# Symmetry analysis of current-induced switching of antiferromagnets

Hikaru Watanabe[*] and Youichi Yanase

*Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan*

Antiferromagnets are robust to external electric and magnetic fields, and hence are seemingly uncontrollable. Recent studies, however, realized the electrical manipulations of antiferromagnets by virtue of the antiferromagnetic Edelstein effect. We present a general symmetry analysis of electrically switchable antiferromagnets based on group-theoretical approaches. Furthermore, we identify a direct relation between switchable antiferromagnets and the ferrotoroidic order. The concept of ferrotoroidic order clarifies the unidirectional nature of switchable antiferromagnets and provides a criterion for the controllability of antiferromagnets. The scheme paves a way for perfect writing and reading of switchable antiferromagnets.

*Introduction.* The spin degree of freedom is highly controllable and has opened a new paradigm of electronics [1–4]. Especially, spin manipulation by an interplay with other degrees of freedom such as charge and valley is attracting much interest in the field of spintronics. The concept is widespread in condensed matter physics, e.g., superconductors [5] and topological materials [6].

Recently, the possibility of manipulating antiferromagnets has been recognized [7–11], while most spintronics devices are based on ferromagnets. The antiferromagnet, which in itself has neither electric nor magnetic polarizations, is robust to external electric and magnetic fields, in contrast to ferroelectric or ferromagnetic material-based devices. Hence, antiferromagnets are considered to be a new candidate for a nonvolatile memory device [12].

The switching of antiferromagnets has been explored in various ways: by using a spin-transfer torque or spin current in heterostructures [13–15] or magnetoelectric effects in bulk antiferromagnets [16]. In particular, manipulation by an electric current has significantly promoted the controllability of antiferromagnets. The switching mechanism utilizes current-induced antiferroic magnetization, namely, the antiferromagnetic (AFM) Edelstein effect. Following a theoretical proposal submitted independently by some groups [17,18], experimental realizations of switching have been achieved in CuMnAs [12] and $Mn_2Au$ [19].

The key to the AFM Edelstein effect is a *locally* noncentrosymmetric crystalline structure, in which the local symmetry of certain sites has no parity symmetry in spite of a globally centrosymmetric crystalline symmetry. The sublattice degree of freedom plays an important role in such systems. The parity symmetry is preserved since the atoms in each sublattice are interchanged by the parity operation. Then, spin-momentum locking arises in a sublattice-dependent manner although uniform spin-momentum locking is forbidden due to the globally centrosymmetric crystal symmetry [20–24]. Accordingly, nonequilibrium *antiferroic* spin polarization is

induced under electric current. This is an analog to the Edelstein effect, that is, current-induced ferroic spin polarization [25–28].

The switchable AFM order, which shares the same symmetry with current-induced antiferroic magnetization, breaks both of inversion and time-reversal symmetry although it preserves translational symmetry. It is noteworthy that the combined symmetry of parity and time-reversal operations, namely, $\mathcal{PT}$ symmetry, is preserved. The $\mathcal{PT}$ symmetry forbids electric polarization and net magnetization and ensures invulnerability to external electric or magnetic fields.

The properties of the AFM Edelstein effect have been investigated in previous studies [17,18,29–31]. The general criterion for determining electrically switchable antiferromagnets, however, remains unclear. An incomplete understanding of switchable antiferromagnets has disrupted further explorations of candidate materials except for the existing candidates [12,19].

In this Rapid Communication, we present a general criterion for the current-induced switching of antiferromagnets. A symmetry analysis based on the magnetic representation theory and the Aizu species clarifies what kind of AFM order can be manipulated by the AFM Edelstein effect. Furthermore, the analysis links switchable antiferromagnets and ferrotoroidic order. Thus, this work not only identifies many candidate materials but also provides a clear viewpoint of AFM spintronics. In the following, we do not discuss the effect of spin-transfer torque and spin current, since we focus on bulk antiferromagnets which are insensitive to surfaces.

