odd filling factor ν turned out to be dependent on ν , suggesting the entanglement between the spin degree of freedom and the orbital motion of the electrons in the regime of the integer quantum Hall effect. This suggestion is further supported, as all the experimental data are well described, when a Dresselhaus-type spin-orbit interaction is introduced into the Hamiltonian of a single electron in the quantizing magnetic field. Surprisingly, such excellent agreement was observed even in the case of tilted magnetic fields. The fitting procedure allowed us to determine the strength of the Dresselhaus spin-orbit term in a 2D electron system and to extract the fundamentally important Dresselhaus constant for bulk AlAs. Unexpectedly, not only was single spin resonance observed around even fillings, it tended to split into two well-resolved lines. Yet this finding remains a mystery and highlights the need for further experimental and theoretical efforts.

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Coupling between the spin degree of freedom and the particle motion is traditionally referred to as a spin-orbit (SO) interaction that has played a fundamental role in a vast number of physical phenomena at the forefront of contemporary condensed matter research, including anomalous [1,2] and spin Hall effects [3,4], topological insulators [5,6], and Majorana fermions [7,8]. The SO interaction in two-dimensional (2D) electron systems (2DES) formed in various semiconductor heterostructures requires the lack of a center of inversion and hence stems either from the inversion asymmetry of the correspondent bulk semiconductors and is then referred to as Dresselhaus spin-orbit coupling [9] or it originates from the asymmetry of the structure along the growth direction and is referred to as the Rashba spin-orbit coupling [10]. The asymmetry of the atomic bonds at the heterointerface is consistently reported to induce a spin-orbit interaction as well [11–13].

A spin-orbit interaction may greatly affect the physics of the quantum Hall effect (QHE) at both fractional and integer fillings. For example, spin-orbit coupling modifies the ground state of QHE by mixing the Landau levels with different orbital and spin numbers [10,14–16] and under certain conditions even favors the formation of an essentially new ground state at odd fillings, namely, the helical spin state [16]. The impact of SO coupling on the QHE ground state and excitation spectra was extensively investigated theoretically [10,14–25], yet only a limited number of experimental studies

exist [26–29]. The main aim of the present Rapid Communication is to experimentally explore further the spin-orbit coupling effects in QHE with the aid of electron spin resonance, one of the most fruitful techniques for studying the magnetic properties of two-dimensional electrons.

The experiments were carried on a 2DES formed in a 4.5-nm AlAs quantum well grown in the [001] direction with the aid of molecular beam epitaxy (MBE). Such semiconductor heterostructures possess several unique physical properties that render them particularly interesting. First, the quality of such structures is good enough for a well-developed QHE to be observed [30]. Second, the electrons in such structures tend to occupy the single out-of-plane valley located at the X points of the Brillouin zone [31,32] and are characterized by a large effective mass [32], $m = 0.28 m_0$ isotropic in the 2D plane. Thus electrons populating a typical narrow AlAs quantum well are by far heavier than the electrons confined in conventional GaAs-based heterostructures. As a result, the ratio between the characteristic Coulomb and Fermi energy is by far larger and the many-body effects are significantly more pronounced as well. Furthermore, the spin-orbit interaction is essentially nonzero in the structure under study due to the lack of inversion symmetry even in the bulk AlAs. The strength of the spin-orbit interaction was revealed to be comparable both in narrow AlAs quantum wells under study and in typical GaAs heterostructures, as it will be demonstrated later in this Rapid Communication. Thus, narrow AlAs quantum wells represent a unique physical playground with a combination of large electron-electron correlations and relatively strong spin-orbit coupling. Note that such a combination was

