Polarization of the spontaneous magnetic field and magnetic fluctuations in $s + is$ anisotropic multiband superconductors

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We show that multiband superconductors with broken time-reversal symmetry can produce spontaneous currents and magnetic fields in response to the local variations of pairing constants. Considering the iron pnictide superconductor $Ba_{1-x}K_xFe_2As_2$ as an example we demonstrate that both the point-group symmetric $s + is$ state and the C_4-symmetry-breaking $s + id$ states produce, in general, the same magnitudes of spontaneous magnetic fields. In the $s + is$ state these fields are polarized mainly on an *ab* crystal plane, whereas in the $s + id$ state their *ab*-plane and *c*-axis components are of the same order. The same is true for the random magnetic fields which are produced by the order parameter fluctuations near the critical point of the time-reversal symmetry-breaking phase transition. Our findings can be used as a direct test of the $s + is/s + id$ dichotomy and the additional discrete symmetry-breaking phase transitions with the help of muon spin-relaxation experiments.

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I. INTRODUCTION

Superconducting states with spontaneously broken time-reversal symmetry (BTRS) recently have been the focus of interest. First, such states have been studied in connection with the chiral *p*-wave order parameter in the superfluid ^{3}He *A* phase [1] and the Sr_2RuO_4 superconducting compound [2]. More recently, $s + id$ and $s + is$ states have been suggested as the candidate order parameters in multiband iron pnictide compounds [3–9]. A recent experiment [10] supports this hypothesis demonstrating the presence of spontaneous currents in the ion-irradiated samples of $Ba_{1-x}K_xFe_2As_2$ in the certain doping level interval.

Spontaneous currents were predicted to exist near impurities in $s + id$ superconducting states which spontaneously break the C_4 crystalline symmetry of the parent compound [3]. As for the $s + is$ states, initially, it has been claimed that a magnetic field can appear only in samples subjected to strain [11]. However, this conclusion was made based on the specific circularly symmetric model of the impurity.

A more general consideration has shown [12,13] that magnetic fields in the $s + is$ state can be generated without strain in the presence of the general-form inhomogeneities of the order parameter. They can be induced, e.g., by the domain wall between $s + is$ and $s - is$ states [14], attached to the sample edge or by any external controllable perturbation, such as the local heating.

Later the particular case of two-dimensional defects elongated along the crystal *c* axis and forming square shapes on the *ab* plane have been studied [15]. In such a system the spontaneous magnetic field generated in the $s + is$ state is several orders of magnitude smaller than in the $s + id$ one. As we show below this difference is not generic, and under more general conditions the magnetic-field amplitudes produced in the two states are of the same order.

The purpose of the present paper is threefold. First, we show that the spontaneous magnetic field is generated both in the $s + is$ and in the $s + id$ states due to the general-form inhomogeneities of the pairing interactions. Such a form of disorder can exist in the sample even without the externally generated defects just due to the spatially inhomogeneous doping level. Second, we demonstrate that, in the general case, when the system is inhomogeneous both on the *ab* plane and in the *c* direction $s + is$ and $s + id$ states yield the same magnitudes of spontaneous fields. However, as shown schematically in Fig. 1 this regime is characterized by the qualitatively different polarizations of the spontaneous field in the $s + id$ and $s + is$ states. This prediction can be used for resolving the $s + id/s + is$ dichotomy in real materials with the help of muon spin-relaxation experiments [10,16,17]. Third, we demonstrate that the order parameter fluctuations near the BTRS phase transition generate random magnetic fields with the critical correlation radius. Thus, the discrete symmetry-breaking phase transition can be revealed through the magnetic-field fluctuations.