*Representation analysis.* We first present a symmetry analysis of magnetic modes induced by the AFM Edelstein effect. The analysis is carried out with the use of a magnetic representation theory [32–35]. We focus on centrosymmetric systems where the $\mathcal{PT}$ symmetry is preserved, though it is straightforward to extend the analysis to noncentrosymmetric systems. Noncentrosymmetric magnets (Ga,Mn)As and $MnSiN_2$ are exemplified in the Supplemental Material [35–38].

The magnetic modes realized by the AFM Edelstein effect do not lead to any translational symmetry breaking, and hence they are characterized by the Néel vector $\boldsymbol{Q} = \boldsymbol{0}$. The formula

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*watanabe.hikaru.43n@st.kyoto-u.ac.jp

of the AFM Edelstein effect is written as

$$\hat{m}^{\mathrm{AF}} = \hat{\kappa}\, \boldsymbol{j}, \tag{1}$$

where the susceptibility tensor $\hat{\kappa}$ has symmetry determined by the crystalline structure [22,30]. Therefore, supposing systems in the paramagnetic phase, we here identify the AFM mode $\hat{m}^{\mathrm{AF}}$ and investigate which components of $\hat{\kappa}$ are allowed.

The allowed magnetic symmetry with $\boldsymbol{Q} = \boldsymbol{0}$ is described by a magnetic representation

$$\Gamma_{\boldsymbol{G}}^{\mathrm{mag}}(\boldsymbol{H}) = \Gamma_{\boldsymbol{G}}^{\mathrm{P}}(\boldsymbol{H}) \otimes \Gamma_{\boldsymbol{G}}^{\boldsymbol{M}}, \tag{2}$$

where $\boldsymbol{G}$ and $\boldsymbol{H}$ denote a crystal group and site-symmetry group of magnetic sites, respectively. $\Gamma_{\boldsymbol{G}}^{\mathrm{P}}$ is a permutation representation of magnetic sites and $\Gamma_{\boldsymbol{G}}^{\boldsymbol{M}}$ is a representation of an axial vector. The basis $\hat{m}^{\mathrm{AF}}$ of allowed magnetic modes is explicitly denoted as $m_{\mu}^{(\tau)}$, where $\tau$ and $\mu$ are indices of the basis of $\Gamma_{\boldsymbol{G}}^{\mathrm{P}}$ and $\Gamma_{\boldsymbol{G}}^{\boldsymbol{M}}$, respectively.

The response formula (1) is explicitly recast,

$$m_{\mu}^{(\tau)} = \kappa_{\tau\mu;\nu}\, j_{\nu}. \tag{3}$$

The coefficient $\kappa_{\tau\mu;\nu}$ is transformed by a symmetry operation $g \in \boldsymbol{G}$ as [39]

$$g(\kappa_{\tau\mu;\nu}) = \sum_{\rho,\lambda,\kappa} \kappa_{\rho\lambda;\kappa} [D^{(\mathrm{P})}(g)]_{\rho\tau} [D^{(\boldsymbol{M})}(g)]_{\lambda\mu} [D^{(j)}(g)]_{\kappa\nu}, \tag{4}$$

where $D^{(\mathrm{P})}$, $D^{(\boldsymbol{M})}$, and $D^{(j)}$ are representation matrices of the sublattice permutation, axial vector, and polar vector, respectively. According to Neumann's principle, the transformed susceptibility tensor should satisfy $g(\hat{\kappa}) = \hat{\kappa}$ [40,41]. Thus, the susceptibility tensor $\hat{\kappa}$ is subject to constraints from the crystal group $\boldsymbol{G}$.

An algebraic calculation by Eq. (4) identifies the symmetry-adapted form of $\kappa_{\tau\mu;\nu}$. To examine the symmetry constraints between magnetic structures and electric currents, it is practical to decompose the representation of the susceptibility tensor $\kappa_{\tau\mu;\nu}$ into irreducible representations of $\boldsymbol{G}$, that is, $\{\Gamma_{\boldsymbol{G}}^{(\alpha)}\}$. The decomposition is obtained as

$$\Gamma_{\boldsymbol{G}}^{\mathrm{mag}}(\boldsymbol{H}) \otimes \Gamma_{\boldsymbol{G}}^{j} = \sum_{\alpha} q_{\alpha} \Gamma_{\boldsymbol{G}}^{(\alpha)}, \tag{5}$$

where a coefficient $q_{\alpha}$ denotes a frequency of $\Gamma_{\boldsymbol{G}}^{(\alpha)}$ in the summation. $\Gamma_{\boldsymbol{G}}^{j}$ is a representation of the polar vector. The coefficient $q_1$ for the identity representation $\Gamma_{\boldsymbol{G}}^{(1)}$ gives the number of independent components of $\hat{\kappa}$.

