Heat transfer statistics in extreme-near-field radiation

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We investigate the full counting statistics of extreme-near-field radiative heat transfer using nonequilibrium Green's function formalism. In the extreme near field, the electron-electron interactions between two metallic bodies dominate the heat transfer process. We start from a general tight-binding electron Hamiltonian and obtain a Levitov-Lesovik-like scaled cumulant-generating function (SCGF) using random-phase approximation to deal with electron-electron interaction. The expressions of heat current and its fluctuation (second cumulant) are obtained from the SCGF. The fluctuation symmetry relation of the SCGF is verified. In the linear response limit (small-temperature gradient), we express the heat current cumulant by a linear combination of lower-order cumulants. The heat current fluctuation is $2k_BT^2$ times the thermal conductance with T being the average temperature in the linear response limit, and this provides an evaluation of heat current fluctuation by measuring the thermal conductance in extreme-near-field radiative heat transfer.

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I. INTRODUCTION

Heat transfer between two bodies in the far-field regime can be well described by Planck's theory of black-body radiation [1]. During the 1970s, experiments in the near field have shown that heat transfer becomes much larger than that being predicted by the Stefan-Boltzmann law with gap sizes smaller than Wien's wavelength [2,3]. Polder and van Hove (PvH) [4] pioneered to give a theoretical description of near-field radiation using Rytov's formulation of fluctuating electrodynamics [5–7]. In the PvH theory, the contributions of heat transfer are mainly from evanescent modes which vanish in the far field. Experimentalists have reduced the gap sizes from orders of 1 μm [8–10] to several tens of nanometers, resulting in heat transfer enhancement from several folds to thousands of folds compared to the corresponding far-field results [11–17]. And these experimental results can be well predicted by fluctuating electrodynamics. Researchers now can reduce gap sizes to within a few nanometers [18–24] or even down to a few Ångströms [23,24] and study the extreme-near-field radiative heat transfer (eNFRHT). In this extreme near field, the propagating field represented by the vector potential is not important and heat transfer is dominated by the scalar potential, i.e., the instantaneous Coulomb interaction. There have been several works on this [25–31], including using the formalism of the nonequilibrium Green's function (NEGF) to deal with heat radiation mediated by electron-electron interaction [27–31] or dipole-dipole interaction [23,32]. Analytical results for near-field heat radiation beyond the dipolar effects have also been presented [33,34].

Electronic current fluctuations in mesoscopic conductors have received intensive investigations and are very important in characterizing the correlations in quantum transport [35]. In order to fully characterize a quantum transport process, people usually employ the formalism of full counting statistics (FCS), which yields not only average current and current fluctuation (the second cumulant) but also the higher-order cumulants [36–58]. FCS for heat and electronic transport in mesoscopic conductors has many applications. For example, entanglement entropy can be accessed by series of the charge cumulants [59,60]. Gallavotti-Cohen symmetry of the generating function in FCS can reveal the symmetry of a nonequilibrium system and can gives the fluctuation theorem of a physical quantity [61–65]. Analogously, due to both thermal and quantum fluctuations, radiative heat transfer between two bodies is stochastic in nature and subject to fluctuations as well. The fluctuation of the heat flux of black-body radiation in the far-field regime was studied by Einstein in 1909 [66], and the fluctuation theorem of black-body radiation has also been recently reported [67]. In the near-field regime, fluctuations of radiative heat transfer have been investigated using the fluctuating electrodynamics [68]. A FCS investigation of near-field heat transfer has yet to be undertaken, and when it is, then the lacuna shall be filled.

In this work, we investigate the heat transfer statistics in the extreme near field dominated by the electron-electron interaction between two metallic bodies. Since obtaining the generating function using the NEGF for heat conduction has been extensively reported [46–49,57,58], we adopt the NEGF formalism which has been used to study the heat current in near-field heat radiation [27–30] to study FCS. The formalism of the NEGF can also give an atomistic description of a system. We start from a general tight-binding Hamiltonian in the presence of Coulomb interaction and obtain the partition function using a path integral in the time domain. Random-phase approximation (RPA) is employed in order to deal with the Coulomb interaction. By introducing a counting parameter, we obtain the modified Hamiltonian together with the modified evolution operator. The generating function is obtained by involving the partition function with the counting field and then the normalization condition. The scaled cumulant-

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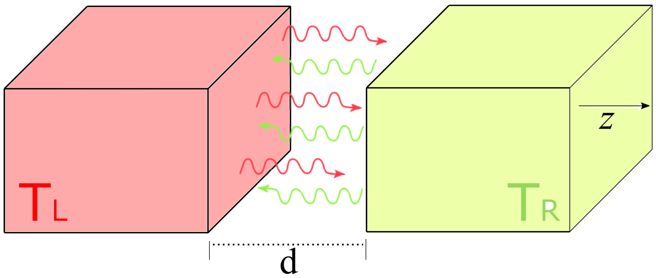