II. GENERAL FIELD STRUCTURE

Here we develop general treatment of spontaneous magnetic fields in BTRS states further considering inhomogeneities created by the spatial variation of pairing constants in the minimal three-band microscopic model [6,18,19] with three distinct superconducting gaps Δ_{1-3} residing in different bands. The pairing which leads to the BTRS state is dominated by the competition of two interband repulsion channels $\eta_{1,2} > 0$ described by the following coupling matrix:

$$\hat{\Lambda} = -\nu_0 \begin{pmatrix} 0 & \eta_1 & \eta_2 \\ \eta_1 & 0 & \eta_2 \\ \eta_2 & \eta_2 & 0 \end{pmatrix}. \quad (1)$$

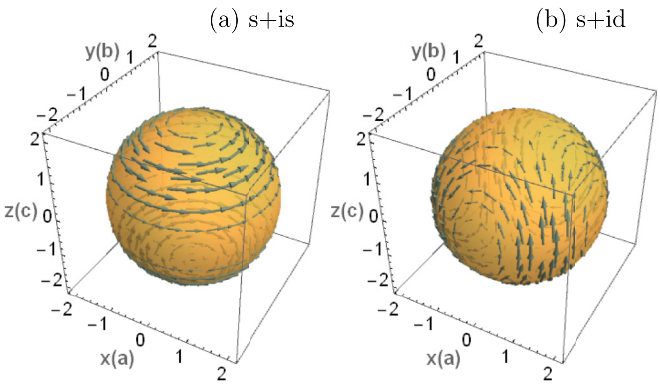

FIG. 1. The arrows show the spontaneous magnetic fields generated by rotationally symmetric three-dimensional (3D) inhomogeneity of the interband phase difference $\theta_{13} = \theta_{13}(r)$ as given by Eq. (2). (a) The anisotropic $s + is$ state with $\gamma_{13}^x = \gamma_{13}^y = 2\gamma_{13}^z$ and (b) the $s + id$ state with $\gamma_{13}^z = 0$ and $\gamma_{13}^x = -\gamma_{13}^y$. The scale of the magnetic field is the same in (a) and (b). The field is plotted on the spherical surface $r =$ const. The crystal anisotropy axis is z. The field is rotationally symmetric in the $s + is$ state and changes sign under the C_4 rotation in the $s + id$ state.

We assume for simplicity that the density of states ν_0 is the same in all superconducting bands. This model can be used for both the $s + is$ and the $s + id$ states. In the former case $\Delta_{1,2}$ corresponds to the gaps at the hole pockets, and Δ_3 is the gap at the electron pockets so that $u_{hh} = \nu_0\eta_1$ and $u_{eh} = \nu_0\eta_2$, respectively, are the hole-hole and electron-hole interactions [6,19]. The same model (1) can be used to describe the $s + id$ states, but there, Δ_1 and Δ_2 describe gaps in the $(0, \pm\pi)$ and $(\pm\pi, 0)$ electron pockets, respectively. In the hole pocket the gap is Δ_3 so that $u_{eh} = \nu_0\eta_1$ and $u_{ee} = \nu_0\eta_2$ are electron-hole and electron-electron interactions, respectively [13,20].

The inhomogeneities of pairing interactions in the model (1) produce spatially varying gap amplitudes $|\Delta_i|$ and phases θ_i. Their gradients can generate spontaneous magnetic fields according to the modified London expression in multiband superconductors [12],

$$\boldsymbol{B} = -4\pi\nabla \times \left(\hat{\lambda}_L^2 \boldsymbol{j}\right) + \frac{1}{\tilde{e}N}\sum_{k>i} \nabla \times (\hat{\gamma}_{ki}\nabla\theta_{ki}), \tag{2}$$

where we use the units with $\hbar = c = 1$. The interband phase differences are $\theta_{ki} = \theta_k - \theta_i$, N is the number of superconducting bands.

