Rapid Communications

Evolution of the transmission phase through a Coulomb-blockaded Majorana wire

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We present a study of the transmission of electrons through a semiconductor quantum wire with strong spin-orbit coupling in proximity to an s-wave superconductor, which is Coulomb blockaded. Such a system supports Majorana zero modes in the presence of an external magnetic field. Without superconductivity, phase lapses are expected to occur in the transmission phase, and we find that they disappear when a topological superconducting phase is stabilized. We express tunneling through the nanowire with the help of effective matrix elements, which depend on both the fermion parity of the wire and the overlap with Bogoliubov–de Gennes wave functions. Using a modified scattering matrix formalism, that allows for including electron-electron interactions, we study the transmission phase in different regimes.

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Introduction. Majorana zero modes (MZMs) are localized zero-energy states that can arise in topological superconductors [1,2]. In the last decade, they have attracted much attention, because they are promising candidates for the realization of quantum computation [3–5]. Recent progress [6–14] suggests that MZMs can be realized experimentally, and that they can be detected by electric conductance measurements.

The motion of electrons through a mesoscopic device is characterized by a transmission matrix T. In the simplest case, it reduces to a complex number. From the Landauer formula the conductance is proportional to the square of its absolute value. Its quantum mechanical phase determines how electrons moving along different trajectories interfere. A typical interference experiment consists of two arms that electrons can travel through, one containing the device, the other one acting as a reference arm. When the arms join, the electrons interfere due to different relative phases. The phase in the reference arm can be adjusted by means of the Aharonov-Bohm effect, leading to oscillations of the total conductance depending on a magnetic flux within the interferometer loop [15–36].

In this Rapid Communication, we study the transmission phase when the device is a quantum dot made of a topological superconductor that can host MZMs [3,37–42]. In particular, in the Coulomb-blockade regime, it has been predicted that the transmission phase is sensitive to the presence of MZMs [43–48]. For concreteness, we consider a semiconducting nanowire with Rashba-type spin-orbit coupling covered by a metallic superconductor such as aluminum [40–42] (see Fig. 1).

We find that the trivial and topological regime can be distinguished by the presence or absence of phase lapses, where the phase exhibits an abrupt change of π. The presence or absence of phase lapses may be understood as originating from the spatial symmetry of Bogoliubov–de Gennes (BdG) wave-function amplitudes and the way they evolve as we scan through consecutive Coulomb-blockade peaks. In neighboring peaks, the dominant contribution to the transmission amplitude switches between being electron type and hole type. In the topological regime, the spatial symmetry of the effective p-wave pairing leads to an opposing inversion symmetry of these amplitudes and thus the absence of phase lapses. In contrast, in the nontopological regime, the doubling of the number of Fermi points invalidates this argument, and hence may introduce phase lapses. We point out that this depends on the spin polarization and discuss how the transmission phase is influenced by external parameters, with and without breaking an effective time-reversal symmetry that may occur in these wires. We also explicitly distinguish between cases where consecutive Coulomb-blockade peaks are dominated by tunneling through the same level or through consecutive levels.

In the Coulomb-blockade regime, the transmission depends on the total number N_0 of electrons in the quantum dot, which is composed of the semiconducting wire and the superconducting coating. If the dot is capacitively coupled to a gate with voltage V_G, then its energy includes a charging term $H_c = E_c N_0^2/2 - eV_G N_0$. At small bias voltage, an electron may only tunnel if there is no energy cost for allowing the electron into the dot, i.e., if the charging terms for N_0 and N_0+1 are roughly equal. This occurs when the gate voltage eV_G takes certain discrete values E_{N_0}. As a function of gate voltage, the transmission $T(V_G)$ has sharp conductance peaks at these values. By considering only two resonances, and by assuming them to be independent of each other, these resonances can be described by a Breit-Wigner form obtained from the retarded Green's function of the wire [49]. Between two resonances, $E_{N_0} < eV_G < E_{N_0+1}$, we find, taking into account the lack of degeneracy of the quantum dot spectrum (for details and limits of applicability, see