Here, we summarize the symmetry constraints for switchable antiferromagnets. Each irreducible representation $\Gamma_{\boldsymbol{G}}^{(\alpha)}$ has inversion parity, since the crystal group $\boldsymbol{G}$ is centrosymmetric. Although both $\Gamma_{\boldsymbol{G}}^{\boldsymbol{M}}$ and $\Gamma_{\boldsymbol{G}}^{j}$ are representations of vector quantities, they have opposite parity. Thus, the permutation representation $\Gamma_{\boldsymbol{G}}^{\mathrm{P}}$ should comprise odd-parity irreducible representations to satisfy $q_1 \neq 0$. This means that a locally noncentrosymmetric property of magnetic sites is required for the AFM Edelstein effect, as mentioned in previous studies [17,18,30]. Furthermore, the magnetic representation $\Gamma_{\boldsymbol{G}}^{\mathrm{mag}}$ should comprise a polar representation $\Gamma_{\boldsymbol{G}}^{j}$. Owing to the time-reversal even/odd ($\mathcal{T}$-even/$\mathcal{T}$-odd) nature of the

representation $\Gamma_{\boldsymbol{G}}^{\mathrm{P}}/\Gamma_{\boldsymbol{G}}^{\boldsymbol{M}}$, the magnetic mode $m_{\mu}^{(\tau)}$ is $\mathcal{T}$ odd and leads to time-reversal symmetry breaking.

From the above analysis we conclude that the current-induced magnetic structure is polar and magnetic ($\mathcal{T}$ odd). It follows that the switchable AFM order by the AFM Edelstein effect contains a *toroidal moment* $\boldsymbol{T}$ [42]. We stress that the Néel vector is $\boldsymbol{Q} = \boldsymbol{0}$. Thus, all the switchable AFM order is regarded as a ferroic toroidal order, namely, ferrotoroidic order. This is a criterion of materials for AFM spintronics.

The toroidic nature of switchable antiferromagnets is intuitively understood by the fact that the electric current gives rise to a shift of the Fermi surface and produces "polarization" in momentum space. Such polarization shares the same symmetry with the toroidal moment as we have shown in the group-theoretical classification [43].

*Aizu species.* Regarding the switchable AFM order as ferrotoroidic order, we may clarify the possibility of AFM domain switching by making use of the *Aizu species*.

In general, a phase transition reduces the symmetry operations of a disordered phase. The symmetry relation between the disordered and ordered phases is formulated by a group-theoretical method. By supposing the crystal group $\boldsymbol{G}(\boldsymbol{K})$ in the disordered (ordered) phase, the coset decomposition of $\boldsymbol{G}$ by $\boldsymbol{K}$ is obtained as

$$\boldsymbol{G} = g_1 \boldsymbol{K} + g_2 \boldsymbol{K} + \cdots g_N \boldsymbol{K}, \tag{6}$$

where $g_1 \in \boldsymbol{K}$ and $g_j \notin \boldsymbol{K}\,(j \neq 1)$. $N$ is the order of $\boldsymbol{G}$ divided by that of $\boldsymbol{K}$. A domain state $s_1$, which is invariant to the symmetry operations of $\boldsymbol{K}$, is transformed into other domain states by symmetry operations of $g_j \boldsymbol{K}\,(j \neq 1)$. Therefore, the coset decomposition (6) shows the relation between domain states $\{s_j\}$,

$$s_j = g_j g_i^{-1} s_i, \tag{7}$$

where the domain $s_j$ is invariant to the symmetry operations of $\boldsymbol{K}_j = g_j \boldsymbol{K} g_j^{-1}$.

