Surface plasmon polaritons sustained at the interface of a nonlocal metamaterial

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Studying basic physical effects sustained in metamaterials characterized by specific constitutive relation is a research topic with a long-standing tradition. Besides intellectual curiosity, it derives its importance from the ability to predict observable phenomena that are, if found with an actual metamaterial, a clear indication on its properties. Here, we consider a nonlocal (strong spatial dispersion), lossy, and isotropic metamaterial and study the impact of the nonlocality on the dispersion relation of surface plasmon polaritons sustained at an interface between vacuum and such metamaterial. For that, Fresnel coefficients are calculated and appropriate surface plasmon polaritons existence conditions are being proposed. Predictions regarding the experimentally observable reflection from a frustrated internal reflection geometry are being made. A different behavior for TE and TM polarization is observed. Our work unlocks opportunities to seek for traces of the nonlocality in experiments made with nowadays metamaterials.

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I. INTRODUCTION

Metamaterials are artificial optical materials that are made from, mostly, periodically arranged inclusions. They are studied out of scientific interest but also because they unlock in perspective interesting applications [1]. Among many examples, we mention perfect lenses [2], invisibility cloaks [3,4], and electromagnetic black holes [5]. One of the main problems in investigating metamaterials with complicated and densely packed geometries concerns their effective description [6]. The goal of finding this description, known as homogenization, is to associate the actual metamaterial to a homogeneous material that has the same optical response. Optical response here means that once the material is homogenized, it can be considered in other geometries and optical settings and the description of the interaction of electromagnetic fields with this homogeneous material continues to be the same as if the actual metamaterial would have been considered. The process of homogenization can be considered as a two step process. First, a suitable constitutive relation is chosen that is expected to cover all emerging effects. Second, by choosing one among many possible technical means, the actual material parameters are retrieved. Occasionally, a particular temporal dispersion, i.e., a frequency dependence, can additionally be assumed for the effective material properties, e.g., the Drude formula for the permittivity when free electrons are considered and the Lorentz formula for bound states, e.g., if the metals additionally form loops, and support resonances such as in a ring resonator. The functional dependence is motivated by basic phenomenological modeling [7]. The availability of such a functional dependence is useful in some numerical schemes, e.g., in the finite-difference time domain method, or when discussing the effects to be supported in metamaterials without being forced to consider a specific implementation.

In the cases where the inclusions are much smaller than the excitation wavelength, local constitutive relations turned out to be fully sufficient [8,9]. While considering for simplicity isotropic materials with no electromagnetic coupling, the well-known Drude and/or Lorentz models for permeability and permittivity do frequently emerge for the frequency dependence of the effective material properties [10]. These material properties have singularities only in the lower half of the complex frequency plane and are with that causal. The emergence of such models can be explained by considering on phenomenological grounds the inclusions to cause either a response associated to free electrons, e.g., in straight wire elements [11,12], or a response associated to a harmonic oscillator, e.g., in a small metallic or dielectric particle that is driven into a resonant optical response, in a split ring [13,14], or any other complicated inclusion that has been suggested in the past [15–17].

However, when the metamaterials are operated at wavelengths that are not much longer than the size of the inclusion but only gently longer or the inclusions themselves show a strong coupling to their neighbors, a local description at the effective level fails to capture the properties of the metamaterial [18]. In some cases, this was even shown to be relevant in the long wavelength limit [19]. An electric field at one point in space can then induce a response at a distant point in space. To improve the description, it is therefore necessary to go beyond the local description and take into account the nonlocality; that is also called a strong spatial dispersion.

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Recently, a model has been proposed [20] introducing specific nonlocal constitutive relations that read as

$$\mathbf{D}(\mathbf{r},\omega) = \boldsymbol{\epsilon}\mathbf{E}(\mathbf{r},\omega) + \nabla\times\boldsymbol{\gamma}\nabla\times\mathbf{E}(\mathbf{r},\omega) + \nabla\times\nabla\times\boldsymbol{\eta}\nabla\times\nabla\times\mathbf{E}(\mathbf{r},\omega) \quad (1)$$

and

$$\mathbf{B}(\mathbf{r},\omega) = \mathbf{H}(\mathbf{r},\omega), \quad (2)$$

with material parameters $\boldsymbol{\epsilon}$, $\boldsymbol{\gamma}$, and $\boldsymbol{\eta}$. In the case of an isotropic medium, as it is assumed in the later calculations, these parameters are scalar functions of the frequency. $\boldsymbol{\gamma}$ is directly related to the local permeability and thus can be replaced by a function of $\boldsymbol{\mu}$. For that reason it is called a weak spatial dispersion. If a formulation of the constitutive relation is mathematically equivalent and physically indistinguishable from a formulation without any derivatives, we call it local. In our case, that happens to be possible with what we call the weak spatial dispersion, where the dispersion relation and interface conditions are equivalent to the ones in the case of local constitutive relations being considered. In contrast, $\boldsymbol{\eta}$ is associated to a strong spatial dispersion; it is a nonlocal material parameter. The constitutive relations above allow for a rigorous mathematical derivation of interface conditions that extend the known conditions from basic electrodynamics [20]. Therefore, not just light propagation in bulk material can be described but also functional elements thereof.