Ref. [50]),

$$T_{\sigma\sigma}(V_G) = \sum_{\substack{N=N_0,\\ N_0+1}} \frac{\rho_F \lambda_{\sigma L}(N)\lambda^*_{\sigma R}(N)}{eV_G - E_N + i\pi\rho_F \sum_{\sigma'}[|\lambda_{\sigma' L}(N)|^2 + |\lambda_{\sigma' R}(N)|^2]} + O\left(\frac{\rho_F^2\lambda^4}{(eV_G)^2}\right), \quad (1)$$

with $\lambda \sim \lambda_{\sigma,L/R}$. Here, σ denotes the spin of the transmitted electron, and we do not consider spin-flip processes as they do not contribute to interference. We will assume tunneling proceeds through Bogoliubov quasiparticles. The resonances then occur at the effective single-particle energy $E_{N_0} = N_0 E_c + \epsilon_{n_{\min}}$, with $\epsilon_{n_{\min}}$ denoting the lowest Bogoliubov quasiparticle energy. This captures the behavior of the system near a resonance. If we assume that the incoming electron tunnels via a single state in the wire, then the complex quantities $\lambda_{\sigma L}(N)$ and $\lambda_{\sigma R}(N)$ can be identified with the coupling of this state to the left and the right lead. Finally, ρ_F is the density of states at the Fermi energy in the leads. For a given gate voltage, we always include the two neighboring resonances with $E_{N_0} < eV_G < E_{N_0+1}$.

When the gate voltage V_G is swept across a resonance, the phase of the transmission changes by π according to Eq. (1). However, when increasing the voltage further, towards the next resonance, a phase lapse has often been seen in many interferometer studies over the years [18,20,23,30,35,51–53]. In particular, this will happen when two subsequent resonances have coefficients with equal phase, $\arg[\lambda_{\sigma L}(N)\lambda^*_{\sigma R}(N)] = \arg[\lambda_{\sigma L}(N+1)\lambda^*_{\sigma R}(N+1)]$, as the denominators in Eq. (1) differ by a relative minus sign. On the other hand, phase lapses are absent when the sign of the coefficients changes from one resonance to the other, such that $\arg[\lambda_{\sigma L}(N)\lambda^*_{\sigma R}(N)] - \arg[\lambda_{\sigma L}(N+1)\lambda^*_{\sigma R}(N+1)] = \pm\pi$ [54].

In the following, we study the evolution of the transmission phase for different regimes of the nanowire system. The interference setup is illustrated in Fig. 1. To avoid the phase rigidity effect in the presence of time-reversal symmetry [26], we include a reservoir in the setup.

Model. The electrons in the wire are described by the BdG Hamiltonian

$$\mathcal{H} = \left[-\frac{\hbar^2}{2m}\partial_y^2 - \mu - i\alpha_R\sigma_x\partial_y\right]\tau_z - B_z\sigma_z - B_x\sigma_x + \Delta\tau_x, \quad (2)$$

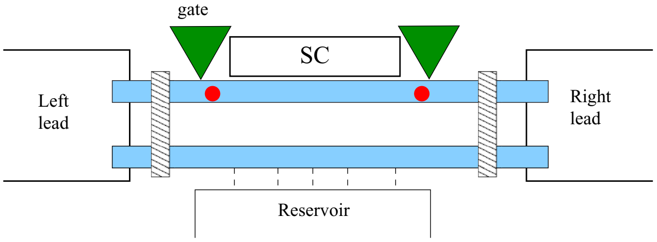

FIG. 1. Schematic setup. We consider two wires (azure rectangles) connected to leads to the left and right. The shaded rectangles denote tunneling bridges, and the triangles gates which control the coupling of the Majorana to the leads. The upper wire is in proximity to a conventional s-wave superconductor (SC in the figure). The red circles represent Majorana zero modes. We couple the reference arm of the interferometer to a reservoir to avoid the phase rigidity effect (coupling indicated by dashed lines) [26].