Here the London penetration depth is given by $\hat{\lambda}_L^{-2} = \sum_k \hat{\lambda}_k^{-2}$ and $\hat{\gamma}_{ki} = \hat{\lambda}_L^2(\hat{\lambda}_k^{-2} - \hat{\lambda}_i^{-2})$, where $\hat{\lambda}_k$'s are, in general, the tensor coefficients characterizing the contribution of each band to the Meissner screening. In the clean limit they can be expressed as follows:

$$\hat{\lambda}_k^{-2} = 8\pi\rho\tilde{e}^2\hat{K}_k|\Delta_k|^2, \tag{3}$$

where $\hat{K}_k = \langle \boldsymbol{v}_k\boldsymbol{v}_k \rangle$ is the anisotropy tensor, $\boldsymbol{v}_k$ is the Fermi velocity in the kth band normalized to the certain band-independent characteristic velocity $\bar{v}_F$. We normalize the gaps by $T_c/\sqrt{\rho}$, where $\rho = \sum_n \pi T_c^3\omega_n^{-3} \approx 0.1$, T_c is the critical temperature, and the magnetic field is by $B_0 = T_c\sqrt{\nu_0/\rho}$, which is close to the thermodynamic critical field at zero temperature [21]. The length is normalized by the Cooper pair size $\xi_0 = \bar{v}_F/T_c$, and we introduce the dimensionless Cooper pair charge as $\tilde{e} = 2e\xi_0^2 B_0$.

In contrast to the usual London electrodynamics, the multicomponent superconducting systems can generate spontaneous magnetic fields due to the second term in Eq. (2) acting as a source according to the mechanisms described below. *First*, the source term in Eq. (2) is nonzero if $\boldsymbol{\nabla}\theta_{ki} \neq 0$, and tensors $\hat{\gamma}_{ki}$ are constant in space but anisotropic. This scenario is generic for the $s + id$ state when all three components $\gamma_{ki}^{x,y,z}$ are different. The $s + is$ state is isotropic on the ab plane, but there is anisotropy on the ca and cb planes $\gamma_{ki}^x = \gamma_{ki}^y \neq \gamma_{ki}^z$. In this case the ab-plane inhomogeneities are decoupled from the magnetic field [15] so that $B_z = 0$. However, in general, the systems are inhomogeneous along the c-axis direction as well, which yields the magnetic response $B_{x,y}$ of the same magnitude as the $s + id$ state.

The general field structures produced by the 3D inhomogeneity in the $s + is/s + id$ states can be found using Eq. (2). In Fig. 1 we show the spontaneous field produced by the interband phase difference modulation $\theta_{13}(r)$. In the $s + is$ case only $B_{x,y} \neq 0$, whereas in the $s + id$ state the field has all components.

Second, the component $B_z \neq 0$ can be generated even in the $s + is$ state [12–15]. According to Eq. (2), for that we need simultaneously $\nabla_{x,y}\theta_{ki} \neq 0$ and $\nabla_{x,y}\gamma_{ki}^{x,y} \neq 0$ with the additional requirement that these gradients are noncollinear to each other. Therefore the B_z component in the $s + is$ state is significantly smaller than in the $s + id$ state where only $\nabla_{x,y}\theta_{ki} \neq 0$ is needed.

Therefore the largest spontaneous field in the $s + is$ case appears in the direction perpendicular to the anisotropy axis, whereas in $s + id$ all components are of the same order. One can distinguish between these states by analyzing the polarization of spontaneous magnetic fields with the help of the muon spin-relaxation techniques [10,16,17]. Below we illustrate these conclusions using more detailed calculations close to T_c using the Ginzburg-Landau (GL) theory.

III. GINZBURG-LANDAU CALCULATION

To go beyond the local approximation we can calculate spontaneous magnetic fields using GL theory derived for the $s + is/s + id$ states [13,20] corresponding to the model (1).

The general free-energy density, normalized to B_0^2, is given by $F = F_s + B^2/8\pi$ where the GL free energy describing both the $s + is$ and the $s + id$ states is given by

$$F_s = \sum_{j=1}^{2}\left((\hat{\boldsymbol{\Pi}}\psi_j)^*\hat{k}_{jj}(\hat{\boldsymbol{\Pi}}\psi_j) + \alpha_j|\psi_j|^2 + \frac{\beta_j}{2}|\psi_j|^4\right) + 2(\hat{\boldsymbol{\Pi}}\psi_1)^*\hat{k}_{12}\hat{\boldsymbol{\Pi}}\psi_2 + \gamma|\psi_1|^2|\psi_2|^2 + \delta\psi_1^{*2}\psi_2^2 + \text{c.c.}, \tag{4}$$

where $\hat{\boldsymbol{\Pi}} = \boldsymbol{\nabla} - i\tilde{e}\boldsymbol{A}$. This model is formulated in terms of the two order parameters ψ_1 and ψ_2 which are related to the individual gap functions within separate bands as $(\Delta_1, \Delta_2, \Delta_3) = (\zeta\psi_2 - \psi_1, \zeta\psi_2 + \psi_1, \psi_2)$, where $\zeta = (\eta_1 - \sqrt{\eta_1^2 + 8\eta_2^2})/4\eta_2$.

