Phenomenology of chiral Dzyaloshinskii-Moriya interactions in strained materials

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We use phenomenological symmetry arguments to demonstrate that while chiral magnetic (Dzyaloshinskii-Moriya) interactions are conventionally restricted by symmetry to appear in a limited set of noncentrosymmetric materials, they may arise in a material with any symmetry when coupled to a strain field. We derive point-group specific free-energy functionals that capture the relationship between an applied strain field and chiral magnetic couplings, demonstrating how strain may offer highly selective control over magnetic textures. Finally, we discuss several examples of common strain configurations that may lead to out-of-plane modulation in the magnetic moment of a quasi-two-dimensional film as required for applications in magnetic devices.

I. INTRODUCTION

The Dzyaloshinskii-Moriya (DM) interaction between magnetic atoms is of fundamental importance to the properties of many magnetic materials, leading to phenomena such as helimagnetism [1], weak ferromagnetism in antiferromagnetic systems [2], and magnetic skyrmions [3,4]. The DM interaction can be described by an antisymmetric coupling between spins, conventionally expressed as a $D_{r,r'} \cdot [S(r) \times S(r')]$ term in a spin Hamiltonian describing spins located at r and r' interacting through the DM vector D [5], or phenomenologically as an antisymmetric coupling in the magnetic order parameters [1,2]. The DM interaction has been studied extensively in a range of materials, such as MnSi [6], FeGe [7,8], and others [9–11]. However, the antisymmetric form of this interaction restricts which components of spin may appear in the DM terms of the energy [2,5]; notably the chiral Dzyaloshinskii-Moriya (cDM) interaction cannot contribute to the macroscopic properties of a centrosymmetric material. The requirement that the host crystal be noncentrosymmetric greatly restricts the space of materials where the effects of the DM interaction can be studied and utilized.

To expand the space of materials where the DM interaction may arise, it is necessary to break the symmetries of the crystal which forbid antisymmetric magnetic couplings. Recent work on interfacial DM effects [12–16] has relied on symmetry breaking at a surface or in a multilayer to observe DM couplings. However, this approach cannot be generalized outside the space of thin films and generally relies on the precise control of material interfaces. One way to systematically achieve symmetry breaking in the bulk of a material is through the application of a strain field. Furthermore, unlike materials with an intrinsic cDM interaction, strain-coupled cDM effects can be in principle dynamically controlled through

mechanical deformation. In this work, we demonstrate that strain gradients (flexomagnetism), and sometimes even homogeneous strain (piezomagnetism), allow for cDM couplings to arise in materials belonging to any point group. Based on symmetry arguments, we derive possible couplings between magnetic order parameters and strain within the three-dimensional (3D) crystallographic point groups and thus reveal how strain and its gradient can lead to specific types of cDM interactions. Our analysis does not rely on a specific atomistic mechanism leading to a cDM interaction, but rather focuses on whether an effective cDM interaction can generally arise [17]. Finally, we provide examples of two relevant strain fields which can lead to cDM interactions in intrinsically DM-inactive materials. Our analysis expands upon previous reports of strain and geometry-coupled magnetic phenomena [18–24], and provides a general roadmap for controlling chiral magnetic interactions via strain in materials of all symmetries.

II. FORMALISM

To determine which magnetic and strain couplings may contribute to the free energy of a material with a particular point group, we construct a general free-energy density functional in terms of symmetry-invariant polynomials of the magnetic order parameters $\{m^{(\alpha)}, m^{(\beta)}, \ldots\}$, strain e and their gradients [25–31]. The free-energy density functional $\mathcal{F}$ can be written as

$$\mathcal{F}(\mathcal{B}) = \mathcal{F}(\mathcal{B}_0) + \sum_{b_i \in \mathcal{B}} \frac{\partial \mathcal{F}}{\partial b_i} b_i + \frac{1}{2} \sum_{b_i, b_j \in \mathcal{B}} \frac{\partial^2 \mathcal{F}}{\partial b_i \partial b_j} b_i b_j + \cdots,$$

where $\mathcal{B} = \{m^{(\alpha)}, \boldsymbol{\nabla} m^{(\alpha)}, m^{(\beta)}, \boldsymbol{\nabla} m^{(\beta)}, \ldots, e, \boldsymbol{\nabla} e\}$ is a joint field consisting of the individual field variables for the magnetization order parameters, strain, and their gradients. The derivative terms $\partial \mathcal{F}/\partial b_i$, $\partial^2 \mathcal{F}/\partial b_i \partial b_j$, ... are evaluated in the reference state denoted by $\mathcal{B}_0$ and thus must obey the symmetry of the point group of the homogeneous unstrained, nonmagnetized material. To enforce these symmetry constraints, we rewrite the free-energy functional

