Rapid Communications

Entangled Pauli principles: The DNA of quantum Hall fluids

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A formalism is developed for the rigorous study of solvable fractional quantum Hall parent Hamiltonians with Landau-level mixing. The idea of organization through "generalized Pauli principles" is expanded to allow for root-level entanglement, giving rise to "entangled Pauli principles." Through the latter, aspects of the effective field theory description become ingrained in exact microscopic solutions for a great wealth of phases for which no similar single Landau-level description is known. We discuss in detail braiding statistic, edge theory, and rigorous zero-mode counting for the Jain-221 state as derived from a microscopic Hamiltonian. The relevant root-level entanglement is found to feature an Affleck-Kennedy-Lieb-Tasaki (AKLT)-type matrix-product ground states structure associated with an emergent SU(2) symmetry.

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Introduction. The fractional quantum Hall (FQH) regime exhibits an astonishing wealth of interacting topological phases. A rich theoretical framework describing such phases has historically nucleated around a construction principle for holomorphic lowest Landau-level (LL) wave functions [1] and fruitful generalizations to the nonholomorphic, multi-LL situation, with optional subsequent lowest-LL projection [2]. This variational principle has proven invaluable in driving the development of field-theoretic descriptions of both the bulk and the edge physics and their intimate relation [3,4]. One may take the point of view that a complete many-body theory of any correlated phase of matter requires, in addition to the aforementioned ingredients, a microscopic Hamiltonian granting analytic access to its low energy sector, reproducing key aspects of the field-theoretic description of such a phase. Such "parent Hamiltonians" do exist for many [5–9] FQH liquids but lack for even more. Notably, to our knowledge, they are absent for most Jain states [10], which are regarded fundamental both theoretically and experimentally.

In this Rapid Communication, we argue that the lack of microscopic Hamiltonians stabilizing representative variational wave functions for FQH phases stems from complexities associated with nonholomorphic variational states. These include unprojected Jain states [2] and more general "parton" constructions [10,11]. In these cases, lowest-LL projection leads to sufficiently intractable wave functions to preclude the construction of parent Hamiltonians. Moreover, the unprojected, multi-LL variational states still lack many "analytic clustering" properties that were instrumental in the construction of parent Hamiltonians for many lowest-LL states [5–7]. For these reasons, even in those cases where parent Hamiltonians have been proposed for multi-LL states, rigorous analytic results are usually lacking. This is particularly true for zero-mode counting, from which the case for incompressibility at special filling factors is usually made. We will develop principles to study the zero-mode properties of frustration-free multiple-LL parent Hamiltonians on the same footing as for similar single-LL Hamiltonians. Our second-quantized framework deemphasizes analytic clustering properties [12], which are arguably less useful in the multi-LL situation, as we will demonstrate. This lack of emphasis on analytic properties, in favor of a "guiding-center-based" description, was recently advocated for various reasons [13–19]. Our approach connects with the topical investigation of frustration-free lattice Hamiltonians and their matrix-product ground states (MPS), with the important additional feature that it extends to nonlocal lattice Hamiltonians and, in principle, MPS of infinite bond dimension [20–22].

The heart of our framework consists in further elaboration on the concept of a "generalized Pauli principle" (GPP), various guises of which play an important role in discussing the structure of single-LL wave functions [23–32]. Our extension not only provides a foundation based on Hamiltonian principles but also generalizes to multiple LLs. The latter will naturally lead to what we coin "entangled Pauli principles" (EPPs), which, in addition to the now familiar rules for GPPs, permit MPS-like entanglement at "root state level" encoding the quantum fluid's "DNA." We argue this generalization to be key in yielding microscopic Hamiltonian descriptions to possibly all FQH phases. We demonstrate our approach in detail for the parent Hamiltonian of the Jain-221 state [33]. By rigorously establishing the zero-mode structure of this Hamiltonian, we make direct contact both with bulk topological and edge conformal properties. As a by-product, this affords a case where simple two-body interactions stabilize a non-Abelian FQH state, in contrast to better known higher-body, single-LL cases [34,35].