Domain properties of the ordered phase are classified by the Aizu species [44–50], the ensemble of pairs of $\boldsymbol{G}$ and $\boldsymbol{K}$ written as $\boldsymbol{G}\,F\,\boldsymbol{K}$. In the Aizu species classification, the species $\boldsymbol{G}\,F\,\boldsymbol{K}$ is characterized by physical quantities such as electric polarization, magnetization, strain, and toroidal moments. In the case of ferrotoroidic order, we first assign a domain $s_1$ with a toroidal moment $\boldsymbol{T}^{(1)}$. Correspondingly, we obtain the toroidal moment

$$\boldsymbol{T}^{(j)} = g_j \boldsymbol{T}^{(1)} \tag{8}$$

for another domain $s_j$. The number of possible toroidal moments is determined by a given species, since the species $\boldsymbol{G}\,F\,\boldsymbol{K}$ imposes the algebraic relation between domains as Eq. (7). Therefore, the Aizu species $\boldsymbol{G}\,F\,\boldsymbol{K}$ is classified as full/partial/zero toroidic, when the domain states are completely/partially/not distinguishable by the toroidal moment $\boldsymbol{T}$. The classification is summarized in Table I.

In a full- or partial-toroidic species, the symmetry-adapted field for the toroidal moment $\boldsymbol{T}$, that is, electric current $\boldsymbol{j}$, energetically distinguishes the domain states completely or partially. The electric current acts on the AFM moment such that the toroidal moment arising from the AFM mode is aligned along the injected current. Therefore, classification

TABLE I. The classification of Aizu species $G\,F\,K$ based on toroidal moments $\{T^{(i)}\}$.

| $\{T^{(i)}\}$ | $G\,F\,K$ |
|---|---|
| $T^{(i)} \neq T^{(j)}$ for all $i, j$ | Full toroidic |
| $T^{(i)} = T^{(j)}$ for some but not all $i, j$ | Partial toroidic |
| $T^{(i)} = T^{(j)} = 0$ for all $i, j$ | Zero toroidic |

based on the Aizu species for the ferrotoroidic order clarifies the AFM domains which are controllable by the electric current $j$. With a pair of the crystal group $G$ and the group for the AFM state $K$, the feasibility of the electrical switching of AFM domains is determined by referring to the toroidic property of the Aizu species [49].

The switchable antiferromagnets should belong to the full- or partial-toroidic species. We have identified candidate materials for the switchable AFM order and show a part of the list in Table II. In the Supplemental Material, we can find more candidates and more detailed information [35]. In the following, we apply our symmetry analysis to some antiferromagnets and reveal the toroidic property of the AFM state.

*Full-toroidic case.* As an example of the ferrotoroidic case, we discuss the tetragonal CuMnAs [61] where AFM domain switching has been demonstrated [12]. The crystal group is $4/mmm$ which is represented as $G = 4/mmm1'$ in magnetic point group notation. The AFM phase is specified by $K = mmm'\langle x \rangle$, where the symbol $\langle x \rangle$ means the twofold rotation symmetry along the $x$ axis. Correspondingly, the Aizu species is denoted by

$$4/mmm1'\,F\,mmm'\langle x \rangle. \tag{9}$$

TABLE II. List of candidate materials. The table lists metallic or semiconducting compounds, crystal point group (PG), direction of toroidal moment ($T$), Néel temperatures ($T_N$), and references (Ref.). More candidates are shown in Supplemental Material [35].

