# Photon drag of a Bose-Einstein condensate

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Kepler's observation of comet tails initiated the research on the radiation pressure of celestial objects and 250 years later they found new incarnation after the Maxwell's equations were formulated to describe a plethora of light-matter coupling phenomena. Further, quantum mechanics gave birth to the photon drag effect. Here, we develop a microscopic theory of this effect which can occur in a general system containing Bose-Einstein–condensed particles, which possess an internal structure of quantum states. By analyzing the response of the system to an external electromagnetic field we find that such a drag results in a flux of particles constituting both the condensate and the excited states. We show that in the presence of the condensed phase, the response of the system acquires steplike behavior as a function of the electromagnetic field frequency with the elementary step determined by the internal energy structure of the particles.

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## I. INTRODUCTION

Ponderomotive force of light acting on atoms, molecules, and other particles results in a momentum transfer between light and matter [1,2]—the phenomenon referred to as the radiation pressure. Historically, the hypothesis of radiation pressure was for the first time suggested by J. Kepler in the beginning of XVII century in an attempt to explain why the tails of comets point away from the Sun. In frameworks of classical electrodynamics, the radiation pressure was considered by J. C. Maxwell in 1870. It was shown to be closely related to the light scattering and absorption by particles.

In the framework of the quantum mechanics, radiation pressure is a result of the momentum transfer from a photon to a system, for instance, an atom [3] or a molecule. In condensed matter, light pressure results in a current of charge carriers and it is called the photon drag effect (PDE). The first theory of this phenomenon was based on electron-photon interaction mediated by an acoustic phonon [4,5]. Charge carriers, such as free electrons and holes, can absorb radiation by means of interaction with an electromagnetic field (EMF), and they are forced to move in a direction of the wave vector of light. PDE has been extensively studied in semiconductors [6–8], dielectrics [9], metals [10,11], mono- and multilayer graphene [12,13], carbon nanotubes [14], topological insulators [15], two-dimensional (2D) electron gas [16–18], and other systems.

According to classical description of the PDE in semiconductors, the drag current reads $\mathbf{j}(\omega) \sim \mathbf{k}\alpha(\omega)I$, where $\mathbf{k}$ is the photon wave vector, $I = cE^2/8\pi$ is the intensity of the electromagnetic wave, and $\alpha(\omega)$ is the absorption coefficient of light by the charge carriers. Evidently, the frequency dependence of the drag current is determined by the spectrum of the absorption coefficient. In the majority of cases, this dependence is monotonous or resonant if the frequency of the EMF $\omega$ is close to the energy of quantum transitions in the system. However, it is not a general rule.

In this paper, we study the effect of radiation pressure on a purely quantum system of bosons containing particles in a condensed quantum state. We will show that in a Bose-Einstein condensate (BEC), the drag current of bosons and thus the response of the system become steplike. It is a universal phenomenon which can be possibly observed in atomic and solid-state condensates, thus we will consider a general model of a Bose gas, in which each boson possesses an internal structure of quantum states, which is essential for the theory developed below. The spectrum of a single boson with an eigenfunction $|\eta, \mathbf{p}\rangle$ reads

$$\varepsilon_\eta(\mathbf{p}) = \varepsilon(\mathbf{p}) + \Delta_\eta, \tag{1}$$

where $\varepsilon(\mathbf{p}) = \mathbf{p}^2/2M$ is a kinetic energy of the particle center-of-mass motion, and $\Delta_\eta$ is the energy spectrum of the internal motion. It can be a spectrum of an atom (in a cold atomic condensate system) or the energy of the relative motion of an electron and a hole constituting an exciton (in excitonic BECs). Here the index $\eta$ stands for the whole set of quantum numbers which characterize the internal spectrum of the particle and the value $\eta = 0$ refers to the lowest energy state (ground state) of the internal spectrum of Bose particles and all the energies will be measured from $\Delta_{\eta=0}$. We will assume that before being irradiated, the system is in the Bose-Einstein condensed state $|\eta, \mathbf{p}\rangle$ where all the particles are in the ground $\eta = 0$ state of their internal motion, zero kinetic energy $\mathbf{p} = 0$ of their center-of-mass motion without a dipole moment.

