Tuning the in-plane Fulde-Ferrell-Larkin-Ovchinnikov state in a superconductor/ferromagnet/normal-metal hybrid structure by current or magnetic field

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Temperature-induced transition of thin superconductor/ferromagnet/normal-metal (S/F/N) hybrid structure to an in-plane Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state is accompanied by vanishing of effective inverse magnetic field penetration depth Λ^{-1} [S. V. Mironov *et al.*, Phys. Rev. Lett. **121**, 077002 (2018)]. Here we show that Λ^{-1} goes to zero only in the limit of zero magnetic field $H \to 0$ and at any finite parallel H or in-plane current I it is finite and positive in the FFLO state, which implies a diamagnetic response. We demonstrate that Λ^{-1} has a nonmonotonic dependence on H and I not only in the parameter range corresponding to the FFLO phase domain but also in its vicinity. We find that for S/F/N/F/S structures with certain thicknesses of F layers there is a temperature-, current-, and magnetic-field-driven transition to and out of the FFLO phase with a simultaneous jump of Λ^{-1}.

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I. INTRODUCTION

In superconductor/ferromagnet (S/F) bilayers the proximity-induced odd-frequency spin-triplet superconducting component in the F layer gives a *negative* contribution to the square of the inverse London penetration depth λ^{-2} [1–6], which is a coefficient in the relation between superconducting current density and vector potential: $\mathbf{j} = -c\mathbf{A}/4\pi\lambda^2$. At some parameters this contribution can exceed the positive contribution from the singlet superconducting component in the S and F layers and makes the effective inverse magnetic field penetration depth $\Lambda^{-1} = \int_0^d \lambda^{-2}(x)dx$ (d is the thickness of the bilayer) negative, which implies a paramagnetic response of the whole structure. In Ref. [7] it is argued that the state with $\Lambda^{-1} < 0$ is unstable, and is found that the S/F bilayer transits to the in-plane Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state as $\Lambda^{-1} \to +0$. In a recent work [8] it was predicted that such an in-plane FFLO state could emerge at a temperature much below the critical one; it is characterized by an unusual current-phase relation and can be realized in the S/F/N trilayer with realistic parameters, where N is a low-resistivity normal metal (Au, Ag, Cu, or Al), S is a disordered superconductor with large residual resistivity in the normal state (NbN, WSi, NbTiN, etc.), and F is an ordinary ferromagnet (Fe, CuNi, etc.).

Motivated by these results and the expected unusual electrodynamic response of the FFLO state in the S/F/N trilayer with rather normal parameters, easily realizable with modern experimental techniques, we theoretically study the effect of the parallel magnetic field H and in-plane current I on the FFLO state in the S/F/N trilayer and the S/F/N/F/S symmetric pentalayer. We find that $\Lambda^{-1} = 0$ only in the limit $H, I \to 0$ and it is positive for any finite magnetic field or current, which means that S/F/N trilayer has a diamagnetic response. The parallel magnetic field and in-plane current suppress proximity-induced odd-frequency triplet superconductivity in F/N layers, and Λ^{-1} increases in weak magnetic field (current). The same effect exists for a trilayer close to the FFLO phase domain ($\Lambda^{-1} \neq 0$ at $H, I = 0$) due to the contribution of the triplet component to Λ^{-1}. In the pentalayer the FFLO phase domain is smaller due to competition of the FFLO state with the π state (well known for S/F/S trilayers [9]), but there are temperature-, magnetic-field-, and current-driven transitions from π to the FFLO state with a considerable change in Λ^{-1}.

The structure of the paper is as follows. In Sec. II we present our theoretical model. In Sec. III we show our results for the effect of parallel magnetic field and in-plane current on Λ^{-1} in the S/F/N trilayer being in the FFLO state or in the state with a large contribution from the odd-frequency triplet component of Λ^{-1}. In Sec. IV we consider different types of the $\pi \to$ FFLO transitions in S/F/N/F/S structures and their influence on the screening properties. Section V contains a brief summary.