FIG. 1. Model for extreme-near-field radiative heat transfer between two vacuum-gapped semi-infinite sides meditated by Coulomb interaction.

generating function (SCGF) is expressed in the energy domain and is reminiscent of Levitov-Lesovik's formula [36–38]. From the SCGF, one can get the average heat current, the fluctuations, and even higher-order cumulants. The fluctuation symmetry in the heat radiation system is verified, and one can also relate the heat current fluctuation with the thermal conductance in the linear response limit. In the Sec. III, using a simple two-dot model, we show the relative difference of the current fluctuation evaluated in the linear response limit and its corresponding value at finite temperature differences and gap distances.

II. THEORETICAL FORMALISM

A. Model and Hamiltonian

For the extreme-near-field radiative heat transfer system, we consider two parallel aligned cubic lattices described by a tight-binding Hamiltonian (see Fig. 1). The two sides are maintained local thermal equilibria with different temperatures, and they exchange heat through the vacuum gap with distance d via electron-electron interaction. The roughness of the surfaces can also be taken take care of here, since the formalism presented below is atomistic.

One can partition the system Hamiltonian as

$$H = H_{0L} + H_{0R} + V_L + V_R + V_{LR}, \tag{1}$$

where

$$H_{0\alpha} = \sum_{m\in\alpha,n\in\alpha} c_m^\dagger h_{mn} c_n, \tag{2}$$

$$V_\alpha = \frac{e_0^2}{2} \sum_{m\in\alpha,n\in\alpha} c_m^\dagger c_m v_{mn} c_n^\dagger c_n, \tag{3}$$

$$V_{LR} = e_0^2 \sum_{m\in L,n\in R} c_m^\dagger c_m v_{mn} c_n^\dagger c_n, \tag{4}$$

with $\alpha = L(R)$ representing the left (right) side and e_0 the elementary charge. $H_{0\alpha}$ is the noninteracting Hamiltonian and V_α is the Coulomb interaction in side α. V_{LR} is the Coulomb interaction between the electrons on the left side and the right side. The front coefficient $1/2$ in V_α is to avoid the double counting. $c_m^{(\dagger)}$ are the annihilation (creation) operators on the left side or the right side. h_{mn} is the on-site energy for $m = n$ and the hopping constant for $m \neq n$. The Hamiltonian can also be written in a compact form,

$$H = \sum_{mn} c_m^\dagger h_{mn} c_n + \frac{e_0^2}{2} \sum_{mn} c_m^\dagger c_m v_{mn} c_n^\dagger c_n. \tag{5}$$

Throughout this work, the left side is set warmer than the right side so that $T_L > T_R$ with $\Delta T = T_L - T_R$.

B. Partition function

We assume that the Coulomb interaction between the left side and the right side is absent at time $t = 0$, so that the initial density matrix of the whole system at $t = 0$ is the direct product of each subsystem and is expressed as $\rho(0) = \rho_L \otimes \rho_R$. After time $t = 0$, the interaction between the left side and the right side is turned on and the system evolves to time t under the evolution operator $U(t,0) = \mathbb{T}\exp[-i\int_0^t H(t')dt'/\hbar]$, where $\mathbb{T}$ is the time ordering operator on the Keldsyh contour. Since we let t go into infinity and consider the steady state of heat transfer between two bodies, the initial system state does not influence any steady-state physical quantities. The partition function of the whole system without any source field or counting field is written as $Z(t) = \mathrm{Tr}[\rho(0)U^\dagger(t,0)U(t,0)]/\mathrm{Tr}\rho(0)$ and is exactly 1. In the next subsection, the generating function is obtained by considering the counting field in the partition function. Using a path integral on the Keldysh contour, the partition function can be expressed as [69]

$$Z(t) = \frac{1}{\mathrm{Tr}\rho(0)} \int \mathcal{D}[\bar\phi\phi] \exp[i\mathcal{S}_0 + i\mathcal{S}_{\rm int}], \tag{6}$$

with $\mathcal{S}_0$ representing the action of the free electron lattice,

$$\mathcal{S}_0 = \int_C d\tau \sum_{mn} \bar\phi_m G^{-1}_{mn} \phi_n, \tag{7}$$

and $\mathcal{S}_{\rm int}$ representing the Coulomb interaction,

$$\mathcal{S}_{\rm int} = -\frac{e_0^2}{2} \int_C d\tau \sum_{mn} \bar\phi_m \phi_m v_{mn} \bar\phi_n \phi_n. \tag{8}$$

In the above expressions, $\bar\phi_m$ and ϕ_m are the fermionic Grassmann variables, and G^{-1} is the inverse electronic Green's function [69]. We have set $\hbar$ to 1. The integration is over the Keldysh contour C. From now on, the time on the Keldysh contour is denoted as Greek letters, and real time is denoted using Latin letters.