Parent Hamiltonian. Consider the n-LL projected "Trugman-Kivelson" interaction for fermions,

$$H_{\mathrm{TK}} = \sum_{i<j} P_n \partial_{z_i} \partial_{\bar{z}_i} \delta(z_i - z_j)\delta(\bar{z}_i - \bar{z}_j) P_n, \qquad (1)$$

where $z_i = x_i + iy_i$ is the coordinate of the ith particle, and $\bar{z}_i$ its complex conjugate. For general projection P_n onto the subspace spanned by the lowest n LLs, this interaction is positive (semi-)definite. If the n-LLs are energetically quenched [36], as is in multilayer graphene [33,37,38], the ground states of the resulting Hamiltonian can be characterized as zero-energy modes (zero modes). For any n, the wave functions of such zero modes will have at least second-order zeros as pairs of particles coalesce into the same point. For both $n = 1$ and $n = 2$, this is equivalent to the polynomial wave function being divisible by the Laughlin-Jastrow factor $\prod_{i<j}(z_i - z_j)^2$. This was realized early on for $n = 1$ [5,6] and leads to the stabilization of the 1/3-Laughlin state and its quasihole excitations. The $n = 2$ case was extensively discussed recently [39]. For $n \geqslant 3$, zero modes can only be characterized as polynomials belonging to the ideal generated by $(z_i - z_j)^2$ and $(\bar{z}_i - \bar{z}_j)^2$ for some fixed $i \neq j$, in addition to being antisymmetric. This makes the characterization of all possible zero modes considerably more challenging. For the case $n = 3$, we will establish that the space of all zero modes is linearly generated by all wave functions of the form

$$\psi = \prod_{i<j}(z_i - z_j)D_1 D_2, \tag{2}$$

where D_1 and D_2 are the polynomial (in $\{z_i, \bar{z}_i\}$) parts of two Slater determinants each comprised of lowest and first excited LL states, and we omit obligatory Gaussian factors. It is easy to see that states of the form (2) are zero modes of the $n=3$ Hamiltonian. The "Jain-221" state, where $D_1 = D_2$ is the Slater determinant of smallest possible angular momentum in the first two LLs for given particle number N, was conjectured to be the densest zero mode [33]. We will show that the set of all possible wave functions of the form (2) is overcomplete and establish rules for the selection of a complete set of zero modes as an EPP on dominance patterns.

Entangled Pauli principle. Our starting point is a second-quantized form of Eq. (1) for $n = 3$, in a disk geometry, which we present in the general [12] form

$$\hat{H}_{\mathsf{TK}} = \sum_J \sum_{\lambda=1}^{8} E_\lambda \mathcal{T}_J^{(\lambda)\dagger} \mathcal{T}_J^{(\lambda)}. \tag{3}$$

The $\mathcal{T}_J^{(\lambda)}$ annihilate a pair of particles of angular momentum $2J$, with $J = 0, \frac{1}{2}, 1, \ldots$, $\mathcal{T}_J^{(\lambda)} = \sum_{x,m_1,m_2} \eta^\lambda_{J,x,m_1,m_2} c_{m_1,J-x} c_{m_2,J+x}$ and Eq. (3) may be viewed as a weighted (by E_λ) sum over eight two-particle projection operators at each J. Note that x is (half-odd) integer if J is (half-odd) integer, and $c_{m,j}$ destroys a fermion in the mth LL, $m = 0, 1, 2$, at angular momentum ("site") $j \geqslant -m$. The η symbols and the positive E_λ can be efficiently derived for general n [40], and are given for $n = 3$ in [41]. Consider the Slater-determinant decomposition of any N-particle zero-mode

$$|\psi\rangle = \sum C_{m_1,j_1;\cdots;m_N,j_N} c^\dagger_{m_1,j_1} \cdots c^\dagger_{m_N,j_N} |0\rangle \equiv \sum C_S |S\rangle . \tag{4}$$

