Magnetic resonance in a high-frequency flow of a two-dimensional viscous electron fluid

P. S. Alekseev
Ioffe Institute, 194021 St. Petersburg, Russia

(Received 26 February 2018; revised manuscript received 13 September 2018; published 29 October 2018)

Two-dimensional electrons in high-quality nanostructures at low temperatures can form a viscous fluid. We develop a theory of high-frequency magnetotransport in such a fluid. The time dispersion of viscosity should be taken into account at the frequencies about and above the rate of electron-electron collisions. We show that the shear viscosity coefficients as functions of magnetic field and frequency have the resonance at the frequency equal to the doubled cyclotron frequency. We demonstrate that such a resonance manifests itself in the plasmon damping. Apparently, the predicted resonance is also responsible for the peaks and features in photoresistance and photovoltage, recently observed on the ultra-high-mobility GaAs quantum wells. The last fact should be considered as important evidence for forming a viscous electron fluid in such structures.

DOI: 10.1103/PhysRevB.98.165440

I. INTRODUCTION

In materials with enough weak disorder a viscous fluid consisting of phonons or conductive electrons can be formed at low temperatures. For the realization of such a hydrodynamic regime, the interparticle collisions conserving momentum must be much more intensive than any other collisions which do not conserve momentum. This idea was proposed many years ago for three-dimensional (3D) materials with strong phonon-phonon and electron-phonon interactions [1,2]. The hydrodynamic regime of thermal transport in liquid helium and dielectrics was studied in sufficient detail [3]. However, in those years there existed not enough pure conductors where the hydrodynamic regime of charge transport could be realized.

Recently, the crisp fingerprints of forming a viscous electron fluid and the realization of hydrodynamic charge transport were discovered in novel ultra-high-quality materials: in the two-dimensional (2D) monovalent layered metal $PdCoO_2$ [4], the 3D Weyl semimetal WP_2 [5], graphene [6,7], and GaAs quantum wells [8–11]. These experimental discoveries were accompanied by an extensive development of theory [12–30]. The brightest of such phenomena is the giant negative magnetoresistance effect, which was discovered in the best-quality GaAs quantum wells [8–11] and in the Weyl semimetal WP_2 [5].

The story of the giant negative magnetoresistance was rather nontrivial. Most of the conventional bulk transport theories predict either absent or parabolic positive magnetoresistance. The most well-known bulk mechanism for negative magnetoresistance is the weak localization effect, which leads to a relatively small negative magnetoresistance in very weak fields for materials with enough strong disorder. The giant negative magnetoresistance effect, which is the decrease in resistance by one to two orders of magnitude in moderate magnetic fields, seemed outstanding, surprising, and mysterious during 5 years after its discovery [31].

A possible explanation for the giant negative magnetoresistance was proposed within the hydrodynamic model taking into account the dependence of the electron viscosity coefficients on the magnetic field and temperature [16]. First, in Ref. [16] a viscous flow of an electron fluid was considered in a long sample with straight boundaries. In this case, the regime of transport critically depends on the type of electron scattering on longitudinal sample edges. If the edges are rough and scattering of electrons on them is diffusive, the Poiseuille flow is formed, and the magnetoresistance is proportional to the diagonal viscosity that leads to giant negative magnetoresistance. If the electron scattering on the edges is completely specular, then the sample resistance is zero. Second, in Ref. [16] it was noted that the presence [32] of macroscopic oval defects in a high-mobility sample can also lead to realization of the viscous magnetotransport. Indeed, in vicinities of the defects the hydrodynamic velocity cannot have a component in the direction perpendicular to the defect edge. A slowdown of the flow occurs due to the viscous transfer of the longitudinal component of the electron momentum in the transversal direction from the regions between the defects to the regions which are immediately in front of the defects. As a result, resistance is again proportional to viscosity. It can be seen that within this picture it is not important whether electron scattering on boundaries of defects is diffuse or specular.

In Ref. [26] a crossover between the hydrodynamic and the ballistic regimes of magnetotransport in a long sample with rough boundaries was numerically examined. In Ref. [27] the ballistic regime of the transport of interacting particles in a long sample was studied in the framework of the analytical model, and it was shown that in the ballistic regime the magnetoresistance is also negative and can be independent of temperature. High-frequency flows of a viscous 2D electron fluid in samples with the Corbino and Hall geometries in a zero magnetic field were studied in Refs. [28–30]. It was shown that in the Hall samples the regions where the electron flow is governed by viscosity are formed near the sample edges [29,30].