 

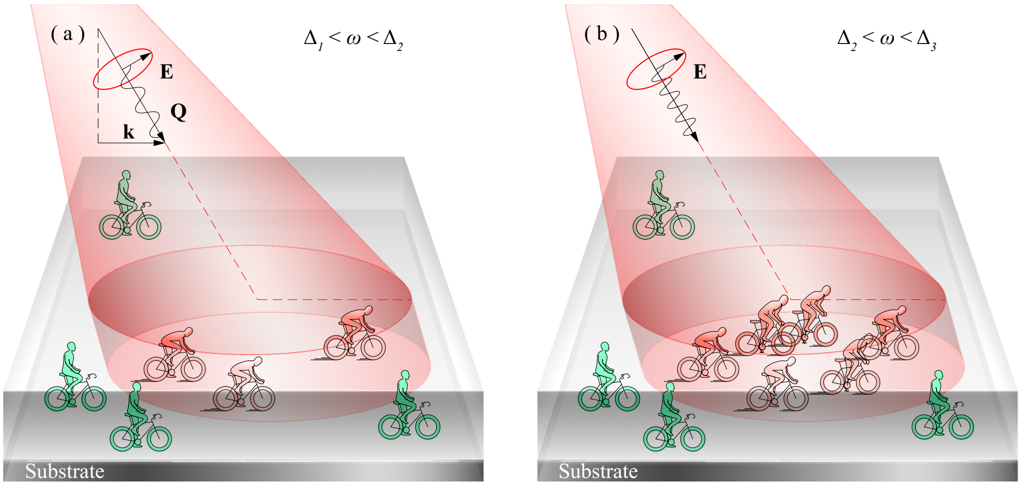


FIG. 1. System schematic. Bosons (presented by bicycles) are exposed to an electromagnetic field. It results in their flux in the direction collinear with the in-plane projection **k** of the wave vector of light **Q**. The current jumps at certain frequency values. For example, above the second threshold $\omega > \Delta_2$ (b) the current is larger than below it (a) since more bosons take part in the current.

## II. MODEL

Let us consider a system of bosons exposed to an EMF (Fig. 1) with the wavelength exceeding the size of a particle thus allowing us to use a dipole approximation. The electric field then depends on the center-of-mass coordinate **r** only, $\mathbf{E}(x) = \mathbf{E}_0 e^{i\mathbf{kr}-i\omega t} + \mathbf{E}_0^* e^{-i\mathbf{kr}+i\omega t}$, and the light-matter coupling can be described by the matrix elements $\mathbf{d}_{21}\cdot\mathbf{E}$. Here the indices 1, 2 stand for the ground and excited quantum states of the internal particle motion, $|1\rangle \equiv |\eta = 0\rangle$ and $|2\rangle \equiv |\eta \neq 0\rangle$. Then $\mathbf{d}_{12} = \langle 1|\mathbf{d}|2\rangle$ is a matrix element of the dipole moment operator of the particle. For simplicity, we assume that initially the particles do not possess a dipole moment, $\mathbf{d}_{11} = \mathbf{d}_{22} = 0$, and $\varepsilon_1(\mathbf{p}) = \varepsilon(\mathbf{p})$, $\varepsilon_2(\mathbf{p}) = \varepsilon(\mathbf{p}) + \Delta_\eta$ are the energies of the ground and excited states, correspondingly.