Performing the Hubbard-Stratonovich transformation [69,70] by introducing the real scalar field Ψ_m, one can reduce the four-particle interaction exactly in terms of an effective electron-photon interaction that is expressed as

$$\exp(i\mathcal{S}_{\rm int}) = \int \mathcal{D}[\Psi] \exp\left\{ i\int_C d\tau \left[\frac{1}{2}\sum_{mn} \Psi_m \left(\frac{v^{-1}}{e_0^2}\right)_{mn} \Psi_n - \sum_m \Psi_m \bar\phi_m \phi_m \right]\right\}, \tag{9}$$

where v^{-1} is the inverse Coulomb interaction matrix. The integration measure $\int \mathcal{D}[\Psi]$ is normalized such that $\int \mathcal{D}[\Psi] \exp\{i\int_C d\tau \frac{1}{2}\Psi(v^{-1}/e_0^2)\Psi\} = 1$. The partition function $Z(t)$ could be further simplified by integrating out the fermionic Grassmann variables with the following relation,

$$\int \mathcal{D}[\bar{\phi}\phi] \exp\left[i \int_C d\tau \sum_{mn} \bar{\phi}_m G^{-1}_{mn} \phi_n - i \sum_m \Psi_m \bar{\phi}_m \phi_m\right] = \det(-iG^{-1} + i\Psi), \tag{10}$$

where $\Psi = \mathrm{diag}[\Psi^+, -\Psi^-]$ is diagonal in the Keldysh, time and the lattice space. Using $\det M = \exp[\mathrm{Tr}\ln M]$, the partition function Z has the form

$$Z \simeq \int \mathcal{D}[\Psi] \exp\left\{i \int_C d\tau \mathrm{Tr}\left[\frac{1}{2}\Psi\left(\frac{v^{-1}}{e_0^2}\right)\Psi - i\ln(1 - G\Psi)\right]\right\}, \tag{11}$$

where some front coefficients are ignored at this moment and are taken into consideration by using the normalization condition of the generating function when discussing FCS in the next subsection. Using the relations $\mathrm{Tr}\ln(1 - M) = -\sum_{j=1} M^j/j$ and $\mathrm{Tr}(G\Psi) = 0$ from the fact $G^{++}_{mn}(t_1, t_1) = G^{--}_{mn}(t_1, t_1)$ [69], one can perform RPA, i.e., expanding the partition function to the second order of the scalar field, and obtain

$$Z \simeq \int \mathcal{D}[\Psi] \exp\left\{\frac{i}{2} \int_C d\tau\ \mathrm{Tr}\left[\Psi\left(\frac{v^{-1}}{e_0^2}\right)\Psi + i(G\Psi G\Psi)\right]\right\}. \tag{12}$$

Owing to the fact that Ψ is diagonal, we have

$$\mathrm{Tr}(G\Psi G\Psi) = \mathrm{Tr}\left[\Psi\begin{pmatrix} G^{++}G^{++} & G^{+-}G^{-+} \\ G^{-+}G^{+-} & G^{--}G^{--}\end{pmatrix}\Psi\right]. \tag{13}$$

Then we can rewrite the partition function in the form

$$Z \simeq \int \mathcal{D}[\Psi] \exp\left\{\frac{i}{2} \int_C d\tau\ \mathrm{Tr}\left[\Psi\left(\frac{v^{-1}}{e_0^2}\right)\Psi - \Psi\Pi\Psi\right]\right\}, \tag{14}$$

by introducing the photon self-energy Π. Keldysh components of the photon self-energy have the expressions

$$\Pi^{ab}_{mn}(t_1, t_2) = -ie_0^2 G^{ab}_{mn}(t_1 - t_2) G^{ba}_{nm}(t_2 - t_1), \tag{15}$$

with $a = -$ and $+$, $b = -$ and $+$, and m and n belonging to the same side. We also adopt the commonly used notations $\Pi^<$ and $\Pi^>$ to denote the lesser and greater photon self-energies with the identities $\Pi^< \equiv \Pi^{+-}$ and $\Pi^> \equiv \Pi^{-+}$.

Finally, by integrating out the real scalar field, we get the partition function in the form of a Fredholm determinant in the time domain:

$$Z(t) \simeq \frac{1}{\sqrt{\det(v^{-1} - \Pi)}} = \left\{\det\left[\begin{pmatrix} v_L & v_{LR} \\ v_{RL} & v_R\end{pmatrix}^{-1} - \begin{pmatrix} \Pi_L & 0 \\ 0 & \Pi_R\end{pmatrix}\right]\right\}^{-1/2}, \tag{16}$$

where the determinant is over both the contour time and the lattice space. In the above formula, v_α is the Coulomb interaction on the same side, and $v_{LR} = v_{RL}$ is the Coulomb interaction between the left side and the right side. The Dyson equation of the photon Green's function is expressed as $D^{-1} = v^{-1} - \Pi$ or

$$\begin{pmatrix} D_{LL} & D_{LR} \\ D_{RL} & D_{RR}\end{pmatrix}^{-1} = \begin{pmatrix} v_L & v_{LR} \\ v_{RL} & v_R\end{pmatrix}^{-1} - \begin{pmatrix} \Pi_L & 0 \\ 0 & \Pi_R\end{pmatrix}. \tag{17}$$

The Dyson equation in this form holds in both time and energy space.