| Compounds | PG | $T$ | $T_N$ | Ref. |
|---|---|---|---|---|
| PrMnSbO | $4/mmm$ | | $35 < T < 230$ | [51] |
| | | $\{T_x, T_y\}$ | $35$ | [51] |
| NdMnAsO | $4/mmm$ | | $23 < T < 359$ | [52,53] |
| | | $\{T_x, T_y\}$ | $23$ | [52,53] |
| DyB$_4$ | $4/mmm$ | $\{T_x, T_y\}$ | $12.7 < T < 20.3$ | [54–56] |
| ErB$_4$ | $4/mmm$ | $\{T_x, T_y\}$ | $13$ | [54,55,57] |
| Mn$_2$Au | $4/mmm$ | $\{T_x, T_y\}$ | $>1000$ | [58] |
| FeSn$_2$ | $4/mmm$ | $\{T_x, T_y\}$ | $93 < T \lesssim 378$ | [59,60] |
| | | $\{T_x, T_y\}$ | $93 \lesssim T < 378$ | [59,60] |
| CuMnAs | $4/mmm$ | $\{T_x, T_y\}$ | $480$ | [61] |
| U$_3$Ru$_4$Al$_{12}$ | $6/mmm$ | $T_z$ | $9.5$ | [62,63] |
| CaMn$_2$Bi$_2$ | $\bar{3}m$ | $\{T_x, T_y\}$ | $154$ | [64] |
| SrMn$_2$Sb$_2$ | $\bar{3}m$ | $\{T_x, T_y\}$ | $110$ | [65] |
| Gd$_5$Ge$_4$ | $mmm$ | $T_z$ | $127$ | [66,67] |
| UCu$_5$In | $mmm$ | $T_y$ | $25$ | [68] |
| YbAl$_{1-x}$Fe$_x$B$_4$ | $mmm$ | $T_x$ | | [69] |

TABLE III. The characterization of the Aizu species of CuMnAs [49]. "F," "P," and "Z" represent full, partial, and zero, respectively.

| $4/mmm1'\,F\,mmm'\langle x \rangle$ | $\hat{\epsilon}$ | $P$ | $M$ | $T$ |
|---|---|---|---|---|
| | P | Z | Z | F |

The coset decomposition (6) is obtained as

$$G = I\,K + P\,K + C_{4z}^+\,K + S_{4z}^+\,K, \tag{10}$$

where $I$, $P$, and $C_{4z}^+(S_{4z}^+)$ are the identity operation, the parity operation, and the fourfold (improper) rotation, respectively. The domain $s_1$ with the polar axis $x$ possesses the toroidal moment $T^{(1)} = T\hat{x}$. Accordingly, the toroidal moment of the domain $s_2 = Ps_1$ is obtained as

$$T^{(2)} = P(T\hat{x}) = -T\hat{x}. \tag{11}$$

Similarly, $T^{(3)} = T\hat{y}$ and $T^{(4)} = -T\hat{y}$ are obtained for the domains $s_3$ and $s_4$, respectively. Thus, all the domains of CuMnAs have different toroidal moments, and the Aizu species (9) is actually full toroidic. Therefore, the AFM state can be completely manipulated by the electric current.

We summarize the properties of the Aizu species (9) in Table III. The species (9) is zero electric and zero magnetic, and hence AFM domains can hold neither electric polarization $P$ nor magnetic polarization $M$. These constraints are consistent with the $\mathcal{PT}$ symmetry preserved in the AFM state. On the other hand, the species is partial elastic. It follows that the AFM domains are partially controllable by stress which is the conjugate field to the strain $\hat{\epsilon}$. Thus, the Aizu species analysis is also useful to elucidate the possibility of an indirect switching of the AFM state.

To support the Aizu species analysis, we conduct a representation analysis. The magnetic Mn ions are positioned in the crystallographic site with a noncentrosymmetric site-symmetry group $H = 4mm$, and CuMnAs is locally noncentrosymmetric. The magnetic representation is obtained as

$$\Gamma_G^{\mathrm{mag}}(H) = \Gamma_G^P(H) \otimes \Gamma_G^M \tag{12}$$

$$= A_{2g} + E_g + A_{1u} + E_u. \tag{13}$$

Then, the product representation (5) comprises $\Gamma_G^{(1)} = A_{1g}$, since the $E_u$ mode in Eq. (13) is included in the polar representation $\Gamma_G^j = A_{2u} + E_u$. Thus, the AFM Edelstein effect is allowed. The correspondence between the toroidal moment and the AFM order is clarified by the projection operator method [32,35]. By the projection operator associated with the basis of $E_u$, the AFM moment aligned along the $\pm x$ axis is revealed to have a toroidal moment $\pm T\hat{y}$. Hence, the in-plane electric current $\pm j_y$ stabilizes the AFM state as shown in Fig. 1. Similarly, the electric current $\pm j_x$ stabilizes the AFM moment along the $\pm y$ axis which has the toroidal moment $\pm T\hat{x}$. Thus, the representation theory is consistent with the Aizu species analysis.