The system response to a pressure of the external EMF is a current of particles which is determined by the light absorption coefficient. The BEC-EMF interaction Hamiltonian reads

$$H_I = \mathbf{d}_{21}\cdot\mathbf{E}_0 \sum_{\mathbf{p}} c^\dagger_{\eta,\mathbf{p}+\mathbf{k}}(t) a_{\mathbf{k}}(t) c_{0,\mathbf{p}}(t) + \text{H.c.}, \tag{2}$$

where $c_{\eta,\mathbf{p}}(t) = c_{\eta,\mathbf{p}}(0)\exp[-i\varepsilon_\eta(\mathbf{p})t]$ and $a_{\mathbf{k}}(t) = a_{\mathbf{k}}(0)\exp(-i\omega t)$ are the annihilation operators for the Bose particle and EMF photon, respectively. The theoretical description of BEC is based on the Bogoliubov theory of a weakly interacting Bose gas [19]. It requires that the Bose gas is diluted, $na^d \ll 1$, where $n$ is the particle concentration, $a$ is a characteristic scale (scattering length in cold atoms), and $d$ is a system dimensionality. In order to describe the dynamics of the BEC, we will use the Gross-Pitaevskii equation. In its framework, low-energy excitations of the BEC represent Bogoliubov quasiparticles (bogolons) with the dispersion $\omega_{\mathbf{p}} = \sqrt{\varepsilon_{\mathbf{p}}(\varepsilon_{\mathbf{p}} + 2gn_c)} = sp\sqrt{1+(p\xi)^2}$, where $s = \sqrt{gn_c/M}$, $\xi = 1/(2Ms)$ are the sound velocity and the healing length, $g$ is the interparticle interaction strength, $n_c$ is the density of particles in the BEC. In a long-wavelength limit $\xi p \ll 1$ (that is equivalent to $\varepsilon_{\mathbf{p}} \ll gn_c$) the dispersion law of the bogolons becomes linear, $\omega_{\mathbf{p}} = sp$. We will consider $T = 0$ thus disregarding the processes of thermal excitation of bogolons. Further we present $c_{0,\mathbf{p}}$ in the form [19]

$$c_{0,\mathbf{p}}(t) = c_{0,0}\delta(\mathbf{p}) + u_{\mathbf{p}} b_{\mathbf{p}}(t) + v_{\mathbf{p}} b^\dagger_{-\mathbf{p}}(t), \tag{3}$$

where $c_{0,0}$ describes the particles in the BEC state with zero momentum and $|c_{0,0}|^2 = n_c$. Here $u_{\mathbf{p}}$ and $v_{\mathbf{p}}$ are the Bogoliubov transformation coefficients and $b_{\mathbf{p}}(t) = b_{\mathbf{p}}(0)\exp(-i\omega_{\mathbf{p}}t)$ are Bogoliubov excitation operators. Substituting Eq. (3) into (2), we can come up with several principal quantum channels of the EMF absorption.

## III. RESULTS

The first term in Eq. (3) substituted in (2) describes a transition of a Bose particle from the BEC to an excited state $\eta \neq 0$ with the energy conservation law $\omega = \varepsilon_{\mathbf{k}} + \Delta_\eta \approx \Delta_\eta$; see Fig. 2, transitions I. Beside these, there exists another type of transition described by the second and third terms in Eq. (3). They can be referred to as the *Belyaev processes* [20,21] and happen when the light absorption is accompanied by not only excitation of the particle but also the emission or

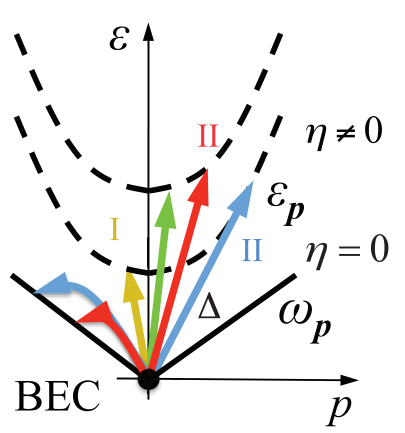


FIG. 2. Excitation spectrum of the internal degrees of freedom of the boson during absorption of a quantum of EMF.