In this paper we develop a theory of nonstationary hydrodynamic transport of a 2D viscous electron fluid in a magnetic field [33]. We derive the Navier-Stokes equation for an ac viscous flow taking into account the time dispersion of viscosity. The obtained frequency-dependent viscosity coefficients have a resonance at the frequency equal to the doubled electron cyclotron frequency $\omega = 2\omega_c$. Herewith the other harmonics of the cyclotron resonance are absent. This resonance is a very special type of the high-order cyclotron resonance related to the viscosity effect. It can be regarded as a novel type of magnetic resonance, which is characteristic for viscous charged fluids and has the following physical nature. A viscous flow is controlled by the diffusivelike transfer of the electron momentum, which is accompanied by the presence of the viscous stress. The last varies in magnetic field as a product of two components of the electron velocity, thus it oscillates with the doubled cyclotron frequency (in the absence of any other fields).

We demonstrate that the proposed *viscous resonance* manifests itself in the damping coefficient of magnetoplasmons. Thus it can be observed in absorption of microwave radiation by an electron fluid (specific details of the way of manifestation of the viscous resonance in samples with different widths have been recently studied in Ref. [34]). We argue that, apparently, the viscous resonance is also responsible for the peaks and features at $\omega = 2\omega_c$ in the photoresistance and the photovoltaic effects, recently observed on the best-quality GaAs quantum wells [35–37].

In this way, we conclude that the possible observation of the viscous resonance in Refs. [35–37] together with the giant negative magnetoresistance evidence of forming a viscous electron fluid in moderate magnetic fields in the ultrapure GaAs quantum wells.

II. VISCOUS FLOW IN A MAGNETIC FIELD

The momentum flux density tensor (per one particle) is defined as $\Pi_{ij}(\mathbf{r},t) = m\langle v_i v_j\rangle$, where m is the electron mass, $\mathbf{v} = (v_x, v_y)$ is the velocity of a single electron, and the angular brackets stand for averaging over the electron velocity distribution at a given time t and point $\mathbf{r} = (x, y)$. The hydrodynamic velocity in this notation is $V_i(\mathbf{r},t) = \langle v_i\rangle$. The values $\mathbf{V}$ and Π_{ij} are proportional to the first and second angular harmonics (by the electron velocity vector $\mathbf{v}$) of the electron distribution function $f(\mathbf{v};\mathbf{r},t)$ (see the discussions in Refs. [38–41]).

If electrons weakly interact between themselves and can be regarded as an almost ideal Fermi gas, the hydrodynamic approach can be used when the characteristic space scale, L, of changing of $\mathbf{V}(\mathbf{r},t)$ is far greater than, at least, one of the following lengths: the electron mean free path relative to electron-electron collisions $l_{ee} = v_F\tau_{ee}$; the electron cyclotron radius $R_c = v_F/\omega_c$; the length of the path that the free electron passes during the characteristic period of changing of $\mathbf{V}(\mathbf{r},t)$, $l_\omega = v_F/\omega$. Here v_F is the Fermi velocity, τ_{ee} is the electron-electron scattering time (its exact definition will be clarified below), ω_c is the cyclotron frequency, and ω is the characteristic frequency of a flow. If one of these conditions is satisfied, then inside the regions of size L the quasiequilibrium distribution of electrons is formed, and the flow can be described by the values $\mathbf{V}$ and Π_{ij}. Below we construct the motion equations for $\mathbf{V}$ and Π_{ij} following the approach developed in Refs. [16,42].

The equation for the hydrodynamic velocity in the zero magnetic field is as follows:

$$m\frac{\partial V_i}{\partial t} = -\frac{\partial \Pi_{ij}}{\partial x_j} - \frac{mV_i}{\tau} + eE_i. \quad (1)$$

Here e is the electron charge, τ is the momentum relaxation time related to electron scattering on disorder or phonons [43], and the summation over repeating indices is assumed. The momentum flux density tensor Π_{ij} is equal to $P\delta_{ij} - \sigma_{ij}$, where P is the pressure in the fluid, δ_{ij} is the Kronecker δ symbol, and σ_{ij} is the viscous stress tensor [39].

For slow flows which vary at a timescale much greater than the time of relaxation of the inequilibrium part of the momentum flux density tensor, Π_{ij} is given by [39]

$$\Pi^{(0)}_{ij} = P\delta_{ij} - m\left[\eta\left(V_{ij} - \frac{1}{2}\delta_{ij}V_{kk}\right) + \frac{\zeta}{2}\delta_{ij}V_{kk}\right], \quad (2)$$

where $V_{ij} = \partial V_i/\partial x_j + \partial V_j/\partial x_i$ and η and ζ are the shear and the bulk viscosity coefficients. For the Fermi gas the last is relatively small: $\zeta \sim (T/\varepsilon_F)^2\eta$ [47], where T is the temperature and ε_F is the Fermi energy. In this regard, we will neglect the bulk viscosity in further considerations.