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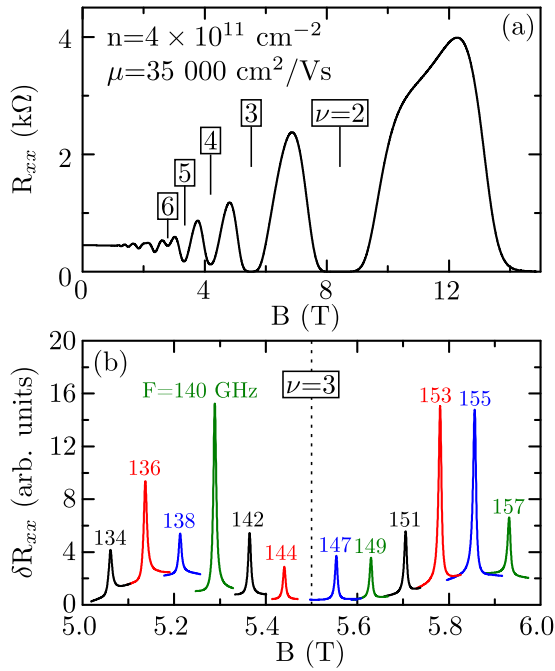


FIG. 1. (a) Magnetoresistance of the sample at a temperature of $T = 0.5$ K. The positions of several designated fillings are demonstrated. (b) Typical spin resonance lines measured around a filling factor $\nu = 3$. The correspondent microwave frequency is denoted near each resonance peak. The temperature was equal to 0.5 K.

reported to introduce a variety of new phenomena including unconventional magnetism, spin liquids, and strongly correlated topological phases [33].

A standard Hall bar was lithographically formed on the sample and indium contacts were deposited and annealed to the 2D electron system. Typical low-temperature sheet densities and electron mobilities were equal to $4 \times 10^{11} \text{ cm}^{-2}$ and $\mu = 3.5 \times 10^4 \text{ cm}^2/\text{Vs}$, respectively, and the electron concentration could be varied by the sample illumination. The sample was placed inside of the ^3He pot of the cryostat with a superconducting magnet. The experiments were carried out at a temperature of 0.5 K and in a magnetic field up to 15 T. The typical longitudinal magnetoresistance R_{xx} of the channel measured at a temperature of 0.5 K is shown in Fig. 1(a).

The conventional method of ESR detection in 2DES was utilized [34]. This approach was first introduced back in 1983 and is based on the high sensitivity of the 2DES magnetoresistance to the absorption of microwave radiation in the QHE regime. Spin resonance is then observed as a sharp peak in the R_{xx} at a fixed microwave frequency. A double lock-in technique was applied to increase the signal-to-noise ratio. A detailed description of the experimental procedure may be found in our previous work [35]. Typical spin resonances observed around a filling factor of $\nu = 3$ are depicted in Fig. 1(b). Correspondent microwave frequencies are indicated near each ESR line.

The dynamic polarization of nuclear spins complicates the precise measurements of the electron g -factor by shifting the position and changing the shape of the ESR line [36]. Yet all the experimental data obtained in the structure under study revealed no signs of nuclear polarization. The possible reason

for this is the decreased strength of the hyperfine interaction [37] in AIAs.

All of the observed resonance lines could be perfectly fitted by narrow Lorentzian curves, and this enabled us to determine the magnetic field position B of the spin resonance. Given the magnetic field position B of the spin resonance at a fixed microwave frequency F , the electron g -factor can be straightforwardly extracted as $g = hF/\mu_B B$, where h is the Planck constant and μ_B is the Bohr magneton. The sign of the electron g -factor cannot be determined with the aid of ESR, yet the positive sign of the extracted g -factor was assumed to be due to the positive g -factor close to the free-electron value of $\approx +2$ in the bulk AIAs [38,39].

The resulting dependencies of the electron g -factor on the magnetic field and filling factor are presented in Figs. 2(a) and 2(b) for three 2D electron densities $n = 1.6$ (solid squares), 2.8 (solid circles), and $4.0 \times 10^{11} \text{ cm}^{-2}$ (open squares). In the limit of large magnetic fields around unity filling the g -factor is almost independent on the magnetic field and its magnitude is almost the same for all the densities studied, $g_0 = 1.876$. This value agrees well with the previously reported measurements of the g -factor tensor in a 15-nm AIAs quantum well [35]. We would like to highlight that in wide AIAs quantum wells the in-plane valleys are occupied instead of the out-of-plane valleys as in the narrow ones [32]. Hence, one of the in-plane components of the g -tensor in a 15-nm well corresponds to the out-of-plane one in the structure under study.