General arguments [12,39] imply that there are "nonexpandable" Slater determinants $|S\rangle$ in such an expansion that are pivotal in the analysis of any zero mode of Eq. (3). These are those states $|S\rangle$ in Eq. (4) with nonzero C_S that cannot be obtained from a $|S'\rangle$ with nonzero $C_{S'}$ through an inward-squeezing [23] process: $|S\rangle \neq c^\dagger_{m_1,j_1} c^\dagger_{m_2,j_2} c_{m_2',j_2+x} c_{m_1',j_1-x} |S'\rangle$, where $j_1 < j_2$, $x > 0$. We define the state obtained from the zero-mode (4) by keeping only the nonexpandable part as the "root state" $|\psi_{\mathrm{root}}\rangle$ of $|\psi\rangle$. The root state is closely related to the thin torus limit [26,27,29,30,42], and is generally subject to simple rules usually known as GPPs in the single-LL context. We will show that the zero-mode condition leads to a generalization thereof in the present case, which we call EPP.

We begin by demonstrating that a state $|S\rangle$ in $|\psi_{\mathrm{root}}\rangle$ may not have a double occupancy at any given j. Otherwise, $|\psi_{\mathrm{root}}\rangle = \sum_{m_1,m_2} \alpha_{m_1,m_2} c^\dagger_{m_1,j} c^\dagger_{m_2,j} |\tilde{S}\rangle + |\mathrm{rest}\rangle$, with $|\mathrm{rest}\rangle$ being orthogonal to each of the leading terms, and $|\tilde{S}\rangle$ an $N-2$ particle Slater determinant with no j mode occupied. The zero-mode condition amounts to [12,39] $\mathcal{T}_J^{(\lambda)} |\psi\rangle = 0$ for all J, λ. Then, in $0 = \langle\psi|\mathcal{T}_{J=j}^{(\lambda)\dagger}|\tilde{S}\rangle = \sum_{x,m_1,m_2} (\eta^\lambda_{J,x,m_1,m_2})^* \langle\psi|c^\dagger_{m_2,J+x} c^\dagger_{m_1,J-x}|\tilde{S}\rangle$, the $x \neq 0$ terms must already give zero, otherwise the $x = 0$ terms would by definition not appear in $|\psi_{\mathrm{root}}\rangle$. One thus obtains the eight conditions

$$\sum_{m_1,m_2} \eta^\lambda_{J,0,m_1,m_2} \alpha_{m_1,m_2} = 0 \quad (\lambda = 1, \ldots, 8). \tag{5}$$

Since there are only three independent numbers $\alpha_{m_1,m_2} = -\alpha_{m_2,m_1}$, and the $x = 0$ η symbols are sufficiently [41] linearly independent, one finds that all α_{m_1,m_2} vanish. One can similarly rule out triple occupancies in $|\psi_{\mathrm{root}}\rangle$. Likewise, one may evaluate possibilities for nearest-neighbor occupancies in $|\psi_{\mathrm{root}}\rangle$. Applying the same method to the similar expression (J half-odd integer) $|\psi_{\mathrm{root}}\rangle = \sum_{m_1,m_2} \beta_{m_1,m_2} c^\dagger_{m_1,J-(1/2)} c^\dagger_{m_2,J+(1/2)} |\tilde{S}\rangle + |\mathrm{rest}\rangle$, there are eight constraints on the nine constants β_{m_1,m_2},

$$\sum_{m_1,m_2} \eta^\lambda_{J,1/2,m_1,m_2} \beta_{m_1,m_2} = 0 \quad (\lambda = 1, \ldots, 8). \tag{6}$$