II. MODEL

To study the superconducting properties of S/F/N and S/F/N/F/S structures we use the one-dimensional Usadel equation [10] for normal g and anomalous f quasiclassical Green's functions. With the standard angle parametrization $g = \cos\Theta$ and $f = \sin\Theta\exp(i\varphi)$ the Usadel equations in different layers can be written as

$$\frac{\hbar D_S}{2}\frac{\partial^2\Theta_S}{\partial x^2} - \left(\hbar\omega_n + \frac{D_S}{2\hbar}q^2\cos\Theta_S\right)\sin\Theta_S + \Delta\cos\Theta_S = 0, \tag{1}$$

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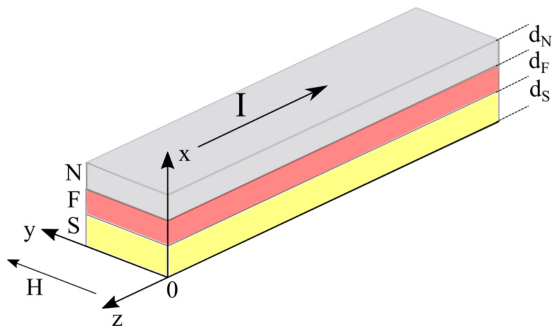

FIG. 1. Schematic representation of the S/F/N structure under consideration with transport current I or placed in parallel magnetic field H.

$$\frac{\hbar D_F}{2}\frac{\partial^2\Theta_F}{\partial x^2}-\left((\hbar\omega_n+ih)+\frac{D_F}{2\hbar}q^2\cos\Theta_F\right)\sin\Theta_F=0, \tag{2}$$

$$\frac{\hbar D_N}{2}\frac{\partial^2\Theta_N}{\partial x^2}-\left(\hbar\omega_n+\frac{D_N}{2\hbar}q^2\cos\Theta_N\right)\sin\Theta_N=0, \tag{3}$$

where subscripts S, F, and N refer to superconducting, ferromagnetic, and normal layers, respectively. Here D is the diffusion coefficient for the corresponding layer, h is the exchange field in the F layer, $\hbar\omega_n=\pi k_BT(2n+1)$ are the Matsubara frequencies (n is an integer number), $q=\nabla\varphi+2\pi\mathbf{A}/\Phi_0$ is the quantity that is proportional to supervelocity $v_s=\hbar q/m$ directed in the z direction (see Fig. 1), φ is the phase of the order parameter, $\mathbf{A}$ is the vector potential, and $\Phi_0=\pi\hbar c/|e|$ is the magnetic flux quantum. The x axis is oriented perpendicular to the surface of the S layer according to Fig. 1. Δ is the superconducting order parameter, which satisfies to the self-consistency equation

$$\Delta\ln\left(\frac{T}{T_{c0}}\right)=2\pi k_BT\sum_{\omega_n>0}\mathrm{Re}\left(\sin\Theta_S-\frac{\Delta}{\hbar\omega_n}\right), \tag{4}$$

where T_{c0} is the critical temperature of a single S layer (film) in the absence of magnetic field. These equations are supplemented by the Kupriyanov-Lukichev boundary conditions between layers [11]

$$D_S\frac{d\Theta_S}{dx}\bigg|_{x=d_S-0}=D_f\frac{d\Theta_F}{dx}\bigg|_{x=d_S+0},$$
$$D_F\frac{d\Theta_F}{dx}\bigg|_{x=d_S+d_F-0}=D_N\frac{d\Theta_N}{dx}\bigg|_{x=d_S+d_F+0}. \tag{5}$$

For the sake of simplicity we assume that the barrier between layers does not exist, and therefore, Θ is a continuous function of x. For interfaces with the vacuum we use the boundary condition $d\Theta/dx=0$. In the π state we add the condition $\Theta=0$ in the middle of the pentalayer structure.

We assume that the thickness of the whole structure is much smaller than the London penetration depth λ of the single S layer and neglect the effect of screening on the vector potential and magnetic field. For the chosen direction of the applied magnetic field (see Fig. 1) we use the following vector potential: $\mathbf{A}=(0,0,-Hx)$ in the case of the trilayer and $\mathbf{A}=(0,0,-H(x-d_S-d_F-d_N/2))$ in the case of the pentalayer.

To calculate the supercurrent density we use the following expression:

$$j=\frac{2\pi k_BT}{e\rho}q\sum_{\omega_n>0}\mathrm{Re}(\sin^2\Theta), \tag{6}$$

where ρ is the residual resistivity of the corresponding layer. From Eq. (6) and the London relation $j=-cA/4\pi\lambda^2$, one can find the expressions for the square of the inverse London penetration depth,

$$\frac{1}{\lambda^2(x)}=\frac{16\pi^2k_BT}{\hbar c^2\rho}\sum_{\omega_n>0}\mathrm{Re}(\sin^2\Theta), \tag{7}$$

and for the inverse effective penetration depth,

$$\Lambda^{-1}=\int_0^d\frac{dx}{\lambda^2(x)}, \tag{8}$$

where the total thickness $d=d_S+d_F+d_N$ for the S/F/N structure and $d=2d_S+2d_F+d_N$ for the S/F/N/F/S structure. In the case of a thin S film Λ coincides with the Pearl penetration depth [12].