with the second quantized form $\hat{H}_{\rm D} = \frac{1}{2}\int dy\, \Psi^\dagger\mathcal{H}\Psi$, where $\Psi = (\psi_\uparrow, \psi_\downarrow, \psi_\downarrow^\dagger, -\psi_\uparrow^\dagger)$ is a Nambu vector of electron operators. The coefficient α_R denotes a Rashba-type spin-orbit coupling while B_x and B_z are Zeeman energies arising from a magnetic field in the xz plane. The proximity to the s-wave superconductor induces a pairing energy Δ, which we choose to be real. Finally, the chemical potential is denoted by μ, and the Pauli matrices σ_i and τ_i act in spin- and particle-hole space, respectively. Particle-hole symmetry $\{\mathcal{H}, P\} = 0$ is described by $P = \sigma_y\tau_y K$, where K denotes complex conjugation. Additionally, we have an antiunitary reflection symmetry $[\mathcal{H}, \tilde{\Pi}] = 0$ defined on BdG wave functions Φ as $(\tilde{\Pi}\Phi)(y) = K\Phi(L-y)$, where L is the length of the wire. For $B_x = 0$, the Hamiltonian also has an effective time-reversal symmetry $\tilde{T} = \sigma_z K$ with $\tilde{T}^2 = +1$ and $[\mathcal{H}, \tilde{T}] = 0$, which puts it into the BDI symmetry class of topological superconductors [55]. For suitable parameters, the wire hosts MZMs [40–42].

The spatial form of the MZM wave function depends on the number of particles in the nanowire $N_{\rm w}$ via the chemical potential μ. However, the total number of particles N_0 in the dot is larger than $N_{\rm w}$, because electrons may also reside in the superconductor. In general, the semiconductor wire has a much larger level spacing than the superconductor, such that at equilibrium most of the electron density will be accommodated in the superconductor. Hence the tunneling from the leads to the hybrid superconductor-wire system is through the same wire level for several subsequent resonances. Thus, tunneling amplitudes are mainly determined by wave functions in the wire, but the charge of additional Cooper pairs mostly counts towards the charge in the superconductor, not the wire. We model this with a charging term

$$\hat{H}_{c,0} = \tfrac{1}{2}E_0\hat{N}_0^2 + \tfrac{1}{2}E_{\rm w}\hat{N}_{\rm w}^2 - eV_G\hat{N}_0 - E_{\rm M}\hat{N}_0\hat{N}_{\rm w}. \quad (3)$$

Here, the charging energy E_0 refers to the whole dot, $E_{\rm w}$ to the wire, and $E_{\rm M}$ denotes the coupling between them. Replacing $\hat{N}_{\rm w}$ by $N_{\rm w} = \langle\hat{N}_{\rm w}\rangle$, and minimizing the Hamiltonian with respect to this expectation value, we obtain $N_{\rm w} = (E_{\rm M}/E_{\rm w})\langle N_0\rangle$ and the effective charging Hamiltonian $\hat{H}_c$ mentioned before Eq. (1) with $E_c = E_0 - E_{\rm M}^2/E_{\rm w}$. In this way, we can also apply our model when tunneling occurs through subsequent wire levels. We refer to the regime $E_{\rm M}/E_{\rm w} \ll 1$ as tunneling through the same level in consecutive peaks, and to the regime $\hat{N}_{\rm w} = \hat{N}_0$ as tunneling through consecutive levels in consecutive peaks.

In order to calculate the transmission phase numerically, we discretize the Hamiltonian $\hat{H}_{\rm D}$ [defined below Eq. (2)] on a lattice with hard-wall boundary conditions. The BdG equations that emerge from $H_{\rm D}$ can then be solved for a given chemical potential which is determined self-consistently for a given total number of particles, and the number of particles in the wire [50]. The BdG wave functions thus obtained are the ones we use to deduce the tunneling amplitudes. For this, we supplement the dot Hamiltonian $\hat{H}_D$ with a description of the leads in terms of

$$\hat{H}_{\rm L} = \sum_{\substack{k,\sigma \\ \alpha = L,R}} \epsilon_{k\alpha} c^{\dagger}_{k\sigma\alpha} c_{k\sigma\alpha},$$