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in terms of the symmetry-invariant polynomials of b_i,

$$\mathcal{F}(\mathcal{B}) = \sum_n \sum_k J_k^{(n)} V_k^{(n)},$$

where the $V_k^{(n)}$ basis function is the kth symmetry-invariant polynomial of order n consisting of the components of $\mathcal{B}$, and $J_k^{(n)}$ is a scalar coefficient. The basis functions are defined as

$$V_k^{(n)} = \sum_{\sigma \in \Omega(\sigma_k^{(n)})} c_\sigma \prod_{i \in \sigma} b_i,$$

where Ω defines the orbit of $\sigma_k^{(n)}$, the kth symmetrically unique product of order parameters of order n in terms of products of b_i and coefficients c_σ. By this construction, each basis function groups symmetrically equivalent terms and thereby guarantees that $\mathcal{F}$ is invariant under symmetry operations in the point group. The coefficients $J_k^{(n)}$ can be similarly expressed in terms of the nth derivatives of $\mathcal{F}$, but as both $J_k^{(n)}$ and the derivatives of $\mathcal{F}$ typically appear as empirical parameters fit to experimental data or quantum-mechanical calculations, their exact relationship to each other is not important.

To obtain the full basis set $\{V_k^{(n)}\}$ up to order n, we follow the procedure described by Thomas and Van der Ven [30,32]. We first derive a symmetry-adapted set of field variables for $\mathcal{B}$, as described in the Supplemental Material, Note 1 [33], which separates the components of $\mathcal{B}$ into independent subspaces whose interactions with each other can be efficiently evaluated. We then project out the symmetrically invariant component of every nth-order monomial of $\mathcal{B}$ using the Reynolds operator $\mathcal{R}^{(P)} = \sum_{\hat{p}_i \in P} U^{\mathcal{B},\hat{p}_i}/|P|$ for the desired point group P which contains $|P|$ symmetry operations $\hat{p}_i$, where the operator $U^{\mathcal{B},\hat{p}_i}$ describes the action of $\hat{p}_i$ on $\mathcal{B}$. Finally, we use Gram-Schmidt orthogonalization to construct an orthonormal basis set of symmetry-invariant polynomial basis functions. Note that since any magnetic order parameter must obey time-reversal symmetry, the symmetry group must include time reversal. We restrict our analysis to systems which may undergo a second-order transition to a paramagnetic state [2,34] and work with the "grey" point groups, defined as the direct product of the crystallographic symmetry operators and the time-reversal operator.

Within a free-energy functional $\mathcal{F}(m^{(\alpha)}, \nabla m^{(\alpha)}, m^{(\beta)}, \nabla m^{(\beta)}, \dots)$, the DM interaction appears to first order in two forms: the inhomogeneous or chiral DM (cDM) interaction and the homogeneous DM (hDM) interaction [1,2,35]. The cDM interaction is an antisymmetric coupling between a magnetic order parameter $m^{(\alpha)}$ and its gradient, yielding terms in the form of a Lifshitz invariant $w_{kn}^{(\alpha)} = \epsilon_{ijk} m_i^{(\alpha)} \partial m_j^{(\alpha)}/\partial r_n$, where ϵ_{ijk} is the permutation tensor and summation over repeated indices is implied. These terms impart a chirality dependence to the free energy based on a rotation of $m^{(\alpha)}$ about the r_k direction propagating along the r_n axis of the crystal. One may refer to w_{kk} terms as "helicoid" interactions and w_{kn}, $n \neq k$ terms as "cycloid" interactions [35], which represent magnetic textures similar to those found in Bloch and Néel domain walls respectively as shown schematically in Fig. 1(a). The hDM interaction is an antisymmetric coupling between two different magnetic order parameters $m^{(\alpha)}$ and $m^{(\beta)}$, yielding terms of the form $h_k^{(\alpha\beta)} = \epsilon_{ijk} m_i^{(\alpha)} m_j^{(\beta)}$. A notable example of this interaction is the emergence of "weak

ferromagnetism" due to spin canting in antiferromagnetic systems in the case where $m^{(\alpha)}$ is the total magnetization and $m^{(\beta)} = s_1 - s_2$ is the difference in the magnetic moment vectors of the antiferromagnetic sublattices s_1 and s_2 [2].

