


Temperature dependence of the magnetoresistance peak in highly disordered superconductors

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Highly disordered superconductors have a rich phase diagram. At a moderate magnetic field (B) the samples go through the superconductor-insulator quantum phase transition. In the insulating phase, the resistance increases sharply with B up to a magnetoresistance peak beyond which the resistance drops with B . In this paper we follow the temperature (T) evolution of this magnetoresistance peak. We show that as T is reduced, the peak appears at lower B 's approaching the critical field of the superconductor-insulator transition. Due to experimental limitations we are unable to determine whether the $T = 0$ limiting position of the peak matches that of the critical field or is at comparable but slightly higher B . We show that, although the peak appears at different B values, its resistance follows an activated T dependence over a large T range with a prefactor that is very similar to the quantum of resistance for Cooper pairs.

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I. INTRODUCTION

Highly disordered superconductors undergo a superconductor-insulator quantum phase transition (SIT) [1–3] driven by experimentally tunable parameters such as B [4–6], disorder strength [7], carrier density [8], or sample thickness [1,9]. The B driven insulating phase, which emerges above a critical B (B_c), exhibits a pronounced peak in the magnetoresistance (MR) [10]. $R(T)$ measured at relatively high T 's in the MR peak has a characteristic activation T similar to the $B = 0$ superconducting T_c [11]. Therefore the MR peak is typically associated with a state where Cooper pairs persist above B_c but become spatially localized. This view of a Cooper-pair insulator is supported by several theoretical and experimental studies [12–18] and some works consider the MR peak itself as the transition point (or crossover) between a bosonic insulator closely above B_c and a fermionic insulator at high B 's [10,19,20].

The B driven insulating phase in our system can be separated into two distinguishable regions that show different B and T dependences. To distinguish between these regimes we first write the $R(T)$ dependence of our insulator as

$$R(T) = R_0 \exp(T_0/T)^\gamma, \quad (1)$$

where R_0 is a constant, $k_B T_0$ is the energy characterizing the conduction process, and $\gamma \in [0, 1]$ (when $\gamma = 1$ the conduction is termed activated). In our system, at B 's slightly above B_c , R increases with increasing B and R has either an activated T dependence (for $1 \text{ K} > T > 200 \text{ mK}$) or a novel T dependence where R seems to diverge at a finite T (T^*) [21] [the $R(T)$ follows a phenomenological fit similar to equation (1) but with $T \rightarrow (T - T^*)$]. The second transport regime appears at high B 's (typically above 6 T) where R has a subactivated T dependence ($\gamma < 1$) and, at a constant T , R decreases with increasing B . The transition between these two regimes occurs in the vicinity of the MR peak.

In this work we provide a systematic investigation of the T dependence of the MR peak. We show that, at low T 's, the B where the MR peak appears (B_{peak}) decreases rapidly. Extrapolating our lower- T B_{peak} raises the possibility that in the limit $T \rightarrow 0$, $B_{\text{peak}} \rightarrow B_c$ [22]. Furthermore, studying the R values at the MR peak we show that, although measured at different B 's, $R(B_{\text{peak}})$ has an activated T dependence over a wide T range. The prefactor of this activation is near the quantum of resistance for Cooper pairs.

II. EXPERIMENTAL RESULTS

For this study we used data obtained from 23 different thin films of highly disordered amorphous indium oxide (α -InO). The samples were deposited by e-gun evaporation of high purity In_2O_3 pellets (at a typical rate of $1.5 \text{ \AA}/\text{sec}$) onto a Si/SiO₂ substrate in an oxygen rich environment (typically 1.5×10^{-5} Torr). Most samples were Hall-bar shaped and the thickness of samples, measured while evaporating using a crystal monitor, ranged between 28 and 40 nm. The contacts of samples were either Ti/Au contacts prepared via optical lithography prior to the In_2O_3 evaporation or pressed indium.