C. Full counting statistics

The statistical behaviors of the heat transfer on a specific side are all encoded in the probability distribution $P(\Delta\epsilon, t)$ of the transferred energy $\Delta\epsilon = \epsilon_t - \epsilon_0$ between an initial time $t = 0$ and time t. The generating function $\mathcal{Z}(\lambda, t)$ with the counting field λ is defined as

$$\mathcal{Z}(\lambda, t) \equiv \langle e^{i\lambda\Delta\epsilon}\rangle = \int P(\Delta\epsilon, t) e^{i\lambda\Delta\epsilon} d\Delta\epsilon. \tag{18}$$

To investigate statistical behaviors of the transferred energy from the left side, we could focus on the energy operator which is actually the free Hamiltonian H_{0L}. Under the two-time measurement scheme [43,71], the generating function of transferred energy can be expressed over the Keldysh contour as [43,48,49,51]

$$\mathcal{Z}(\lambda, t) = \mathrm{Tr}\left\{\rho(0)\mathcal{T}_C \exp\left[-\frac{i}{\hbar}\int_C H_\gamma(t')dt'\right]\right\}\Big/\mathrm{Tr}\rho(0) = \mathrm{Tr}\{\rho(0)U^\dagger_{\lambda/2}(t,0)U_{-\lambda/2}(t,0)\}/\mathrm{Tr}\rho(0), \tag{19}$$

with the modified evolution operator

$$U_\gamma(t, 0) = \mathbb{T}\exp\left[-\frac{i}{\hbar}\int_0^t H_\gamma(t')dt'\right], \tag{20}$$

where $\gamma = -\lambda/2$ on the forward contour branch and $\gamma = \lambda/2$ on the backward contour branch. Here the modified evolution operator is expressed by the modified Hamiltonian

$$H_\gamma = e^{i\gamma H_{0L}} H e^{-i\gamma H_{0L}} = \widetilde{H}_{0L} + H_{0R} + \widetilde{V}_L + V_R + \widetilde{V}_{LR}. \tag{21}$$

The tilde over the Hamiltonians means that the annihilation (creation) operators $c^{(\dagger)}_m$ on the left side in Eqs. (2)–(4) are replaced with $c^{(\dagger)}_m(t_\gamma)$, with $t_\gamma = \hbar\gamma$, and $c_m(t_\gamma) = e^{i\gamma H_{0L}} c_m e^{-i\gamma H_{0L}}$ for $m \in L$. This replacement only affects the electronic Green's function on the left side with a time shift for lesser and greater components in the partition function, which means $G^{ab}_{mn}(t_1 - t_2) \to G^{ab}_{mn}(t_1 - t_2 - (a - b)\lambda/2)$, with $m, n \in L$.

Considering the counting field and the normalization condition, we arrive at the generating function being expressed as

$$\mathcal{Z}(t) = \frac{\sqrt{\det(v^{-1} - \Pi)}}{\sqrt{\det(v^{-1} - \widetilde{\Pi})}}, \tag{22}$$

with the transformed photon self-energy on the left side being expressed as

$$\widetilde{\Pi}^{ab}_{mn}(t_1, t_2) = -ie_0^2 G^{ab}_{mn}\left(t_1 - t_2 - (a - b)\frac{\lambda}{2}\right) \times G^{ba}_{nm}\left(t_2 - t_1 - (b - a)\frac{\lambda}{2}\right), \tag{23}$$

for $m, n \in L$. The photon self-energy on the right side remains unchanged. By using the Dyson equation, Eq. (17), and defining $\bar{\Pi} = (\widetilde{\Pi} - \Pi)$, we have

$$\mathcal{Z}(t) = \frac{1}{\sqrt{\det[1 - D_{LL}\bar{\Pi}_L]}}. \tag{24}$$

By letting $\lambda = 0$, we get a vanishing $\bar{\Pi}$ and can verify the normalization condition of the generating function. In the long time limit, we can Fourier transform Eq. (24) into the energy domain with the form

$$\lim_{t\to\infty} \ln \mathcal{Z}(t) = -\frac{t}{2}\int \frac{d\omega}{2\pi} \mathrm{Tr}\ln[1 - D_{LL}\bar{\Pi}_L]. \tag{25}$$