There is a unique solution to these equations which thus determines any nearest-neighbor pair in $|\psi_{\mathrm{root}}\rangle$ to be in a certain entangled state. In evaluating constraints at root level for pairs further separated, we must also take into account inward-squeezed configurations of the pair. Writing $|\psi\rangle = \sum_{m_1,m_2} \gamma_{m_1,m_2} c^\dagger_{m_1,J-1} c^\dagger_{m_2,J+1} |\tilde{S}\rangle + \alpha_{m_1,m_2} c^\dagger_{m_1,J} c^\dagger_{m_2,J} |\tilde{S}\rangle + |\mathrm{rest}\rangle$, where the first term is nonexpandable, we obtain eight conditions on the 12 constants γ_{m_1,m_2}, $\alpha_{m_1,m_2} = -\alpha_{m_2,m_1}$. After eliminating the latter, these result in five conditions on the γ_{m_1,m_2}:

$$\sum_{m_1,m_2} \Omega^\mu_{J,m_1,m_2} \gamma_{m_1,m_2} = 0 \quad (\mu = 1, \ldots, 5), \tag{7}$$

with Ω a function of the η's at $x = 0, 1/2$. The constraints derived so far require any two particles in a root state to be entangled when in configurations ...11... or ...101..., where 0 denotes an empty site, 1 denotes a single occupancy (in any LL), and consecutive entries denote states with consecutive j. We now ask what these constraints imply for clusters of more than two particles.

Emergent SU(2) symmetry. Let us apply to $|\psi_{\mathrm{root}}\rangle$ a nonunitary (but invertible) single-particle transformation $\hat{V}$ such that

$c^\dagger_{m,j} = \hat{V}^{-1} d^\dagger_{m-1,j} \hat{V} = v_{m,s_z} d^\dagger_{s_z,j}$, where $s_z = 0, \pm 1$ is interpreted as the SU(2) label of a spin-1 particle, as detailed in [41]. In the new basis, Eq. (6) requires any nearest-neighbor 11 pair in $\hat{V}\,|\psi_{\text{root}}\rangle$ to form a singlet. Clearly, then, it cannot be entangled with any other particle. This is consistent with Eqs. (6) and (7) only if any such pair is separated by at least two zeros from any other particle in $|\psi_{\text{root}}\rangle$. Moreover, Eq. (7) takes on a form implying that any 101 configuration is orthogonal to the spin-2 sector. The satisfiability of this condition for N particles separated by individual empty sites is tantamount to the problem of finding ground states of an open AKLT chain [43]. To label such a structure, we use the notation $\ldots 1_{\sigma_L} 0101 \ldots 0101_{\sigma_R} \ldots$ where $\sigma_{L,R} = \pm$ denote the boundary spin-1/2 degrees of freedom of an AKLT ground state. Aside from the aforementioned entangled 11 and 101 blocks, a root state may have singly occupied sites surrounded by at least two empty sites on either side. Such sites may be in any of the three LLs, or in any "spin state" after the $\hat{V}$ map. We denote such configurations by $\ldots 001_{s_z} 00 \ldots$. All of these observations imply that a complete set of (rotated) root states is afforded by product states of entangled units of the 11 and $1_{\sigma_L} 0 \ldots 01_{\sigma_R}$(AKLT) type, and of 1_{s_z} units, all separated by at least two empty sites. We refer to the resulting patterns as "dominance patterns" compatible with an EPP.

The SU(2) structure discussed here is not limited to the root state, but emerges in the full zero-mode sector of the Hamiltonian [44]. Indeed, we identified global SU(2) generators $S_\nu,\ \nu = x, y, z$ that leave the zero-mode subspace invariant [41]. Consequently, zero modes can be organized into irreps of this SU(2) symmetry, as suggested by the root structure and associated dominance patterns.