The magnetic order parameters $m^{(\alpha)}, m^{(\beta)}, \dots$ are generally defined as $m^{(\alpha)} = \sum c_i^{(\alpha)} s_i$ where $\{s_i\}$ are the local magnetic moments of the sublattices in the material and the coefficients c_i are chosen such that the order parameters are orthogonal and span irreducible subspaces of the point group of the material [2,31,36]. In the special case of total magnetization m ($c_i = 1$), permutation of $\{s_i\}$ leaves m invariant, meaning that total magnetization transforms as an axial vector, with $U_{ij}^{m,\hat{p}} = (\det \boldsymbol{R})\boldsymbol{R}_{ij}$, where $\boldsymbol{R}$ is the Cartesian rotation matrix associated with symmetry operation $\hat{p}$. Correspondingly, the gradient of total magnetization ∇m transforms as a rank-2 pseudotensor with $U_{(i'j')(ij)}^{\nabla m,\hat{p}} = (\det \boldsymbol{R})\boldsymbol{R}_{i'i}\boldsymbol{R}_{j'j}$, where (ij) represents the $m_{i,j} = \partial m_i/\partial r_j$ component of ∇m.

In this work, we consider the simplest case of a DM interaction: the emergence of cDM couplings $w_{kn} = \epsilon_{ijk} m_i \partial m_j/\partial r_n$ in the total magnetization m, where we drop the superscripts (α) for the sake of clarity. In this case, the coefficients multiplying the w_{kn} terms are the components $\boldsymbol{D}_{kn}$ of the Dzyaloshinskii tensor, and the various couplings between strain and w_{kn} yield the possible contributions of strain to this micromagnetic term. This situation where m is a good order parameter is relevant to materials which are low-temperature ferromagnets or helimagnets and for practical applications in magnetic materials and skyrmionics [37–41]. More generally, this case provides an informative example of how strain and strain gradients can give rise to antisymmetric couplings in high-symmetry materials. Because total magnetization is a universal magnetic order parameter whose symmetry behavior depends on only the point group of a material, these couplings provide a sufficient condition for the possibility of strain-coupled cDM based on point-group symmetry alone.

The free energy associated with the reversible mechanical deformation of a solid defined by the vector field u is characterized by strains and strain gradients. Strain is a measure of the symmetric component of the first derivative of u, parametrized by $E_{ab} = 1/\sqrt{|P(ab)|} \sum_{ij=P(ab)} \partial u_i/\partial r_j$ for $a \leqslant b$ where $P(ab)$ is the set of unique permutations of $\{a, b\}$. We choose to normalize the strain terms using the Frobenius norm, equivalent to conventional Kelvin notation [30]. Strain gradient can then be defined in terms of the Cartesian derivatives of E, with $E_{ab,k} = \partial E_{ab}/\partial r_k$. While alternative formulations of strain gradient exist, they are formally equivalent to $E_{ab,k}$ [27,42] and can be related to each other using identities given in the Supplemental Material, Note 1 [33]. The symmetry operators for strain and strain gradient are similar to those of second- and third-rank tensors:

$$U_{(a'b')(ab)}^{E,\hat{p}} = \sqrt{\frac{|P(ab)|}{|P(a'b')|}} \sum_{i'j'=P(a'b')} \boldsymbol{R}_{i'a}\boldsymbol{R}_{j'b},$$