In this contribution, we continue a long-standing tradition in metamaterials research where basic physical effects are explored for a specific material with properties coming from a model assumption. On the one hand, such research is intellectually appealing, since effects that were predicted with those materials constitute a major driving force to finally identify materials that offer these properties. This may have started with the seminal work by Veselago [21]. There, he simply assumed a material with a dispersive permittivity and a dispersive permeability and afterwards studies observable optical effects. Contemporary examples would be the field of transformation optics or the suggestion for a perfect lens [22–24]. On the other hand, with this kind of consideration we can predict experimentally observable features that would be a conclusive evidence to decide whether a particular constitutive relation is indeed a viable effective description of a metamaterial. This shows the possibility of a way from a purely theoretical approach to ultimately a potential experimental verification thereof.

Here, we investigate propagating surface plasmon polaritons (SPP) at the interface between an ordinary material and a nonlocal metamaterial that exhibits a negative refractive index in some frequency range of interest. A list of metamaterials that have been designed to undergo such properties can be found in [25]. Our contribution is particularly inspired by the work of Ruppin [26] that pioneered the study of surface waves sustained at the interface between metamaterials described by local constitutive relations and ordinary media. Specifically, we investigate the dispersion relation of surface waves and the reflection to be observed in a frustrated total internal reflection geometry and how these are affected by a strong nonlocality for both transverse electric and transverse magnetic polarizations. While we focus on metallic structures that undergo a negative index material, a similar study was made in Refs. [27,28] for dielectric structures described with a nonlocal, hydrodynamic model for the electric susceptibility. There, the authors found that the difference between the transversal and longitudinal (nonlocal) susceptibilities yields a great impact on the reflection minima for the p polarization. With our contribution here, we point to possible traces of nonlocality observable in experiment.

Our manuscript is structured such that we introduce in the next section the material models we consider. We study then the interface Fresnel equations, which need to be known in order to derive the dispersion relation of the propagating SPPs. In Sec. IV, we study the dispersion relation in the presence of nonlocality where we particularly emphasize the question how the onset of a weak nonlocality changes the dispersion relation of the propagating SPP sustained at the interface between vacuum and the metamaterial. In a last step, experimental features are predicted as observable in an attenuated total reflection setup. Finally, we conclude on our research.

II. MATERIAL MODELS AND DISPERSION RELATIONS

In order to model a metamaterial with negative index behavior, we consider a homogeneous and isotropic metamaterial with a Drude permittivity

$$\epsilon(\omega) = 1 - \frac{\omega_\mathrm{p}^2}{\omega(\omega + \mathrm{i}\Gamma_\epsilon)} \quad (3)$$

and a permeability described by a Lorentz model

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + \mathrm{i}\Gamma_\mu\omega}, \quad (4)$$

with parameters ω_p, ω_0, Γ_ϵ, Γ_μ, and F, which will be commented on below. By rewriting the fields to

$$\mathbf{D}'(\mathbf{r},\omega) = \mathbf{D}(\mathbf{r},\omega) + \nabla\times\mathbf{Q}(\mathbf{r},\omega) \quad (5)$$

and

$$\mathbf{H}'(\mathbf{r},\omega) = \mathbf{H} - \mathrm{i}k_0\mathbf{Q}(\mathbf{r},\omega), \quad (6)$$

the parameter $\gamma(\omega)$ that is associated to a weak spatial dispersion can be expressed through $\mu(\omega)$:

$$\gamma(\omega) = \frac{\mu(\omega) - 1}{\mu(\omega)k_0^2}, \quad (7)$$

with $k_0 = \omega/c$, by choosing $\mathbf{Q}(\mathbf{r},\omega) \overset{!}{=} -\gamma\nabla\times\mathbf{E}(\mathbf{r},\omega)$.

Introducing strong spatial dispersion via the parameter η, we make a model assumption and set its frequency dependence to

$$\eta(\omega) = \frac{GF\omega^2}{\omega_0^2 - \omega^2 - \mathrm{i}\Gamma_\eta\omega}, \quad (8)$$

where G is a parameter scaling the strength of the nonlocality. With that we connect the nonlocality to the magnetic response. This seems to be meaningful as in the resonance an enhanced light-matter interaction can be observed that will cause the nonlocal response. Moreover, the choice above assures all results being physical. Other assumptions like a nondispersive quantity would not fulfill this requirement and lead to unphysical statements, e.g., an energy velocity exceeding the speed