Because we neglect the variation of H due to screening we simply use the Helmholtz free energy,

$$F_H=\pi N(0)k_BT\sum_{\omega_n\geqslant0}\int\mathrm{Re}\{\hbar D[(\nabla\Theta)^2+\sin^2\Theta(q/\hbar)^2]-4(\hbar\omega_n+ih)(\cos\Theta-1)-2\Delta\sin\Theta\}dx. \tag{9}$$

In numerical calculations we use dimensionless units. The magnitude of the order parameter is normalized in units of k_BT_{c0}, length is in units of $\xi_c=\sqrt{\hbar D_S/k_BT_{c0}}$, the free energy is in units of $F_0=N(0)(k_BT_{c0})^2\xi_c$. The magnetic field is measured in units of $H_0=\Phi_0/2\pi\xi_c^2$, and the effective penetration depth is in units of $\Lambda=\lambda_0^2/d_S$, where λ_0 is the London penetration depth of the single S layer at zero temperature.

To find the effective penetration depth Λ^{-1}, we numerically solve Eqs. (1)–(4), using Kupriyanov-Lukichev boundary conditions (5). In calculations we assume that the density of states on the Fermi level $N(0)$ is the same for all layers and therefore the ratio of resistivities is inversely proportional to the ratio of corresponding diffusion coefficients. To reduce the number of free parameters we also assume that the resistivities of the S layer and F layers are equal, i.e., $\rho_S/\rho_F=1$, which roughly corresponds to parameters of real S and F films. Because formation of a FFLO state in the S/F/N structure needs a large ratio of resistivities between the N layer and S layers, we use $\rho_S/\rho_N=150$ in our calculations, which is close to the parameters of real materials [8]. For example, for the pair NbN/Al the ratio ρ_S/ρ_N can be as large as 400 [13], while for the pair NbN/CuNi $\rho_S/\rho_F\sim1.5$ [14]. The exchange field of the ferromagnet h is assumed to be of the order of the Curie temperature T_{curie} (for example, in CuNi [14] $h\sim13k_BT_{c0}$).

The in-plane FFLO state can be realized as a FF-like state [in this case $f(z)\sim\exp(iq_0z)$] or as a LO-like state [in this case $f(z)\sim\cos(q_0z)$ near T^{FFLO}] . In addition to one-dimensional (1D) calculations we also numerically solved a

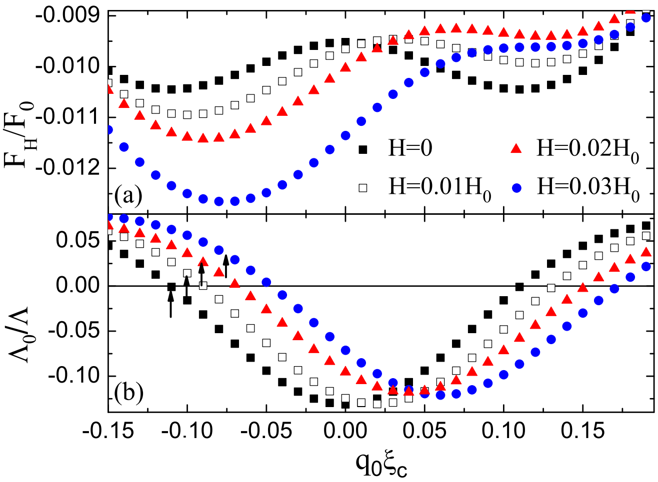

FIG. 2. Dependence of (a) the free energy F_H and (b) inverse effective penetration depth Λ^{-1} on q_0 for the S/F/N trilayer being in the in-plane FFLO state at different values of the parallel magnetic field. The arrows indicate the values of Λ^{-1} corresponding to the left minimum of the free energy. We use the following parameters of the system: $h = 5k_BT_{c0}$, $d_S = 1.1\xi_c$, $d_F = 0.5\xi_c$, $d_N = \xi_c$, and $T = 0.3T_{c0}$.

