Low-temperature resistance in metals without inversion center

V. P. Mineev
Univ. Grenoble Alpes, CEA, INAC, PHELIQS, GT, F-38000 Grenoble, France
and Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia

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The well known quadratic low-temperature dependence of resistance in ordinary metals habitually serves as the criterium of applicability of the Landau Fermi liquid theory to the description of electron liquid in concrete material. Such a type of behavior is determined by momentum relaxation due to the electron-electron scattering. Here I consider this problem in the metals without inversion center. It is shown that the corresponding scattering time at temperatures much smaller than the spin-orbit coupling is practically temperature independent.

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I. INTRODUCTION

Normal metal resistivity at low temperatures is usually described by the well known formula

$$\rho = \rho_0 + AT^2. \tag{1}$$

This dependence derived by Landau and Pomeranchuk [1] is determined by the relaxation time due to electron-electron scattering

$$\tau_{ee} \propto \frac{1}{T^2}, \tag{2}$$

which is faster than the case of electron-phonon interaction at temperatures much lower than the Debye temperature

$$\tau_{eph} \propto \frac{1}{T^5}. \tag{3}$$

The Landau-Pomeranchuk's relaxation is caused by the interaction between electrons. They, owing to the Pauli principle, can scatter with each other only in a narrow energy layer of the order of temperature near the Fermi surface. The realization of dependence (1) in a concrete material is commonly used as the direct indication of applicability of the Landau Fermi-liquid theory, and vice versa, a deviation from this dependence stimulates the search of some physical mechanism responsible for non-Fermi-liquid behavior.

In neutral Fermi liquid like normal ^{3}He the low-temperature relaxation due to quasiparticle-quasiparticle scattering has the same $1/T^2$ dependence that determines the temperature dependence of viscosity, thermal conductivity [2,3], and the longitudinal spin-diffusion coefficient [4]. The latter takes place even in ^{3}He polarized by an external magnetic field H where the Fermi spheres of quasiparticles with opposite spins have different radii. Nevertheless, during and after the scattering processes spin-up and spin-down quasiparticles remain at a distance of the order of temperature T from their Fermi surfaces.

Quite a different situation is realized for the spin diffusion in the direction perpendicular to an external magnetic field [5,6]. Here, the scattering processes involve all the states between two Fermi surfaces, and the relaxation time acquires the following temperature dependence

$$\tau_\perp \propto \frac{1}{(2\pi T)^2 + (gH)^2}, \tag{4}$$

where g is the ^{3}He nuclei gyromagnetic ratio. This result has been obtained [7] making use of the kinetic equation for the matrix distribution function of Fermi particles [5,8,9] in the frame of Landau Fermi-liquid theory and does not originate no matter which violation of it.

Recently there was interest in substances without the center of inversion [10]. In this case the spin-orbit coupling lifts the spin degeneracy of the electron energy bands. The initial Fermi surface splits on two Fermi surfaces for the electrons characterized by the opposite helicity that is the spin projection on the direction of the momentum dependent vector of spin-orbit coupling. The difference of the Fermi momenta of these Fermi surfaces Δk_F is determined by the value of spin-orbital coupling. Like the transverse spin relaxation in the spin-polarized ^{3}He one can expect that in noncentrosymmetric metals the relaxation time due to electro-electron interaction have the following form

$$\tau_{ee} \propto \frac{1}{(2\pi T)^2 + (v_F \Delta k_F)^2}. \tag{5}$$

Here we derive the formula for low-temperature conductivity in metals without inversion center determined by the momentum relaxation due to the electron-electron scattering. It has more complex structure than that given by the simple relation $\sigma \propto \tau_{ee}$. However, it is also determined by the combination similar to the denominator of Eq. (5). The spin-orbital band splitting $v_F \Delta k_F$ is directly expressed through the corresponding splitting of de Haas-van Alphen magnetization oscillation frequencies [11]. Determined experimentally the typical magnitude of band splitting in many noncentrosymmetric metals is of the order of hundreds Kelvin [12–14]. This is much less than the Fermi energy but comparable with the Debye temperature. Hence at temperatures significantly lower than the Debye temperature the T^2 term in the denominator of Eq. (5) is proved to be unobservable. The calculations in the paper are performed under assumption $v_F \Delta k_F \ll \varepsilon_F$.

The paper is organized as follows. Section II contains the basic notions of the electron energy spectrum and the equilibrium distribution in metals without inversion. In Sec. III the formula for the low-temperature conductivity in noncentrosymmetric metals is derived. The solution of the kinetic equation used in this derivation is obtained in the Appendix at the end of the paper. The Appendix is divided into two parts: The first contains the kinetic equation and its general properties, and the second one is devoted to the derivation of deviation of distribution function from equilibrium under an electric field. The conclusion and discussion of obtained results are given in Sec. IV.

II. EQUILIBRIUM DISTRIBUTION OF ELECTRONS

The spectrum of noninteracting electrons in a metal without inversion center is:

$$\hat{\varepsilon}(\mathbf{k})-\mu\hat{1}=\xi(\mathbf{k})\hat{1}+\boldsymbol{\gamma}(\mathbf{k})\cdot\boldsymbol{\sigma}, \tag{6}$$