Using Eqs. (1) and (2), one obtains the Navier-Stokes equation in the linear by $\mathbf{V}$ regime,

$$\frac{\partial \mathbf{V}}{\partial t} = \frac{e}{m}\mathbf{E} - \frac{\mathbf{V}}{\tau} - \nabla P + \eta\,\Delta\mathbf{V}. \quad (3)$$

In this paper we take into account the compressibility of the electron fluid. Thus one needs to supplement Eq. (3) by the gas equation of state $P = P(n)$ (here n is the electron density) and by the continuity equation. The last in the linear regime has the form

$$\frac{\partial n}{\partial t} + n_0 \mathrm{div}\,\mathbf{V} = 0, \quad (4)$$

where n_0 is the unperturbed electron density.

The value given by Eq. (2) is attained during the time τ_{ee} as described by the Drude-like equation,

$$\frac{\partial \Pi_{ij}}{\partial t} = -\frac{1}{\tau_{ee}}\big(\Pi_{ij} - \Pi^{(0)}_{ij}\big). \quad (5)$$

Here τ_{ee} is the time of relaxation of the second angular moment (by the electron velocity) of the electron distribution function. As a rule, it is related to electron-electron scattering. Hydrodynamic effects are significant for an electron fluid in a solid if the scattering on disorder or phonons is much less intensive than electron-electron scattering: $\tau_{ee} \ll \tau$ [1]. Formulas (1), (2), and (5) are the whole system of equations describing nonstationary flows of a 2D viscous electron fluid in a zero magnetic field.

For a high-frequency flow with characteristic frequencies ω compared to $1/\tau_{ee}$ the relation between $\Pi_{ik}(\mathbf{r},t)$ and $V_{ik}(\mathbf{r},t)$ is nonlocal by time. Owing to linearity of all equations, we can decompose all the values by the time harmonics proportional to $e^{-i\omega t}$. For each pair of harmonic $\mathbf{V}(\mathbf{r},\omega)e^{-i\omega t}$ and $\Pi_{ij}(\mathbf{r},\omega)e^{-i\omega t}$ we obtain from Eqs. (1), (2), and (5) the relations between the amplitudes $\mathbf{V}(\mathbf{r},\omega)$ and $\Pi_{ij}(\mathbf{r},\omega)$. These relations have the same form as Eqs. (2) and (3) but contain the amplitude $\mathbf{E}(\mathbf{r},\omega)$ of the electric-field harmonic instead of $\mathbf{E}(\mathbf{r},t)$ and the frequency-dependent viscosity coefficient $\eta(\omega)=\eta/(1-i\omega\tau_{ee})$ instead of η.

Now let us consider a viscous electron flow in a magnetic field $\mathbf{B}$ perpendicular to the 2D layer. In this paper we are interested in the regime of moderate magnetic fields when quantum effects do not play a significant role in the motion of individual electrons and electron fluid as a whole. Therefore, we consider sufficiently small nonquantizing magnetic fields, satisfying the inequality $\hbar\omega_c \ll T \ll \varepsilon_F$. Herewith the relation among the parameters ω_c, ω, and τ_{ee} can be arbitrary. The theory of high-frequency transport in Fermi systems in quantizing magnetic fields was developed, for example, in Ref. [48].

In the presence of a magnetic field additional terms will appear in the equations for $\partial V_i/\partial t$ and $\partial \Pi_{ij}/\partial t$ since now the quantities $\langle v_i\rangle$ and $\langle v_i v_j\rangle$ will change in time not only due to collisions and the electric-field force, but also due to the magnetic-field force. The last force for each electron is $(eB/c)\epsilon_{ikz}v_k$, where ϵ_{lik} is the unit antisymmetric tensor and z is the direction of magnetic-field $\mathbf{B}$. For the averaged products of the velocity components in the presence of only magnetic field $\mathbf{B}$ we have as follows:

$$\frac{\partial\langle v_i\rangle}{\partial t} = \omega_c\epsilon_{ikz}\langle v_k\rangle,$$
$$\frac{\partial\langle v_i v_j\rangle}{\partial t} = \omega_c(\epsilon_{ikz}\langle v_k v_j\rangle + \epsilon_{jkz}\langle v_i v_k\rangle). \quad (6)$$

The terms (6) should be added to the right-hand side of Eqs. (1) and (5) [49],

$$m\frac{\partial V_i}{\partial t} = -\frac{mV_i}{\tau} - \frac{\partial \Pi_{ij}}{\partial x_j} + eE_i + \omega_c\epsilon_{ikz}V_k,$$
$$\frac{\partial \Pi_{ij}}{\partial t} = -\frac{\Pi_{ij}-\Pi_{ij}^{(0)}}{\tau_{ee}} + \omega_c(\epsilon_{ikz}\Pi_{kj}+\epsilon_{jkz}\Pi_{ik}). \quad (7)$$