In Fig. 1(a) we follow the T dependence of the insulating peak by displaying R vs B of sample AD8a1mm at $T \in (0.06, 1.2) \text{ K}$. The continuous lines were measured by 4 probe ($R < 100 \text{ K}\Omega$ data, excitation current of 0.1 nA) and 2 probe ($R > 100 \text{ K}\Omega$, excitation voltage of $10 \mu\text{V}$) techniques. The black triangles connected via dashed lines mark R extracted from 2 probe dc current-voltage characteristics. The red circles mark the MR peak of each isotherm (see Supplemental Material [23] Sec. S2 for details of our peak detection algorithm). The difference between full and empty circles will be explained in a following section. Upon decreasing T , B_{peak} increases slightly down to 0.6 K and then decreases rapidly.

This drop in B_{peak} is puzzling because, as mentioned above, the MR peak is associated with the process of termination of localized Cooper pairs [10,19,20]. As the superconducting order parameter, which is nonzero locally in a Cooper-pair

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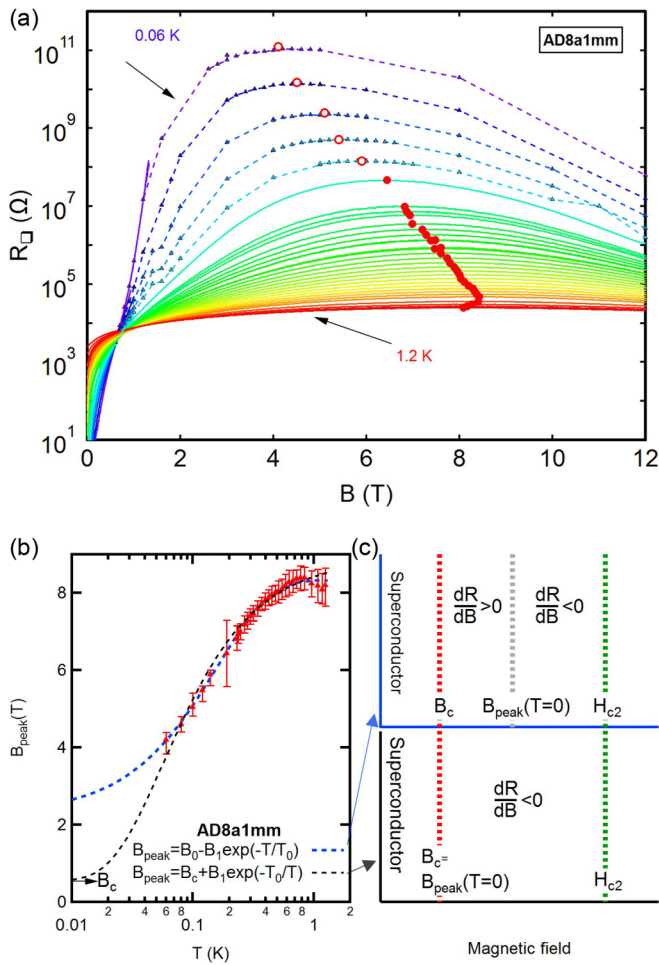


FIG. 1. T evolution of the MR peak. (a) R (log scale) vs B of sample AD8a1mm. The color coding stands for different isotherms ranging from 60 mK (purple) to 1.2 K (red). Full lines correspond to data extracted via 2 and 4 terminal lock-in measurements. Triangles connected via dashed lines correspond to data extracted from the DC I - V characteristics. The red circles mark the MR peak at each T . (b) B_{peak} vs T (semilog). The red triangles are B_{peak} extracted from the data of (a). The dashed lines are fits to different phenomenological functional forms; the main difference between these forms is that according to the blue line $B_{\text{peak}}^0 = 2.16T > B_c$ and according to the black line $B_{\text{peak}}^0 = B_c$. (c) Two possible $T \rightarrow 0$ phase diagrams. The top, blue, (bottom, black,) diagram corresponds to the blue (black) fit in (b). The main difference between the phase diagrams is that the bottom diagram predicts that $\frac{dR}{dB}(T \rightarrow 0) < 0$ throughout the insulating phase.

insulator, diminishes with increasing T one would expect that at low T 's local superconductivity will withstand higher B 's therefore one would expect B_{peak} to increase at low T 's and approach $H_{c2} \sim 14$ T [17] as $T \rightarrow 0$. Our results show otherwise; B_{peak} decreases as $T \rightarrow 0$.