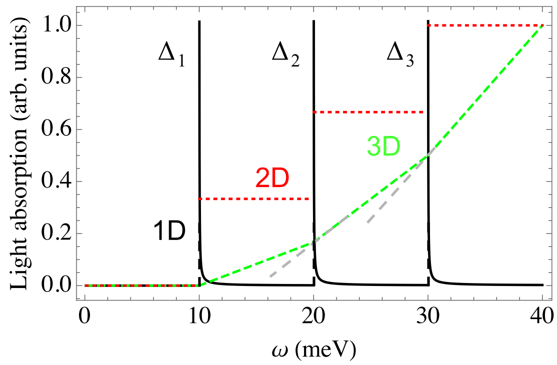


FIG. 3. Spectrum of the normalized light absorption coefficient for different system dimensionality: one dimesion (black solid), two dimensions (red dotted), and three dimensions (green dashed). Dashed gray lines show the change of the slope of the green dashed curve with the increase of $\omega$, manifesting a steplike threshold behavior in 3D case.

absorption of a Bogoliubov excitation of the condensate; see Fig. 2 , transitions II. The corresponding energy conservation law reads $\omega = \varepsilon_{\mathbf{p}+\mathbf{k}} + \Delta_\eta + \omega_{-\mathbf{p}}$, where $\omega_\mathbf{p}$ is the bogolon dispersion. Further analysis shows that such processes result in the steplike behavior of the BEC response.

Indeed, the probability of the absorption of a photon is proportional to $\sum_\mathbf{p} v_\mathbf{p}^2 \delta(\omega - \varepsilon_{\mathbf{p}+\mathbf{k}} - \Delta_\eta - \omega_{-\mathbf{p}})$ which can be found by the Fermi golden rule using Eqs. (2) and (3). In the long-wavelength limit, $\omega_\mathbf{p} \approx sp$, and accounting for the fact that $sp \gg \varepsilon_{\mathbf{p}+\mathbf{k}}$ and $v_\mathbf{p}^2 \sim \omega_\mathbf{p}^{-1}$, in the 2D case we find the thresholdlike behavior of the light absorption coefficient:

$$\sum_\eta |\mathbf{d}_{\eta 1} \cdot \mathbf{E}_0|^2 \int_0^\infty \frac{pdp}{\omega_\mathbf{p}} \delta(\omega - \Delta_\eta - \omega_{-\mathbf{p}}) \sim \sum_\eta |\mathbf{d}_{\eta 1} \cdot \mathbf{E}_0|^2 \theta[\omega - \Delta_\eta]. \quad (4)$$

Thus, taking into account the internal structure of the particles leads to the "quantization" of the response of the system to the external light pressure in a 2D condensate. For the 1D condensate the photon absorption is proportional to $\sum_\eta |\mathbf{d}_{\eta 1} \cdot \mathbf{E}_0|^2 \theta[\omega - \Delta_\eta](\omega - \Delta_\eta)^{-1}$ whereas in the 3D case the absorption coefficient qualitatively behaves as $\sum_\eta |\mathbf{d}_{\eta 1} \cdot \mathbf{E}_0|^2 \theta[\omega - \Delta_\eta](\omega - \Delta_\eta)$. Figure 3 shows the normalized (to unity) spectra of these functions. In one dimension, the current acquires a comblike form, whereas in three dimensions it takes a form of a broken straight line. The most intriguing results occur for the 2D system: the light absorption coefficient demonstrates the *steplike* behavior with increase of the external EM field frequency as is evident from Eq. (4). (Here we disregard the broadening of the peaks and steps due to finite lifetime of the particles and show only the principal picture.)