where $\xi(\mathbf{k})=\varepsilon(\mathbf{k})-\mu$ denotes the spin-independent part of the spectrum measured relative to the chemical potential μ, $\hat{1}$ is the unit 2×2 matrix in the spin space, $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ are the Pauli matrices. The second term in Eq. (6) describes the spin-orbit coupling whose form depends on the specific noncentrosymmetric crystal structure. The pseudovector $\boldsymbol{\gamma}(\mathbf{k})$ satisfies $\boldsymbol{\gamma}(-\mathbf{k})=-\boldsymbol{\gamma}(\mathbf{k})$ and $g\boldsymbol{\gamma}(g^{-1}\mathbf{k})=\boldsymbol{\gamma}(\mathbf{k})$, where g is any symmetry operation in the point group $\mathcal{G}$ of the crystal. A more detailed theoretical description of noncentrosymmetric metals is presented in the paper [15]. We shall work with isotropic spectrum $\varepsilon(\mathbf{k})=\frac{k^2}{2m}$ and

$$\boldsymbol{\gamma}(\mathbf{k})=\gamma\mathbf{k}, \tag{7}$$

compatible with the 3D cubic crystal symmetry. Here γ is a constant. The calculations with anisotropic spectrum and $\boldsymbol{\gamma}(\mathbf{k})$ corresponding to other crystal symmetries are much more cumbersome but do not qualitatively change the results.

The eigenvalues of the matrix (6) are

$$\xi_{\pm}(\mathbf{k})=\xi(\mathbf{k})\pm|\boldsymbol{\gamma}(\mathbf{k})|. \tag{8}$$

There are two Fermi surfaces determined by the equations

$$\xi_{\pm}(\mathbf{k})=0 \tag{9}$$

with different Fermi momenta

$$k_{F\pm}=\mp m\gamma+\sqrt{2m\mu+(m\gamma)^2} \tag{10}$$

and common value of the Fermi velocity

$$\mathbf{v}_F=\frac{\partial(\varepsilon\pm\gamma k)}{\partial\mathbf{k}}|_{k=k_{F\pm}}=\hat{\mathbf{k}}\sqrt{\frac{2\mu}{m}+\gamma^2}, \tag{11}$$

here $\hat{\mathbf{k}}$ is the unit vector along momentum $\mathbf{k}$.

The matrix of equilibrium electron distribution function is

$$\hat{n}_0=\frac{n_++n_-}{2}\hat{1}+\frac{n_+-n_-}{2|\boldsymbol{\gamma}|}\boldsymbol{\gamma}\cdot\boldsymbol{\sigma}, \tag{12}$$

where

$$n_{\pm}=\frac{1}{e^{\xi_{\pm}}+1} \tag{13}$$

are the Fermi functions. Near the corresponding Fermi surfaces the dispersion laws are

$$\xi_{\pm}\approx v_F(k-k_{F\pm})=\varepsilon-\mu_{\pm}, \tag{14}$$

with

$$\varepsilon=v_Fk,\quad \mu_{\pm}=v_Fk_{F\pm},\quad \mu_+-\mu_-=-2mv_F\gamma. \tag{15}$$

III. CONDUCTIVITY

The matrix of nonequilibrium distribution function is the sum of the scalar and the spinor parts

$$\hat{n}=\tfrac{1}{2}[f\ \hat{1}+\mathbf{g}\cdot\boldsymbol{\sigma}]. \tag{16}$$

The current density is

$$\begin{aligned}\mathbf{j}&=e\,Tr\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{\partial\hat{\varepsilon}(\mathbf{k}_1)}{\partial\mathbf{k}_1}\hat{n}=\frac{1}{2}e\,Tr\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\left(\frac{\partial\xi(\mathbf{k}_1)}{\partial\mathbf{k}_1}+\frac{\partial\boldsymbol{\gamma}(\mathbf{k}_1)\cdot\boldsymbol{\sigma}}{\partial\mathbf{k}_1}\right)(\delta f(\mathbf{k}_1)\hat{1}+\delta\mathbf{g}(\mathbf{k}_1)\cdot\boldsymbol{\sigma})\\&=e\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\left(\frac{\mathbf{k}_1}{m}\delta f(\mathbf{k}_1)+\gamma\delta\mathbf{g}(\mathbf{k}_1)\right).\end{aligned} \tag{17}$$

Here,

$$\delta f=f-(n_++n_-), \tag{18}$$

$$\delta\mathbf{g}=\mathbf{g}-\frac{\boldsymbol{\gamma}}{|\boldsymbol{\gamma}|}(n_+-n_-) \tag{19}$$

are the deviations of scalar and spinor distribution functions from the equilibrium distribution function. The derivation of them from the kinetic equation performed under assumption $v_F\Delta k_F\ll\varepsilon_F$ is presented in Appendix B. They are

$$\delta f(\mathbf{k}_1)=-\frac{4e}{\pi\tilde{W}_0m^3I_1(\varepsilon_1)}\mathbf{E}\cdot\mathbf{v}_1\frac{\partial(n_{1+}+n_{1-})}{\partial\varepsilon_1}, \tag{20}$$

$$\delta\mathbf{g}(\mathbf{k}_1)=-\frac{4e}{\pi m^3[\tilde{W}_0I_1(\varepsilon_1)-W_0I_2]}\left[\mathbf{E}\cdot\mathbf{v}_1\frac{\partial(n_{1+}-n_{1-})}{\partial\varepsilon_1}\hat{\mathbf{k}}_1+\frac{(n_{1+}-n_{1-})}{|\mathbf{k}_1|}(\mathbf{E}-(\mathbf{E}\cdot\hat{\mathbf{k}}_1)\hat{\mathbf{k}}_1)\right], \tag{21}$$

where the integrals $I_1(\varepsilon_1)$, I_2 are given by Eqs. (B21) and (B24). Substituting these expressions in Eq. (17) we obtain