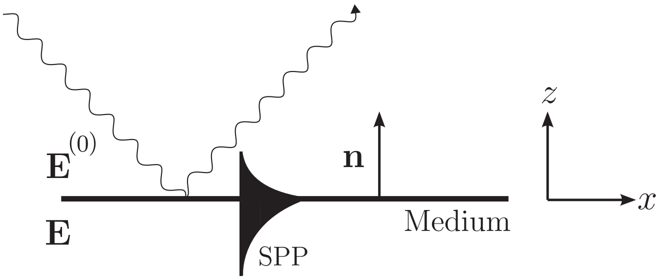

FIG. 1. Schema of surface polaritons sustained at an interface. The interface at $z = 0$ divides two extended half-spaces. One is vacuum while the other one is made initially from an isotropic homogeneous medium with permittivity ϵ and permeability μ. Later, also a more complicated constitutive relation is considered. The light is not transmitted but excites surface plasmon polaritons (SPP) on the interface. For this purpose, it is required to have a wave vector component in the direction of the interface that is larger than the length of the wave vector in vacuum.

of light in vacuum. The resonance frequency of the material is chosen to be $\omega_0 = 4$ GHz and the plasma frequency to be $\omega_\mathrm{p} = 10$ GHz. Further, we set $F = 0.56$ and the damping terms shall be $\Gamma_\epsilon = 0.03\,\omega_\mathrm{p}$ and $\Gamma_\mu = \Gamma_\eta = 0.03\,\omega_0$. For the local material parameters, the same values were used by Ruppin in [26]. The exact values are of no importance and only serve the purpose to make everything definite.

We restrict ourselves to a plane geometry where the half-space below the x axis is filled with the metamaterial and the half-space above it with vacuum, as illustrated in Fig. 1. Because the material described with the constitutive relation in Eqs. (1) and (2) is modeled as a homogeneous material and the vacuum is trivial, the eigenmodes and hence the fields can be described by plane waves

$$\mathbf{E}^{(i)}(\mathbf{r}) \propto \exp\left[\mathrm{i}\left(hx + k_z^{(i)} z\right)\right]. \tag{9}$$

$\mathbf{E}^{(0)}(\mathbf{r})$ shall be the field in the vacuum with

$$\left(k_z^{(0)}\right)^2 = k_0^2 - h^2 \tag{10}$$

being the corresponding dispersion relation. For the metamaterial with a strong spatial dispersion, characterized by the constitutive relation given in Eq. (1), the dispersion relation reads as

$$\left(k_z^{(i)}\right)^2 = -h^2 + p \pm \sqrt{p^2 - q}, \tag{11}$$

with $q = \epsilon/\eta$ and $p = (2k_0^2\eta\mu)^{-1}$ [20].

This relation has four mathematical solutions in total, two of which are physical. We denote these two with the indexes $i = 1$ and $i = 2$. Note that they only differ in the sign in front of the square root. The others result in amplitudes diverging towards infinity due to the positive imaginary part of the wave numbers. In the lossless case, for the existence of SPPs, it is usually required that the wave vector components away from the interface are purely imaginary, so that the wave is evanescent in that direction. However, in the more realistic case of an absorptive medium as considered here, this condition is not feasible [29]. Instead, we only require the radiation away from the interface to be very strongly dampened, so

$$\left|\mathrm{Im}\left(k_z^{(i)}\right)\right| > \left|\mathrm{Re}\left(k_z^{(i)}\right)\right|. \tag{12}$$

Using this condition, SPP with small radiative losses can also be discussed.

III. FRESNEL COEFFICIENTS

As mentioned above, interface conditions have been derived for the model of nonlocality discussed here. Using these, the corresponding Fresnel equations have also been found [20]. They read

$$\mathbf{F}_\mathrm{TM}\begin{pmatrix} E_z^\mathrm{R} \\ E_z^{(1)} \\ E_z^{(2)} \end{pmatrix} = -E_z^\mathrm{I}\begin{pmatrix} k_z^\mathrm{I} \\ 1 \\ 0 \end{pmatrix} \tag{13}$$

for TM polarization with the TM Fresnel matrix

$$\mathbf{F}_\mathrm{TM} \equiv \begin{pmatrix} k_z^\mathrm{R} & -k_z^{(1)} & -k_z^{(2)} \\ 1 & -\epsilon & -\epsilon \\ 0 & \eta k_z^{(1)}|\mathbf{k}^{(1)}|^2 & \eta k_z^{(2)}|\mathbf{k}^{(2)}|^2 \end{pmatrix}, \tag{14}$$

where $|\mathbf{k}^{(i)}|^2 = (k_z^{(i)})^2 + h^2$, and

$$\mathbf{F}_\mathrm{TE}\begin{pmatrix} E_x^\mathrm{R} \\ E_x^{(1)} \\ E_x^{(2)} \end{pmatrix} = -E_x^\mathrm{I}\begin{pmatrix} 1 \\ k_z^\mathrm{I} \\ 0 \end{pmatrix} \tag{15}$$

for TE polarization, where we introduced the TE Fresnel matrix

$$\mathbf{F}_\mathrm{TE} \equiv \begin{pmatrix} 1 & -1 & -1 \\ k_z^\mathrm{R} & k_z^{(1)}A_1 & k_z^{(2)}A_2 \\ 0 & \eta k_z^{(1)} & \eta k_z^{(2)} \end{pmatrix}, \tag{16}$$