Let us develop the quantum field theory calculations for 2D BEC accounting for the finite lifetimes of the bosons in excited states $\eta \neq 0$. The evolution of the system is studied by the following equation:

$$i\partial_t \Psi(x) = \begin{pmatrix} \varepsilon_1(\mathbf{p}) - \mu + g|\psi_1|^2 & \mathbf{d}_{12}\mathbf{E} \\ \mathbf{d}_{21}\mathbf{E} & \varepsilon_2(\mathbf{p}) \end{pmatrix} \Psi(x), \quad (5)$$

where the spinor $\Psi(x) = (\psi_1^*(x), \psi_2^*(x))^T$ describes the condensate particles $\psi_1(x)$ and the excited particles $\psi_2(x)$; $\mu$ is a chemical potential. The drag current of the particles reads

$$\mathbf{j}_c = \frac{i}{2M} \sum_{l=1,2} \langle \psi_l \nabla_\mathbf{r} \psi_l^* - \psi_l^* \nabla_\mathbf{r} \psi_l \rangle_t, \quad (6)$$

where $\langle \ldots \rangle_t$ stands for the time averaging and $l = 1$ corresponds to the contribution of the BEC component $\psi_1(x)$ and $l = 2$ ($\eta = 2, 3 \ldots$) is the contribution of the excited states $\psi_2(x)$. Considering the EMF $\mathbf{E}(x)$ as a perturbation, we can replace $\psi_1(x) \to \psi_0 + \delta\psi_1(x)$ and $\psi_2(x) \to \delta\psi_2(x)$, where $\psi_0$ describes the BEC state, with $n_c = |\psi_0|^2$. Linearizing Eq. (5) gives the following system of equations:

$$\hat{\mathcal{G}}_0^{-1} \delta\hat{\psi}_1(x) = -\mathbf{E}(x)\hat{\mathbf{d}}\delta\hat{\psi}_2(x), \quad (7)$$

$$\hat{\mathfrak{G}}_0^{-1} \delta\hat{\psi}_2(x) = -\mathbf{E}(x)\hat{\mathbf{d}}^*(\hat{\psi}_0 + \delta\hat{\psi}_1(x)), \quad (8)$$

where

$$\hat{\mathbf{d}} = \begin{pmatrix} \mathbf{d}_{12} & 0 \\ 0 & \mathbf{d}_{12}^* \end{pmatrix}, \ \delta\hat{\psi}_i(x) = \begin{pmatrix} \delta\psi_i(x) \\ \delta\psi_i^*(x) \end{pmatrix}; \quad (9)$$

$$\hat{\mathcal{G}}_0^{-1} = \begin{pmatrix} i\partial_t - \varepsilon_\mathbf{p} - gn_c & -gn_c \\ -gn_c & -i\partial_t - \varepsilon_\mathbf{p} - gn_c \end{pmatrix},$$

$$\hat{\mathfrak{G}}_0^{-1} = \begin{pmatrix} i\partial_t - \varepsilon_\mathbf{p} - \Delta_\eta & 0 \\ 0 & -i\partial_t - \varepsilon_\mathbf{p} - \Delta_\eta \end{pmatrix}.$$

Substituting the formal solution of Eq. (8) into (7) yields an integrodifferential equation

$$\hat{\mathcal{G}}_0^{-1} \delta\hat{\psi}_1(x) = \mathbf{E}(x)\hat{\mathbf{d}} \int dx_1 \hat{\mathfrak{G}}_0(x - x_1)\mathbf{E}(x_1)\hat{\mathbf{d}}^*(\hat{\psi}_0 + \delta\hat{\psi}_1(x_1)). \quad (10)$$