$$\mathbf{j}=\frac{4e^2}{3\pi m^3}\left\{v_F\frac{N_{0+}k_{F+}+N_{0-}k_{F-}}{m\tilde{W}_0 I_1(\mu_+)}+\gamma\left[\frac{N_{0+}v_F}{[\tilde{W}_0 I_1(\mu_+)-W_0 I_2]}-\frac{N_{0-}v_F}{[\tilde{W}_0 I_1(\mu_-)-W_0 I_2]}-2\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{n_{1+}-n_{1-}}{k_1[\tilde{W}_0 I_1(\varepsilon_1)-W_0 I_2]}\right]\right\}\mathbf{E}, \tag{22}$$

where $N_{0\pm}=\sqrt{2m^3\mu_\pm}/2\pi^2$ are the density of states at the corresponding Fermi momenta. Thus, the conductivity is

$$\sigma=\frac{4e^2}{3\pi m^3}\left\{v_F\frac{N_{0+}k_{F+}+N_{0-}k_{F-}}{m\tilde{W}_0 I_1(\mu_+)}+\gamma\left[\frac{N_{0+}v_F}{[\tilde{W}_0 I_1(\mu_+)-W_0 I_2]}-\frac{N_{0-}v_F}{[\tilde{W}_0 I_1(\mu_-)-W_0 I_2]}-2\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{n_{1+}-n_{1-}}{k_1[\tilde{W}_0 I_1(\varepsilon_1)-W_0 I_2]}\right]\right\}. \tag{23}$$

It consists of two parts: The first line corresponds to the linear in field deviation of the scalar part of the distribution function whereas the second line originates from the deviation of the spinor part of the distribution function.

In the absence of spin-orbit interaction $\gamma=0$

$$I_1(\mu_+=\mu_-)=2(\pi T)^2,\quad I_2=0, \tag{24}$$

we come to the Landau-Pomeranchuk temperature dependence of conductivity

$$\sigma=\frac{4e^2v_F^2N_0}{3\pi^3m^3\tilde{W}_0}\frac{1}{T^2}. \tag{25}$$

On the other hand at finite Fermi radii splitting $|\mu_+-\mu_-|\gg T$ the integrals

$$I_1(\mu_+)=I_1(\mu_-)\propto I_2\propto(\mu_+-\mu_-)^2 \tag{26}$$

are practically temperature independent and the conductivity is determined by the temperature independent Eq. (23).

IV. CONCLUDING REMARKS

We have derived the low-temperature conductivity in the metals without inversion center determined by the momentum relaxation due to electron-electron scattering. Unlike the center-symmetric metals the conductivity in the materials with the space parity violation proved to be temperature independent so long as $T\ll v_F\Delta k_F\ll\varepsilon_F$. The physical reason for this behavior is that in noncentrosymmetric metals the processes of relaxation are determined by all the electronic states in between the Fermi surfaces split by the spin-orbit coupling.

At $\gamma=0$ the Landau-Pomeranchuk electron-electron scattering rate determining the conductivity Eq. (25) is

$$\frac{1}{\tau_{ee}}\sim\frac{V^2}{\varepsilon_F^2}\frac{T^2}{\varepsilon_F}, \tag{27}$$

where V is the amplitude of screened short range potential of the electron-electron interaction. At low temperatures, when the quasiparticles energy ξ counted from the Fermi energy is of the order of temperature, the inequality $1/\tau_{ee}\propto\xi^2/\varepsilon_F\ll\xi$ serves as the base of the Landau Fermi liquid theory.

The corresponding scattering rate at finite Fermi radii splitting $|\mu_+-\mu_-|\gg T$ determining the conductivity Eq. (23) is

$$\frac{1}{\tau_{ee}}\sim\frac{V^2}{\varepsilon_F^2}\frac{(\mu_+-\mu_-)^2}{\varepsilon_F}. \tag{28}$$

So, the condition $1/\tau_{ee}\propto(\mu_+-\mu_-)^2/\varepsilon_F\ll\xi$ is fulfilled for not arbitrary small ξ values. Thus, the problem of a lifetime of quasiparticles interacting by the short range potential in metals without inversion center demands further investigation.

The presented calculations made for the noncentrosymmetric metals with cubic symmetry. In this case the spin degeneracy is completely lifted, and the low-temperature dependence of resistivity due to electron-electron scattering is

$$\rho_{ee}-\rho_{ee}(T=0)\propto T^2. \tag{29}$$

In less symmetric materials the degeneracy of states is also lifted everywhere besides isolated points where the two split Fermi surfaces touch each other as this is the case in the metals with tetragonal or tetrahedral symmetry (see Ref. [15]). This residual degeneracy can lead to the negligibly small temperature dependent correction to the T^2 temperature dependence of resistivity originating from the electron-electron scattering.

In a metal without inversion center the total resistivity at zero temperature consists of two parts originating from resistivity due to electron-electron scattering and due to electron scattering on impurities

$$\rho=\rho_{ee}(T=0)+\rho_{\rm imp}. \tag{30}$$

The resistivity due impurity scattering is proportional to impurity concentration $\rho_{\rm imp}\propto n_{\rm imp}$. Thus, the zero temperature resistivity due to electron-electron scattering $\rho_{ee}(T=0)$ can be experimentally found by the measuring of low temperature resistivity $\rho(n_{\rm imp})$ at several finite impurity concentrations with subsequent taking the formal limit

$$\rho_{ee}(T=0)=\rho(n_{\rm imp}\to 0). \tag{31}$$

APPENDIX A: KINETIC EQUATION

The kinetic equation for the matrix distribution function of electrons derived by V. P. Silin [8] is