The coefficients of the gradient terms in Eq. (4) are combined from the anisotropy tensors characterizing each superconducting band as follows:

$$\hat{k}_{11} = \rho(\hat{K}_1 + \hat{K}_2), \tag{5}$$

$$\hat{k}_{22} = \rho[\zeta^2(\hat{K}_1 + \hat{K}_2) + \hat{K}_3], \tag{6}$$

$$\hat{k}_{12} = \zeta\rho(\hat{K}_2 - \hat{K}_1). \tag{7}$$

The difference between $s + is$ and $s + id$ symmetries is determined by the structure of the mixed-gradient coefficients (7) on the ab plane. That is, for the $s + is$ state, $K_i^x = K_i^y \equiv K_i^{xy}$ so that $k_{12}^x = k_{12}^y \equiv k_{12}^{xy}$. For the $s + id$ state, $K_{1,2}^x \neq K_{1,2}^b$, but $K_1^x = K_2^y$ so that $k_{12}^x = -k_{12}^y \equiv k_{12}^{xy}$. Despite having quite different properties on the ab plane both states are characterized by the anisotropy on the ca and cb planes determined by the coefficients $K_i^z \neq K_i^x, K_i^y$.

In the $s + is$ state this anisotropy provides linear coupling between magnetic field and pairing constant inhomogeneities. For that at least two bands should have different anisotropies, otherwise the problem can be rescaled to the fully isotropic one when only the nonlinear coupling is possible yielding much smaller spontaneous currents.

The other coefficients in GL expansion (4) are expressed in terms of the pairing constants (1) as

$$\alpha_1 = -2(G_0 - G_1 + \tau), \tag{8}$$

$$\alpha_2 = -(1 + 2\zeta^2)(G_0 - G_2 + \tau), \tag{9}$$

$$\beta_1 = 2, \qquad \beta_2 = 1 + 2\zeta^4, \tag{10}$$

$$\gamma = 4\zeta^2, \qquad \delta = 2\zeta^2, \tag{11}$$

where $\tau = 1 - T/T_c$, $G_1 = 1/\eta_1$, and $G_2 = (\eta_1 + \sqrt{\eta_1^2 + 8\eta_2^2})/4\eta_2^2$ are the positive eigenvalues of the matrix (1) $\hat{\Lambda}^{-1}$ and $G_0 = \min(G_1, G_2)$.

The Ginzburg-Landau model is valid in the vicinity of T_c when both order parameters $|\psi_1|$ and $|\psi_2|$ are small. In this regime the bulk BTRS state appears for the close values of pairing constants due to the following reason. In the homogeneous state both order parameters appear simultaneously when $\eta_1 = \eta_2$. In the case of the finite detuning, one of the order parameters nucleates first. For example, when $\eta_1 > \eta_2$, the ψ_1 state nucleates at T_c since $G_0 = G_1 < G_2$. Then the bulk critical temperature of the BTRS transition, that is, nucleation of ψ_2 in this case can be found from the relation $\alpha_2 = -2\zeta^2|\psi_1|^2$, which is equivalent to $\tau = G_2 - G_1 + \zeta^2|\psi_1|^2$. The we have the restriction $|G_1 - G_2| \leqslant \tau$ so that $|1 - \eta_1/\eta_2| \leqslant \tau$. The inhomogeneous BTRS state however occurs for much higher amplitudes of the pairing constant variations because the additional order parameter nucleates locally at the regions where $\eta_1 \approx \eta_2$.