with $A_i = [-\mu^{-1} + \eta k_0^2|\mathbf{k}^{(i)}|^2]$. With these equations, the reflection coefficients

$$r^\mathrm{TM} = \frac{\epsilon k_z^\mathrm{I}\left[h^2 + \left(k_z^{(1)}\right)^2 + \left(k_z^{(2)}\right)^2 + k_z^{(1)}k_z^{(2)}\right] - k_z^{(1)}k_z^{(2)}\left(k_z^{(1)} + k_z^{(2)}\right)}{\epsilon k_z^\mathrm{R}\left[h^2 + \left(k_z^{(1)}\right)^2 + \left(k_z^{(2)}\right)^2 + k_z^{(1)}k_z^{(2)}\right] - k_z^{(1)}k_z^{(2)}\left(k_z^{(1)} + k_z^{(2)}\right)} \tag{17}$$

for TM polarization and

$$r^\mathrm{TE} = \frac{\mu k_z^\mathrm{I}\left(k_z^{(1)} + k_z^{(2)}\right) + h^2 - k_z^{(1)}k_z^{(2)} - \eta\mu k_0^2|\mathbf{k}^{(1)}|^2|\mathbf{k}^{(2)}|^2}{\mu k_z^\mathrm{R}\left(k_z^{(1)} + k_z^{(2)}\right) + h^2 - k_z^{(1)}k_z^{(2)} - \eta\mu k_0^2|\mathbf{k}^{(1)}|^2|\mathbf{k}^{(2)}|^2} \tag{18}$$

for TE polarization have been calculated. The reflection coefficients of a local medium can be obtained from this in the limit $\eta \to 0$. One of the $k_z^{(i)}$ is divergent in this limit; without restriction let it be $k_z^{(2)}$. Now, using $\lim_{\eta\to 0} k_z^{(i)}\eta = 0$, $\lim_{\eta\to 0}(k_z^{(2)})^2\eta = (k_0^2\mu)^{-1}$, and $\lim_{\eta\to 0}(k_z^{(1)})^2\eta = 0$ leads to

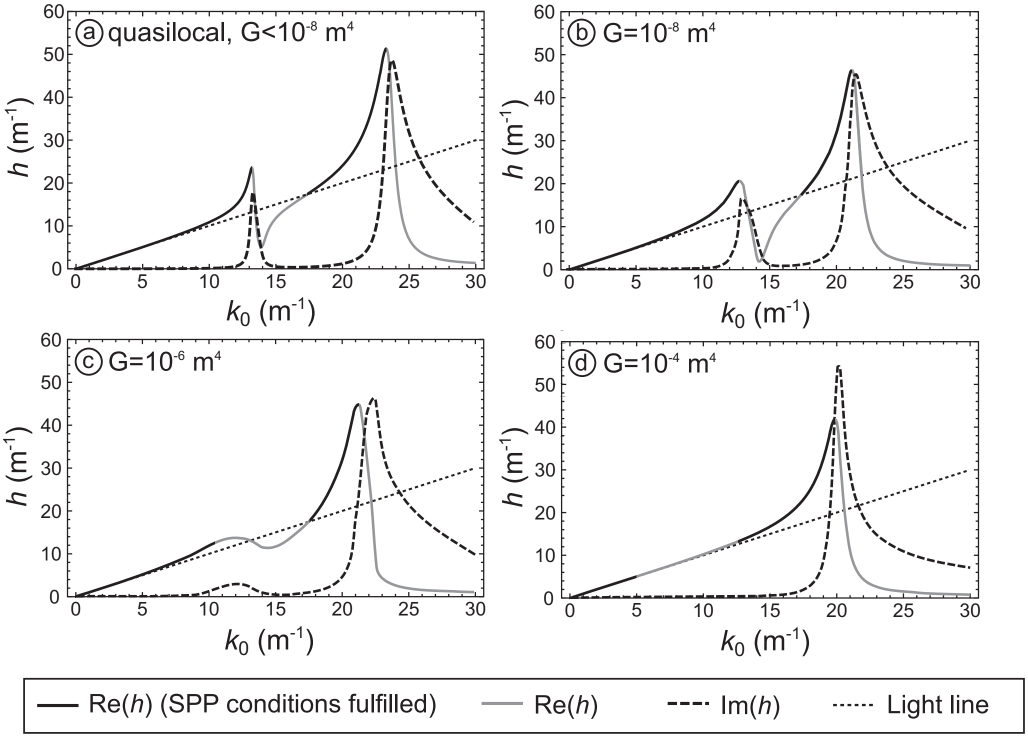

FIG. 2. SPP dispersion curve for TM polarized light.