*Partial-toroidic case.* Next, we discuss a partially controllable AFM state of U$_3$Ru$_4$Al$_{12}$, which belongs to a partial-toroidic species.

U$_3$Ru$_4$Al$_{12}$ crystallizes in a hexagonal structure ($G = 6/mmm$). The magnetic uranium ions form a kagome lattice [62,63]. Interestingly, the compound shows a compensated
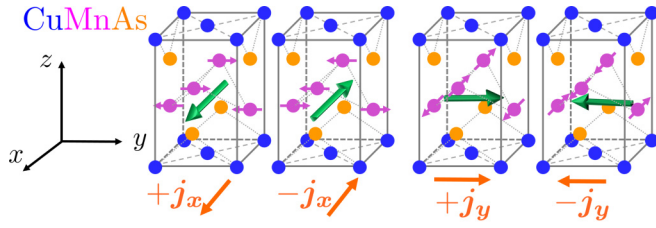
FIG. 1. The correspondence between the AFM domain and the toroidal moment in CuMnAs. The green- (purple-) colored arrows represent the toroidal (magnetic) moments. The electric current stabilizing each domain is depicted.
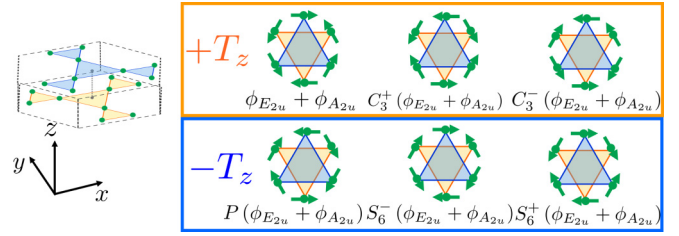


FIG. 2. The uranium sites in $U_3Ru_4Al_{12}$ (left) and the possible AFM domains (right panels) [63]. $C_3^{\pm}(S_6^{\pm})$ denotes the threefold (improper) rotation. Each of domains has a toroidal moment $\pm T\hat{z}$.

and noncollinear AFM order by which the threefold rotation symmetries are broken [63]. The Aizu species is given by

$$6/mmm1' \, F \, mmm'\langle z \rangle, \tag{14}$$

where $\langle z \rangle$ means the twofold rotation symmetry along the $z$ axis. The species is partial toroidic as shown in Table IV and allows the domain states to be partially controllable by the electric current. Following the algebraic relation between the domain states, half of the six domains host the same toroidal moment $\boldsymbol{T} \parallel \hat{z}$ which can be inverted by the out-of-plane electric current $j_z$.

We also present a representation analysis. The U atoms are positioned in crystallographic sites with the site-symmetry group $\boldsymbol{H} = mm2$. The magnetic representation is obtained as

$$\Gamma_G^{\text{mag}}(\boldsymbol{H}) = A_{2g} + B_{1g} + B_{2g} + A_{1u} + A_{2u} + B_{1u}$$
$$+ 2E_{1g} + E_{2g} + E_{1u} + 2E_{2u}, \tag{15}$$

which comprises polar representations $A_{2u}$ and $E_{1u}$. The basis of $A_{2u}$ ($E_{1u}$) can be taken as a toroidal moment $\boldsymbol{T} \parallel \hat{z}$ ($\boldsymbol{T} \parallel \{\hat{x}, \hat{y}\}$), and the AFM Edelstein effect is actually allowed when $\boldsymbol{j} \parallel \hat{z}$ ($\boldsymbol{j} \parallel \{\hat{x}, \hat{y}\}$).

The magnetic order of $U_3Ru_4Al_{12}$ [63] is represented by the $A_{2u}$ and $E_{2u}$ irreducible representations. These representations are odd parity, and the former (latter) is polar (nonpolar). By representing one of the magnetic domains by the basis $\phi_{E_{2u}} + \phi_{A_{2u}}$, the other AFM domains are labeled as depicted in Fig. 2. The $\phi_{A_{2u}}$ mode corresponds to the toroidal moment $T_z$, and hence the electric current $j_z$ enables the switching between the AFM domains having different $\phi_{A_{2u}}$ components. On the other hand, the domains with the same $\phi_{A_{2u}}$ components cannot be switched by the electric current. Thus, the representation analysis is consistent with the Aizu species analysis in a partial-toroidic case of $U_3Ru_4Al_{12}$. To control the AFM domains perfectly, we may use the magnetopiezoelectric effect, which is explained in Supplemental Material [31,35,43].