Below, we consider the spontaneous magnetic field produced by the superconducting currents generated by the inhomogeneities of the pairing constant $\eta_2 = \eta_2(\boldsymbol{r})$.

At first let us consider the two-dimensional inhomogeneities on the ab plane so that $\eta_1 = 1$ and

$$\eta_2(x, y) = 1 + 0.5\,\sin(x/2)\sin(y/2). \tag{12}$$

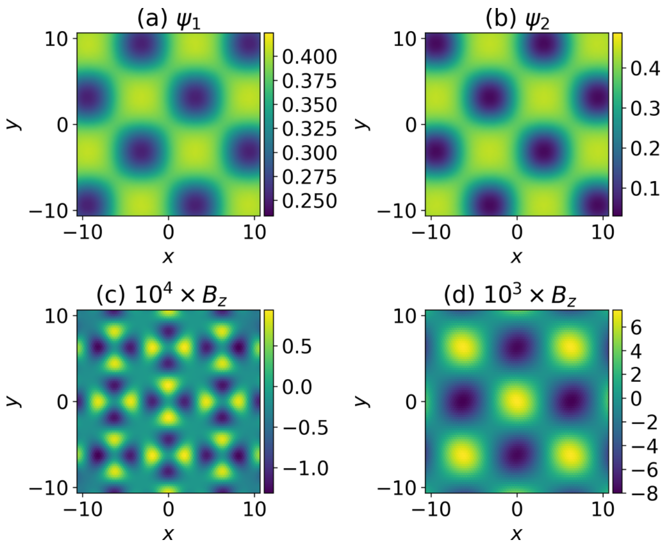

FIG. 2. (a) and (b) Order parameter modulation produced by the ab-plane inhomogeneities of the form (12). The corresponding spontaneous field B_z is shown for (c) the $s + is$ state with $K_1^{xy} = 1$, $K_2^{xy} = 1.5$, and $K_3^{xy} = 0.5$, and (d) the $s + id$ state with $K_1^x = K_2^y = 1$, $K_1^y = K_2^x = 1.5$, and $K_3^{xy} = 0.5$. The GL parameter is $\tilde{e} = 1/4$, $\tau = 0.2$, and the field is normalized to $\tau B_0/\tilde{e}$.

This model allows for demonstrating differences between the linear and the nonlinear mechanisms of the spontaneous current generation where the former takes place for $s + id$ and the latter is $s + is$ pairings. The magnetic field produced by ab inhomogeneities has only the z component. The calculated distributions of $B_z = B_z(x, y)$ are shown in Figs. 2(c) and 2(d) where the magnetic field is given in units of $\tau B_0/\tilde{e}$ which has an order of the upper critical field H_{c2} at a given temperature. One can see that, for one and the same set of parameters, the $s + is$ state yields the spontaneous magnetic-field response about 10^2 times smaller than $s + id$, which is consistent with the results obtained before [15].

Except for the special case of the ab-plane inhomogeneities, in general, the $s + is$ and $s + id$ states produce the magnetic fields of comparable amplitudes. To demonstrate this, we compare responses produced by the pairing constant variation given by

$$\eta_2 = 1 + 0.5e^{-(x^2+y^2)/8}, \quad s + id \text{ state}, \tag{13}$$

$$\eta_2 = 1 + 0.5e^{-(x^2+z^2)/8}, \quad s + is \text{ state}. \tag{14}$$

The former inhomogeneity (13) corresponds to the ab-plane defect, whereas the latter (14) is the ca-plane defect. To obtain spontaneous fields produced by ca-plane defects in the $s + is$ case we assume that there is the ca-plane anisotropy set by the choice of coefficient ratio in different bands $K_1^{xy} = 1$, $K_2^{xy} = 1.5$, $K_3^{xy} = 0.5$, $K_1^z = 2$, $K_2^z = 3$, $K_3^z = 1$. Such a system yields the magnetic-field component $B_y(x, z)$ shown in Fig. 3(a).