where α labels the left and right leads, respectively, σ denotes spin, and $c_{k\sigma\alpha}$ ($c^{\dagger}_{k\sigma\alpha}$) annihilates (creates) an electron in lead α. The tunnel coupling between leads and dot is described by

$$\hat{H}_{\rm T} = \sum_{\substack{m\sigma \\ \alpha = L,R}} t_{\alpha m\sigma} c^{\dagger}_{\sigma}(y_\alpha) d_m + \text{H.c.}, \tag{4}$$

where $c_\sigma(y_\alpha)$, $c^{\dagger}_{\sigma}(y_\alpha)$ are lead operators evaluated in the vicinity of lead α, d_m ($d^{\dagger}_m$) annihilates (creates) an electron in the mth wire state with energy ϵ_m, and $t_{\alpha m\sigma}$ is the tunneling matrix element between lead α and the mth wire state. The tunneling matrix elements are identical in magnitude, while their phase is set to be the phase of $\varphi_{m\sigma}(y)$, the eigenstates of the wire part $\hat{H}_D$ for $\Delta = 0$, close to the ends of the wire.

The wire operators d_m can now be expressed in terms of BdG operators β_n and $\beta^{\dagger}_n$. Assuming the gap in the superconductor is greater than in the wire, an unpaired electron must enter the wire, occupying the lowest BdG quasiparticle state for N_0 odd. Denoting this state by $n_{\rm min}$ we thus write $2\beta^{\dagger}_{n_{\rm min}}\beta_{n_{\rm min}} - 1 = (-1)^{N_0}$ in the ground state. Higher quasiparticle states are not occupied in the ground state, so $\beta^{\dagger}_n\beta_n = 0$ for $n \neq n_{\rm min}$. Following Ref. [43], we project the Hamiltonian to the states with N_0 and $N_0 + 1$ particles and introduce new fermionic operators f_n. The tunneling part then becomes [50]

$$H_{\rm T} = \frac{1}{2} \sum_{\substack{\alpha = L,R \\ n\sigma}} \lambda_{n\sigma\alpha} c^{\dagger}_{\sigma}(y_\alpha) f_n + \text{H.c.}, \tag{5}$$

with effective tunneling matrix elements

$$\lambda_{n_{\rm min}\sigma\alpha} = \lambda^{u}_{n_{\rm min}\sigma\alpha} + \lambda^{v}_{n_{\rm min}\sigma\alpha} - (-1)^{N_0}\left[\lambda^{u}_{n_{\rm min}\sigma\alpha} - \lambda^{v}_{n_{\rm min}\sigma\alpha}\right], \tag{6}$$

and $\lambda_{n\sigma\alpha} = 2\lambda^{u}_{n\sigma\alpha}$ for $n \neq n_{\rm min}$. Here,

$$\lambda^{u}_{n\sigma\alpha} = \sum_m t_{\alpha m\sigma} \int dy\, \varphi^{*}_{m\sigma}(y) u_{n\sigma}(y), \tag{7}$$

$$\lambda^{v}_{n\sigma\alpha} = \sum_m t_{\alpha m\sigma} \int dy\, \varphi^{*}_{m\sigma}(y) v^{*}_{n\sigma}(y), \tag{8}$$

where $u_{n\sigma}(y)$ and $v_{n\sigma}(y)$ are the BdG wave functions found by solving the BdG equation corresponding to $\hat{H}_{\rm D}$. To obtain the transmission amplitude, we apply the scattering matrix formalism [56,57]. The S matrix is $S(\epsilon) = \mathbb{1} - 2\pi i W^{\dagger}[\epsilon\mathbb{1} - H_D + i\pi W W^{\dagger}]^{-1} W$, where $\mathbb{1}$ denotes the unit matrix and W the coupling matrix obtained from $\hat{H}_{\rm T}$. If we assume that for each N_0, tunneling proceeds only via the Bogoliubov quasiparticle with the lowest energy, we find, at zero temperature and at the Fermi level, that the transmission is given by Eq. (1) with tunneling amplitudes $\lambda_{\sigma\alpha}(N_0) = \lambda_{n_{\rm min}\sigma\alpha}/2$ for $\alpha = L, R$.