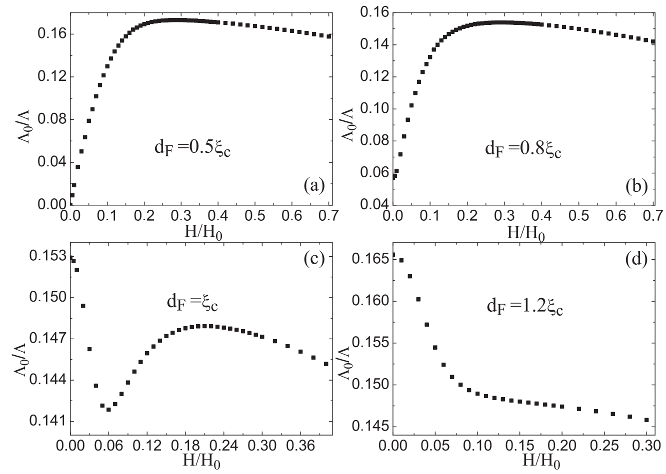

FIG. 3. Dependence of the inverse effective penetration depth of the magnetic field Λ^{-1} in the S/F/N trilayer on the parallel magnetic field H at different thicknesses of the F layer d_F: (a) $0.5\xi_c$ (FFLO state), (b) $0.8\xi_c$, (c) ξ_c, and (d) $1.2\xi_c$. The other parameters of the trilayer are the following: $h = 5k_BT_{c0}$, $d_S = 1.1\xi_c$, $d_N = \xi_c$, and $T = 0.2T_{c0}$.

two-dimensional Usadel equation (in the x and z directions in Fig. 1) with the boundary conditions along the z direction $f(z=0) = f(z=\pi/q_0) = 0$ and found that such a LO-like state has an energy (per unit of volume) larger than the FF-like state at any $q_0 = \partial\varphi/\partial z$. Therefore, throughout our paper under the FFLO state we assume a FF-like state $f(z) \sim \exp(iq_0z)$ and solve the 1D problem in the x direction.

III. S/F/N TRILAYER

Let us first consider the S/F/N trilayer. As shown in [8], in the case $\rho_N \ll \rho_S$ there is a range of parameters for which the in-plane FFLO phase appears below the critical temperature $T^{\text{FFLO}} < T_c$. In the FFLO phase the effective penetration depth $\Lambda^{-1} = 0$ as $H \to 0$, which signals the vanishing of the magnetic response at $T \leqslant T^{\text{FFLO}}$. Here we calculate the effect of finite H and I on Λ^{-1}.

In Fig. 2(a) we show the dependence $F_H(q_0)$ of the trilayer below the FFLO transition temperature T^{FFLO}. One can see that in the absence of the external field two states with $q_0 \neq 0$ have a minimal energy. Both states correspond to $\Lambda^{-1} = 0 = \partial F_H/\partial q_0$ [Fig. 2(b)]. The parallel magnetic field H breaks the symmetry $F_H(q_0)$ and leads to the increase in and furthermore disappearance of one of the energy minimums [right minimum in Fig. 2(a)]. Corresponding to this, the minimum state has a negative value of Λ^{-1} at $H > 0$ and, according to the arguments suggested in Ref. [7], should be considered an unstable one. Indeed, one can show that the term corresponding to the contribution of the kinetic energy to F_H is proportional to $\Lambda^{-1}q^2$. When $\Lambda^{-1} < 0$, it is energetically favorable to have nonzero supervelocity $\sim q$ (if it were zero) or to increase it (if it were finite), which makes such a state with negative Λ^{-1} unstable. To see how this instability evolves in time and what the finite state is, one should solve the three-dimensional (3D) problem in our case (taking into account $q \neq 0$ in all directions), which is out of the scope of the present research. Further, we consider only the state with $\Lambda^{-1} \geqslant 0$, corresponding to the left minimum of the dependencies $F_H(q_0)$ in Fig. 2(a).

The field dependence of $\Lambda^{-1} \geqslant 0$ is present in Fig. 3(a). One can see that Λ^{-1} nonmonotonically changes with the field. The increase of Λ^{-1} at relatively weak magnetic field is connected to two effects. The first one is the suppression of the superconducting correlations (including a triplet one) in the N layer by magnetic field, and we find that it gives the main contribution to the increase in Λ^{-1}. Besides that there is a slight enhancement off the singlet superconductivity in the S layer because weak magnetic field decreases the supervelocity $\sim q = q_0 + 2\pi A/\Phi_0$ in the S layer, and it also enhances Λ^{-1}. The second effect is responsible for the enhancement of T_c^{FFLO} (see Fig. 4) by applied field; previously, this effect was predicted for the S/F bilayer in the FFLO state in Ref. [15]. Note that the found enhancement of T_c is rather small for an S/F/N trilayer with realistic parameters.

Sufficiently large magnetic field destroys proximity-induced superconductivity in F/N layers, and Λ^{-1} reaches the maximum value [see Fig. 3(a)]. The following decrease of Λ^{-1} is explained by a gradual increase of $q \sim A$ in the S layer and the gradual suppression of $|\Delta|$ as usual in the S film. These results show that Λ^{-1} is finite and positive at any finite H for the S/F/N trilayer in the FFLO state. One may also conclude that the magnetic response is diamagnetic and nonlinear even at $H \to 0$ because Λ^{-1} changes from zero to the finite value.