As in the case of a zero magnetic field, we first consider the case of slow flows when the characteristic frequencies of $\mathbf{V}(\mathbf{r},t)$ are small in comparison with ω_c and $1/\tau_{ee}$. Putting $\partial\Pi_{ij}/\partial t = 0$, we find from Eqs. (2) and (7) the values of Π_{ij} as a linear combination of the values of $\Pi_{ij}^{(0)}$ and, thus, of P and V_{ij},

$$\Pi_{ij} = P\delta_{ij} - \sigma_{ij},$$
$$\sigma_{ij} = m\left[\eta_{xx}\left(V_{ij}-\frac{1}{2}\delta_{ij}V_{kk}\right) + \frac{\eta_{xy}}{2}\epsilon_{ikz}V_{kj}\right], \quad (8)$$

where η_{xx} and η_{xy} are the stationary shear viscosity coefficients of the 2D electron fluid in the magnetic field {see Ref. [16] and Eq. (10) for the case $\omega=0$}.

With the help of Eqs. (7) and (8), we arrive at the Navier-Stokes equation of the compressible 2D electron fluid in a magnetic field at low frequencies, which differs from Eq. (3) by the change in η on η_{xx} and the appearance the two magnetic terms $\omega_c[\mathbf{V}\times\mathbf{e}_z]$ and $\eta_{xy}[\Delta\mathbf{V}\times\mathbf{e}_z]$ [16].

Second, we consider the case of a high-frequency flow when the characteristic frequencies ω are compared to ω_c and $1/\tau_{ee}$. As in the case of a zero magnetic field, we decompose the values of $\mathbf{V}(\mathbf{r},t)$ and $\Pi_{ij}(\mathbf{r},t)$ by the time harmonics proportional to $e^{-i\omega t}$. As a final result, we arrive at the Navier-Stokes equation for amplitude $\mathbf{V}(\mathbf{r},\omega)$ of each velocity harmonic,

$$-i\omega\mathbf{V} = \frac{e}{m}\mathbf{E}(\mathbf{r},\omega) + \omega_c[\mathbf{V}\times\mathbf{e}_z] - \frac{\mathbf{V}}{\tau} - \frac{\nabla P}{m} + \eta_{xx}(\omega)\Delta\mathbf{V} + \eta_{xy}(\omega)[\Delta\mathbf{V}\times\mathbf{e}_z], \quad (9)$$

where the viscosity coefficients depend on the magnetic field and the frequency as follows:

$$\eta_{xx}(\omega) = \eta\frac{1-i\omega\tau_{ee}}{1+\left(-\omega^2+4\omega_c^2\right)\tau_{ee}^2 - 2i\omega\tau_{ee}},$$
$$\eta_{xy}(\omega) = \eta\frac{2\omega_c\tau_{ee}}{1+\left(-\omega^2+4\omega_c^2\right)\tau_{ee}^2 - 2i\omega\tau_{ee}}. \quad (10)$$

It is seen that at $\omega_c \gg 1/\tau_{ee}$ the viscosity coefficients $\eta_{xx}(\omega)$ and $\eta_{xy}(\omega)$ exhibit a resonance at $\omega = 2\omega_c$. Indeed, the internal frequency of rotation of the value $\Pi_{ij} = m\langle v_i v_j\rangle$ is the doubled cyclotron frequency $2\omega_c$ (see Fig. 1). Thus when frequency ω of the variation of a flow is close to the internal frequency $2\omega_c$, the resonance occurs. It is not just a second harmonic of the one-particle cyclotron resonance as it is related not to the motion of individual electrons but to the motion of the momentum flux of the electron ensemble (see Fig. 1). Such resonance is a special type of the high-order cyclotron resonance of collective electron motion related to the viscosity effect in a magnetic field, and so it can be named *the viscous resonance*.