Braiding statistics. Recently, multi-LL wave functions have been discussed on the torus [45]. If the dominance patterns established here are understood as "thin torus (TT) patterns," there exists a well-defined "coherent state" method to associate braiding statistics to the excitations of the underlying state [46–51]. In this regard, we first observe that if we discard the subscripts $\sigma_{R,L}$ and s_z in the dominance patterns satisfying the EPP, the resulting reduced patterns of 1's and 0's satisfy the GPP associated with TT/dominance patterns of the $\nu = 1/2$ Moore-Read (MR) Pfaffian state: There are no more than two 1's in any four adjacent sites. In particular, the densest such patterns, $\ldots 11001100 \ldots$ and $\ldots 10101010 \ldots$, signify the sixfold torus degeneracy of the MR state in the usual way [30]. We assume that the EPP remains meaningful on the torus and governs TT limits of zero modes of Eq. (1), and that the usual assumptions about adiabatic continuity [26] into the TT limit hold. Then, in the presence of periodic boundary conditions, the discussion of ground state degeneracy carries over from the MR case, and the torus degeneracy of the $n = 3$ Hamiltonian will be six. However, any charge-1/4 quasihole excitation, represented by the familiar domain walls between 1010 and 1100 patterns, will carry an additional spin-1/2 described by a σ label. So long as we fix the state of this spin (say, $\uparrow$) for all quasiholes, the coherent state method will make the same predictions for the statistics as in the MR case [47,50]. That is, one finds that each quasihole carries a Majorana fermion, and braiding two such quasiholes is described by an operator $\theta_{ij} = \exp[i\theta_m - (-1)^m \frac{\pi}{4}\gamma_i\gamma_j]$, where γ_k is the Majorana operator of the kth quasihole, and θ_m is a phase only determined up to one of eight possible values by the coherent state method, as reported earlier for the $\nu = 1$ bosonic MR state [47,50]. Elsewhere we will show that, for the fermions, the method yields $\theta_m = \frac{m\pi}{4},\ m = 0, \ldots, 7$. This is consistent with $\theta = \frac{\pi}{4}$ [52] for the $\nu = 1/2$ MR state, but it seems possible that the 221 state discussed here realizes a different allowed phase which, presumably, can be determined from the conformal field theory (CFT) proposed in [11,53,54]. The SU(2) symmetry discussed above can, however, be used to argue that this phase does not depend on the spin state of the quasiholes, and the full braid operator is given simply by $\theta_{ij} X_{ij}$, where X_{ij} exchanges the spin of the ith and jth quasiholes.

Zero-mode counting and edge physics. General principles [12,39,41] imply that at any angular momentum L, the number of possible dominance patterns sets an upper bound on the number of linearly independent zero modes. This bound was derived as a necessary condition on root states (the EPP). As such it applies to a large class of Hamiltonians of the form of Eq. (1), and can be generalized to Hamiltonians with different number of terms, internal degrees of freedom, or multibody interactions. That there are, however, indeed as many zero modes as admitted by the EPP depends strongly on the details of the Hamiltonian. To establish this for the $n = 3$ Hamiltonian (1), we must show that to each dominance pattern allowed by the EPP, there is a zero mode with the corresponding root state. We show in [41] that indeed, for every dominance pattern one can construct one such zero mode from the states (2). This then necessarily yields a complete set of zero modes. It is easy to show that the (odd N) Jain-221 state has $|\psi_{\text{root}}\rangle$ corresponding to the densest possible (minimum angular momentum) pattern consistent with the EPP: $10011001100110011\ldots$ (the leading orbital may not be entangled [41]). This establishes that the Jain-221 state is the densest possible zero mode, since there are no allowed dominance patterns at higher filling factor, or smaller L at given N. Note that the topological shift on the sphere, which further distinguishes candidate $\nu = 1/2$ states and in principle relates to Hall viscosity [55,56], is likewise efficiently encoded in this pattern. The existence of a densest filling factor (here: 1/2) permitting zero modes usually hints at incompressibility. This is particularly so if the edge theory encoded in the zero-mode counting is a unitary rational CFT. Using patterns, we have full control over zero-mode counting. Let $\mathcal{N}(\Delta L)$ be the number of zero modes of Eq. (1) at angular momentum ΔL relative to the ground state, where $\Delta L \ll N$. Detailed counting for the number of zero modes at $\Delta L = 3$ in terms of patterns is shown in Table I. One may ask [57,58] if $\mathcal{N}(\Delta L)$ agrees with the number of states having ΔL energy quanta in some CFT. In the presence of suitable chemical potential terms, one may find [39] complete agreement, for $\Delta L \ll N$, between the degeneracies of some CFT Hamiltonian and of the total angular momentum operator $\hat{L}$ within the zero-mode sector of a special Hamiltonian, for any fixed particle number N (N being identified with a suitable conserved quantity of the CFT). For $\Delta L \leqslant 4$, we verified such agreement between the mode counting determined by our EPP and the mode counting in a 1 + 1D edge theory of the form [11,53]