As the magnetic field is decreased, the electron g -factor experiences well-resolved jumps around each even filling factor. Such behavior is observed for all the 2DES densities studied. Unexpectedly, spin resonance was detectable even at the exactly even filling factors where the total spin of the 2D electrons should normally be zero. Furthermore, in the vast region around the even filling factors, ESR tended to split into two well-resolved peaks, whereas a more usual single resonance line was observed around odd fillings. A typical two-peak structure of the ESR measured in the vicinity of $\nu = 2$ at a fixed microwave frequency of 215.9 GHz is demonstrated in Fig. 2(c). Such extraordinary behavior of the ESR near the even fillings strikingly contradicts the experimental data reported for conventional GaAs heterostructures [40–43]. The g -factor calculated from the position of the left peak corresponds to the value of the g -factor at $\nu = 3$, whereas the right peak is characterized with g close to the $\nu = 1$ value. This finding indicates the complicated structure of the $\nu = 2$ integer quantum Hall effect (IQHE) state in the system under study. Note that this effect may stem from the influence of the strong electron-electron correlations, as the e - e interaction may introduce Landau level mixing [44]. Yet a full understanding of this effect needs further theoretical and experimental efforts.

Let us focus on the odd fillings instead. Around each odd filling the g -factor continuously increased with a decrease of the magnetic field. Particularly exciting is the fact that the value of the g -factor at the exactly odd filling factor ν demonstrated a strong monotonic dependence on ν : g rises as ν is increased [see Fig. 2(b)]. This finding represents the key feature that distinguishes the g -factor behavior measured from what was observed in conventional GaAs-based

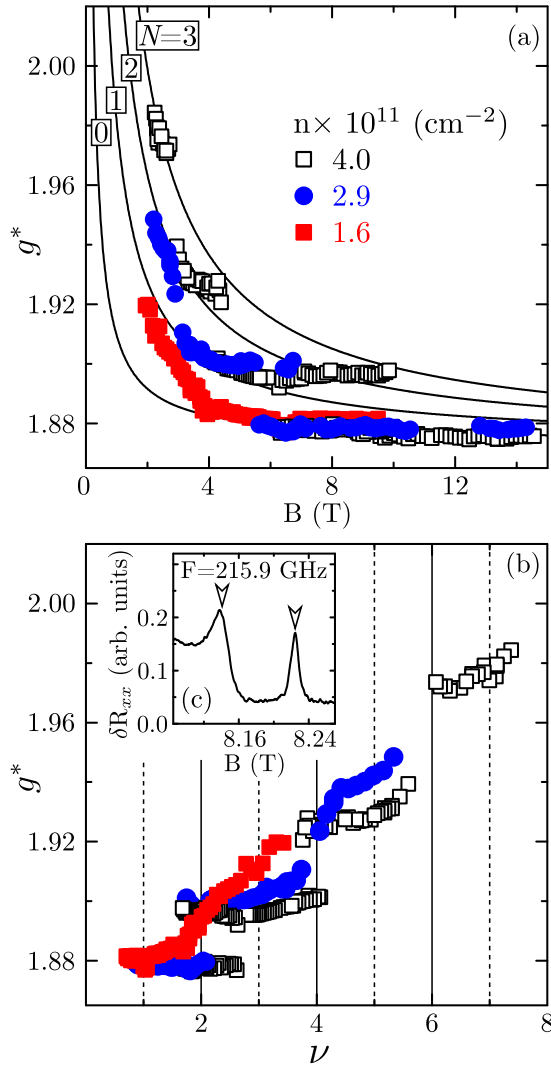


FIG. 2. The dependence of the electron g -factor on the magnetic field and on the filling factor is demonstrated in (a) and (b), respectively, for a number of 2D electron densities $n = 1.6$ (solid squares), 2.8 (solid circles), and $4.0 \times 10^{11} \text{ cm}^{-2}$ (open squares). The magnetic field was aligned perpendicular to the quantum well plane. Solid lines in (a) represent the fit of the experimental data with Eq. (1). (c) Typical ESR signal in the vicinity of the filling factor of 2. The double-peak structure of ESR is clearly resolved. The microwave frequency was fixed at 215.9 GHz.

heterostructures [40–43], and implies the entanglement between the electron spin and the orbital motion of the electron in the regime of the quantum Hall effect. Formally speaking, the spin-orbit interaction is not diagonal in the Landau states basis, induces couplings between the spin-split Landau levels with different numbers N and spin projections, and, as a result, modifies the spin splitting of the electron.