$$U_{(a'b',k')(ab,k)}^{\nabla E,\hat{p}} = \sqrt{\frac{|P(ab)|}{|P(a'b')|}} \sum_{i'j'=P(a'b')} \boldsymbol{R}_{i'a}\boldsymbol{R}_{j'b}\boldsymbol{R}_{k'k},$$
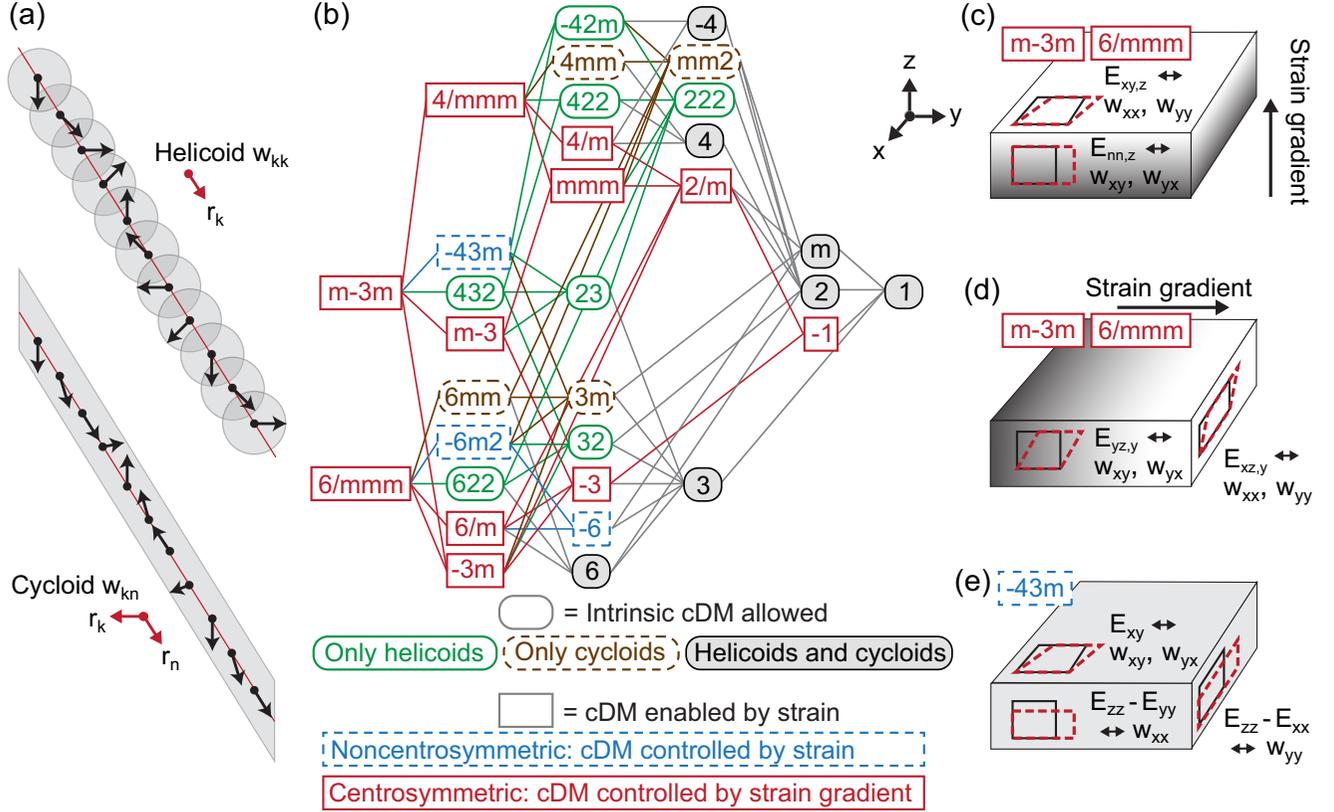
FIG. 1. (a) Helicoid and cycloid spin textures comprising the two variants of cDM interactions in total magnetization m. (b) A map of cDM interactions in materials across all 32 point groups. Point groups where cDM interactions are intrinsically allowed are denoted by ovals and color-coded by whether symmetry allows for helicoidal (green) or cycloidal (brown) cDM terms in the free energy, or both (black). Point groups where cDM interactions are enabled by a strain field are denoted by rectangles and color-coded by whether cDM interactions couple to homogeneous strain (blue) or only strain gradients (red). The connections between point groups indicate group-subgroup relationships and illustrate which point groups can be reached by symmetry-breaking operations. (c,d) The couplings between elastic strains and strain gradients and cDM interactions which yield a modulation in m_z (the out-of-plane component of total magnetization) of a $m\bar{3}m$ or $6/mmm$ material, for out-of-plane and in-plane gradients in strain respectively. (e) Couplings between homogeneous strain and cDM interactions yielding a modulation in m_z for a $\bar{4}3m$ material.