Results. Before discussing the results in detail let us note that the presence of the superconductor may influence the visibility of the interference. Indeed, when the superconductor acts as a normal conductor, its level spacing is small, and we expect a rather large suppression of the interference and weak localization correction to the conductance [58]. Our main results for realistic parameter regimes [59] are shown in Fig. 2. In the normal phase, the wire shows phase lapses when tunneling proceeds through the same level [Fig. 2(a)]. When tunneling occurs through consecutive levels, phase lapses are partially absent. In the trivial regime, superconductivity does not change the pattern. We now consider the topologically nontrivial regime. When electrons tunnel through the same level, we can distinguish the normal conducting and the superconducting regime by the presence or absence of phase lapses. For tunneling through consecutive levels phase lapses are always absent.

Interestingly, the transmission is sensitive to the spin polarizations of the MZMs at the ends of the wire. Defining $E_{\rm SO} = m\alpha_R^2/(2\hbar^2)$, we find that, in the regime $B_z \gg E_{\rm SO}$, both ends have the same spin polarization, in the z direction. For $B_z \ll E_{\rm SO}$, the ends have orthogonal polarizations along the y axis, so electrons in the two arms of the interferometer will have orthogonal polarizations, leading to a suppression of the interference signal.

Note that the effective tunneling matrix elements alternate between $\lambda^{u}_{n\alpha}$ and $\lambda^{v}_{n\alpha}$ when summing over N_0. Furthermore, the BdG wave functions, of a Hamiltonian invariant to the spatial symmetry operation ($x \to -x$) with respect to the wire's midpoint, are either symmetric or antisymmetric under this operation, such that only either the even or odd elements in the sums (7) and (8) are nonzero. We therefore expect the existence of phase lapses to be connected with the inversion (anti)symmetry of BdG wave functions. Moreover, the Hamiltonian (2) is invariant under the inversion symmetry $\Pi = \tilde{\Pi}\tilde{T}$. This implies that $u_{n\uparrow}(y)$ and $v_{n\downarrow}(y)$ behave in the same way under inversion and likewise for $u_{n\downarrow}(y)$ and $v_{n\uparrow}(y)$. On the other hand, $u_{n\uparrow}(y)$ and $u_{n\downarrow}(y)$, and $v_{n\uparrow}(y)$ and $v_{n\downarrow}(y)$ behave in an opposite way under inversion. Naively, we would then expect no phase lapses, regardless of whether the wire is in the topological or trivial regime. In the latter case, however, the dominant particle- and holelike processes have opposite spin, such that products such as $\lambda^{u}_{n\uparrow L}\lambda^{u*}_{n\uparrow R}$ and $\lambda^{v}_{n\downarrow L}\lambda^{v*}_{n\downarrow R}$ with the same sign dominate the transmission, and phase lapses occur. This is unlike the topological regime where the spins are mostly polarized.

We now study the transmission phase in the topological regime for the case where the BDI time-reversal symmetry is broken by a magnetic field, $B_x \neq 0$ [60–62]. We consider a homogeneous wire longer than the localization length of the Majorana wave functions, in the regime $B_x \ll E_{\rm SO} \ll B_z$. Here, we can linearize the Hamiltonian (2) and find [50]

$$\tilde{\mathcal{H}} = -i\hbar v_F \partial_y s_z \tau_z + \varepsilon s_z + \tilde{\Delta}\tau_x s_z, \tag{9}$$

acting on BdG wave functions $\Phi = [u_1, u_2, v_2, v_1]^T$, where u_1, v_1 and u_2, v_2 correspond to right and left movers. The Pauli matrices s_μ operate on these, while τ_μ act on particles and holes. They have opposite velocities $\pm v_F$, but the magnetic field along the wire axis causes an energy shift ε. The effective pairing $\tilde{\Delta}$ is obtained from the relation $\varepsilon/\tilde{\Delta} = B_x/\Delta$. Since a hard-wall boundary reflects left into right movers, we obtain $u_1(0) = -u_2(0)$ and $v_1(0) = -v_2(0)$. Then, the