The Keldysh space dimension can be eliminated by writing the term $\mathrm{Tr}\ln(1-M)$ as $\ln\det(1-M)$ and then using the identity (assume A is invertible)

$$\det\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det A \det(D - CA^{-1}B). \tag{26}$$

The SCGF $\mathcal{F}(\lambda) = \lim_{t\to\infty} \ln \mathcal{Z}(t)/t$ is expressed as

$$\mathcal{F}(\lambda) = -\frac{1}{2}\int \frac{d\omega}{2\pi} \mathrm{Tr}\ln\big[1 - D^r_{LR}\Pi^<_R D^a_{RL}\bar{\Pi}^>_L - D^r_{LR}\Pi^>_R D^a_{RL}\bar{\Pi}^<_L\big], \tag{27}$$

with $\bar{\Pi}^{</>}_L = \widetilde{\Pi}^{</>}_L - \Pi^{</>}_L$.

The Fourier transformation of Eq. (23) enables us to obtain the expression of transformed self-energy in the energy domain as

$$\widetilde{\Pi}^{ab}_{mn}(\omega) = -ie_0^2\int \frac{dE}{2\pi} G^{ab}_{mn}(E)G^{ba}_{nm}(E-\hbar\omega)e^{i(a-b)\lambda\hbar\omega/2}, \tag{28}$$

where E is the unit of energy and ω is the angular frequency. In the local equilibrium approximation, the electrons are maintained in an equilibrium state, so that $G^<_{mn}(E) = iA_{mn}(E)f_\alpha(E)$ and $G^>_{mn}(E) = iA_{mn}(E)[f_\alpha(E) - 1]$, with the electronic spectral function $A_{mn}(E) = -2\mathrm{Im}[G^r_{mn}(E)]$ and $m, n \in \alpha$. Using the relation $f_\alpha(E)[f_\alpha(E-\hbar\omega) - 1] = N_\alpha(\omega)[f_\alpha(E) - f_\alpha(E-\hbar\omega)]$ with the Bose-Einstein distribution $N_\alpha(\omega) = 1/[e^{\beta_\alpha\hbar\omega} - 1]$ at the temperature $T_\alpha = 1/(k_B\beta_\alpha)$, one has

$$\Pi^<_{mn}(\omega) = -iN_\alpha(\omega)A_{\Pi mn}(\omega), \tag{29}$$

with

$$A_{\Pi mn}(\omega) = e_0^2\int \frac{dE}{2\pi}[f_\alpha(E) - f_\alpha(E-\hbar\omega)]A_{mn}(E)A_{nm}(E-\hbar\omega). \tag{30}$$

Then the lesser and greater photon self-energies can be written as

$$\Pi^<_\alpha(\omega) = N_\alpha(\omega)\big[\Pi^r_\alpha(\omega) - \Pi^a_\alpha(\omega)\big] = 2iN_\alpha(\omega)\mathrm{Im}\big[\Pi^r_\alpha(\omega)\big], \tag{31}$$

$$\Pi^>_\alpha(\omega) = [N_\alpha(\omega) + 1]\big[\Pi^r_\alpha(\omega) - \Pi^a_\alpha(\omega)\big]. \tag{32}$$

From Eqs. (29)–(31), the retarded photon self-energy is obtained with the form

$$\Pi^r_{mn}(\omega) = -ie_0^2\int \frac{dE}{2\pi}\big[G^r_{mn}(E)G^<_{nm}(E-\hbar\omega) + G^<_{mn}(E)G^a_{nm}(E-\hbar\omega)\big]. \tag{33}$$

We finally arrive at the expression of the SCGF as

$$\mathcal{F}(\lambda) = -\int_0^\infty \frac{d\omega}{2\pi}\ln(1 - \mathcal{T}(\omega)\{(e^{i\lambda\hbar\omega} - 1)N_L(\omega) \times [1 + N_R(\omega)] + (e^{-i\lambda\hbar\omega} - 1)N_R(\omega)[1 + N_L(\omega)]\}), \tag{34}$$

where the transmission coefficient is

$$\mathcal{T}(\omega) = 4\mathrm{Tr}\big\{D^r_{LR}(\omega)\mathrm{Im}\big[\Pi^r_R(\omega)\big]D^a_{RL}(\omega)\mathrm{Im}\big[\Pi^r_L(\omega)\big]\big\}. \tag{35}$$

The SCGF is reminiscent of Levitov-Lesovik's formula for electronic transport [36–38]. The front coefficient 1/2 in Eq. (27) is missing in Eq. (34), because the contributions from the positive and negative angular frequencies are the same for the heat current.