III. RESULTS AND DISCUSSION

We now proceed to evaluate the form of the cDM interaction in m in the free-energy density across all point groups. This portion of the energy functional, $\mathcal{F}_{cDM}$, is an addition to the traditional terms appearing in the total magnetic free-energy functional $\mathcal{F}$ [4]—at minimum, direct exchange $A \sum_{i,j} (\partial m_i / \partial r_j)^2$ for some exchange constant A, and magnetocrystalline anisotropy $\mathcal{F}_{ani}$, which is symmetry dependent and enumerated for all points groups in the Supplemental Material, Note 3 [33]. For the cDM interaction, while we focus on a limited number of examples in this discussion, complete tables of symmetry-invariant basis functions governing the cDM and strain-coupled cDM interactions are available in the Supplemental Material, Notes 4 and 5 respectively [33]. Note that while the basis sets given in the Supplemental

Material are equivalent, those presented in the main text are reformatted for clarity.

In most noncentrosymmetric point groups, $\mathcal{F}_{cDM}$ can be nonzero without any additional symmetry breaking from strain. For example in the 432 and 23 point groups, the allowed cDM interaction consists of helicoids described by a single term, $\mathcal{F}_{cDM}^{432} = J^{cDM}(w_{11} + w_{22} + w_{33})$ for some scalar parameter J^{cDM}, equivalent to the previously proposed expression for this point group, $J^{cDM}(m \cdot \nabla \times m)$ [43]. The cDM interaction is similarly limited to helicoids in 622, 32, $\bar{4}2m$, 422, and 222. In 6mm, 3m, 4mm, and mm2 the allowed cDM interactions are of purely cycloid type, and finally, in 6, 3, $\bar{4}$, 4, m, and 2 both interaction types are possible. In these materials, although the behavior of the cDM interaction is generally determined by the structure of the material itself, strain couplings provide a route to introducing anisotropies in the Dzyaloshinskii tensor which have been recently discussed as a route towards controlling emergent phase behavior, such as skyrmion vs antiskyrmion formation [44]. We refer the reader to the Supplemental Material, Note 5 [33], for a complete list of these couplings.

TABLE I. Basis functions coupling cDM interactions to strain gradients within the $m\bar{3}m$ point group.

$$V_1^c = \sqrt{3}/3\,\epsilon_{ijk}w_{ij}E_{kk,k}$$
$$V_2^c = \sqrt{6}/6\,\epsilon_{ijk}w_{ij}(E_{ii,k} + E_{jj,k})$$
$$V_3^c = \sqrt{6}/6\,\epsilon_{ijk}w_{ij}(E_{ki,i} + E_{kj,j})$$
$$V_4^c = \sqrt{6}/6\,\epsilon_{ijk}(w_{ij} + w_{ji})E_{ii,k}$$
$$V_5^c = \sqrt{6}/6\,\epsilon_{ijk}(w_{ij} + w_{ji})E_{ki,i}$$
$$V_6^c = \sqrt{3}/3\,\epsilon_{ijk}w_{ii}E_{ij,k}$$

In centrosymmetric point groups such as $m\bar{3}m = 432 \otimes i$, the presence of inversion symmetry forbids the cDM interaction from arising in the absence of a secondary source of symmetry breaking. A strain field may lift this restriction, offering precise control over the character of the cDM interaction through strain engineering. To understand how strain fields couple to magnetic interactions, it is informative to examine the free-energy functionals of the highest symmetry $m\bar{3}m$ and $6/mmm$ point groups as all other point groups are subgroups of $m\bar{3}m$ or $6/mmm$ and thus allow any couplings seen here. Generating the basis set of symmetry-invariant polynomials of $\{m, \nabla m, e, \nabla e\}$, we find that while homogeneous strain does not break inversion symmetry, strain gradients give rise to cDM terms in the free-energy functional allowing for $\mathcal{F}_{\text{cDM}}$ to be nonzero under all symmetries. The cDM component of the free-energy density has the form $\mathcal{F}_{\text{cDM}} = \sum_i J_i^{\text{cDM}} V_i$ where J_i^{cDM} are scalar parameters. For the $m\bar{3}m$ and $6/mmm$ point groups, abbreviated as c and h, the basis functions V_i are defined in Tables I and II respectively. In both cases, any gradient of axial strain and gradients of shear strain in the plane of shear ($V_{1...5}^c$, $V_{1...11}^h$) give rise to cycloid terms. Gradients of shear strain orthogonal to the shear plane

TABLE II. Basis functions coupling cDM interactions to strain gradients within the $6/mmm$ point group, where we assign the sixfold axis to be oriented along r_3.