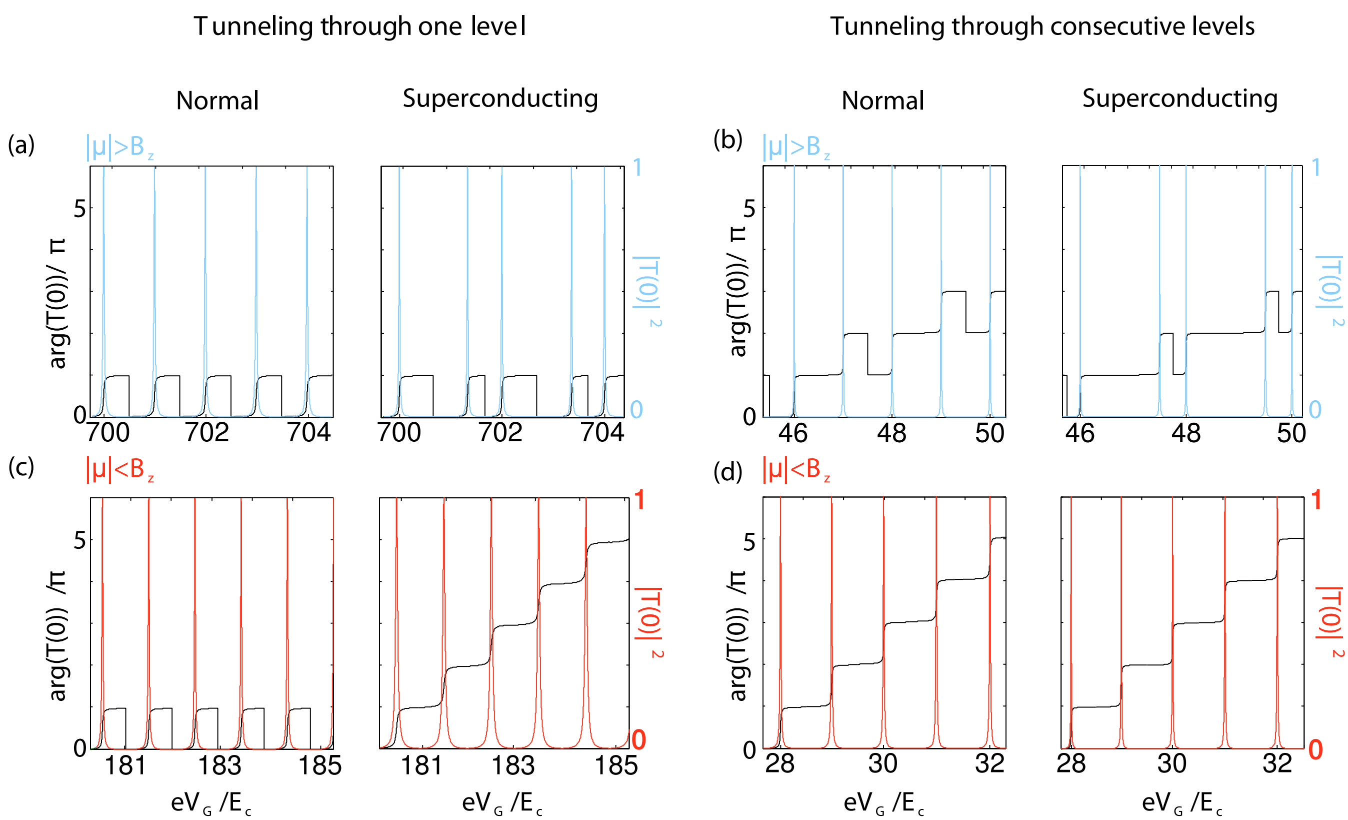

FIG. 2. Plots of the the transmission phase $\arg(T_{\uparrow\uparrow}+T_{\downarrow\downarrow})/\pi$ (black) and amplitudes $|T_{\uparrow\uparrow}|^2+|T_{\downarrow\downarrow}|^2$ (blue and red), at zero temperature and at the Fermi level calculated using the S matrix, for a quantum wire with tunneling through the same level in consecutive peaks [(a) and (c)], and tunneling through consecutive levels in consecutive peaks [(b) and (d)]. For the former, $E_{\rm M}/E_{\rm w}=0.05$, such that only every 20th electron entering the hybrid system of a wire and superconductor, enters the wire, while for the latter, $E_{\rm M}/E_{\rm w}=1$. In the presence of a superconducting gap Δ, the wire can either enter a trivial [(a) and (b)] or a topological [(c) and (d)] phase, depending on the number of particles $N_{\rm w}$ in the wire. All plots are calculated by discretizing the BdG Hamiltonian (2) on a one-dimensional (1D) lattice with 500 sites with parameters $L=2.5\ \mu$m, $m=0.02m_e$, $\tilde{t}=\hbar^2/(2ma^2)=80$ meV, $u_0=\alpha_R/a=0.05\tilde{t}$, $B_z=0.003\tilde{t}$, $B_x=0$, and $E_c=3\Delta$. Here, a is the lattice spacing, $\tilde{t}$ the hopping, and u_0 an effective spin-orbit coupling in the discretized model. The two columns in each panel are results for the normal regime ($\Delta=0$) and the superconducting regime where $\Delta=2.5\delta_F$. The tunneling matrix elements t on the left and right sides have been chosen to be equal and satisfy $\rho_F t^2=0.2\Delta$. The plots have been made by summing the transmission over the spin configurations. The transmission phases have been shifted by integer multiples of π for clarity.

Majorana solution localized at the left end of the wire is given by $\Phi_L(y)\propto(1,-1,-ie^{-i\delta},ie^{-i\delta})^T e^{-(y/\hbar v_F)\sqrt{\tilde{\Delta}^2-\varepsilon^2}}$ with phase $\delta=\arcsin(\varepsilon/\tilde{\Delta})$. Identifying the tunneling amplitude as the component $u_1(0)$, and fixing its phase by requiring particle-hole symmetry, we thus find, at resonance,