The kth cumulant of the heat current $\langle\langle I_h^k\rangle\rangle$ could be calculated by taking the kth derivative of the SCGF, which is $\mathcal{F}(\lambda)$ with respect to $i\lambda$,

$$\langle\langle I_h^k\rangle\rangle = \frac{\partial^k\mathcal{F}(\lambda)}{\partial(i\lambda)^k}\bigg|_{\lambda=0}. \tag{36}$$

The heat current (the first cumulant) bears a Caroli form [27–31],

$$I_h = \int_0^\infty \frac{d\omega}{2\pi}\hbar\omega\mathcal{T}(\omega)[N_L(\omega) - N_R(\omega)]. \tag{37}$$

The heat current fluctuation (the second cumulant) has the following expression,

$$\langle\langle I_h^2\rangle\rangle = \int_0^\infty \frac{d\omega}{2\pi}(\hbar\omega)^2\{\mathcal{T}[N_L(1+N_R) + N_R(1+N_L)] + \mathcal{T}^2[N_L - N_R]^2\}. \tag{38}$$

Applying the relation $N_R(1+N_L) = \exp(\Delta\beta\hbar\omega)N_L(1+N_R)$, with $\Delta\beta = \beta_L - \beta_R$, in Eq. (34), we can verify the following fluctuation symmetry relation,

$$\mathcal{F}(\lambda) = \mathcal{F}(-\lambda - i\Delta\beta). \tag{39}$$

This symmetry relation has already been derived for heat transfer through conductors [57,58,62,63], and it is now verified for eNFRHT where heat transfer through a gap is mediated by Coulomb interaction in the RPA level. This symmetry implies that the backward probability of the transferred energy $-\Delta\epsilon$ from the cold right side to the hot left side is exponentially suppressed with respect to the forward one with the detailed balance relation (also called Gallavotti-Cohen symmetry [72,73])

$$\frac{P(-\Delta\epsilon)}{P(\Delta\epsilon)} = \exp[(\beta_L - \beta_R)\Delta\epsilon], \tag{40}$$

which also implies that

$$\int d\Delta\epsilon P(\Delta\epsilon) = \int d\Delta\epsilon P(-\Delta\epsilon)e^{-\Delta\beta\Delta\epsilon} \equiv \langle e^{-\Delta\beta\Delta\epsilon}\rangle = 1. \tag{41}$$

The above equality, Eq. (40), also holds in black-body radiation in the far field [67].

Now we consider the universal relations for heat current cumulants under a small temperature gradient which is in analogy with the universal relation for particle current cumulants [40,41]. In the linear response regime $\Delta\beta \to 0$, the total derivative of $\mathcal{F}(-\lambda - i\Delta\beta)$ with respect to $\Delta\beta$ has the following expansion,

$$\frac{d^k\mathcal{F}(-\lambda - i\Delta\beta, \Delta\beta)}{d\Delta\beta^k}\bigg|_{\lambda=0} = \sum_{j=0}^{k}\binom{k}{j}\frac{\partial^k\mathcal{F}(i\lambda,\Delta\beta)}{\partial\Delta\beta^{k-j}\partial(i\lambda)^j}\bigg|_{\lambda=0}, \tag{42}$$

where we have written the dependence of $\Delta\beta$ of the SCGF explicitly out on both sides. Since $\mathcal{F}(\lambda = 0, \Delta\beta) = 0$, the left-hand side of Eq. (42) vanishes due to Eq. (39). The last term in the summation of Eq. (42) is the kth derivative of the SCGF with respect to the counting field $i\lambda$, which is actually $\langle\langle I_h^k\rangle\rangle$ in the linear response limit. Then we have the relation

$$\langle\langle I_h^k\rangle\rangle_l = -\sum_{j=1}^{k-1}\binom{k}{j}\frac{\partial^{k-j}\langle\langle I_h^j\rangle\rangle_l}{\partial\Delta\beta^{k-j}}, \tag{43}$$

in which the heat current cumulant is expressed by a linear combination of lower-order heat current cumulants. Here, we have added the subscript "l" in $\langle\langle I_h^k\rangle\rangle$ to distinguish it from the one calculated from Eq. (36). By specifying $k = 2$, we can relate the heat current fluctuation with the heat current through $\langle\langle I_h^2\rangle\rangle_l = -2\partial I_h/\partial\Delta\beta$, which leads to

$$\langle\langle I_h^2\rangle\rangle_l = 2k_B T^2 G_h, \tag{44}$$

where the average temperature $T = (T_L + T_R)/2$ and the thermal conductance $G_h \equiv \partial I_h/\partial\Delta T$, with $\Delta T = T_L - T_R$.