Even if Λ^{-1} is positive at $H = 0$ and the trilayer is not in the FFLO state, the dependence $\Lambda^{-1}(H)$ may be nonmonotonic due to the contribution of the triplet component to Λ^{-1}. In Figs. 3(b)–3(d) we demonstrate it by varying the thickness of the F layer and keeping parameters of the trilayer constant. With increasing d_F the contribution of the triplet component to Λ^{-1} decreases, but it stays finite. A small increase of d_F

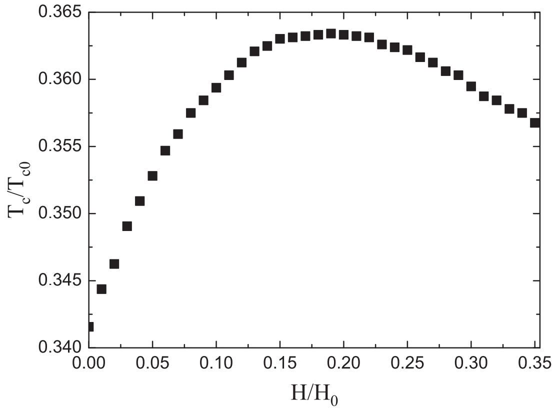

FIG. 4. Dependence of the critical temperature of the S/F/N trilayer on the parallel magnetic field H. The parameters of the system are the following: $h = 25k_BT_{c0}$, $d_S = 1.2\xi_c$, $d_F = 0.16\xi_c$, $d_N = \xi_c$. Using a smaller thickness of the S layer, one can obtain a larger relative change in T_c, but T_c itself goes to lower temperatures. A similar result can be obtained for $h = 5k_BT_{c0}$.

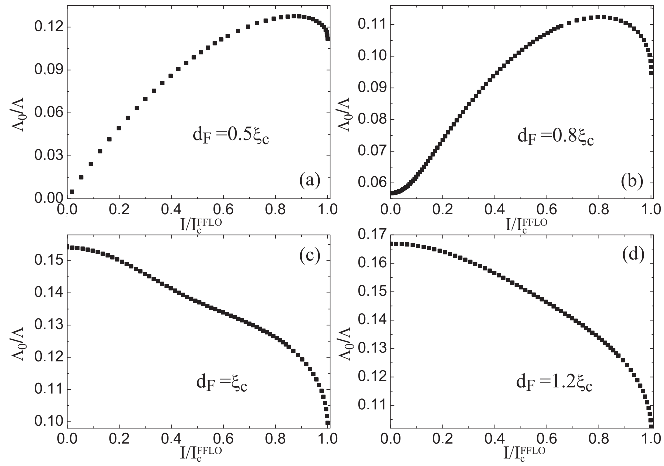

FIG. 5. Dependence of the inverse effective penetration depth Λ^{-1} in the S/F/N trilayer on the in-plane I at different thicknesses of the F layer d_F: (a) $0.5\xi_c$ (FFLO state), (b) $0.8\xi_c$, (c) ξ_c, and (d) $1.2\xi_c$. Current is expressed in units of the critical current of the FFLO state I_c^{FFLO}. The rest of the parameters are the same as in Fig. 3.

[see Fig. 3(b)] drives the system from the FFLO state, but due to a considerable contribution of the triplet component the dependence $\Lambda^{-1}(H)$ resembles the one shown in Fig. 3(a). In Ref. [8] a somewhat related effect was found for the dependence $\Lambda^{-1}(T)$ in the vicinity of the FFLO domain. Consequently, the increase of Λ^{-1} with magnetic field can serve as a precursor of the FFLO state as the increase of Λ^{-1} with increasing temperature [8].

At larger d_F, i.e., farther from the FFLO domain, $\Lambda^{-1}(H = 0)$ increases, and starting from some value of d_F ($\approx 2\sqrt{\hbar D_F/h}$), the inverse penetration depth Λ^{-1} decreases in a weak magnetic field [see Fig. 3(c)]. Our calculations show that the effect is connected to faster decay of the singlet component than the triplet one in the N layer at weak magnetic field. At some larger value of the field [it roughly corresponds to the minimum of the dependence $\Lambda^{-1}(H)$ shown in Fig. 3(c)], the proximity-induced superconductivity gets suppressed more strongly, and Λ^{-1} increases as in Figs. 3(a) and 3(b), and the dependence $\Lambda^{-1}(H)$ has both a minimum and maximum. A further increase of d_F [see Fig. 3(d)] leads to the monotonic decrease of Λ^{-1} in the magnetic field (the triplet component gives a small contribution to Λ^{-1}), and the influence of N layers manifests in the rapid vanishing of Λ^{-1} at relatively weak fields when superconductivity is terminated there.