Expressing $\delta\hat{\psi}_1(x)$ via $\delta\hat{\psi}_2(x)$ using Eq. (7),

$$\delta\hat{\psi}_1(x) = -\int dx_1 \hat{\mathcal{G}}_0(x - x_1)\mathbf{E}(x_1)\hat{\mathbf{d}}\delta\hat{\psi}_2(x_1), \quad (11)$$

we can find the closed system of equations for $\delta\psi_2(x)$:

$$\hat{\mathfrak{G}}_0^{-1} \delta\hat{\psi}_2(x) = -\mathbf{E}(x)\hat{\mathbf{d}}^* \left( \hat{\psi}_0 - \int dx_1 \hat{\mathcal{G}}_0(x - x_1)\mathbf{E}(x_1)\hat{\mathbf{d}}\delta\hat{\psi}_2(x_1) \right). \quad (12)$$

The total drag current can be found using Eqs. (10), (12), and (6) (the details are given in the Supplemental Material [22]).

If the bosons are in the normal phase, their current spectrum represents a set of resonances [22,23]. Instead, in the presence of the BEC, total drag current consists of two components. The first one demonstrates the resonant behavior

$$\mathbf{j}_{c1} = \frac{2n_c \mathbf{k}\tau^2}{M\hbar} \sum_\eta |\mathbf{d}_{1\eta} \cdot \mathbf{E}_0|^2 \times \left[ \frac{1}{1 + 4\tau^2(\omega - \Delta_\eta)^2} - \frac{1}{1 + 4\tau^2(\omega + \Delta_\eta)^2} \right], \quad (13)$$

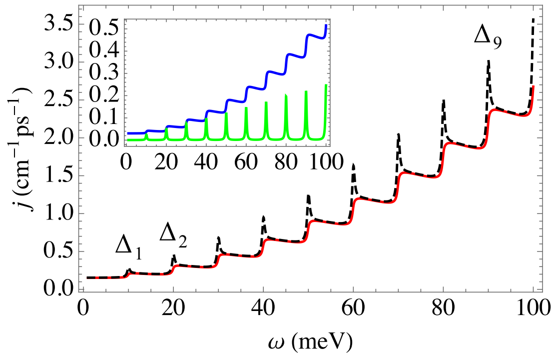


FIG. 4. Spectrum of the total current density $j_c = j_{c1} + j_{c2}$ (main plot). Two components of the current $j_{c1}$ (green curve) and $j_{c2}$ (blue curve) according to Eqs. (13) and (14) (inset) for the parameters $\Delta = 10$ meV, $M = 0.5m_0$, $n_c = 2 \times 10^{13}$ cm$^{-2}$ (red curve), $n_c = 2 \times 10^{14}$ cm$^{-2}$ (black dashed curve), $n_c = 5 \times 10^{13}$ cm$^{-2}$ (inset).

and the second current component has a steplike structure,

$$\mathbf{j}_{c2} = \frac{5\tau\mathbf{k}}{8\pi\hbar^2}\sum_\eta |\mathbf{d}_{1\eta}\cdot\mathbf{E}_0|^2 \times (\arctan[2\tau(\omega+\Delta_\eta)] + \arctan[2\tau(\omega-\Delta_\eta)]), \quad (14)$$

where $\tau$ is the particle lifetime in the excited states, which we take independent of $\eta$ for simplicity. Evidently, both the components (13) and (14) share some similar properties, they are (i) collinear with the momentum of the EMF (since $\sim\mathbf{k}$) and (ii) proportional to the intensity of the EMF, $I \sim |\mathbf{E}_0|^2$.