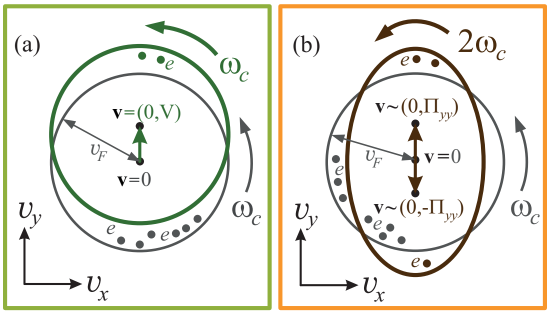

FIG. 1. Schematic of the two distributions $f(\mathbf{v})$ of 2D electrons by their velocities $\mathbf{v}=(v_x,v_y)$. The equilibrium Fermi distributions $f_F(\mathbf{v})$ are shown in gray in both panels (a) and (b). The quasiequilibrium distribution $f_{\mathbf{V}}(\mathbf{v}) = f_F(\mathbf{v}-\mathbf{V})$ with the mean hydrodynamic velocity $\mathbf{V}=(0,V)$ is shown in green in panel (a). In the magnetic field, such a distribution function together with each individual electron rotates with frequency ω_c. The nonequilibrium distribution $f_\Pi(\mathbf{v}) = f_F(\mathbf{v}) + f_2(\mathbf{v})$ with zero mean velocity and the second-harmonic $f_2(\mathbf{v})$ corresponding to a nonzero component Π_{yy} of the momentum flux density tensor Π_{ij} is shown in brown in panel (b). The rotation of individual electrons with the frequency ω_c leads to the rotation of such a distribution function f_Π with frequency $2\omega_c$.

If the interaction between 2D electrons is strong, they must be treated as a Fermi liquid. A preliminary analysis [34], following Ref. [47], shows that, for a sufficiently large value of interaction between the Fermi-liquid quasiparticles, the kinetic equation reduces to the Navier-Stokes equation for a highly viscous electron fluid, which at very high frequencies is just the Newton equation of vibrations of an amorphous medium with damping. The coefficients η and ζ will contain the Landau parameters describing the interaction between quasiparticles. The conditions of applicability of the theory will expand significantly. In particular, Eqs. (9) and (10) will be applicable even at short wavelengths and high frequencies $L \sim l_\omega$.

III. PLASMON DAMPING

The time dispersion of viscosity can manifest itself in damping of the magnetoplasmons. Below we calculate the magnetoplasmon damping coefficient related to viscosity using Eqs. (4), (9), and (10). Herewith, we will not consider the retardation effects which can be important in the region of small wave vectors in some structures (see, for example, Refs. [50,51]).

For the case of waves in the absence of external ac fields, the electric-field $\mathbf{E}(\mathbf{r},\omega)$ in Eq. (9) is induced by the perturbation of the 2D electron density $\delta n = n - n_0$. When we can neglect the retardation effects, we just have $\mathbf{E} = -\nabla\delta\varphi$, where $\delta\varphi$ is related to δn by the electrostatic equations. For the structures with a metallic gate located at the distance d from the 2D layer we have as follows: $\delta\varphi = (4\pi ed/\kappa)\delta n$, where κ is the background dielectric constant. For the structures without a gate the relation between $\delta\varphi(\mathbf{r},t)$ and $\delta n(\mathbf{r},t)$ is given just by the Coulomb law with the charge-density $\varrho(\mathbf{r},z) = e\,\delta n(\mathbf{r})\delta(z)$, where $\delta(z)$ is the δ-function depicting the position of the 2D layer.

We solve together Eqs. (4) and (9), and the electrostatic equation assuming that $\delta n(\mathbf{r},t)$, $\delta\varphi(\mathbf{r},t)$, $\mathbf{V}(\mathbf{r},t) \sim e^{-i\omega t+\mathbf{q}\cdot\mathbf{r}}$. The ratio of the terms $-\nabla P/m$ and $e\mathbf{E}/m$ in Eq. (9) is estimated as a_B/d for the structures with a gate and as $a_B q$ for the ungated structures, where a_B is the Bohr radius. Both these values must be much smaller than unity when the 2D electrostatic equations are applicable. Neglecting the terms describing the relaxation processes, we obtain from Eqs. (4) and (9) the usual formula for the dispersion law of magnetoplasmons. For the gated structures it is as follows:

$$\omega_{0,q} = \sqrt{\omega_c^2 + s^2q^2}, \tag{11}$$

where $s = \sqrt{4\pi e^2 n_0 d/m\kappa}$. The second term under the root in Eq. (11) s^2q^2 is the squared plasmon frequency in the absence of a magnetic field. For the ungated structure it changes on $2\pi e^2 n_0 q/m\kappa$.

The viscosity terms and the terms describing scattering on disorder leads to a small correction to the magnetoplasmon dispersion (11) as well as to the arising of a finite damping: $\omega_q = \omega_{0,q} + \Delta\omega_q - i\Upsilon_q$. The damping coefficient Υ_q takes the form

$$\Upsilon_q = \frac{\omega_c^2 + \omega_{0,q}^2}{2\omega_{0,q}^2}\left[\frac{1}{\tau} + \mathrm{Re}\,\eta_{xx}q^2\right] + \frac{\omega_c}{\omega_{0,q}}\,\mathrm{Im}\,\eta_{xy}q^2. \tag{12}$$

Here the viscosity coefficients $\eta_{xx}(\omega)$ and $\eta_{xy}(\omega)$ are taken at $\omega = \omega_{0,q}$.