The consistent theoretical treatment of the problem is discussed in detail in Ref. [45]. Here, we will present only a concise description of the main results. For a positive sign of the g -factor, the Rashba spin-orbit term was demonstrated to decrease the spin splitting of the Landau levels in contrast to the observed dependencies, whereas the Dresselhaus term was shown to increase the spin splitting and the g -factor, in full

agreement with the experiment. In the model, where only the Dresselhaus term in the form $\beta(k_x\sigma_x - k_y\sigma_y)$ (here, k is the wave vector, and σ is the Pauli matrix) is taken into account, one might deduce the spin splitting of the Landau level with the number N ,

$$g^* \mu_b B = 1/2 \sqrt{(h\omega_c + g_0 \mu_B B)^2 + \frac{8\beta^2}{l_B^2} N} + 1/2 \sqrt{(h\omega_c + g_0 \mu_B B)^2 + \frac{8\beta^2}{l_B^2} (N+1)} - h\omega_c. \quad (1)$$

Here, $h\omega_c$ is the cyclotron energy, g_0 is the bare electron g -factor, and $l_B = \sqrt{\hbar/eB}$ is the magnetic length. In the limit of large Zeeman and cyclotron energies the expression can be reduced to

$$g^* = g_0 + A\beta^2(2N+1)/B, \quad (2)$$

where A is a positive constant. Now one can easily see that this simplified single-electron model captures all the major features of the experimental data. For a fixed Landau level number N the electron g -factor increases with a decrease of B and does not depend on the electron density. Furthermore, the value of g should experience abrupt jumps around even fillings as the number N may be varied only discretely.

This theoretical model was tested quantitatively by approximating all the experimental dependencies for all the 2D densities studied with the aid of Eq. (1). As g_0 was taken from the large magnetic field limit of the experimental data and the cyclotron frequency was taken from our previous work [32], only the single parameter was varied, namely, β , the strength of the spin-orbit interaction. The approximation curves corresponding to a series of fixed N are denoted in Fig. 2(a) by solid lines. The N th curve fits perfectly the experimental data around odd filling $\nu = 2N + 1$, justifying the model utilized, and at the same time substantial discrepancies are observed around neighboring even filling factors where ESR was found to split into two resonance lines.

The parameter β extracted from the fit was equal to 6 meV \AA , the value of the same order of magnitude as observed in conventional GaAs quantum wells [12]. In case we assume that the bulk spin-orbit interaction is solely responsible for the SO coupling in the 2D system, i.e., no contribution from the heterointerfaces is present, then the fundamentally important Dresselhaus constant γ can be estimated as $\gamma = \frac{\beta d^2}{\pi^2} = 1.2 \text{ eV \AA}^3$ for bulk AlAs, where $d = 45 \text{ \AA}$ is the quantum well width. Although this value is more than an order of magnitude smaller than in bulk GaAs [12], still the SO coupling in the 2D electron system is enhanced due to the small width of the quantum well.

The proposed model is analytically solvable only if the magnetic field is aligned perpendicular to the 2D system. Remarkably, Eq. (1) was found to be still applicable even in the case of large tilt angles between the magnetic field and the normal to the 2D plane. The dependencies of the g -factor at the exactly odd filling factors on ν are demonstrated for a number of tilt angles θ in Fig. 3. The orientation of the in-plane component of the magnetic field was fixed to be