*Readout of AFM domains.* Following the symmetry analysis revealing an essential role of ferrotoroidic order for the

TABLE IV. The characterization of the Aizu species of $U_3Ru_4Al_{12}$ [49].

| $6/mmm1' \, F \, mmm'\langle z \rangle$ | $\hat{\epsilon}$ | $\boldsymbol{P}$ | $\boldsymbol{M}$ | $\boldsymbol{T}$ |
|---|---|---|---|---|
| | P | Z | Z | P |

electrical switching of AFM domains, we present a complete readout of the domains using a functionality arising from the unidirectional nature of the ferrotoroidic order.

In an experiment of CuMnAs, an electrical readout of AFM domains has been performed by measuring anisotropic magnetoresistance (AMR) [12]. The AMR, however, cannot completely distinguish domains, because domains with opposite toroidal moments show the same AMR. On the other hand, we can make use of the *unidirectional* property of the ferrotoroidic order to discern the AFM domains in a complete way.

The ferrotoroidic order induces a unidirectional anisotropy in various transport phenomena: In a nonlinear electric conductivity up to the second order $|\boldsymbol{E}|^2$ denoted by

$$j_\mu = \sigma_{\mu\nu} E_\nu + \tilde{\sigma}_{\mu\nu} E_\nu^2, \tag{16}$$

the ferrotoroidic moment $\boldsymbol{T} \parallel \hat{x}_\mu$ gives rise to a finite longitudinal component $\tilde{\sigma}_{\mu\mu}$, which changes sign between domains with opposite toroidal moment [43]. Therefore, the nonlinear conductivity may distinguish the domain states of switchable antiferromagnets in a complete manner. Thus, both the manipulation and detection of AFM states can be electrically carried out.

The nonlinear conductivity $\tilde{\sigma}_{\mu\mu}$ indicates a dichromatic transport. Indeed, dichromatic transport is an emergent physical property induced by the ferrotoroidic order. Although dichromatic transport has been observed in noncentrosymmetric systems under an external magnetic field [70–73] and several ferromagnetic materials [74,75], it may be realized in ferrotoroidic AFM states without a magnetic field. Such dichroism induced by the ferrotoroidic order is tunable by the current $\boldsymbol{j}$ through AFM domain switching. When we vary an electric current, a hysteretic behavior may be observed as a signal of AFM domain switching.

It is noteworthy that the manipulation and detection of the ferrotoroidic domain by a tunable electric current are realizable only in metallic systems, in contrast to previous observations of ferrotoroidic order in magnetic insulators [42,76,77]. The toroidic domain in insulators can be manipulated by making use of the magnetoelectric effect. To be specific, the toroidic domain in insulators is inverted by simultaneously applying both electric and magnetic fields [76,77]. In contrast, the toroidic domains in metals are controllable by only injecting the electric current.

To summarize, we provide a general criterion of electrically switchable antiferromagnets based on the complementary use of the Aizu species and the representation theory. Both approaches unveil the direct correspondence between

switchable AFM states and ferrotoroidic order. The concept of ferrotoroidic order uncovers functionalities of antiferromagnets and gives a clear viewpoint in AFM spintronics. It is desirable for further developments of AFM spintronics to explore the functionalities of various antiferromagnets. On the basis of a symmetry analysis, we provided a list of electrically switchable antiferromagnets, which will be useful for future studies.

Recently, we became aware of an experiment of CuMnAs which demonstrated the switching and reading of the AFM domain states with opposite toroidal moments [78]. The domain states have been distinguished by the nonlinear Hall conductivity which is described by Eq. (16). The toroidal moment $\boldsymbol{T} \parallel \hat{x}_\mu$ gives rise to a transverse nonlinear conductivity $\tilde{\sigma}_{\mu\nu}$ [79], and hence the experimental result [78] is consistent with our symmetry analysis.

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