At high frequencies and high magnetic fields, $\omega, \omega_c \gg 1/\tau_{ee}$, we obtain from Eqs. (10) and (12),

$$\Upsilon_q = \frac{1}{\tau}\frac{w^2+1}{2w^2} + \frac{\eta q^2}{w^2}\frac{w^4 + 13w^2 + 4}{4w^2 + \beta^2(w^2-4)^2}, \tag{13}$$

where $\beta = \omega_c\tau_{ee} \gg 1$ and $w = w(q) = \omega_{0,q}/\omega_c$. Near the resonance of the shear viscosity coefficients, when $w \approx 2$, the value Υ_q takes the form

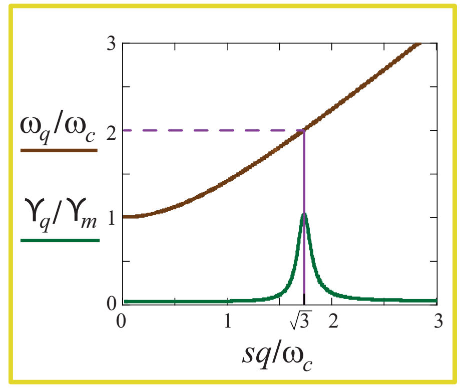

FIG. 2. The magnetoplasmon dispersion law $\omega_{0,q}$ and the damping coefficient Υ_q calculated by Eqs. (11) and (14) for a gated structure. The maximum value of Υ_q, $\Upsilon_m = 5/(8\tau) + 27\eta\omega_c^2/(8s^2)$ is attained at the wave-vector $q_m = \sqrt{3}\omega_c/s$ corresponding to the equality $\omega_{0,q} = 2\omega_c$.

$$\Upsilon_q = \frac{5}{8\tau} + \frac{9\eta q^2}{8(1+\varepsilon^2\beta^2)}. \tag{14}$$

where $\varepsilon = \varepsilon(q) = w(q) - 2$, $\varepsilon \ll 1$. In high-quality structures at low temperatures the inequality $1/\tau \lesssim \eta q^2/\beta^2$ can take place in certain intervals of wave vectors and magnetic fields. Provided this condition, the damping coefficient Υ_q in the resonance is greater than outside the resonance in $\beta^2 \gg 1$ times [see Eq. (14) and Fig. 2].

IV. DISCUSSION AND CONCLUSION

A linear response $\mathbf{V}(\mathbf{r},t)$ on ac electric-field $\mathbf{E}_{ex}(t)$ in a given sample should be calculated from Eqs. (4) and (9) with appropriate boundary conditions. Such a calculation has been performed in Ref. [34] for the flow in a long sample with rough boundaries. The linear response is directly related to the absorbtion of energy from an ac external field. It has been shown in Ref. [34] that the response of the fluid in not too wide samples consists of the two parts. The first is the plasmon contribution, formed by standing magnetoplasmon waves and located in the bulk of the sample. The second is the viscous contribution, located in the narrow near-edge regions and formed by standing waves of the transverse zero sound associated with the time dispersion of viscosity. The viscous resonance manifests itself via both contributions. In the plasmon contribution, the viscous resonance can be observed by the dependence of the width and the amplitude of the plasmonic resonances on ω and ω_c, which are governed by the damping coefficient of magnetoplasmons Υ_q (12). The maximum width of the plasmonic resonance occurs when an integer or half-integer number of plasmon wavelengths $2\pi/q$ is close to the sample width W and ω is close to $2\omega_c$ [34].

It is possible that the viscous resonance is also responsible for the strong peak and features observed at $\omega = 2\omega_c$ in the photoresistance [35,36] and the photovoltaic effects [37] in the high-mobility GaAs quantum wells. Indeed, it was stressed in Ref. [35] that the strong peak in photovoltage and the very well-pronounced giant negative magnetoresistance, explained in Ref. [16] as a manifestation of forming of a viscous flow, are observed in the *same best-quality* GaAs structures. If 2D electrons in such structures form a viscous fluid, than any response of the structure on an ac field (absorption, photovoltage, and photoresistance) must inevitably have peculiarities at the frequency of the viscous resonance.

To construct the theories of the photoresistance and the photovoltaic effects, one should supplement the hydrodynamic equation (9) by the nonlinear terms following Refs. [52,53]. The peak and features at $\omega = 2\omega_c$ in photovoltage and photoresistance were observed in Refs. [35–37] at rather high magnetic fields when the inequality $R_c \ll W$ was fulfilled. A preliminary analysis shows that this justifies the applicability of the Fermi-gas model for the description of hydrodynamics near the viscous resonance. However, the Fermi-gas model outside the resonance, in particular, in small magnetic fields, seems to be irrelevant. Justification of the realization of the hydrodynamic regime outside the resonance has been performed in Ref. [34] by use of the phenomenological Fermi-liquid model.