$$H = \sum_{i=0,1} H_{b,i}(\Phi_i) + H_f(\gamma) - \frac{5}{2} N_0. \tag{8}$$

TABLE I. Survey of all dominance patterns with angular momentum $\Delta L = 3$ above the ground state for odd particle number. The total number of these patterns including "spin degeneracy" allowed by AKLT entanglement or due to isolated occupied sites is 33, in agreement with Table II. The corresponding densest state ($\Delta L = 0$) has the pattern 100110011 ... 110011, where the boundary condition at the left end is explained in [41].

Patterns	Degeneracy
$100...110011001_{s_z}0001_{s_z}$	3×3
$100...1100110001_{\sigma_L}01_{\sigma_R}$	4
$100...11001_{\sigma_L}0101_{\sigma_R}001_{s_z}$	4×3
$100...11001_{\sigma_L}01_{\sigma_R}0011$	4
$100...1_{\sigma_L}0101010101_{\sigma_R}$	4

Here, Φ_i are free chiral bosons of compactification radii $\frac{1}{2}$ and 1, respectively, γ is a Majorana field in the antiperiodic sector, all modes are copropagating, N_i is the winding number of Φ_i, and the parity of the number of occupied Majorana modes must be opposite to $N_0 + N_1$. Except for the chemical potential term, Eq. (8) is the $U(1) \times SU(2)_2$-edge CFT first ascribed to the Jain-221 state in Refs. [11,53,54], notably different from other non-Abelian candidate states at half-filling, such as the Pfaffian [3] or anti-Pfaffian [59,60]. Table II describes the above mode-counting agreement when N_0 is identified with the particle number N.

Conclusion. Our framework enables controlled access to numerous quasiexactly solvable quantum-many-body Hamiltonians with LL mixing. We argued that the ability to deal with LL mixing is *essential* to establish microscopic models for a more comprehensive set of phases in the FQH regime. To give an important and concrete example, a substantial number of results were obtained with special focus on the $n = 3$ LL projected Trugman-Kivelson Hamiltonian: (i) Generalized Pauli principles of lowest-LL model wave functions become "entangled" in the presence of LL degrees of freedom. (ii) This establishes a link between a large class of FQH states, in particular "parton-like" states, and MPS of *finite* bond dimension. The latter are in turn linked to one-dimensional symmetry-protected topological phases; in our example, the Haldane phase [61,62]. (iii) EPPs can be used for efficient and, as we show, *rigorous* zero-mode counting. In particular, they establish densest zero modes, which typically remains the only direct analytic evidence for the incompressible character of certain model FQH states; here, the Jain-221 state.

TABLE II. Number of modes for a given number of "quanta" relative to the ground state. Quanta refers to angular momentum in the case of microscopic zero modes, and energy in the effective edge theory (8). The counting agrees for at least up to four quanta, and for $\Delta L = 3$, is shown in detail in Table I in terms of patterns. The chemical potential in (8) is chosen to give equality between total ground state angular momentum and total edge energy for any $\Delta L \ll N$.

ΔL or ΔE	0	1	2	3	4
N odd	1	4	14	33	77
N even	3	7	22	50	115

(iv) Through direct zero-mode counting, we confirmed a "zero-mode paradigm" for Eq. (1), i.e., the edge theory of Eq. (1) ($n = 3$) is a $U(1) \times SU(2)_2$ CFT. (v) We identified an emergent SU(2) symmetry [44] under which the zero-mode spaces of Eq. (1) and many of its generalizations remain invariant. (vi) We demonstrated how microscopically derived EPP-dominance patterns encode bulk topological properties, notably braiding statistics, which are of Ising/Majorana type for the Jain-221 state.