One can compare it with the qualitatively similar distribution of the B_z component produced by the Gaussian ab-plane inhomogeneity (13) in the $s + id$ state shown in Fig. 3(b). Magnetic signatures of the ca defect in the $s + id$ states are qualitatively similar to that in $s + is$ shown in Fig. 3(a).

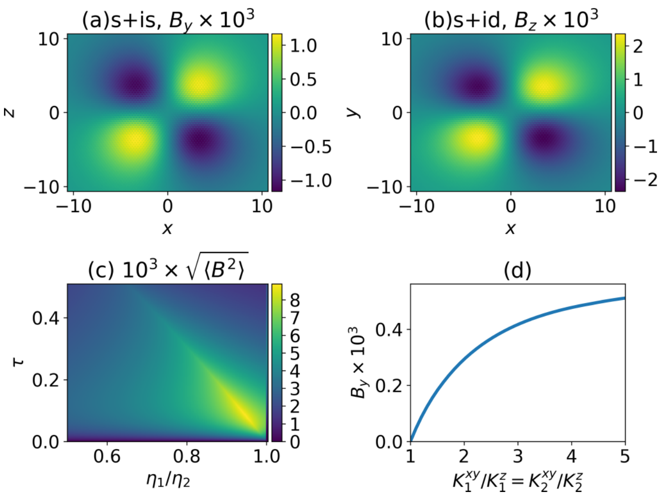

FIG. 3. (a) and (b) Spontaneous fields produced by the inhomogeneities of the pairing constant $\eta_2(\boldsymbol{r})$ (13) and (14) and $\eta_1 = 1$. (a) The ca-plane defect in the $s + is$ superconductor with anisotropy parameters for $s + is$ are $K_1^{xy} = 1$, $K_2^{xy} = 1.5$, $K_3^{xy} = 0.5$, $K_1^z = 2$, $K_2^z = 3$, $K_3^z = 0.5$. (b) The ab-plane defects in the $s + id$ state characterized by $K_1^x = K_2^y = 1$, $K_1^y = K_2^x = 1.5$, $K_3^x = K_3^y = 0.5$. The GL parameter $\tilde{e} = 1/4$ for both cases $\tau = 0.2$. The field is normalized to $\tau B_0/\tilde{e}$. (c) A spontaneous magnetic field on the ab plane produced by the critical fluctuations near the BTRS transitions in the anisotropic $s + is$ state. The field is measured in $(|k_{12}^z - k_{12}^{xy}|/\sqrt{k_{11}k_{22}})(T_c/E_F)B_0$, where E_F is the Fermi energy. The field maximum at each value of η_1/η_2 corresponds to the BTRS critical temperature. (d) The magnitude of the in-plane magnetic field vs the effective mass anisotropy in the $s + is$ superconductors.

On the ca plane the $s + id$ state is described by structurally identical GL equations as the $s + is$ one with the interchange $K_k^{xy} \to K_k^x$.

The 122 iron pnictide compounds have been shown to feature anisotropy which can vary, in rather wide limits, from $K_i^{xy}/K_i^z \approx 1$–3 [22] to $K_i^{xy}/K_i^z \approx 4$–5 [23]. In Fig. 3(d) we show the field amplitude dependence on the degree of anisotropy $K_1^z/K_1^{xy} = K_2^z/K_2^{xy} \neq 1$ and fixed $K_3^z/K_3^{xy} = 1$.

The general analytical expression for the spontaneous field in the case of the 3D inhomogeneity can be obtained using the reduced GL theory in the vicinity of the BTRS transition. It can be constructed assuming the main order parameter to be $\psi_1 = |\psi_1|e^{i\varphi_1}$ with $|\psi_1| = \text{const}$ and introducing the BTRS order parameter $\Theta = -i(\psi_2\psi_1^*)/|\psi_1|$. Then we represent the GL free energy in terms of the gauge-invariant momentum $\boldsymbol{Q} = \boldsymbol{A} - \nabla\varphi_1/\tilde{e}$ and, the real and imaginary parts of the complex order parameter $\Theta = \Theta_r + i\Theta_{im}$ are as follows:

$$\begin{aligned} F(\Theta, \boldsymbol{Q}) &= \tilde{\alpha}_r\Theta_r^2 + \tilde{\alpha}_{im}\Theta_{im}^2 + \frac{\beta_2}{2}|\Theta|^4 \\ &+ \frac{|\nabla \times \boldsymbol{Q}|^2}{8\pi} + |\psi_1|^2\tilde{e}^2\boldsymbol{Q}\hat{k}_{11}\boldsymbol{Q} \\ &+ 2|\psi_1|\tilde{e}\boldsymbol{Q}\hat{k}_{12}(\nabla\Theta_r + \tilde{e}\boldsymbol{Q}\Theta_{im}) \\ &+ (\nabla + i\tilde{e}\boldsymbol{Q})\Theta^*\hat{k}_{22}(\nabla - i\tilde{e}\boldsymbol{Q})\Theta. \end{aligned} \quad (15)$$

Here $\tilde{\alpha}_r = \alpha_2 + |\psi_1|^2(\delta - \gamma)$ and $\tilde{\alpha}_{im} = \alpha_2 + |\psi_1|^2(\delta + \gamma)$. The equation $\tilde{\alpha}_r(T) = 0$ gives the critical temperature of the BTRS transition. In the vicinity of this transition only the variation of Θ_r is important as $\tilde{\alpha}_{im}$ is positive and nonvanishing. Therefore we can describe the time-reversal symmetry-breaking phase transition in terms of the real-valued order parameter Θ_r,

$$\begin{aligned} F(\Theta_r, \boldsymbol{Q}) &= \tilde{e}^2|\psi_1|^2\boldsymbol{Q}\hat{k}_{11}\boldsymbol{Q} + \tilde{e}^2\Theta_r^2\boldsymbol{Q}\hat{k}_{22}\boldsymbol{Q} \\ &+ \frac{|\nabla \times \boldsymbol{Q}|^2}{8\pi}\tilde{\alpha}_r\Theta_r^2 + \frac{\beta_2}{2}\Theta_r^4 + \nabla\Theta_r\hat{k}_{22}\nabla\Theta_r \\ &+ 2\tilde{e}|\psi_1|\boldsymbol{Q}\hat{k}_{12}\nabla\Theta_r. \end{aligned} \quad (16)$$

Note that the real order parameter Θ_r is still coupled to the magnetic field because the superconducting current obtained from functional (16) is given by

$$\boldsymbol{j} = -2|\psi_1|^2\tilde{e}^2\hat{k}_{11}\boldsymbol{Q} - 2\tilde{e}|\psi_1|\hat{k}_{12}\nabla\Theta_r. \quad (17)$$

For simplicity let us assume that the coefficients $\hat{k}_{ii}$ for $i = 1, 2$ are isotropic and the anisotropy is determined by $\hat{k}_{12}$. Then, going to the Fourier-transform $\Theta_r(\boldsymbol{r}) = V\int e^{i\boldsymbol{q}\boldsymbol{r}}\Theta_r(\boldsymbol{q})d^3\boldsymbol{q}/(2\pi)^3$ in the volume V we obtain the magnetic field,

$$\boldsymbol{B}(\boldsymbol{q}) = \Theta_r(\boldsymbol{q})\sqrt{8\pi/k_{11}}(\boldsymbol{q} \times \hat{k}_{12}\boldsymbol{q})/[\lambda(q^2 + \lambda^{-2})], \quad (18)$$

where $\lambda = 1/(\sqrt{8\pi k_{11}}|\psi_1|\tilde{e})$ is the London penetration length. Equation (18) shows that gradients Θ_r with necessity produce the spontaneous magnetic field. They can be induced by the inhomogeneous pairing constant through the spatially varying coefficient $\alpha_r = \alpha_r(\boldsymbol{r})$ in Eq. (16). One can see that in the wide range of parameters $q\lambda \gg 1$ the magnetic-field amplitude is independent of the inhomogeneity scale.