To conclude, we predict the viscous resonance at $\omega = 2\omega_c$ related to the motion of the viscous stress tensor in a magnetic field. This resonance manifests itself in the dependence of the damping of magnetoplasmons on their wave vectors and, probably, in the photoresistance and the photovoltaic effects.

ACKNOWLEDGMENTS

The author wishes to thank Professor M. I. Dyakonov under whose guidance this research was undertaken for the discussions, advice, and support during the course of the work and for his participation in writing the text of the paper. The author also thanks A. P. Dmitriev and I. V. Gorniy for valuable discussions; D. G. Polyakov for turning his attention to Ref. [42]; A. P. Alekseeva, E. G. Alekseeva, I. P. Alekseeva, N. S. Averkiev, A. I. Chugunov, M. M. Glazov, I. V. Krainov, A. N. Poddubny, P. S. Shternin, D. S. Svinkin, and V. A. Volkov for advice and support. The part of this work devoted to the time dispersion of viscosity in the magnetic field (Secs. II and IV) was supported by the Russian Science Foundation (Grant No. 17-12-01182); the part of this work devoted to plasmon damping due to viscosity (Sec. III) was supported by a grant from the Basis Foundation (Grant No. 17-14-414-1).

[1] R. N. Gurzhi, Sov. Phys. Usp. **11**, 255 (1968).
[2] V. L. Gurevich, *Transport in Phonon Systems* (Elsevier, Amsterdam/New York, 1986).
[3] L. P. Pitaevskii, Sov. Phys. Usp. **11**, 342 (1968).
[4] P. J. W. Moll, P. Kushwaha, N. Nandi, B. Schmidt, and A. P. Mackenzie, Science **351**, 1061 (2016).
[5] J. Gooth, F. Menges, C. Shekhar, V. Süß, N. Kumar, Y. Sun, U. Drechsler, R. Zierold, C. Felser, and B. Gotsmann arXiv:1706.05925.
[6] D. A. Bandurin, I. Torre, R. Krishna Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G. H. Auton, E. Khestanova, K. S. NovoseIov, I. V. Grigorieva, L. A. Ponomarenko, A. K. Geim, and M. Polini, Science **351**, 1055 (2016).
[7] R. Krishna Kumar, D. A. Bandurin, F. M. D. Pellegrino, Y. Cao, A. Principi, H. Guo, G. H. Auton, M. Ben Shalom, L. A. Ponomarenko, G. Falkovich, K. Watanabe, T. Taniguchi, I. V. Grigorieva, L. S. Levitov, M. Polini, and A. K. Geim, Nat. Phys. **13**, 1182 (2017).
[8] A. T. Hatke, M. A. Zudov, J. L. Reno, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **85**, 081304 (2012).
[9] R. G. Mani, A. Kriisa, and W. Wegscheider, Sci. Rep. **3**, 2747 (2013).
[10] L. Bockhorn, P. Barthold, D. Schuh, W. Wegscheider, and R. J. Haug, Phys. Rev. B **83**, 113301 (2011).
[11] Q. Shi, P. D. Martin, Q. A. Ebner, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **89**, 201301 (2014).
[12] R. N. Gurzhi, A. N. Kalinenko, and A. I. Kopeliovich, Phys. Rev. Lett. **74**, 3872 (1995).
[13] M. Hruska and B. Spivak, Phys. Rev. B **65**, 033315 (2002).
[14] A. V. Andreev, S. A. Kivelson, and B. Spivak, Phys. Rev. Lett. **106**, 256804 (2011).
[15] M. Mendoza, H. J. Herrmann, and S. Succi, Sci. Rep. **3**, 1052 (2013).
[16] P. S. Alekseev, Phys. Rev. Lett. **117**, 166601 (2016).
[17] L. Levitov and G. Falkovich, Nat. Phys. **12**, 672 (2016); H. Guo, E. Ilseven, G. Falkovich, and L. Levitov, Proc. Natl. Acad. Sci. USA **114**, 3068 (2017).
[18] A. Lucas, Phys. Rev. B **95**, 115425 (2017); A. Lucas and K. C. Fong, J. Phys.: Condens. Matter **30**, 053001 (2018).
[19] F. M. D. Pellegrino, I. Torre, and M. Polini, Phys. Rev. B **96**, 195401 (2017).
[20] P. S. Alekseev, A. P. Dmitriev, I. V. Gornyi, V. Y. Kachorovskii, B. N. Narozhny, M. Schutt, and M. Titov, Phys. Rev. Lett. **114**, 156601 (2015).
[21] G. Y. Vasileva, D. Smirnov, Y. L. Ivanov, Y. B. Vasilyev, P. S. Alekseev, A. P. Dmitriev, I. V. Gornyi, V. Y. Kachorovskii, M. Titov, B. N. Narozhny, and R. J. Haug, Phys. Rev. B **93**, 195430 (2016).
[22] P. S. Alekseev, A. P. Dmitriev, I. V. Gornyi, V. Y. Kachorovskii, B. N. Narozhny, M. Schutt, and M. Titov, Phys. Rev. B **95**, 165410 (2017).
[23] P. S. Alekseev, A. P. Dmitriev, I. V. Gornyi, V. Y. Kachorovskii, and M. A. Semina, Semiconductors **51**, 766 (2017).
[24] P. S. Alekseev, A. P. Dmitriev, I. V. Gornyi, V. Y. Kachorovskii, B. N. Narozhny, and M. Titov, Phys. Rev. B **97**, 085109 (2018).
[25] P. S. Alekseev, A. P. Dmitriev, I. V. Gornyi, V. Y. Kachorovskii, B. N. Narozhny, and M. Titov, Phys. Rev. B **98**, 125111 (2018).
[26] T. Scaffidi, N. Nandi, B. Schmidt, A. P. Mackenzie, and J. E. Moore, Phys. Rev. Lett. **118**, 226601 (2017).
[27] P. S. Alekseev and M. A. Semina, Phys. Rev. B **98**, 165412 (2018).
[28] A. Tomadin, G. Vignale, and M. Polini, Phys. Rev. Lett. **113**, 235901 (2014).
[29] R. Moessner, P. Surówka, and P. Witkowski, Phys. Rev. B **97**, 161112(R) (2018).
[30] M. Semenyakin and G. Falkovich, Phys. Rev. B **97**, 085127 (2018).