We obtain very similar results for the larger value of the exchange field ($h = 25k_BT_{c0}$) with the only difference being that they occur in a much narrower range of d_F with respect to ξ_c, reflecting the smaller value of the characteristic decay length of superconducting correlations in the F layer $\xi_F \sim 1\sqrt{h}$ (results are not shown here). Qualitatively, the same dependencies $\Lambda^{-1}(H)$ [except for the one with two extrema shown in Fig. 3(c)] can be found at fixed d_F when one increases temperature from $T < T^{\mathrm{FFLO}}$ to $T^{\mathrm{FFLO}} < T < T_c$ when the trilayer is driven from the in-plane FFLO to the uniform state but with a still noticeable contribution of triplet superconductivity to Λ^{-1}.

The FFLO state in the S/F/N trilayer can be tuned not only by parallel magnetic field but ny in-plane current too. As in the case of parallel magnetic field, applied current breaks the symmetry $q_0 \rightarrow -q_0$, and in Fig. 5 we show the dependence of $\Lambda^{-1} \geqslant 0$ on the in-plane current for the same parameters as in Fig. 3. External current (supervelocity) suppresses stronger proximity-induced superconductivity in the N layer than the superconductivity in the S layer (like in the S/N bilayer [13]), and Λ^{-1} increases with the current for some d_F [see Figs. 5(a) and 5(b)]. Qualitatively, the results shown in Fig. 5(a) could be found using a modified Ginzburg-Landau equation [16], as done in Ref. [17], where a current-carrying FFLO state was studied. This approach is much simpler than the Usadel equations used here and allows us to obtain an analytical solution for current states, but it has two disadvantages: (i) it cannot be used to study states which are not in the FFLO phase domain, and (ii) it is difficult to relate coefficients in the Ginzburg-Landau functional to microscopic parameters of the S/F/N structure.

IV. $\pi \rightarrow$ FFLO TRANSITION IN S/F/N/F/S STRUCTURES

Let us now discuss symmetric the S/F/N/F/S pentalayer (it can be imagined as a doubled trilayer). Our interest in this system is mainly connected to the existence of the π state, corresponding to the phase difference π between outer S layers, together with the zero state (the uniform or FFLO one) considered in the previous section. We restrict ourselves by considering the uniform π state because in the chosen parameter range the modulated (FFLO) π state is not realized (note that in a recent work [18] such a state was predicted for the S/F/S structure in a certain range of parameters).

The responses of the trilayer and pentalayer on the parallel magnetic field are somewhat different due to different orbital effects produced by H. In the trilayer magnetic-field-induced supervelocity is maximal in the N layer, while in pentalayer it is maximal in S layers. It is the reason why for the pentalayer we did not find an enhancement of T_c by parallel magnetic field (shown in Fig. 4 for the trilayer) and a dependence

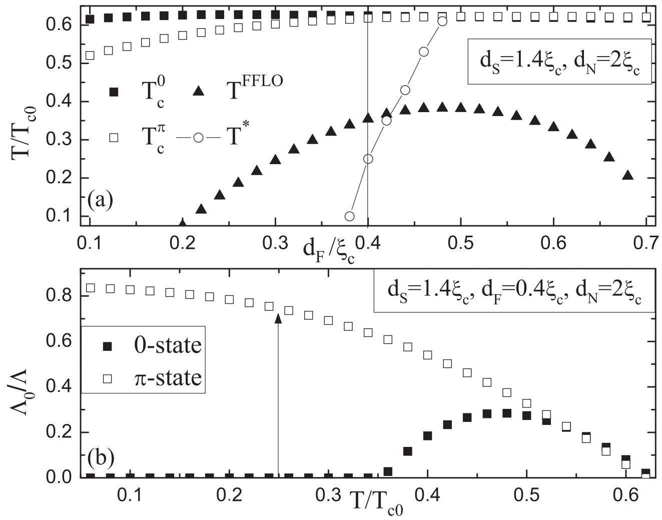

FIG. 6. (a) Dependence of the critical temperature of zero (T_c^0), π (T_c^π), and FFLO ($T^{\rm FFLO}$) states on the thickness of the F layer for the S/F/N/F/S pentalayer. Below temperature T^* the π state is energetically more favorable than the zero state. (b) Temperature dependence of Λ^{-1} for the pentalayer with $d_F = 0.4\xi_c$ being in the zero or π state. The arrow indicates the temperature of the 0-π transition. We use the following parameters: $h = 5k_BT_{c0}$, $d_S = 1.4\xi_c$, $d_N = 2\xi_c$.