In Fig. 1(b) we display B_{peak} vs T . The error bars mark our uncertainty interval of B_{peak} (in Sec. S2 of the Supplemental Material [23] we elaborate on our error determination procedure). In order to study the ground state of the system and the $T = 0$ quantum phase transition it is important to know what is the $T = 0$ limit of the peak, $B_{\text{peak}}^0 \equiv B_{\text{peak}}(T \rightarrow 0)$.

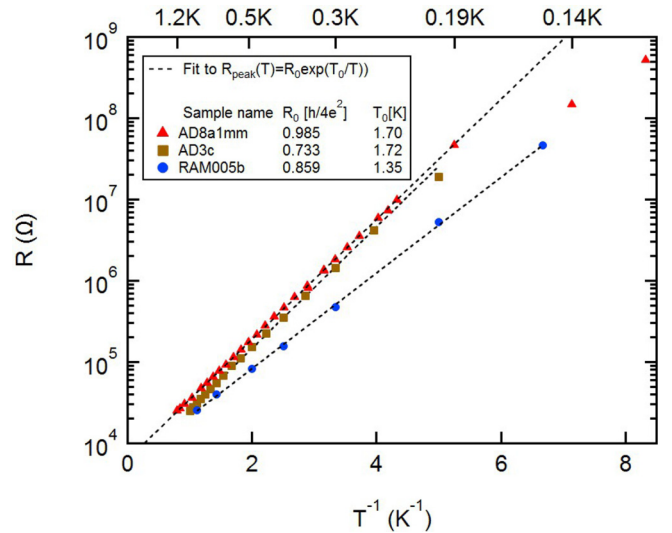


FIG. 2. T activation of MR peak. R_{peak} (log scale) vs T^{-1} of three samples where the red triangles were extracted from the data of Fig. 1(a). The dashed black lines are activated fits to $R_{\text{peak}}(T) = R_0 \exp(T_0/T)$. For sample AD8a1mm we get $R_0 = 0.985 \frac{h}{4e^2}$ and $T_0 = 1.7$ K where the fit holds for $T \in (0.19, 1.2)$ K below which R_{peak} is subactivated.

Because B_{peak}^0 in highly disordered samples requires measuring diverging R 's it is not experimentally measurable. In the absence of a relevant theory we have to fit the B_{peak}^0 data to a phenomenological form and extrapolate it to $T = 0$. The dashed lines in Fig. 1(b) are two such fits (the functional forms are stated in the figure). From the first fit (blue line) we get $B_{\text{peak}}^0 = 2.16T > B_c(T = 0) = 0.55T$ according to which even as $T \rightarrow 0$ there still is a MR peak at $B > B_c$. The dashed black line is an activated fit where we placed the constraint $B_{\text{peak}}^0 = B_c$. In Sec. S3 of the Supplemental Material [23] we perform a statistical analysis and show that both fits are plausible, therefore we cannot rule out the possibility that $B_{\text{peak}}^0 = B_c$. We stress that other functional forms might also describe our observation and the determination of the true $B_{\text{peak}}(T)$ dependence awaits a theoretical model. We note that B_c can also have a weak T dependence; more on this in the discussion section.

In Fig. 1(c) we sketch two possible $T = 0$ phase diagrams, the top (bottom) phase diagram corresponds to the blue (black) fit in Fig. 1(b). In the discussion section we present a third possible phase diagram that arises due to the phenomenon of the finite- T insulator [21].