Figure 4 shows the spectrum of the current density and its components for the parameters taken for bosons in a solid sate (see a detailed discussion in our followup work [24]). We assumed that $\Delta_\eta$ is equidistant, $\Delta_\eta = \eta\Delta$. In general, it is not necessarily the case and one has to consider the selection rules for the internal transitions between quantum states. In calculations we use $\Delta = 10$ meV (in cold atoms $\Delta$ is $\sim$ three orders of magnitude smaller). The transition matrix element was taken to be $|\mathbf{d}_{12}\cdot\mathbf{E}_0| = 0.01\Delta$. Obviously, this value is controlled by the amplitude of the external EMF obeying the condition $|\mathbf{d}_{12}\cdot\mathbf{E}_0| \ll \Delta$ (since the perturbation theory is applicable if the external light is reasonably weak). The most interesting are the terms in the second lines in both the Eqs. (13) and (14) and the sums over the states $\eta$. Evidently, these terms are proportional to the absorption coefficient, which allows us to experimentally study the effect by measuring reflection and transmission coefficients. While Eq. (13) has resonant behavior, Eq. (14) obeys steplike behavior (see Fig. 4, inset). Summing up (13) and (14), we find the total current in the system, $\mathbf{j}_c = \mathbf{j}_{c1} + \mathbf{j}_{c2}$ (see Fig. 4, main plot). Another possibility of experimental observation of the effect is the measurement of the current itself, which can be done in a system of indirect excitons which we considered here [24]. If contact leads are appended to electron or hole layers (separately), an actual electric current can be measured.

## IV. DISCUSSION

In a Bose gas in normal phase at low temperatures $T \ll \Delta_\eta$, the particles mostly occupy the lowest energy state with energy $\varepsilon_\mathbf{p}$. If the system absorbs a quantum of the EMF with frequency $\omega = ck$, energy conservation should be fulfilled, $ck = \varepsilon_{\mathbf{p}+\mathbf{k}} + \Delta_\eta - \varepsilon_\mathbf{p}$, where $\varepsilon_{\mathbf{p}+\mathbf{k}} + \Delta_\eta$ is the energy of the excited state. Due to the smallness of the wave vector of light $\mathbf{k}$, we have $\omega \approx \Delta_\eta$ that determines the resonant structure of photon drag current [22]. In the BEC state ($\mathbf{p} = 0$) the low-lying excitation branch is the Bogoliubov soundlike quasiparticles having the dispersion $\omega_\mathbf{p}$; see Fig. 2. If we disregard the internal degrees of freedom of particles and put $\Delta_\eta = 0$, the bogolons could absorb light only if $ck = sk$. However since $c \gg s$, such processes are forbidden and the condensate itself does not feel the light pressure.

A principally different situation happens if we account for the internal degrees of freedom of the Bose particles. In this case two types of photon-mediated transitions become possible. The first type constitutes transitions caused by excitations of internal states of the Bose particle located in BEC and $ck = \varepsilon_\mathbf{k} + \Delta_\eta$, which at small wave vectors of light simplifies to $\omega = \Delta_\eta$. Such transitions correspond to the component $\mathbf{j}_{c1}$ of the current. It has resonant dependence on the frequency $\omega$.

The second type of transition processes occurs with simultaneous excitation of both condensate density oscillations (Bogoliubov sound-like quasiparticles) and the excitation of individual Bosons into the excited state of internal motion $\eta \neq 0$ with conservation energy law $\omega = \varepsilon_{\mathbf{p}+\mathbf{k}} + \Delta_\eta + \omega_{-\mathbf{p}}$. The latter transitions result in the steplike behavior of the current and can occur only in the presence of BEC.

## V. CONCLUSIONS AND OUTLOOK

We developed a microscopic theory of the radiation pressure in a general system containing Bose-Einstein–condensed particles. We found that under the pressure of an external electromagnetic field there appears a drag flux of particles constituting both the condensate and the excited states. Moreover, in the presence of the condensed phase, this current demonstrates steplike behavior for a 2D system.

This theory is universal. It can be applied to any Bose condensates which possess internal degrees of freedom. Most bosons such as cold atoms, excitons, and exciton-polaritons possess this property. Second, the response of a BEC to external radiation pressure can manifest itself in a number of other phenomena, in which the processes of light absorption play a major role, such as Raman scattering (since the scattering cross section is proportional to the imaginary part of the linear-response function), acoustoelectric effects [25], acoustic drag in condensates of hybrid particles, and detection of “dark” condensates [26].

## ACKNOWLEDGMENTS

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