The above establishes the emergence of non-Abelian topological phases based on a solvable *two*-body interaction, which has potentially interesting implications for trilayer graphene. Our findings straightforwardly generalize to bosons, where Eq. (1) becomes a pure *contact* interaction. It was demonstrated [63], at least for $n = 1$, that such contact interactions in an optical lattice with engineered band structure lead to exactly the same zero modes found in the continuum. Our results thus imply that a controlled route to non-Abelian phases, using only realistic two-body contact interactions, is feasible. Interestingly, many of these findings generalize to $n = 4$, where a new parton state emerges [40] supporting Fibonacci-type anyons that facilitate universal fault-tolerant quantum computation [64]. Furthermore, the emergent SU(2) symmetry discussed here proves paramount to the construction of parent two-body Hamiltonians to *all* (unprojected) Jain states at filling factors $\nu = p/(2pq + 1)$, where only the case $\nu = 2/5$ has so far been discussed [36,39]. We leave these extensions for future work.

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[1] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[2] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989).
[3] G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991).
[4] X.-G. Wen, Int. J. Mod. Phys. B **5**, 1641 (1991).
[5] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).
[6] S. A. Trugman and S. Kivelson, Phys. Rev. B **31**, 5280 (1985).
[7] B. I. Halperin, Helv. Phys. Acta **56**, 75 (1983).
[8] S. H. Simon, E. H. Rezayi, N. R. Cooper, and I. Berdnikov, Phys. Rev. B **75**, 075317 (2007).
[9] S. C. Davenport and S. H. Simon, Phys. Rev. B **85**, 075430 (2012).
[10] J. Jain, Phys. Rev. B **41**, 7653 (1990).
[11] X. G. Wen, Phys. Rev. Lett. **66**, 802 (1991).
[12] G. Ortiz, Z. Nussinov, J. Dukelsky, and A. Seidel, Phys. Rev. B **88**, 165303 (2013).
[13] F. D. M. Haldane, Phys. Rev. Lett. **107**, 116801 (2011).
[14] G. Murthy and R. Shankar, Rev. Mod. Phys. **75**, 1101 (2003).
[15] L. Chen and A. Seidel, Phys. Rev. B **91**, 085103 (2015).

[16] T. Mazaheri, G. Ortiz, Z. Nussinov, and A. Seidel, Phys. Rev. B **91**, 085115 (2015).

[17] C. H. Lee, R. Thomale, and X.-L. Qi, Phys. Rev. B **88**, 035101 (2013).

[18] B. Kang and J. E. Moore, Phys. Rev. B **95**, 245117 (2017).

[19] A. Weerasinghe, T. Mazaheri, and A. Seidel, Phys. Rev. B **93**, 155135 (2016).

[20] J. Dubail, N. Read, and E. H. Rezayi, Phys. Rev. B **86**, 245310 (2012).

[21] M. P. Zaletel and R. S. K. Mong, Phys. Rev. B **86**, 245305 (2012).

[22] B. Estienne, Z. Papić, N. Regnault, and B. A. Bernevig, Phys. Rev. B **87**, 161112 (2013).

[23] B. A. Bernevig and F. D. M. Haldane, Phys. Rev. Lett. **100**, 246802 (2008).

[24] B. A. Bernevig and N. Regnault, Phys. Rev. Lett. **103**, 206801 (2009).

[25] R. Thomale, B. Estienne, N. Regnault, and B. A. Bernevig, Phys. Rev. B **84**, 045127 (2011).

[26] A. Seidel, H. Fu, D.-H. Lee, J. M. Leinaas, and J. Moore, Phys. Rev. Lett. **95**, 266405 (2005).

[27] A. Seidel and D.-H. Lee, Phys. Rev. Lett. **97**, 056804 (2006).

[28] A. Seidel and K. Yang, Phys. Rev. Lett. **101**, 036804 (2008).