$$\frac{\partial\hat{n}_1}{\partial t}-i[\hat{\varepsilon}_1,\hat{n}_1]+\frac{1}{2}\left(\frac{\partial\hat{\varepsilon}_1}{\partial\mathbf{k}_1}\frac{\partial\hat{n}_1}{\partial\mathbf{r}}+\frac{\partial\hat{n}_1}{\partial\mathbf{r}}\frac{\partial\hat{\varepsilon}_1}{\partial\mathbf{k}_1}\right)-\frac{1}{2}\left(\frac{\partial\hat{\varepsilon}_1}{\partial\mathbf{r}}\frac{\partial\hat{n}_1}{\partial\mathbf{k}_1}+\frac{\partial\hat{n}_1}{\partial\mathbf{k}_1}\frac{\partial\hat{\varepsilon}_1}{\partial\mathbf{r}}\right)=\hat{I}, \tag{A1}$$

where $[\hat{\varepsilon}_1,\hat{n}_1]$ is the commutator of $\hat{\varepsilon}_1=\hat{\varepsilon}(\mathbf{k}_1)$ and $\hat{n}_1=\hat{n}(\mathbf{k}_1)$. We put $\hbar=1$. The integral term for particle-particle collisions in the Born approximation was derived by Silin (Ref. [9]) and Jeon and Mullin (Ref. [5]). Including the

Umklapp processes of scattering it is

$$\hat{I} = 2\pi \int d^3\mathbf{k}' \frac{d^3\mathbf{k}''}{(2\pi)^3} \frac{d^3\mathbf{k}_2}{(2\pi)^3} \sum_{\mathbf{m}} \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'') \times \delta\left(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}' - \mathbf{k}'' - \frac{2\pi\mathbf{m}}{a}\right)\hat{F}, \tag{A2}$$

where $\frac{2\pi\mathbf{m}}{a}$ is a vector of reciprocal lattice,

$$\begin{aligned}\hat{F} = {} & \tfrac{1}{2}W_1\{[\hat{n}', (\hat{1} - \hat{n}_1)]_+ Tr((\hat{1} - \hat{n}_2)n'') \\ & - [(\hat{1} - \hat{n}'), \hat{n}_1]_+ Tr(\hat{n}_2(\hat{1} - \hat{n}''))\} \\ & + \tfrac{1}{2}W_2\{[\hat{n}'(\hat{1} - \hat{n}_2)\hat{n}'', (\hat{1} - \hat{n}_1)]_+ \\ & - [(\hat{1} - \hat{n}')\hat{n}_2(\hat{1} - \hat{n}''), \hat{n}_1]_+\}.\end{aligned} \tag{A3}$$

Here $[\hat{A}, \hat{B}]_+$ means the anticommutator, and the following designations $\hat{n}' = \hat{n}(\mathbf{k}')$, $\varepsilon' = \varepsilon(\mathbf{k}')$ etc. are introduced. In the isotropic Fermi liquid like ^{3}He $W_1 = [V(|\mathbf{k}_1 - \mathbf{k}'|)]^2$, $W_2 = -V(|\mathbf{k}_1 - \mathbf{k}'|)V(|\mathbf{k}_1 - \mathbf{k}''|)$ are expressed trough the Fourier transform of the quasiparticles potential of interaction. The latter in concrete metal is unknown, and due to charge screening we put them as the constants: $W_1 = W_0/2$, $W_2 = -W_0/2$.

The matrix of equilibrium electron distribution function is

$$\hat{n}_0 = \hat{P}_+ n_+ + \hat{P}_- n_-, \tag{A4}$$

where

$$\hat{P}_\pm(\mathbf{k}) = \frac{1}{2} \pm \frac{\boldsymbol{\gamma}(\mathbf{k}) \cdot \boldsymbol{\sigma}}{2|\boldsymbol{\gamma}(\mathbf{k})|} \tag{A5}$$

are the projection operators such that

$$\hat{P}_\pm^2 = \hat{P}_\pm, \quad \hat{P}_+\hat{P}_- = \hat{P}_-\hat{P}_+ = 0, \quad \hat{P}_+ + \hat{P}_- = 1.$$

The substitution of equilibrium distribution Eq. (A4) causes the collision integral to vanish. Indeed, substituting Eq. (A4) to Eq. (A3) we obtain

$$\begin{aligned}\hat{F} = \frac{1}{4}W_0 \sum_{\lambda,\mu,\nu,\tau} & \{\left[n'_\lambda(1 - n_{1\mu})(1 - n_{2\nu})n''_\tau - (1 - n'_\lambda)n_{1\mu}n_{2\nu}(1 - n''_{2\tau})\right]\left(\hat{P}'_\lambda\hat{P}_{1\mu} + \hat{P}_{1\mu}\hat{P}'_\lambda\right)Tr\left(\hat{P}_{2\nu}\hat{P}''_\tau\right) \\ & - \left[n'_\lambda(1 - n_{2\mu})n''_\nu(1 - n_{1\tau}) - (1 - n'_\lambda)n_{2\mu}(1 - n''_\nu)n_{1\tau}\right]\left(\hat{P}'_\lambda\hat{P}_{2\mu}\hat{P}''_\nu\hat{P}_{1\tau} + \hat{P}_{1\tau}\hat{P}'_\lambda\hat{P}_{2\mu}\hat{P}''_\nu\right)\}\end{aligned} \tag{A6}$$

The substitution of this expression into Eq. (A2) yields zero because the combination in square parenthesis $[n'_\lambda(1 - n_{1\mu})(1 - n_{2\nu})n''_\tau - (1 - n'_\lambda)n_{1\mu}n_{2\nu}(1 - n''_{2\tau})]$ is equal to zero at arbitrary $\lambda, \mu, \nu, \tau = \pm$ due to the energy conservation $\varepsilon_1 + \varepsilon_2 = \varepsilon' + \varepsilon''$.