It is interesting to consider the $R(T)$ dependence at the MR peak (R_{peak}) itself. In Fig. 2 we display R_{peak} (log scale) vs T^{-1} of three different samples. The dashed black lines are fits to equation (1) with $\gamma = 1$. Although at every T R_{peak} is measured at a different B [see Figs. 1(a) and 1(b)], it can be seen that overall it has an activated behavior between 0.19 and 1.2 K (for sample AD8a1mm). In Sec. S1 of the Supplemental Material [23] we compare between the activated behavior of $R(B_{\text{peak}})$ and $R(B)$ for all constant B 's and show that the fit quality of the peak is better than that of any other constant B . At $T < 0.19$ K R_{peak} deviates from the activated behavior [these points are marked in Fig. 1(a) by empty circles]. We

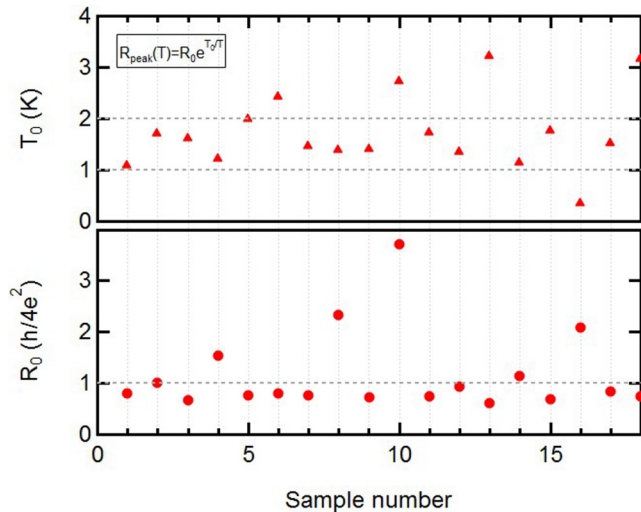


FIG. 3. T_0 and R_0 for various samples. In 18 out of 23 samples we studied, the $R_{\text{peak}}(T)$ fits an activated behavior. Displayed here are T_0 (top) and R_0 (bottom), extracted from the activated fit for these 18 samples. For most samples $T_0 \in (1, 2)$ K, which is the typical range of the superconducting T_c , and R_0 is close to $\frac{h}{4e^2}$.

discuss this low- T deviation in the discussion section. In Sec. S4 of the Supplemental Material [23] we plot R_{peak} vs T^{-1} for 18 different samples that showed an activated peak.

In activated transport the parameters T_0 and R_0 reflect the microscopic state of the conduction process. The fact that we can describe the T evolution of R_{peak} with single T_0 and R_0 indicates that the peak results from a unique microscopic state that evolves through different B 's. The values of the fit parameters, extracted from Fig. 2, hold some information regarding the nature of this maximal R state. We note that the prefactor R_0 is very close to the quantum of resistance for Cooper pairs ($R_Q = \frac{h}{4e^2}$) and the activation T , T_0 , is close to the superconducting T_c at $B = 0$ [1.5 K for sample AD8a1mm, typically in our samples $T_c \in (1, 2)$ K]. In Fig. 3 we display R_0/R_Q (circles, bottom graph) and T_0 (triangles, top graph) for 18 different samples we studied (see Supplemental Material Sec. S3 [23] for a table that includes the sample names and fit parameters values). It can be seen that for most samples $R_0/R_Q \in (0.6-1.15)$ and $T_0 \in (1, 2)$ K. It is important to mention that, as discussed in Ref. [11], T_0 and the superconducting T_c are of the same order but show opposite dependencies on disorder strength. This is demonstrated in Fig. S4(b) of the Supplemental Material [23], where we display T_0 vs B_c of each sample and show that T_0 decreases with B_c (due to instrumental limitations we could not plot T_0 vs T_c [24]).

We would like to stress that the data shown were extracted from 18 samples where the MR peak showed a good activation fit. The samples are very diverse; out of the 18 samples four were insulating at $B = 0$ [25] (these samples still had a T -activated MR peak), and 14 were superconducting with $B_c \in (0.1-4.3)$ T (in Table II of the Supplemental Material [23] we write B_c of the superconducting samples and we mention which samples are insulating at $B = 0$). The area of the different samples varied from $50 \mu\text{m}^2$ to 2mm^2 . We also studied five additional samples where the peak did not show a good activation fit. In these samples R_{peak} had a

reasonable variable range hopping behavior (see discussion section).