[29] E. J. Bergholtz and A. Karlhede, J. Stat. Mech.: Theory Exp. (2006) L04001.

[30] E. J. Bergholtz, J. Kailasvuori, E. Wikberg, T. H. Hansson, and A. Karlhede, Phys. Rev. B **74**, 081308 (2006).

[31] X.-G. Wen and Z. Wang, Phys. Rev. B **77**, 235108 (2008).

[32] X.-G. Wen and Z. Wang, Phys. Rev. B **78**, 155109 (2008).

[33] Y.-H. Wu, T. Shi, and J. K. Jain, Nano Lett. **17**, 4643 (2017).

[34] M. Greiter, X.-G. Wen, and F. Wilczek, Phys. Rev. Lett. **66**, 3205 (1991).

[35] N. Read and E. Rezayi, Phys. Rev. B **59**, 8084 (1999).

[36] E. H. Rezayi and A. H. MacDonald, Phys. Rev. B **44**, 8395 (1991).

[37] E. McCann and V. I. Fal'ko, Phys. Rev. Lett. **96**, 086805 (2006).

[38] Y. Barlas, K. Yang, and A. MacDonald, Nanotechnology **23**, 052001 (2012).

[39] L. Chen, S. Bandyopadhyay, and A. Seidel, Phys. Rev. B **95**, 195169 (2017).

[40] M. Tanhayi Ahari, S. Bandyopadhyay, Z. Nussinov, A. Seidel, and G. Ortiz (unpublished).

[41] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.98.161118 for second quantization of parent Hamiltonian, detailed calculation of entangled Pauli principle, emergent SU(2)-symmetry, braiding statistics, zero mode counting and edge physics.

[42] A. Weerasinghe and A. Seidel, Phys. Rev. B **90**, 125146 (2014).

[43] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Phys. Rev. Lett. **59**, 799 (1987).

[44] C. D. Batista and G. Ortiz, Adv. Phys. **53**, 1 (2004).

[45] S. Pu, Y.-H. Wu, and J. K. Jain, Phys. Rev. B **96**, 195302 (2017).

[46] A. Seidel and D.-H. Lee, Phys. Rev. B **76**, 155101 (2007).

[47] A. Seidel, Phys. Rev. Lett. **101**, 196802 (2008).

[48] A. Seidel and K. Yang, Phys. Rev. B **84**, 085122 (2011).

[49] A. Seidel, Phys. Rev. Lett. **105**, 026802 (2010).

[50] J. Flavin and A. Seidel, Phys. Rev. X **1**, 021015 (2011).

[51] J. Flavin, R. Thomale, and A. Seidel, Phys. Rev. B **86**, 125316 (2012).

[52] C. Nayak and F. Wilczek, Nucl. Phys. B **479**, 529 (1996).

[53] X.-G. Wen, Phys. Rev. B **60**, 8827 (1999).

[54] B. Overbosch and X.-G. Wen, arXiv:0804.2087.

[55] N. Read, Phys. Rev. B **79**, 245304 (2009).

[56] N. Read and E. H. Rezayi, Phys. Rev. B **84**, 085316 (2011).

[57] M. Milovanović and N. Read, Phys. Rev. B **53**, 13559 (1996).

[58] N. Read, Phys. Rev. B **79**, 045308 (2009).

[59] S.-S. Lee, S. Ryu, C. Nayak, and M. P. A. Fisher, Phys. Rev. Lett. **99**, 236807 (2007).

[60] M. Levin, B. I. Halperin, and B. Rosenow, Phys. Rev. Lett. **99**, 236806 (2007).

[61] F. D. M. Haldane, Phys. Rev. Lett. **50**, 1153 (1983).

[62] F. D. M. Haldane, Phys. Lett. A **93**, 464 (1983).

[63] E. Kapit and E. Mueller, Phys. Rev. Lett. **105**, 215303 (2010).

[64] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. D. Sarma, Rev. Mod. Phys. **80**, 1083 (2008).