$$V_1^h = \epsilon_{ij3}w_{i3}E_{j3,3}$$
$$V_2^h = \epsilon_{ij3}w_{3i}E_{j3,3}$$
$$V_3^h = \epsilon_{ij3}w_{i3}E_{33,j}$$
$$V_4^h = \epsilon_{ij3}w_{3i}E_{33,j}$$
$$V_5^h = \sqrt{3}/3\,\epsilon_{ij3}w_{i3}(\sqrt{2}E_{jj,j} + E_{ij,i})$$
$$V_6^h = \sqrt{3}/3\,\epsilon_{ij3}w_{3i}(\sqrt{2}E_{jj,j} + E_{ij,i})$$
$$V_7^h = \sqrt{3}/6\,\epsilon_{ij3}w_{i3}(E_{jj,j} + 3E_{ii,j} - \sqrt{2}E_{ij,i})$$
$$V_8^h = \sqrt{3}/6\,\epsilon_{ij3}w_{3i}(E_{jj,j} + 3E_{ii,j} - \sqrt{2}E_{ij,i})$$
$$V_9^h = (w_{12} - w_{21})E_{33,3}$$
$$V_{10}^h = \sqrt{2}/2\,(w_{12} - w_{21})(E_{22,3} + E_{11,3})$$
$$V_{11}^h = \sqrt{2}/2\,(w_{12} - w_{21})(E_{23,2} + E_{13,1})$$
$$V_{12}^h = \sqrt{2}/2\,(w_{11} + w_{22})(E_{23,1} - E_{13,2})$$
$$V_{13}^h = w_{33}(E_{23,1} - E_{13,2})$$
$$V_{14}^h = \sqrt{2}/2\,(w_{11} - w_{22})E_{12,3} + 1/2(w_{12} + w_{21})(E_{22,3} - E_{11,3})$$
$$V_{15}^h = 1/2(w_{11} - w_{22})(E_{23,1} + E_{13,2}) + 1/2(w_{12} + w_{21})(E_{23,2} - E_{13,1})$$

(V_6^c, $V_{12...15}^h$) lead to helicoid terms. As inversion symmetry is lifted by any strain gradient, these remaining restrictions on the cDM interactions arise from the rotation and rotoinversion axes left unperturbed by the strain field.

While strain gradients are necessary for cDM interactions in centrosymmetric materials, homogeneous strain can still be used to activate and control these magnetic couplings in materials belonging to the $\bar{4}3m$, $\bar{6}m2$, and $\bar{6}$ point groups. These point groups are not centrosymmetric, but the combination of rotation and rotoinversion axes nonetheless forbids the cDM interaction without external symmetry breaking. Thus, since it is not necessary to break inversion symmetry in these point groups, the cDM interaction may arise due to strain alone. For example, $\mathcal{F}_{\text{cDM}}^{\bar{4}3m}$ contains the following two pure-strain terms:

$$V_1^{\bar{4}3m} = \sum_k \epsilon_{ijk}w_{ij}E_{ij},$$
$$V_2^{\bar{4}3m} = \sum_k \epsilon_{ijk}w_{ii}E_{jj},$$

in addition to the six V^c polynomials which depend on strain gradients. Here, shear strains break rotational symmetry and lead to cycloid interactions in the plane of shear, while anisotropic biaxial strains can disrupt mirror planes and thus lead to helicoid terms propagating normal to the plane of strain. In the hexagonal $\bar{6}m2$ and $\bar{6}$ point groups, anisotropic biaxial strain in the basal plane $E_{11} - E_{22}$ and any shear which lifts a rotational symmetry lead to cycloids, while E_{13} or E_{23} shear which breaks the basal plane mirror symmetry can lead to helicoids. This situation where homogeneous strain determines the form of the cDM interaction is of particular interest as homogeneous strain is likely to be easier to control experimentally than a strain gradient. Furthermore, this coupling is likely to readily yield a large response by analogy to the significant effect of strain on magnetic textures in chiral helimagnets [45].

The implication of these couplings is that high-symmetry materials currently considered to be cDM inactive may indeed exhibit cDM interactions if strained in a particular way. Figure 1(b) provides a summary of the cDM interactions which arise intrinsically in each point group, or whether the magnetism is piezomagnetically or flexomagnetically controlled for point groups where symmetry breaking is necessary. The group-subgroup relationships indicated by connections between higher and lower-symmetry point groups illustrate which symmetries need to be lifted to allow a desired type of chiral magnetic behavior.