the correct local limit for both reflection coefficients

$$\lim_{\eta\to 0} r^{\mathrm{TM}} = \frac{\epsilon(\omega)k_z^{\mathrm{I}} - \lim_{\eta\to 0} k_z^{(1)}}{\epsilon(\omega)k_z^{\mathrm{R}} - \lim_{\eta\to 0} k_z^{(1)}} \tag{19}$$

and

$$\lim_{\eta\to 0} r^{\mathrm{TE}} = \frac{k_z^{\mathrm{I}} - \mu(\omega)\lim_{\eta\to 0} k_z^{(1)}}{k_z^{\mathrm{R}} - \mu(\omega)\lim_{\eta\to 0} k_z^{(1)}}. \tag{20}$$

IV. SPP DISPERSION WITH NONLOCALITY

The SPP dispersion relation can be found from the poles of the reflection coefficients in Eqs. (18) and (17) [30] with $k_z^{\mathrm{R}} = -k_z^{\mathrm{I}} = -k_z^{(0)}$. Making use of the simplifications

$$\left(k_z^{(1)}k_z^{(2)}\right)^2 = h^4 + \frac{\epsilon}{\eta} - \frac{h^2}{k_0^2\eta\mu} \tag{21}$$

and

$$\left(k_z^{(1)}\right)^2 + \left(k_z^{(2)}\right)^2 = -2h^2 + \left(k_0^2\eta\mu\right)^{-1}, \tag{22}$$

these equations are solved. The solution formulas are very long and not displayed here for readability. Instead, numerical values as given above have been used to study the effect of the nonlocality on the SPP dispersion in Figs. 2 and 3.

Inequality (12) gives an existence condition for SPP. This, however, is not entirely sufficient. Calculating the ratio of the energy transmitted by each of the waves from the Fresnel equations (15) and (13),

$$\tau_{\mathrm{TM}} = \left|\frac{t_{\mathrm{TM}}^{(2)}}{t_{\mathrm{TM}}^{(1)}}\right|^2 = \left|\frac{k_z^{(1)}\left[h^2 + \left(k_z^{(1)}\right)^2\right]}{k_z^{(2)}\left[h^2 + \left(k_z^{(2)}\right)^2\right]}\right|^2 \tag{23}$$

for TM polarization and

$$\tau_{\mathrm{TE}} = \left|\frac{t_{\mathrm{TE}}^{(2)}}{t_{\mathrm{TE}}^{(1)}}\right|^2 = \left|\frac{h^2 + \left(k_z^{(1)}\right)^2}{h^2 + \left(k_z^{(2)}\right)^2}\right|^2 \tag{24}$$

for TE polarization, we see that if one of the $k_z^{(i)}$ is very large compared to the other one, the associated wave carries a lot less energy. We require $0.01 < \tau_{\mathrm{TE/TM}} < 100$ as an additional condition, such that both waves carry at least 1% of the energy. If this additional condition is not met, the original condition in Eq. (12) is waived for the very large $k_z^{(i)}$. This needs to be done in order to ensure that even for divergent wave numbers, as they occur in the local limit, the SPP conditions proposed here remain meaningful. Finally, due to causality, only points outside the light cone are relevant, i.e., requiring $h > k_0$. The resulting dispersion curves are displayed in Fig. 2 for TM polarization and in Fig. 3 for TE polarization.

Turning to TM polarization first, the quasilocal case shown in Fig. 2(a) exhibits a dispersion relation similar to the one obtained by Ruppin in [26] but for a lossy medium. Quasilocal means that this curve can either be obtained by a very low nonlocality or the classical derivation coming from the purely local wave equation and interface conditions from local constitutive relations. Instead of two divergent branches in the lossless case, it is now one connected curve that exhibits back-bending in place of the divergences. Opposed to the lossless case, not all solutions outside the light cone fulfill the SPP conditions. For a weak nonlocality with $G = 10^{-8}$ m^4 [see Fig. 2(b)], the dispersion relation stays similar compared to the quasilocal case, although a slight broadening with a decrease in height can be observed for the peaks in the dispersion relation. Again, SPPs exist on the upward part of the peaks. This broadening of the peak close to the