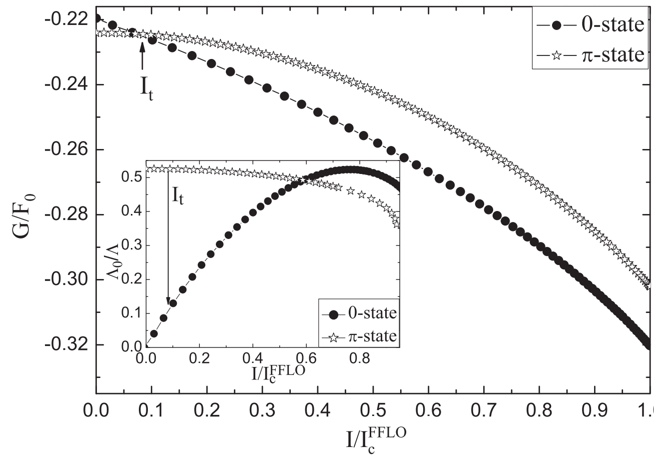

FIG. 7. Current dependence of the Gibbs energy for zero and π states in the S/F/N/F/S pentalayer at temperature $T = 0.2T_{c0}$. Current is expressed in units of the critical current of the FFLO state $I_c^{\rm FFLO}$. The temperature of the 0-π transition $T^* = 0.26T_{c0}$ for the chosen parameters ($h = 5k_BT_{c0}$, $d_S = 1.2\xi_c$, $d_F = 0.4\xi_c$, $d_N = 2\xi_c$). The arrow indicates the transition current I_t when Gibbs energies of π and FFLO states become equal. In the inset we show the dependence of Λ^{-1} on supercurrent in both states.

$\Lambda^{-1}(H)$ like the one shown in Fig. 3(c). In addition, due to the symmetry of the considered pentalayer the parallel magnetic field does not remove the degeneracy of the FFLO state with respect to the sign of q_0 as it does for the trilayer. Despite these differences we find that in the FFLO state and at parameters close to the FFLO phase domain Λ^{-1} increases in the weak magnetic field and decreases in large field, which leads to the maximum in the dependence $\Lambda^{-1}(H)$ like in the trilayer [see Figs. 3(a) and 3(b)]. The only quantitative difference is that in the pentalayer the FFLO state exists in a narrower range of d_F than in the trilayer (with the same parameters) because of its competition with the π state. Further in this section we mainly focus on the temperature- and current-/magnetic-field-driven $\pi \to$ FFLO transition in the symmetric pentalayer.

Figure 6(a) demonstrates the dependence of the critical temperatures of zero (uniform and FFLO) and π (uniform) states on the thickness of the F layers. The temperature dependence of Λ^{-1} for the pentalayer in the FFLO state resembles the dependence the S/F/N structure in the FFLO state (compare with Fig. 3(a) from [8]). In the π state *the same* pentalayer shows a monotonic increase of Λ^{-1} with decreasing temperature, which is typical for a single layer of hybrid S/F or S/F/N structures with no or a negligible contribution of the odd-frequency triplet superconductivity to Λ^{-1}. Similar to the temperature-driven 0-π transition in the S/F/S structures [19,20], such a transition occurs in our pentalayer at $T = T^*$ for some range of d_F. In contrast to the S/F/S structure considered in Ref. [20] in our pentalayer Λ^{-1} increases at the $0 \to \pi$ transition since in the π state there is practically no negative contribution from the triplet component to Λ^{-1}. This difference becomes even more dramatic at the transition from the zero FFLO state to the uniform π state when Λ^{-1} changes from zero to a finite value as temperature decreases [see Fig. 6(b)].

We also find, at fixed temperature $T < T^* < T^{\rm FFLO}$, a current- or magnetic-field-driven transition to the FFLO state. Let us first consider the current-driven transition. In Fig. 7 we show the dependence of the Gibbs energy $G = F_H - (\hbar/2|e|)Iq_0$, which should be used for a current-driven state instead of the Helmholtz free energy [21], on current for the π and FFLO states. We can see that at $I > I_t$ the FFLO state becomes more energetically favorable. Like for the temperature-driven transition, there is a jump in Λ^{-1} (see the inset in Fig. 7) and in q because the transition current $I_t \sim q\Lambda^{-1}$ is the same in both states. The jump in q implies that the $\pi \to$ FFLO transition at $I > I_t$ should be accompanied by the appearance of a transitional electric field, which accelerates the superconducting condensate, and the voltage pulse.