The conservation laws of total particles number $\int \frac{d^3\mathbf{k}_1}{(2\pi)^3} Tr(\hat{n}_1)$, spin $\int \frac{d^3\mathbf{k}_1}{(2\pi)^3} Tr(\boldsymbol{\sigma}\hat{n}_1)$, momentum $\int \frac{d^3\mathbf{k}_1}{(2\pi)^3} Tr(\mathbf{k}_1\hat{n}_1)$, and energy $\int \frac{d^3\mathbf{k}_1}{(2\pi)^3} Tr(\hat{\varepsilon}(\mathbf{k}_1)\hat{n}_1)$ are satisfied. The check of their validity using Eqs. (A1), (A2), and (A3) belongs to that category of calculations which are more easily done independently than by following their development.

APPENDIX B: DEVIATIONS OF DISTRIBUTION FUNCTIONS FROM THE EQUILIBRIUM DISTRIBUTION

In a constant electric field the stationary kinetic equation acquires the form

$$\left(e\mathbf{E} \cdot \frac{\partial}{\partial \mathbf{k}_1}\right)\hat{n}_1 = \hat{I}. \tag{B1}$$

The matrix Fermi distribution function is convenient to represent as the sum of the scalar and the spinor parts

$$\hat{n} = \frac{1}{2}[f\ \hat{1} + \mathbf{g} \cdot \boldsymbol{\sigma}]. \tag{B2}$$

The kinetic equations for this function are

$$\left(e\mathbf{E} \cdot \frac{\partial}{\partial \mathbf{k}_1}\right) f_1 = Tr\hat{I}, \tag{B3}$$

$$\left(e\mathbf{E} \cdot \frac{\partial}{\partial \mathbf{k}_1}\right)\mathbf{g}_1 = Tr(\boldsymbol{\sigma}\hat{I}), \tag{B4}$$

where (see Ref. [5])

$$\begin{aligned}Tr\hat{I} = {} & \frac{\pi}{2}W_0 \int d^3\mathbf{k}' \frac{d^3\mathbf{k}''}{(2\pi)^3} \frac{d^3\mathbf{k}_2}{(2\pi)^3} \sum_{\mathbf{m}} \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'')\delta\left(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}' - \mathbf{k}'' - \frac{2\pi\mathbf{m}}{a}\right) \\ & \times \left\{\left[f' - \frac{1}{2}(f_1 f' + \mathbf{g}_1 \cdot \mathbf{g}')\right]\left[f'' - \frac{1}{2}(f_2 f'' + \mathbf{g}_2 \cdot \mathbf{g}'')\right] - \left[f_1 - \frac{1}{2}(f_1 f' + \mathbf{g}_1 \cdot \mathbf{g}')\right]\left[f_2 - \frac{1}{2}(f_2 f'' + \mathbf{g}_2 \cdot \mathbf{g}'')\right]\right. \\ & \left. - \left[\mathbf{g}' - \frac{1}{2}(f'\mathbf{g}_1 + f_1\mathbf{g}')\right] \cdot \left[\mathbf{g}'' - \frac{1}{2}(f''\mathbf{g}_2 + f_2\mathbf{g}'')\right] + \left[\mathbf{g}_1 - \frac{1}{2}(f'\mathbf{g}_1 + f_1\mathbf{g}')\right] \cdot \left[\mathbf{g}_2 - \frac{1}{2}(f''\mathbf{g}_2 + f_2\mathbf{g}'')\right]\right\},\end{aligned} \tag{B5}$$

$$Tr(\sigma\hat{I}) = \frac{\pi}{2}W_0\int d^3\mathbf{k}'\frac{d^3\mathbf{k}''}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\sum_{\mathbf{m}}\delta(\varepsilon_1+\varepsilon_2-\varepsilon'-\varepsilon'')\delta\left(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}'-\mathbf{k}''-\frac{2\pi\mathbf{m}}{a}\right)$$
$$\times\left\{\left[f''-\frac{1}{2}(f_2f''+\mathbf{g}_2\cdot\mathbf{g}'')\right]\left[\mathbf{g}'-\frac{1}{2}(f'\mathbf{g}_1+f_1\mathbf{g}')\right]-\left[f_2-\frac{1}{2}(f_2f''+\mathbf{g}_2\cdot\mathbf{g}'')\right]\left[\mathbf{g}_1-\frac{1}{2}(f'\mathbf{g}_1+f_1\cdot\mathbf{g}')\right]\right.$$
$$-\left[f'-\frac{1}{2}(f_1f'+\mathbf{g}_1\cdot\mathbf{g}')\right]\left[\mathbf{g}''-\frac{1}{2}(f''\mathbf{g}_2+f_2\mathbf{g}'')\right]+\left[f_1-\frac{1}{2}(f_1f'+\mathbf{g}_1\cdot\mathbf{g}')\right]\left[\mathbf{g}_2-\frac{1}{2}(f''\mathbf{g}_2+f_2\mathbf{g}'')\right]$$
$$\left.+\frac{1}{2}[(\mathbf{g}_1\cdot\mathbf{g}')(\mathbf{g}_2-\mathbf{g}'')-(\mathbf{g}_2\cdot\mathbf{g}'')(\mathbf{g}_1-\mathbf{g}')+(\mathbf{g}'\cdot\mathbf{g}'')(\mathbf{g}_1-\mathbf{g}_2)]\right\}. \tag{B6}$$

In the process of derivation of these equations it was important to keep all the integrations in the collision integral which allows us to demonstrate the vanishing of its imaginary part due to symmetry $\mathbf{k}'\leftrightarrow\mathbf{k}''$.