The fields produced by the rotationally symmetric 3D defect $\Theta_r = \Theta_r(r)$ have the same structure as shown in Fig. 1. Based on the above analysis one can suggest the polarization-sensitive test of the superconducting state symmetry. That is, under general conditions, the spontaneous magnetic field in the $s + is$ state is directed mostly on the ab plane with the typical ratio of components $B_z/B_\perp \sim 10^{-2}$ as one can see comparing Figs. 2(c) and 3(a), where $\boldsymbol{B}_\perp = (B_x, B_y, 0)$. On the other hand, the $s + id$ state produces spontaneous fields which have, in general, all components with the same order $B_z/B_\perp \sim 1$.

IV. CRITICAL MAGNETIC FLUCTUATIONS

The spontaneous magnetic field produced by the order parameter inhomogeneities allows for the direct observation of the critical phenomena and fluctuations near the BTRS phase transition.

From (18) we get the variance of magnetic-field components on the ab plane,

$$\langle B_\perp^2(\boldsymbol{q})\rangle = \langle\Theta_r^2(\boldsymbol{q})\rangle\frac{(k_{12}^z - k_{12}^{xy})^2}{k_{11}}\frac{8\pi\lambda^2q_\perp^2q_z^2}{(\lambda^2q^2 + 1)^2}, \quad (19)$$

where $q_\perp = \sqrt{q_x^2 + q_y^2}$.

For simplicity we consider the limiting case when the cross-coupling gradient terms in the functional (16) are rather small $k_{12}^z, k_{12}^{xy} \ll \sqrt{k_{11}k_{22}}$ when the feedback of the magnetic-field fluctuations can be neglected. Then, fluctuations of the order parameter Θ_r near the BTRS critical temperature can be calculated using the conventional expression [24] $\langle \Theta_r^2(\boldsymbol{q})\rangle = T/[2B_0^2 V(k_{22}q^2 + |\tilde{\alpha}_r|)]$.

Now, we can calculate the average value of the spontaneous magnetic-field amplitude $\langle B_\perp^2\rangle = V\int d^3\boldsymbol{q}\langle B_\perp^2(\boldsymbol{q})\rangle$ using the ultraviolet cutoff on the scale of ξ_0^{-1}. The dependence of the average amplitude $\bar{B}_\perp = \sqrt{\langle B_\perp^2\rangle}$ on system parameters $(T, \eta_1/\eta_2)$ for fixed $\eta_1 = 0.5$ is shown in Fig. 3(c). Using the typical value of $T_c/E_F = 10^{-3}$ one can see that the field amplitude in Fig. 3(c) is about $10^{-5}B_0$ which is of the same order as produced by the $s + is$ state with ab inhomogeneities shown in Fig. 2(c).

The average amplitudes of magnetic-field components $\bar{B}_k^2 = \int B_k^2 n(B_k) dB_k$ can be derived from the magnetic-field distribution function $n(B_k)$ which is a directly measurable experimental quantity. It can be obtained as the Fourier transform of the complex muon spin-polarization function in the time domain [16]. In this way, comparing the signals from muon beams polarized along the c axis and on the ab plane, one can determine whether the system is in the $s + is$ or in the $s + id$ state. Besides that one can distinguish the line of the BTRS phase transition. As shown in Fig. 3(c) the BTRS phase transitions correspond to the distinct maxima of the fluctuating field amplitude.

These spontaneous fields provide therefore the direct access to the previously hidden critical behavior near the discrete symmetry-breaking phase transitions.

V. CONCLUSION

To summarize, we have shown that, in general, the $s + id$ and $s + is$ phases in multiband superconductors can produce spontaneous currents and magnetic fields in response to the spatial inhomogeneities caused by either the fluctuations of the pairing constants or the critical fluctuations of the order parameter components. This is in contrast to the previous predictions. However, the spontaneous field polarization is found to be drastically different in the $s + is$ and $s + id$ states making it possible to distinguish between them experimentally using muon spin-relaxation measurements. The random magnetic fields produced by the scalar order parameter fluctuations can reveal the critical behavior near the BTRS transition, and, in general, any additional discrete-symmetry-breaking phase transition deep in the superconducting state.

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