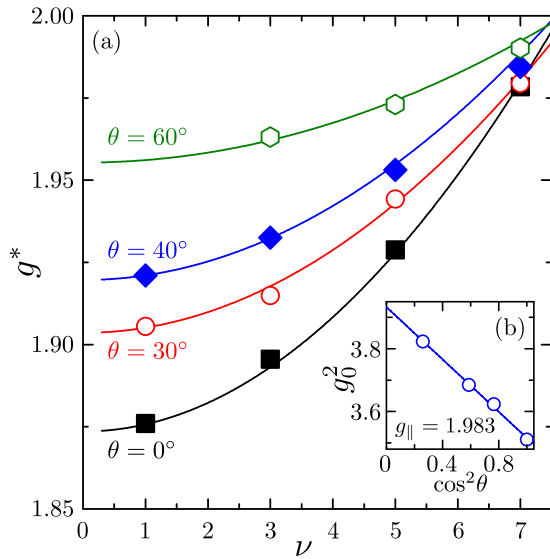


FIG. 3. (a) The electron g -factor measured exactly at the odd fillings for various tilt angles θ of the magnetic field. Solid lines represent the fit of the experimental data with Eq. (1). (b) The dependence of the bare single-electron g -factor squared g_0^2 on the squared cosine of θ is demonstrated in the inset. The in-plane g -factor extracted from the linear approximation to $\theta = 0^\circ$ is indicated.

$B_{\parallel} \parallel [100]$; other orientations were tested as well, yet the results were essentially the same. The calculations according to Eq. (1) are presented in Fig. 3 by solid lines and are in excellent agreement with the experimental results. Note that no fitting parameters were used as the value of β was already known from the experiments in the perpendicular magnetic field. The value of the bare electron g_0 was extracted from the large magnetic field limit and was θ dependent. The dependence of g_0^2 on $\cos^2 \theta$ demonstrated in the inset to Fig. 3 was linear and the extrapolation of this dependence to $\cos \theta = 0$, i.e., $\theta = 90^\circ$, allowed us to determine the in-plane component of the g -factor tensor $g_{\parallel} = 1.983$, a value consistent with the ESR studies of wide AlAs quantum wells [35,46]. This justifies our choice of the bare single-electron g -factor.

An important issue that should be addressed is why no such SO coupling effects have been detected in GaAs quantum wells in the IQHE regime despite the large amount of experimental work reported [40,41,43]. After all, the SO interaction does not entangle the spin-split Landau levels with the same number N and may couple only levels with different N and spin projections. Thus, in agreement with Eq. (1), to estimate

the SO contribution to the g -factor in the IQHE regime, one should compare the strength of the SO coupling with the cyclotron energy, a characteristic energy separation between Landau levels with different N . The cyclotron energy in GaAs is much higher than in AlAs due to the substantially smaller effective mass. As a result, in GaAs quantum wells the SO effects in the IQHE regime are much weaker, despite the SO interaction being practically of the same strength as in the structure under present study.

Note that ESR probes essentially a single-particle g -factor, as the Larmor theorem prohibits any contributions of the electron-electron interactions in the spin resonance frequency. Yet the Larmor theorem was predicted to fail in systems with strong SO coupling [21,22]. Despite the relatively large SO coupling and enhanced strength of e - e interactions due to the large effective mass, no signatures of the e - e interactions were detected, as can be clearly seen from Fig. 2. For example, the electron g -factor around the unity filling factor is almost constant and does not reveal any contributions $\sim \sqrt{B}$.

In conclusion, the spin resonance of 2D electrons was studied in a high-quality 4.5-nm AlAs quantum well in the regime of the integer quantum Hall effect. The electron g -factor, measured with the aid of ESR, turned out to be strongly dependent on the magnetic field with discontinuities around the even filling factors. The value of the g -factor increased with a decrease of the magnetic field around each odd filling. In contrast to conventional GaAs quantum wells, the g -factor at the exactly odd filling factor ν turned out to be dependent on ν in the structure under study. These experimental findings combined imply the entanglement between the spin degree of freedom and the orbital motion of the electrons in the regime of the integer quantum Hall effect. This suggestion is further supported, as all the experimental data are well approximated with the theoretical model when the Dresselhaus-type SO interaction is included into the Hamiltonian of a single electron in a quantizing magnetic field. Surprisingly, such excellent agreement was observed even in the case of tilted magnetic fields. The strength of the spin-orbit interaction in 2D systems was determined and the Dresselhaus constant was extracted for bulk AlAs. ESR tended to split into two well-resolved peaks around even fillings, yet this finding remains a mystery and highlights the need for further experimental and theoretical efforts.

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