III. NUMERICAL CALCULATION

Current fluctuation in an electron transport system is more difficult to experimentally measure compared to the mean current. It is expected that the heat current fluctuation is difficult to measure as well. The linear response relation, Eq. (44), provides us an evaluation of heat current fluctuations using heat conductance. Since the relation of Eq. (44) is obtained in the linear response limit, the actual heat current fluctuation should deviate from the one evaluated in Eq. (38) beyond the linear response limit. To quantify the deviation, we introduce the relative difference for the heat current fluctuation:

$$d_r = |\langle\langle I_h^2\rangle\rangle - \langle\langle I_h^2\rangle\rangle_l|/\langle\langle I_h^2\rangle\rangle. \tag{45}$$

For a general problem of near-field radiation between metal objects at a distance of order nanometers, one can use the recursive Green's function method to get the retarded Green's function in the absence of Coulomb interaction [74,75]. For the situation of an infinitely large surface, the periodic boundary condition can be used so that one can work in the momentum space. Having obtained the Green's functions, the photon self-energies can be obtained through convolution from Eqs. (31)–(33). The photon Green's function can be found through matrix inversion indicated by Eq. (17). To get the electron density of states in one of the surfaces more accurately, the Fock self-energies are incorporated in the noninteracting retarded Green's function [29]. Since the Thomas-Fermi screening length in metals is usually a few lattice spacings, three to five layers are enough for convergency [30]. More calculation details can be found in Refs. [29,30].

For simplicity, we consider a nano-sized capacitor consisting of two quantum dots [27], and each plate can host a charge of 0 or $-Q$. The retarded photon self-energy is calculated as

$$\Pi_\alpha^r(\omega) = -iQ^2\int\frac{dE}{2\pi}\big[G_\alpha^r(E)G_\alpha^<(E-\hbar\omega) + G_\alpha^<(E)G_\alpha^a(E-\hbar\omega)\big], \tag{46}$$

where

$$G_\alpha^r(E) = \big[G_\alpha^a(E)\big]^* = 1/\big[E - \epsilon_\alpha - \Sigma_\alpha^r(E)\big],$$
$$G_\alpha^<(E) = -f_\alpha(E)\big[G_\alpha^r(E) - G_\alpha^a(E)\big], \tag{47}$$

with the Fermi distribution function $f_\alpha(E) = 1/\{\exp[(E-\mu_\alpha)/(k_BT_\alpha)]+1\}$ at temperature T_α and chemical potential μ_α. The self-energies due to electron reservoirs are chosen to follow the Lorentz-Drude model [76] with $\Sigma_\alpha^r(E) = \frac{1}{2}\Gamma_\alpha/(i + E/E_\alpha)$, where Γ_α and E_α are electron reservoir

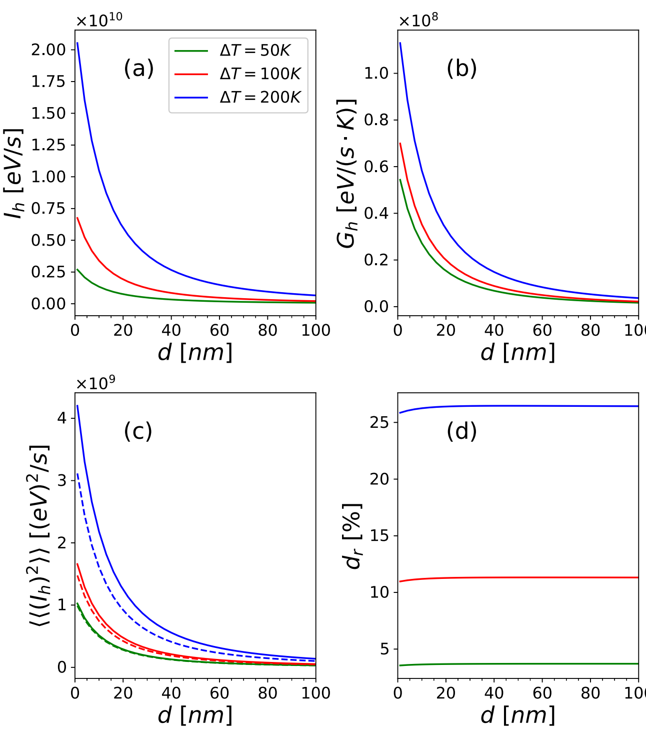

FIG. 2. (a) Heat current, (b) thermal conductance, (c) heat current fluctuation, and (d) the relative difference for heat current fluctuation versus the gap distance for different temperature gradients ΔT with $T_R = 300$ K. In panel (c), the exact heat current fluctuations and the ones in the linear response limit are plotted using solid and dashed lines, respectively. We set the chemical potentials of the electron reservoirs on both sides as $\mu_L = \mu_R = 0$, and we set the quantum dot levels as $\epsilon_L = \epsilon_R = 0$. Other electron reservoir constants are $\Gamma_L = 1$ eV, $\Gamma_R = 0.5$ eV, $E_L = 2$ eV, and $E_R = 1$ eV. The areas of both plates are chosen as $A = 389.4$ nm^2 to be close to the experimental value [23], and $Q = 1.5e_0$.