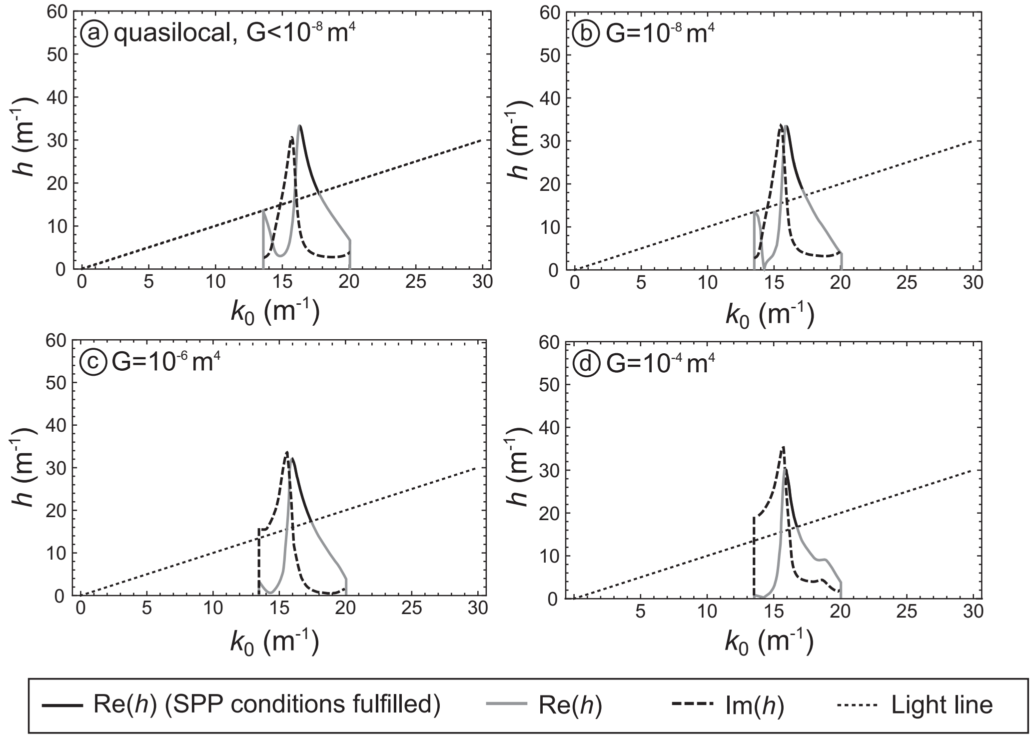

FIG. 3. SPP dispersion curve for TE polarized light.

resonance increases further and further with increasing the nonlocality amplitude G as seen in Figs. 2(b)–2(d). Until the highest $G = 10^{-4}$ m^4 the nonlocality causes the medium to be nondispersive along the surface, as shown in Fig. 2(d). While SPPs continue to be allowed on the upward slope of the higher frequency peak, the lower frequency range where the SPP condition is fulfilled significantly shrinks towards lower frequencies when increasing the nonlocality. In conclusion, the dispersion relations show that, opposed to the local description, an increase in nonlocality strongly decreases the lower frequency range in which SPPs are allowed, while for frequencies greater than ω_0 the effects of nonlocality are very limited. An even further increase of the nonlocality parameter, however, does not alter the shape of the dispersion relation any more, showing a limiting behavior for both very high and very low nonlocalities with one and two SPP branches, respectively.

For TE polarization, the dispersion curves are displayed in Fig. 3. There are only valid solutions to the dispersion relation in the frequency range where the permeability takes negative values. Similar to the TM dispersion relation, a comparison to the lossless case discussed by Ruppin [26] shows that instead of the sharp divergence of the dispersion curve there is a back bending into the peak as shown in Fig. 3(a). SPPs can exist on the downward slope of that peak. Figure 3(b) shows the change when taking into account a small nonlocality with $G = 10^8$ m^4. For all higher values of the nonlocality amplitude G, there are slight changes in the shape of the dispersion curve, but the physically relevant part that lies outside the light cone stays almost exactly the same, just as the frequency range in that the SPP conditions are fulfilled. In summary, one can state that this type of nonlocality does not affect the TE mode of SPPs a lot and opposed to the TM mode, where the changes are quite significant, only causes very little changes to the dispersion relation.

V. ATR SPECTRA

Surface plasmon polariton excitation can be observed using the method of attenuated total reflection (ATR) spectroscopy. A geometry for that was proposed by Otto [31].

It consists of a prism with permittivity ϵ_P, an air layer of thickness d, and the medium to be analyzed, so in this case the metamaterial. Light is sent in at an angle θ that is totally reflected at the prism-air interface. The evanescent waves penetrating the air layer can then excite SPPs at the air-metamaterial interface. This geometry, commonly referred to as the Otto configuration, is illustrated in Fig. 4. Using a transfer matrix formalism [32] and the previously derived reflection coefficients in Eq. (17) and Eq. (18), the reflectivity of such a setup has been calculated for both polarizations. For the metamaterial, the same parameters as above have been used. The permittivity of the prism is chosen to be nondispersive with $\epsilon_P = 3$.

Turning to TM polarization first, with an angle of incidence of $\theta = \pi/4$ and thickness $d = 3$ cm, the spectra in Fig. 5 have been obtained. The quasilocal curve has two SPP peaks, one for each SPP branch, and is identical to the one predicted by Ruppin [26]. For all different magnitudes of the nonlocality, the higher frequency peak stays almost unchanged. Belonging to the higher frequency peak in the SPP dispersion relation (Fig. 2), this observation confirms the prediction that there is no significant change to that peak. The lower frequency peak in the spectrum, however, gets less pronounced and drifts