As an example of these couplings, consider a quasi-2D film with a magnetic configuration that exhibits some modulation in the out-of-plane component of magnetization. Defining the film to lie in the xy plane, the magnetic configurations giving modulations in m_z are the helicoid interactions w_{xx} and w_{yy} and cycloid interactions w_{yx} and w_{xy}. The impact of strain gradients on these interactions can be summarized by two cases: the strain gradient lies in the out-of-plane z direction, or the in-plane y direction. The first case may arise in an epitaxial thin film due to a mismatch in lattice constant, modulus, or thermal-expansion coefficient with the substrate, where the strain decays exponentially over a characteristic length δ ($E_{ij,z} \propto E_{ij} \propto e^{-z/\delta}$) [46–48]. The second case is

representative of a biphasic material with a composition fluctuation along the y direction and an elastic strain linear with composition order parameter ν [$E_{ij,y} \propto \nu \cos(y/\delta)$] [49]. For a centrosymmetric material belonging to $m\bar{3}m$ or $6/mmm$, helicoid interactions require a gradient in shear strain orthogonal to the plane of shear. Thus, w_{xx} and w_{yy} couple to $E_{xy,z}$ in the first case and $E_{xz,y}$ in the second case. Cycloid interactions w_{xy} and w_{yx} couple to in-plane gradients of axial strains and shear gradients lying in the plane of shear: $E_{xx,z}$, $E_{yy,z}$, and $E_{zz,z}$ in the first case and $E_{yz,y}$ in the second. For a noncentrosymmetric material belonging to the $\bar{4}3m$ point group, helicoid interactions may additionally form due to a biaxial strain, essentially scaling with Poisson's ratio, while cycloid interactions may arise due to in-plane shear. These couplings are summarized in Figs. 1(c) and 1(d) for $m\bar{3}m$ and $6/mmm$, and Fig. 1(e) for $\bar{4}3m$.

While the above examples demonstrate that strain and strain gradients may in principle give rise to cDM interactions, an outstanding question is the strength of this coupling. In the case of a coupling to homogeneous strain, there is direct experimental evidence that the magnitude of the induced interaction can be significant. It has been reported that small applied strains ($\approx 0.3\%$) can distort skyrmion lattices by up to 20% in FeGe, a well-known chiral helimagnet [45]. The period of these magnetic modulations is roughly proportional to A/D, the ratio of micromagnetic exchange strength to cDM, implying that the effect of strain can be significant even on the scale of intrinsic cDM. Much fewer data are available regarding the strength of potential couplings between cDM and strain gradients. Beck and Fähnle report that in epitaxial Fe on W, the gradient in misfit strain yields a cDM interaction approximately one to two orders of magnitude weaker than the cDM induced by the Fe/W interface [50]. However, the Fe/W interface yields a very strong cDM interaction ($A/D_{\text{interface}} \approx$ 8 nm) [51], meaning that $A/D_{\text{strain-gradient}} \approx 400$ nm, which is within an order of magnitude of the strength of intrinsic cDM in bulk chiral helimagnets such as FeGe [52]. Thus, we anticipate that in certain materials, strain-gradient induced cDM may yield significant changes in magnetic structure, especially if no other antisymmetric magnetic interactions are present.

IV. CONCLUSION

In this work, we have used phenomenological arguments to demonstrate that the chiral Dzyaloshinskii-Moriya interaction in the total magnetization may arise in materials belonging to any point group as a result of even a relatively simple strain field. We have (1) argued that gradients in strain lift inversion symmetry allowing antisymmetric couplings to form in otherwise centrosymmetric materials, and (2) we have identified three noncentrosymmetric point groups where homogeneous strain alone can control chiral magnetic interactions. Our results also give a complete representation of the strain dependence of the Dzyaloshinskii tensor for all point groups, which illustrates how strain may be used to induce anisotropic cDM interactions. To illustrate how these couplings may arise in real materials, we presented two practically relevant examples of strained materials where, despite the high symmetry of the underlying crystal, the strain field can give rise to relevant cDM interactions. Our discussion highlights how chiral magnetism may be controlled through mechanical forces, especially in situations when large strains and strain gradients can be expected, such as in piezoelectrics, polycrystals, polyphasic materials, nanomaterials, and electrochemical systems [53–59]. We hope this will inspire targeted characterization work via phase-field simulations and mechanomagnetic experiments.

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