Now, integrating over $d^3\mathbf{k}'$ we liquidate the δ function of momenta. Following usual linearization procedure we substitute in the left hand sides of Eqs. (B3) and (B4) the equilibrium distribution Eq. (12), and on the right hand side we leave only the terms linear in deviation from equilibrium distribution. Thus, we obtain the linear integral equations for the functions

$$\delta f = f-(n_++n_-), \tag{B7}$$

$$\delta\mathbf{g} = \mathbf{g}-\frac{\boldsymbol{\gamma}}{|\boldsymbol{\gamma}|}(n_+-n_-). \tag{B8}$$

To establish the parametrical dependence of relaxation time we don't need, however, to solve this difficult problem. For this purpose it is enough to keep only the terms with δf_1 and $\delta\mathbf{g}_1$ neglecting other terms containing $\delta f'$, $\delta\mathbf{g}'$ etc. This is a sort of relaxation time approximation. Thus, we come to the following equations

$$e\mathbf{E}\cdot\mathbf{v}_1\frac{\partial(n_{1+}+n_{1-})}{\partial\varepsilon_1} = -\frac{\pi}{4}W_0\int\frac{d\mathbf{k}''}{(2\pi)^3}\frac{d\mathbf{k}_2}{(2\pi)^3}\delta(\varepsilon_1+\varepsilon_2-\varepsilon'-\varepsilon'')$$
$$\times\left\{\left[f_0'f_0''\left(1-\frac{1}{2}f_{02}\right)+\left(1-\frac{1}{2}f_0'\right)\left(1-\frac{1}{2}f_0''\right)2f_{02}\right]\delta f_1-\mathbf{g}_{02}\cdot\mathbf{g}_0''\delta f_1+(f_0''-f_{02})\mathbf{g}_0'\cdot\delta\mathbf{g}_1\right.$$
$$\left.-(\mathbf{g}_0''-\mathbf{g}_{02})\cdot\mathbf{g}_0'\delta f_1-(2-f_0'-f_0'')\mathbf{g}_{02}\cdot\delta\mathbf{g}_1-(f_0'-f_{02})\mathbf{g}_0''\cdot\delta\mathbf{g}_1\right\}, \tag{B9}$$

$$e\mathbf{E}\cdot\mathbf{v}_1\frac{\partial(n_{1+}-n_{1-})}{\partial\varepsilon_1}\frac{\mathbf{k}_1}{|\mathbf{k}_1|}+\frac{(n_{1+}-n_{1-})}{|\mathbf{k}_1|}\left(e\mathbf{E}-\frac{(e\mathbf{E}\cdot\mathbf{k}_1)\mathbf{k}_1}{\mathbf{k}_1^2}\right)$$
$$=-\frac{\pi}{4}W_0\int\frac{d\mathbf{k}''}{(2\pi)^3}\frac{d\mathbf{k}_2}{(2\pi)^3}\delta(\varepsilon_1+\varepsilon_2-\varepsilon'-\varepsilon'')\left\{\left[f_0'f_0''\left(1-\frac{1}{2}f_{02}\right)+\left(1-\frac{1}{2}f_0'\right)\left(1-\frac{1}{2}f_0''\right)2f_{02}\right]\delta\mathbf{g}_1\right.$$
$$\left.-\mathbf{g}_{02}\cdot\mathbf{g}_0''\delta\mathbf{g}_1+(f_0''-f_{02})\mathbf{g}_0'\delta f_1-(\mathbf{g}_0''-\mathbf{g}_{02})f'\delta f_1-(2\mathbf{g}_{02}-f_0''\mathbf{g}_{02}-f_2\mathbf{g}_0'')\delta f_1-(\mathbf{g}_0'-\mathbf{g}_{02})\cdot\mathbf{g}_0''\delta\mathbf{g}_1\right\}. \tag{B10}$$

Several terms in Eqs. (B9) and (B10) vanishes at integration over directions of momenta. Other subintegrand terms like $\propto(f''-f_2)\mathbf{g}''\cdot\delta\mathbf{g}_1$ cancel out due to antisymmetry in respect of interchange of arguments $\varepsilon_2\leftrightarrow\varepsilon''$. We obtain

$$e\mathbf{E}\cdot\mathbf{v}_1\frac{\partial(n_{1+}+n_{1-})}{\partial\varepsilon_1} = -\frac{\pi}{4}W_0\int\frac{d\mathbf{k}''}{(2\pi)^3}\frac{d\mathbf{k}_2}{(2\pi)^3}\delta(\varepsilon_1+\varepsilon_2-\varepsilon'-\varepsilon'')\left\{f_0'f_0''\left(1-\frac{1}{2}f_{02}\right)+\left(1-\frac{1}{2}f_0'\right)\left(1-\frac{1}{2}f_0''\right)2f_{02}\right\}\delta f_1 \tag{B11}$$

$$e\mathbf{E}\cdot\mathbf{v}_1\frac{\partial(n_{1+}-n_{1-})}{\partial\varepsilon_1}\frac{\mathbf{k}_1}{|\mathbf{k}_1|}+\frac{(n_{1+}-n_{1-})}{|\mathbf{k}_1|}\left(e\mathbf{E}-\frac{(e\mathbf{E}\cdot\mathbf{k}_1)\mathbf{k}_1}{\mathbf{k}_1^2}\right) = -\frac{\pi}{4}W_0\int\frac{d\mathbf{k}''}{(2\pi)^3}\frac{d\mathbf{k}_2}{(2\pi)^3}\delta(\varepsilon_1+\varepsilon_2-\varepsilon'-\varepsilon'')$$
$$\times\left\{f_0'f_0''\left(1-\frac{1}{2}f_{02}\right)+\left(1-\frac{1}{2}f_0'\right)\left(1-\frac{1}{2}f_0''\right)2f_{02}-\mathbf{g}_0'\cdot\mathbf{g}_0''\right\}\delta\mathbf{g}_1. \tag{B12}$$