III. DISCUSSION

In Figs. 1(b) and 1(c) we introduced two phenomenological fits that correspond to two possible $T = 0$ phase diagrams. The main difference between these two fits is whether the $T \rightarrow 0$ limit of B_{peak} is B_c or some other $B > B_c$. We would like to note that, as already discussed above, in the insulating phase close to the SIT, R seems to diverge at a finite T [21], T^* , which depends on B , $T^*(B)$. As T^* is finite over a continuous B range, at nonzero T 's below the maximal T^* R will be infinite over a continuous B range (assuming no other transport mechanism takes over at low T 's) and B_{peak} will not be single valued. This picture will correspond to a third phase diagram where B_{peak}^0 spans over a B range above B_c .

Nonactivated behavior of R_{peak} . Although the activation behavior of the peak was observed in the majority of samples we examined, there were also five samples (out of 23) where R at the MR peak was subactivated with a reasonable variable range hopping fit. Four out of these five samples are in the high disorder limit and one in the opposite, low-disorder, limit. We did not manage to pinpoint a criterion that makes these samples different. In addition, in some of the samples that showed an activated MR peak, there were deviations from activation at sufficiently low T 's. One possible explanation can be that at low T 's, in the highly disordered samples, variable range hopping becomes beneficial and therefore dominates the low T transport. In Sec. S5 of the Supplemental Material [23] we show that the main observations of this paper can be reproduced from a single model of two parallel transport channels where one is activated and the other is subactivated. One built-in result of this model is that at sufficiently low T 's variable range hopping will become beneficial and R_{peak} will become subactivated [see inset of Fig. S6(d)].

The MR peak in other systems. A MR peak is not unique to highly disordered superconductors and was also observed in various systems such as high- T_c superconductors [26–28], Josephson junction arrays [29], granular superconductors [30], quantum hall [31], and complex oxide interfaces [32]. In some high T_c superconductors a MR peak appears only below certain T 's, in addition and in contrast to our findings, in both ρ_{ab} and ρ_c , B_{peak} increases while cooling [26–28]. Josephson junction arrays exhibit multiple MR peaks (and multiple SITs) [29,33] separated by minimas at $f = n/m$ where f is the magnetic flux per cell in units of the magnetic flux quantum ($\Phi_0 = h/2e$) and n, m are integers. It seems that in these arrays B_{peak} has no significant T dependence. A further comparison to other systems is beyond the scope of this work.

T dependence of B_c . In Fig. 4 we replot the data of Fig. 1(a) focusing on the vicinity of the crossing of different isotherms. Clearly there is no single B_c where all isotherms coincide. Instead there are multiple crossing points and the crossing B between successive isotherms (B_x) has a clear T dependence. In the inset of Fig. 4 we plot B_x vs T for the same three samples displayed in Fig. 2 (see Fig. S5(c) of the Supplemental Material [23] for $B_x(T)$ of additional samples).

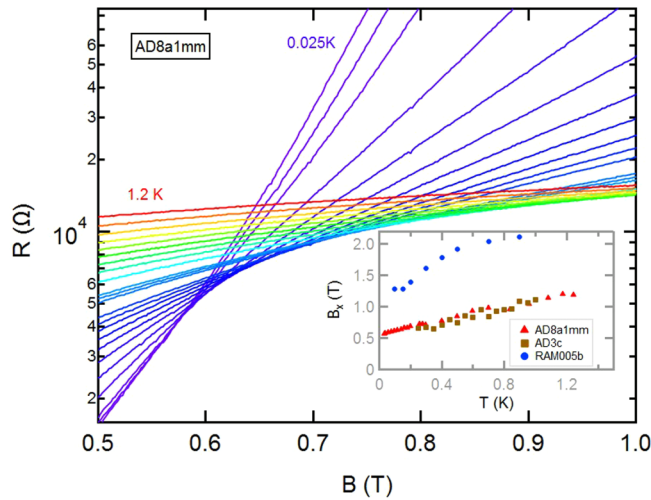


FIG. 4. T dependence of B_c . R (log scale) vs B of sample AD8a1mm where we plot the same data of Fig. 1(a) focusing on the crossing point. The color coding marks different isotherms. It can be seen that the crossing between two consecutive isotherms varies with T . Inset: B_x vs T for three different samples where the red triangles correspond to the main figure (AD8a1mm) and the blue and brown data are of the same samples as in Fig. 2.