Our calculations show that near T^* the transition occurs at sufficiently small currents $I_t \ll I_c^{\rm FFLO}$, where $I_c^{\rm FFLO}$ is the critical current of the FFLO state (it corresponds to the maximal possible superconducting current flowing along the pentalayer in the FFLO state), which is close to I_c^π of the π state. With decreasing temperature I_t increases but stays smaller than I_c for both FFLO and π states, which makes current-driven transition possible at all temperatures $0 < T < T^*$. Note that there is also a transition from a π to zero uniform state when $T^* > T^{\rm FFLO}$, but it requires larger currents and exists in a narrow temperature interval below T^*.

The parallel magnetic field affects the superconductivity differently in the FFLO and π states, which at temperatures $T < T^* < T^{\rm FFLO}$ can result in the field-driven $\pi \to$ FFLO transition (Fig. 8). Like for the current-driven transition, here we also have a jump in Λ^{-1} (see the inset in Fig. 8), while the dependence $\Lambda^{-1}(H)$ in the FFLO state resembles the

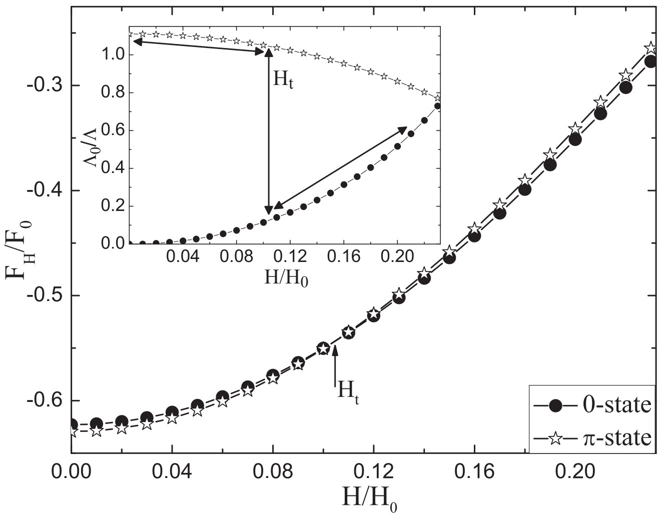

FIG. 8. Dependence of the free energy of the S/F/N/F/S pentalayer on parallel magnetic field H in the FFLO and π states at temperature $T = 0.2T_{c0}$. In the inset we show the dependence of Λ^{-1} on parallel magnetic field H in π and FFLO states. The arrow indicates the field of the $\pi \rightarrow$ FFLO transition. Parameters are the same as in Fig. 6(b).

one for the S/F/N trilayer [compare with Fig. 3(a)]. Because energies of π and FFLO states are rather close, one needs a relatively large magnetic field to make the FFLO state more energetically favorable (see Fig. 8). Unlike the trilayer, in the S/F/N/F/S pentalayer at a certain field H_{c1} vortices can emerge. Using the expression valid for the S film with thickness d_S, $H_{c1} \sim \Phi_0/d_S^2$ [22], and replacing thickness d_S by the total thickness of the pentalayer, we obtain $H_{c1} \simeq 0.2H_0$ for the parameters in Fig. 8. This estimation explains our choice of maximal magnetic field in Fig. 8. To study the effect of vortices one needs the solution of the 3D problem, which is out of the scope of our paper.

V. SUMMARY

We have studied the effect of parallel magnetic field and in-plane current on screening properties of thin S/F/N and S/F/N/F/S structures in or close to the FFLO state. In the parameter region corresponding to the formation of the in-plane FFLO phase, the effective inverse magnetic field penetration depth Λ^{-1} is positive at any finite magnetic field/current, and $\Lambda^{-1} \rightarrow 0$ as $H, I \rightarrow 0$, which implies a diamagnetic response of such structures. Due to the suppression of the triplet superconductivity in F/N layers by magnetic field/current the dependence $\Lambda^{-1}(H)/(I)$ has unusual field/current dependence not only in the FFLO state but also for parameters close to the FFLO phase domain; Λ^{-1} increases in weak fields/currents and reaches a maximal value at finite H/I. We also found that the parallel magnetic field not only controls the screening properties of the FFLO state but can also drive the S/F/N/F/S pentalayer from the uniform π state to the in-plane FFLO state, which is accompanied by a giant change in Λ^{-1}. The same transition can be induced by an in-plane current or by changing the temperature.

Experimentally, the predicted effects could be verified, for example, by the two-coil technique [13,23–25], which allows us to measure Λ^{-1} of thin superconducting structures directly. Potentially found results could be used in the magnetic field sensors (due to the strong magnetic field dependence of Λ^{-1}) or in kinetic inductance detectors of electromagnetic radiation or particles [26] when local heating of the heterostructure due to absorbed energy may considerably change Λ^{-1} (see, for example, Fig. 6), which determines the kinetic inductance of the sample.

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