[31] See the website of the conferences devoted to the transport phenomena in the ultra-high-mobility 2D nanostructure [https://www.coulomb.univ-montp2.fr/MIRO-and-all-that?lang=en].
[32] L. Bockhorn, I. V. Gornyi, D. Schuh, C. Reichl, W. Wegscheider, and R. J. Haug, Phys. Rev. B **90**, 165434 (2014).
[33] This paper was presented at the seminar of the Institute of Nanotechnology (Karlsruhe Institute of Technology, Karlsruhe, Germany, 2017; for the abstract of the talk, Negative magnetoresistance in viscous flow of two-dimensional electrons, see https://www.int.kit.edu/calendar.php/event/33486) and at the Russian conference on semiconductor physics (Ekaterinburg, Russia, 2017; for the abstract of the talk, High-frequency magnetotransport in a viscous two-dimensional electron liquid, see http://semicond2017.imp.uran.ru, p. 333. A result similar to Eq. (10) was independently obtained in Ref. [19]. However, the existence of the resonance at $\omega = 2\omega_c$ was not noticed and discussed in that work.
[34] P. S. Alekseev and A. P. Alekseeva (unpublished).
[35] Y. Dai, R. R. Du, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **105**, 246802 (2010).
[36] A. T. Hatke, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **84**, 241304 (2011).
[37] M. Bialek, J. Lusakowski, M. Czapkiewicz, J. Wrobel, and V. Umansky, Phys. Rev. B **91**, 045437 (2015).
[38] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981).
[39] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1987).
[40] M. S. Steinberg, Phys. Rev. **109**, 1486 (1958).
[41] Y. M. Aliev, J. Appl. Mech. Tech. Phys. **3**, 11 (1965).
[42] A. N. Kaufman, Phys. Fluids **3**, 610 (1960).
[43] For the temperature dependence of the electron-phonon scattering rates in GaAs quantum wells, see Refs. [44–46].
[44] V. Karpus, Sov. Phys. Semicond. **20**, 6 (1986).
[45] P. S. Alekseev, M. S. Kipa, V. I. Perel, and I. N. Yassievich, JETP **106**, 806 (2008).
[46] M. S. Keepa, P. S. Alekseev, and I. N. Yassievich, Semiconductors **44**, 198 (2010).
[47] I. M. Khalatnikov and A. A. Abrikosov, Sov. Phys. JETP **6**, 84 (1958).
[48] M. Genske and A. Rosch, Phys. Rev. A **92**, 062108 (2015).
[49] In the second part of Eq. (7) we assume that the electric-field force is far smaller than the magnetic-field force as it usually takes place in the magnetotransport experiments.
[50] V. I. Fal'ko and D. E. Khmel'nitskii, Sov. Phys. JETP **68**, 1150 (1989).
[51] V. A. Volkov and A. A. Zabolotnykh, Phys. Rev. B **89**, 121410 (2014).
[52] M. B. Lifshits and M. I. Dyakonov, Phys. Rev. B **80**, 121304 (2009).
[53] Y. M. Beltukov and M. I. Dyakonov, Phys. Rev. Lett. **116**, 176801 (2016).