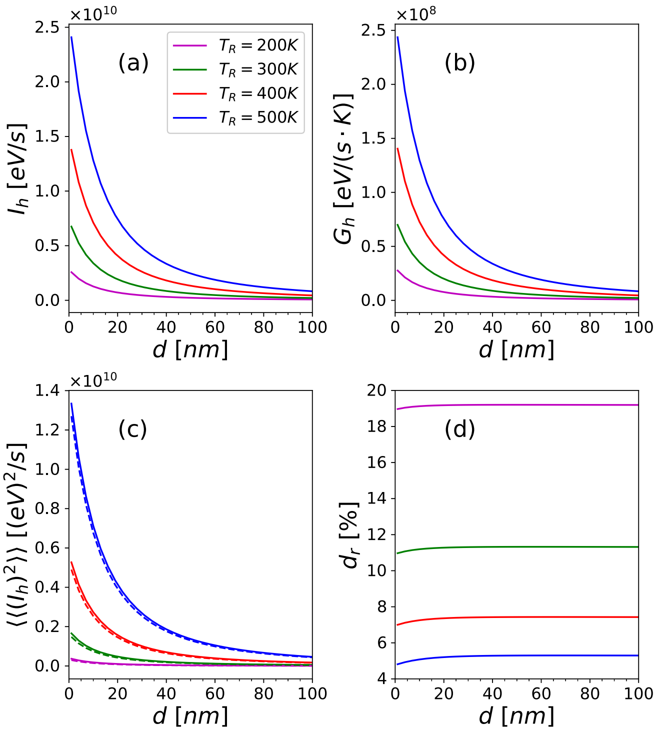

FIG. 3. (a) Heat current, (b) thermal conductance, (c) heat current fluctuation, and (d) the relative difference for heat current fluctuation versus the gap distance by varying T_R with $\Delta T = 100$ K. Heat current fluctuations in the linear response limit are plotted in dashed lines in panel (c). Other parameters are the same as those in Fig. 2.

constants. The Coulomb interaction matrix for the capacitor is [30]

$$v^{-1} = \begin{pmatrix} C & -C \\ -C & C \end{pmatrix}, \tag{48}$$

where the capacitance of the parallel plate is $C = \epsilon_0 A/d$, with A being the plate area and ϵ_0 being the vacuum dielectric constant. The photon retarded Green's function D^r_{LR} is then obtained from the Dyson equation, Eq. (17), with the form [27]

$$D^r_{LR} = (D^a_{RL})^* = \left[\Pi^r_L \Pi^r_R / C - (\Pi^r_L + \Pi^r_R)\right]^{-1}. \tag{49}$$

Heat current and fluctuation is calculated from Eqs. (37) and (38), respectively. Thermal conductance is obtained by numerically differentiating the heat current.

We plot the heat current, the thermal conductance, the heat current fluctuation, and the relative difference for the heat current fluctuation versus the vacuum gap distance by varying temperature gradient ΔT in Fig. 2 and by varying T_R in Fig. 3. In Figs. 2(c) and 3(c), the exact heat current fluctuations and the ones in the linear response limit are plotted using solid and dashed lines, respectively. A nondivergent heat current is found with $d \to 0$. The $1/d^2$ divergence at short distance is the result of using a local dielectric function in the framework of fluctuating electrodynamics [19,77]. Since in our approach, we do not use such an approximation, the heat current is found to be convergent at zero distances. Figures 2(b) and 3(b) demonstrate the behaviors of thermal conductance with respect to temperatures, i.e., that thermal conductance increases with increasing temperature gradient or average temperature. One can clearly see that the relative differences d_r increase with increasing temperature gradients. The linear response approximation, Eq. (44), becomes less accurate with decreasing average temperature, as shown in Fig. 3(d). One can see from Figs. 2(d) and 3(d) that the relative difference is not sensitive to the vacuum gap distance.

IV. CONCLUSION

In this work, using the nonequilibrium Green's function formalism, we have obtained the scaled cumulant-generating function (SCGF) of the heat transfer in the extreme-near-field radiation. Random-phase approximation has been used in dealing with the electron-electron interaction which meditates the heat exchange between two bodies. We have verified the fluctuation symmetry of the SCGF and demonstrated that the probability for energy flown from the cold side to the hot side is exponentially suppressed. Both heat current and its fluctuations are obtained from the SCGF, and the heat current is in a Caroli form. The heat current cumulant is shown to be expressed by a linear combination of lower-order cumulants in the linear response limit. A specific case of this is that heat current fluctuation is proportional to thermal conductance in the linear response limit. We numerically show the deviations of fluctuations evaluated in the linear response limit from its value. The evaluation of fluctuation from thermal conductance becomes poorer with larger temperature gradients and lower average temperatures, and the relative difference is not sensitive to the gap distance.

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