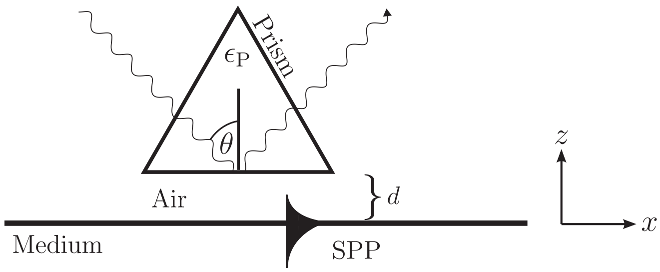

FIG. 4. Schema of the Otto configuration for attenuated total reflection (ATR) spectroscopy. The angle of incidence θ is chosen to be the angle of total internal reflection resulting in an evanescent wave in the air layer, that couples to the SPPs in the medium at the air-metamaterial interface, where $z = 0$.

off to lower frequencies. This is associated with shrinking of the frequency range that the SPP conditions are fulfilled for with increasing nonlocality. Correspondingly, the frequencies that SPP can be excited are lower and lower. Additionally, the decreasing depth of the peak implies that the coupling strength to the SPP mode gets less with an increase in nonlocality. The very small peak seen in between the two SPP peaks is due to the excitation of bulk polaritons [26]. With an increase in nonlocality, this peak also becomes less pronounced.

The spectrum for TE polarization is shown in Fig. 6. The parameters used here are $\theta = \pi/3$ for the angle of incidence and $d = 1$ cm for the distance between the prism and the metamaterial. Again, the quasilocal curve is identical to Ruppin's work. The minimum at higher frequencies of that curve is due to the excitation of SPP, while the one at lower frequencies is due to the excitation of bulk polaritons. An increase of the nonlocality leads to the bulk polariton peak moving to lower frequencies and becoming less deep. The SPP peak keeps its form and position independent of the nonlocality. This is in compliance with the dispersion relation (Fig. 3), which predicted no significant effect despite the increasing nonlocality.

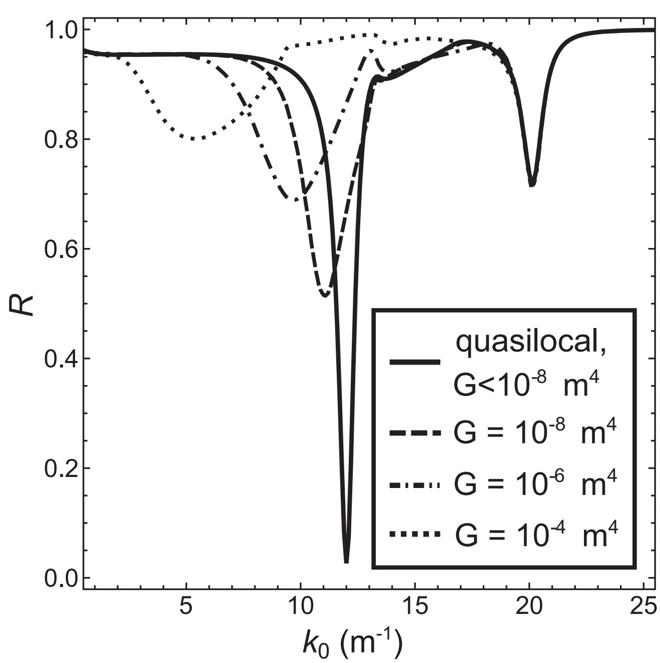

FIG. 5. Reflection coefficient of TM polarized light calculated for Otto configuration with $d = 3$ cm and $\theta = \pi/4$.

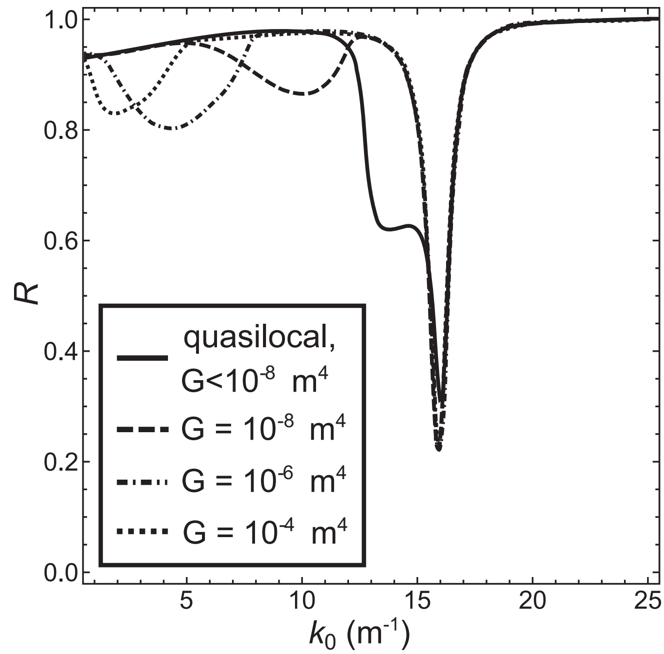

FIG. 6. Reflection coefficient of TE polarized light calculated for Otto configuration with $d = 1$ cm and $\theta = \pi/3$.