Following the procedure of Ref. [3], reproduced in Ref. [16] in a somewhat different manner, we reexpress the integration over $\mathbf{k}''$ and $\mathbf{k}_2$ as

$$d^3\mathbf{k}''d^3\mathbf{k}_2\approx\frac{m^3}{2\cos(\theta/2)}d\varepsilon''d\varepsilon_2d\varepsilon'\sin\theta d\theta d\phi d\phi_2. \tag{B13}$$

Here θ is the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$, ϕ is the azimuthal angle of $\mathbf{k}_2$ around direction $\mathbf{k}_1$, and ϕ_2 is the angle between the planes $(\mathbf{k}_1,\mathbf{k}_2)$ and $(\mathbf{k}',\mathbf{k}'')$. Due to the Fermi surfaces separation this formula is valid within an accuracy of the terms of the order of

$\gamma k_F/\mu$. Taking into account the δ function of energies we can integrate over ε' and using the equality $\varepsilon' = \varepsilon_1 + \varepsilon_2 - \varepsilon''$, we come to the equations

$$e\mathbf{E}\cdot\mathbf{v}_1\frac{\partial(n_{1+}+n_{1-})}{\partial\varepsilon_1} = -\frac{\pi}{4}\tilde{W}_0 m^3\int d\varepsilon'' d\varepsilon_2\left\{f_0' f_0''\left(1-\frac{1}{2}f_{02}\right)+\left(1-\frac{1}{2}f_0'\right)\left(1-\frac{1}{2}f_0''\right)2f_{02}\right\}\delta f_1, \tag{B14}$$

$$\begin{aligned} e\mathbf{E}\cdot\mathbf{v}_1\frac{\partial(n_{1+}-n_{1-})}{\partial\varepsilon_1}\frac{\mathbf{k}_1}{|\mathbf{k}_1|} &+ \frac{(n_{1+}-n_{1-})}{|\mathbf{k}_1|}\left(e\mathbf{E}-\frac{(e\mathbf{E}\cdot\mathbf{k}_1)\mathbf{k}_1}{\mathbf{k}_1^2}\right) \\ &= -\frac{\pi}{4}\tilde{W}_0 m^3\int d\varepsilon'' d\varepsilon_2\left\{f_0' f_0''\left(1-\frac{1}{2}f_{02}\right)+\left(1-\frac{1}{2}f_0'\right)\left(1-\frac{1}{2}f_0''\right)2f_{02}\right\}\delta\mathbf{g}_1 \\ &+ \frac{\pi}{4}W_0 m^3\int d\varepsilon'' d\varepsilon_2\int\frac{\cos\frac{\theta}{2}d\theta d\phi d\phi_2}{(2\pi)^6}(\mathbf{g}_0'\cdot\mathbf{g}_0'')\delta\mathbf{g}_1, \end{aligned} \tag{B15}$$

where $\tilde{W}_0 = W_0\int\cos\frac{\theta}{2}d\theta d\phi d\phi_2/(2\pi)^6$. These equations contain the integral

$$\begin{aligned} I_1(\varepsilon_1) &= \int d\varepsilon'' d\varepsilon_2\left\{f_0' f_0''\left(1-\frac{1}{2}f_{02}\right)+\left(1-\frac{1}{2}f_0'\right)\left(1-\frac{1}{2}f_0''\right)2f_{02}\right\} \\ &= \frac{1}{2}\sum_{\lambda,\mu,\nu}\int d\varepsilon'' d\varepsilon_2\{n_\lambda' n_\mu''(1-n_{2\nu})+(1-n_\lambda')(1-n_\mu'')n_{2\nu}\}, \end{aligned} \tag{B16}$$

where the indices $\lambda, \mu, \nu = \pm$.

Let us calculate one particular term in this sum

$$\begin{aligned} \frac{1}{2}\int d\varepsilon'' d\varepsilon_2 n_+' n_+''(1-n_{2+}) &= \frac{1}{2}T^2\int dxdy\frac{1}{e^{t_++x-y}+1}\frac{1}{e^y+1}\frac{1}{e^{-x}+1} \\ &= \frac{1}{2}T^2\int dx\frac{t_++x}{e^{t_++x}+1}\frac{1}{e^{-x}+1} = \frac{1}{2}T^2\frac{\pi^2+t_+^2}{2}n(t_+). \end{aligned} \tag{B17}$$

Here

$$x = \frac{\varepsilon_2-\mu_+}{T},\quad y = \frac{\varepsilon''-\mu_+}{T},\quad t_+ = \frac{\varepsilon_1-\mu_+}{T}, \tag{B18}$$

and

$$n(t_+) = \frac{1}{e^{t_+}+1} \tag{B19}$$

is the Fermi distribution function. Similar integration in all the other terms yields

$$\begin{aligned} I_1(\varepsilon_1) &= \frac{1}{2}\sum_{\lambda,\mu,\nu}\int d\varepsilon'' d\varepsilon_2\{n_\lambda' n_\mu''(1-n_{2\nu})+(1-n_\lambda' 0(1-n_\mu'')n_{2\nu}\} \\ &= \frac{1}{4}T^2\{[\pi^2+t_+^2]n(t_+)+[\pi^2+(t_+-\kappa)^2]n(t_+-\kappa)+[\pi^2+(t_++\kappa)^2]n(t_++\kappa)+[\pi^2+t_-^2]n(t_-)+[\pi^2+t_+^2]n(t_+) \\ &\quad +[\pi^2+(t_--\kappa)^2]n(t_--\kappa)+[\pi^2+(t_-+\kappa)^2]n(t_-+\kappa)+[\pi^2+t_-^2]n(t_-)\}+(t_\pm\to-t_\pm,\kappa\to-\kappa) \\ &= \frac{1}{4}T^2\{[\pi^2+t_+^2]+[\pi^2+(t_+-\kappa)^2]+[\pi^2+(t_++\kappa)^2]+[\pi^2+t_-^2] \\ &\quad +[\pi^2+t_+^2]+[\pi^2+(t_--\kappa)^2]+[\pi^2+(t_-+\kappa)^2]+[\pi^2+t_-^2]\}, \end{aligned} \tag{B20}$$