A T dependence of B_x is an experimental result that was observed in several systems near their critical point. Near the SIT it was reported in highly disordered superconductors [4,20] and high- T_c superconductors [26,27] and near the superconductor-metal transition in complex oxide interfaces [34] and ultrathin Ga films [35]. As can be seen, in the high disorder limit of α -InO films (samples with $B_x \ll H_{c2} \simeq 14$ T in Figs. 4 and S5(c) of the Supplemental Material [23] and in Ref. [4]) $\frac{dB_x}{dT} > 0$ and the $T \rightarrow 0$ limit of B_x seems to be

a finite positive value. On the other hand, in systems where superconductivity terminates with a metallic or weakly insulating phase such as α -InO films in the less disordered limit (see Ref. [36]), high- T_c superconductors [26,27] and in some systems near the superconductor-metal transition [34,35] the trend is opposite as $\frac{dB_x}{dT} < 0$ (as expected if we replace B_x with H_{c2} [37]). We note that in our study and in the references where we managed to identify the T dependences of both B_x and B_{peak} [4,26,27] $\frac{dB_x}{dT}$ and $\frac{dB_{\text{peak}}}{dT}$ have the same sign [38].

The T variation of B_x poses an ambiguity in defining B_c which results in several similar but nonequivalent definitions [39]. This ambiguity limits the ability to use duality arguments or to perform scaling analysis over wide T ranges near the QCP. In this work we defined B_c as $B_c \equiv B_x(T \rightarrow 0)$. For sample AD8a1mm this results in $B_c = 0.55$ T.

In summary, we have shown that, as $T \rightarrow 0$, B_{peak} decreases which raises questions regarding the value of B_{peak}^0 and raises the interesting possibility that for sufficiently disordered samples $B_{\text{peak}}^0 = B_c$. We have shown that although the MR peak appears at different B 's, R_{peak} is typically activated and can be described with single R_0 and T_0 suggesting that this maximally resistive state is a single microscopical state that emerges at lower and lower B 's. The similarities of R_0 to R_Q and of T_0 to the superconducting T_c shows that this unique insulating state is tightly related to the microscopic superconducting nature of the insulating phase.

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- [23] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevB.98.184515> for additional technical information, parameters of different samples, and for a simplified phenomenological model of parallel transport channels.
- [24] We would have liked to plot T_0 vs T_c of all samples but, unfortunately, due to instrumental limitations we do not have T_c of most samples. This is because we are measuring in a dilution refrigerator that is limited in T 's above 1.2 K. We define T_c as $R(T_c) = \frac{R_{\text{normal}}}{2}$ therefore to measure T_c of 1–2 K we need to first reach a sufficiently high T where we can measure the normal state resistance (typically $> 2\text{K}$). While reaching these T 's is possible, it demands a special effort and was not done in the majority of past experiments from which we extracted the data displayed in Fig. 3.
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- [38] Except for Ref. [20] where both B_{peak} and B_c are only weakly T dependent and it seems that one out of the three samples they study disobeys our observation.
- [39] For example, in Ref. [4] B_c was defined by interpolating the $R(B)$ isotherms and finding B such that $\frac{dR}{dT}$ becomes zero. Such a definition could not be applied to Ref. [36] where $R(T)$ measurements at different B 's measured in the superconducting phase were nonmonotonic and reached a maximum at different T 's, therefore there were multiple values of B such that $\frac{dR}{dT} = 0$. Therefore they defined B_c by finding the first plotting T of each maximum (T_{max}) vs B and extrapolated it to $T_{\text{max}} = 0$. In Ref. [45] it is defined by extracting a characteristic energy scale and finding B where it vanishes (characteristic energy scales vanish near quantum critical points [3]).
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