VI. CONCLUSIONS

Concluding, we have discussed the Fresnel equations for an interface between a nonlocal, homogeneous, and isotropic metamaterial and vacuum and derived expressions for the reflection coefficients for both TE and TM polarizations. Further, we proposed appropriate existence conditions for SPP in lossy, nonlocal media. Using these results, we obtained the dispersion relation for surface plasmon polaritons and discussed the effect of a gradually increasing nonlocality on it. We observed that the nonlocality has no significant effect in the case of TE polarized light. For TM polarized light, however, it leads to the collapse of the lower frequency dispersion peak to a nondispersive form, while the higher frequency peak shows very little change. The lower frequency range where SPPs were allowed in the local case shrinks significantly. These observations were backed up by calculating and discussing the ATR spectrum in an Otto configuration. With that, we outlined a possible way of investigating and verifying nonlocal material models. On the basis of a model assumption, we were able to extract distinct characteristic effects of the nonlocality on the dispersion relation of surface plasmon polaritons. Further, we showed how our approach can be used to predict experimental consequences of nonlocality, ultimately offering a potential future path from a solely mathematical approach to actual physical evidence.

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[1] N. Engheta, *Advances in Electromagnetics of Complex Media and Metamaterials* (Springer, Dordrecht, 2002), pp. 19–37.
[2] S. Guenneau and S. A. Ramakrishna, C. R. Phys. **10**, 352 (2009).
[3] M. G. Silveirinha, A. Alù, and N. Engheta, Phys. Rev. B **78**, 075107 (2008).
[4] A. Monti, A. Alù, A. Toscano, and F. Bilotti, Photonics **2**, 540 (2015).
[5] E. E. Narimanov and A. V. Kildishev, Appl. Phys. Lett. **95**, 041106 (2009).
[6] A. Alù, Phys. Rev. B **84**, 075153 (2011).
[7] M. G. Silveirinha, Phys. Rev. B **75**, 115104 (2007).
[8] V. A. Markel and J. C. Schotland, Phys. Rev. E **85**, 066603 (2012).
[9] V. A. Markel and I. Tsukerman, Phys. Rev. B **88**, 125131 (2013).
[10] A. Serdiukov, I. Semchenko, S. Tertyakov, and A. Sihvola, *Electromagnetics of Bi-anisotropic Materials–Theory and Application* (Gordon and Breach Science Publishers, New York, 2001), Vol. 11.
[11] M. G. Silveirinha, Phys. Rev. E **73**, 046612 (2006).
[12] Y. Zhao, P. A. Belov, and Y. Hao, Opt. Express **14**, 5154 (2006).
[13] P. A. Belov and C. R. Simovski, Phys. Rev. E **72**, 026615 (2005).
[14] G. Wang, Y. Lei, and W. Zhang, Phys. Lett. A **379**, 491 (2015).
[15] G. Du, Z. Wang, H. Li, Y. Zhang, and H. Chen, Phys. Lett. A **372**, 730 (2008).
[16] C. Menzel, C. Rockstuhl, T. Paul, F. Lederer, and T. Pertsch, Phys. Rev. B **77**, 195328 (2008).
[17] W. Wu, E. Kim, E. Ponizovskaya, Y. Liu, Z. Yu, N. Fang, Y. Shen, A. Bratkovsky, W. Tong, C. Sun, X. Zhang, S.-Y. Wang, and R. Williams, Appl. Phys. A **87**, 143 (2007).
[18] C. Menzel, T. Paul, C. Rockstuhl, T. Pertsch, S. Tretyakov, and F. Lederer, Phys. Rev. B **81**, 035320 (2010).
[19] P. A. Belov, R. Marqués, S. I. Maslovski, I. S. Nefedov, M. Silveirinha, C. R. Simovski, and S. A. Tretyakov, Phys. Rev. B **67**, 113103 (2003).
[20] K. Mnasri, A. Khrabustovskyi, C. Stohrer, M. Plum, and C. Rockstuhl, Phys. Rev. B **97**, 075439 (2018).
[21] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[22] H. Chen, C. T. Chan, and P. Sheng, Nat. Mater. **9**, 387 (2010).
[23] J. B. Pendry and S. A. Ramakrishna, J. Phys.: Condens. Matter **15**, 6345 (2003).
[24] J. B. Pendry, Opt. Express **11**, 755 (2003).
[25] C. M. Soukoulis, S. Linden, and M. Wegener, Science **315**, 47 (2007).
[26] R. Ruppin, Phys. Lett. A **277**, 61 (2000).
[27] R. J. Churchill and T. G. Philbin, Phys. Rev. B **94**, 235422 (2016).
[28] R. J. Churchill and T. G. Philbin, Phys. Rev. B **95**, 205406 (2017).
[29] R. Warmbier, G. S. Manyali, and A. Quandt, Phys. Rev. B **85**, 085442 (2012).
[30] D. Barchiesi, Eur. J. Phys. **33**, 1345 (2012).
[31] A. Otto, Z. Phys. **216**, 398 (1968).
[32] K. Ohta and H. Ishida, Appl. Opt. **29**, 1952 (1990).