where we used the notation $\kappa = (\mu_+-\mu_-)/T$. Finally we obtain

$$I_1(\varepsilon_1) = 2(\pi T)^2+(\varepsilon_1-\mu_+)^2+(\varepsilon_1-\mu_-)^2+(\mu_+-\mu_-)^2. \tag{B21}$$

Thus, the deviation of the scalar part of distribution function from its equilibrium value is

$$\delta f(\mathbf{k}_1) = -\frac{4e}{\pi\tilde{W}_0 m^3 I_1(\varepsilon_1)}\mathbf{E}\cdot\mathbf{v}_1\frac{\partial(n_{1+}+n_{1-})}{\partial\varepsilon_1}. \tag{B22}$$

The second integral in Eq. (B15) is

$$I_2 = \int d\varepsilon'' d\varepsilon_2 \int \frac{\cos\frac{\theta}{2} d\theta d\phi d\phi_2}{(2\pi)^6} (\mathbf{g}'_0 \cdot \mathbf{g}''_0)$$
$$= \int d\varepsilon'' d\varepsilon_2 \int \frac{\cos\frac{\theta}{2} d\theta d\phi d\phi_2}{(2\pi)^6} \frac{\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}'' - 2\pi\mathbf{m}/a}{|\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}'' - 2\pi\mathbf{m}/a|} \cdot \frac{\mathbf{k}''}{|\mathbf{k}''|} (n'_{0+} - n'_{0-})(n''_{0+} - n''_{0-}). \tag{B23}$$

The analytic calculation of this integral is impossible. However, at $\mu^+ \to \mu_-$ it tends to zero as $\propto(\mu^+ - \mu_-)^2$. We will treat this integral as energy and momentum independent constant

$$I_2 \approx (\mu^+ - \mu_-)^2. \tag{B24}$$

Thus, the deviation of the spinor part of the distribution function from its equilibrium value is

$$\delta\mathbf{g}(\mathbf{k}_1) = -\frac{4e}{\pi m^3[\tilde{W}_0 I_1(\varepsilon_1) - W_0 I_2]} \left[\mathbf{E} \cdot \mathbf{v}_1 \frac{\partial(n_{1+} - n_{1-})}{\partial\varepsilon_1} \hat{\mathbf{k}}_1 + \frac{(n_{1+} - n_{1-})}{|\mathbf{k}_1|} (\mathbf{E} - (\mathbf{E} \cdot \hat{\mathbf{k}}_1)\hat{\mathbf{k}}_1) \right], \tag{B25}$$

where $\hat{\mathbf{k}}_1 = \frac{\mathbf{k}_1}{|\mathbf{k}_1|}$.

[1] L. Landau and I. Pomeranchuk, Zh. Eksp. Teor. Fiz. **7**, 379 (1937) [Phys. Z. Sowietunion **10**, 649 (1936)].

[2] I. Y. Pomeranchuk, Zh. Eksp. Teor. Fiz. **20**, 919 (1950).

[3] A. A. Abrikosov and I. M. Khalatnikov, Rep. Prog. Phys. **22**, 329 (1959).

[4] D. Hone, Phys. Rev. **121**, 669 (1961).

[5] J. W. Jeon and W. J. Mullin, J. Phys. (France) **49**, 1691 (1988).

[6] A. E. Meyerovich, Spin polarized Phases of ^{3}He, in *Helium Three*, edited by W. P. Halperin and L. P. Pitaevskii (Elsevier Science Publishers, Amsterdam, 1990), p. 757.

[7] V. P. Mineev, Phys. Rev. B **69**, 144429 (2004).

[8] V. P. Silin, Zh. Eksp. Teor. Fiz. **33**, 1227 (1957) [Sov. Phys. JETP **6**, 945 (1958)].

[9] V. P. Silin, *Vvedenie v Kineticheskuyu Teoriu Gasov* (Nauka, Moscow, 1971).

[10] E. Bauer and M. Sigrist (eds.), *Non-Centrosymmetric Superconductors: Introduction and Overview*, in Lecture Notes in Physics, Vol. 847 (Springer, Heidelberg, 2012).

[11] V. P. Mineev and K. V. Samokhin, Phys. Rev. B **72**, 212504 (2005).

[12] T. Terashima, M. Kimata, S. Uji, T. Sugawara, N. Kimura, H. Aoki, and H. Harima, Phys. Rev. B **78**, 205107 (2008).

[13] Y. Onuki, A. Nakamura, T. Uejo, A. Teruya, M. Hedo, T. Nakama, F. Honda, and H. Harima, J. Phys. Soc. Jpn. **83**, 061018 (2014).

[14] A. Maurya, H. Harima, A. Nakamura, Y. Shimizu, Y. Homma, DeXin Li, F. Honda, Y. J. Sato, and D. Aoki, J. Phys. Soc. Jpn. **87**, 044703 (2018).

[15] V. P. Mineev and M. Sigrist, "Introduction to Superconductivity in Metals without Inversion Center," p. 129 in Ref. [10].

[16] G. Beym and C. Pethick, in *Physics of Liquid and Solid Helium*, edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1978), Vol. 2, Chap. 2.