$$\arg(T_{\uparrow\uparrow})=\arcsin(B_x/\Delta)+\pi\cdot\text{integer},\tag{10}$$

which is independent of the total wire length. This equation implies that upon breaking BDI symmetry, the interference pattern will not be at an extremum at zero flux. The phase shift will, however, be identical for consecutive peaks. To understand the phase shift $\arcsin(B_x/\Delta)$, we use an antiunitary reflection symmetry of the Hamiltonian (2) and denote the BdG wave functions by $\Phi(y)=[u_\uparrow,u_\downarrow,v_\downarrow,-v_\uparrow]^T$. The left Majorana solution satisfies $\mathcal{H}\Phi_L=0$ and $P\Phi_L=\Phi_L$, such that $\Phi_R(y)=\Phi_L(L-y)^*$ is the Majorana solution at the right end. For a finite but sufficiently long wire, these will hybridize slightly to give the lowest-energy Bogoliubov quasiparticle $\Phi\propto(\Phi_L\pm i\Phi_R)$, where the relative prefactor is fixed to $\pm i$ because the coupling Hamiltonian has particle-hole symmetry. Thus, the transmission amplitude is fully determined by the left Majorana solution, $T_{\uparrow\uparrow}\propto u_\uparrow(y_L)u_\uparrow^*(y_R)=\mp iu_{\uparrow,L}(y_L)^2$, and does not depend on the wire length if the BdG equation does not explicitly depend on it.

Conclusion. We have studied the transmission of electrons through a hybrid system consisting of a quantum wire and an s-wave superconductor in the Coulomb-blockade regime in a magnetic field and with strong spin-orbit coupling. In this regime the system supports zero-energy MZMs. We consider only tunneling through a single Bogoliubov quasiparticle. In this case we found that the existence of phase lapses in the transmission phase depends on the superconducting gap and on whether the wire is in the topological or trivial phase. We expect that it should be possible to measure this effect in interference experiments.

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